IASTU Condensed Matter Seminar
July, 2015

Spontaneous Loop Currents and Emergent Gauge Fields in Optical Lattices

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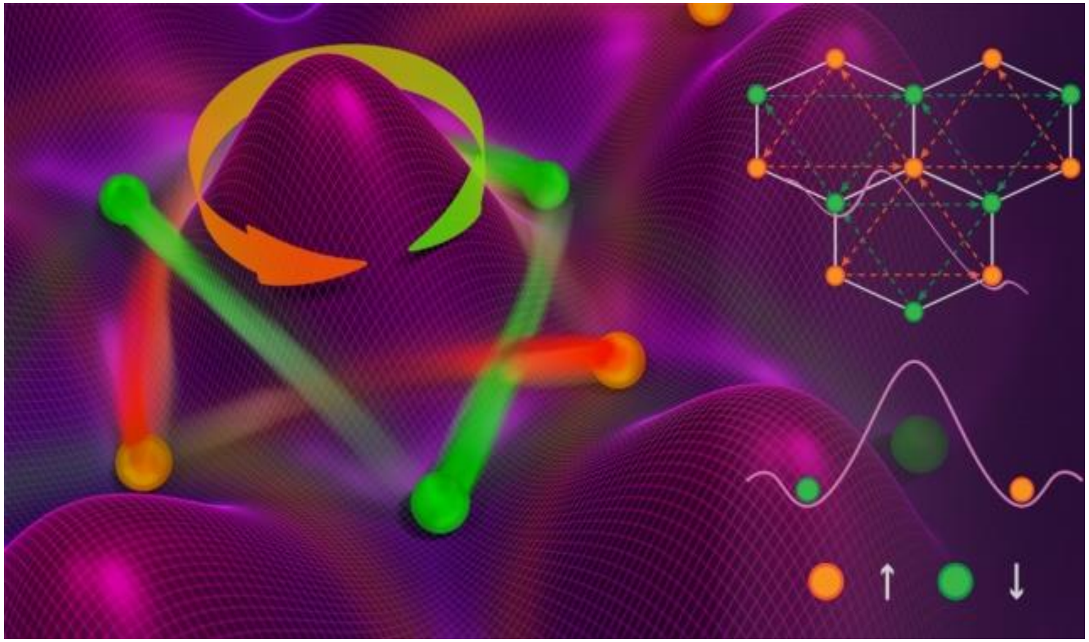


[Figure from JQI website]

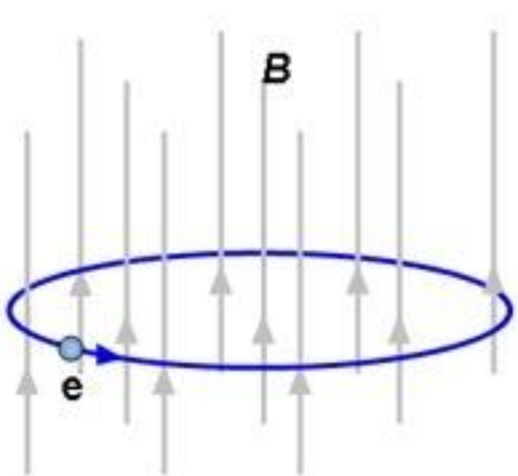
Gauge fields and Quantum Hall states


Klitzing

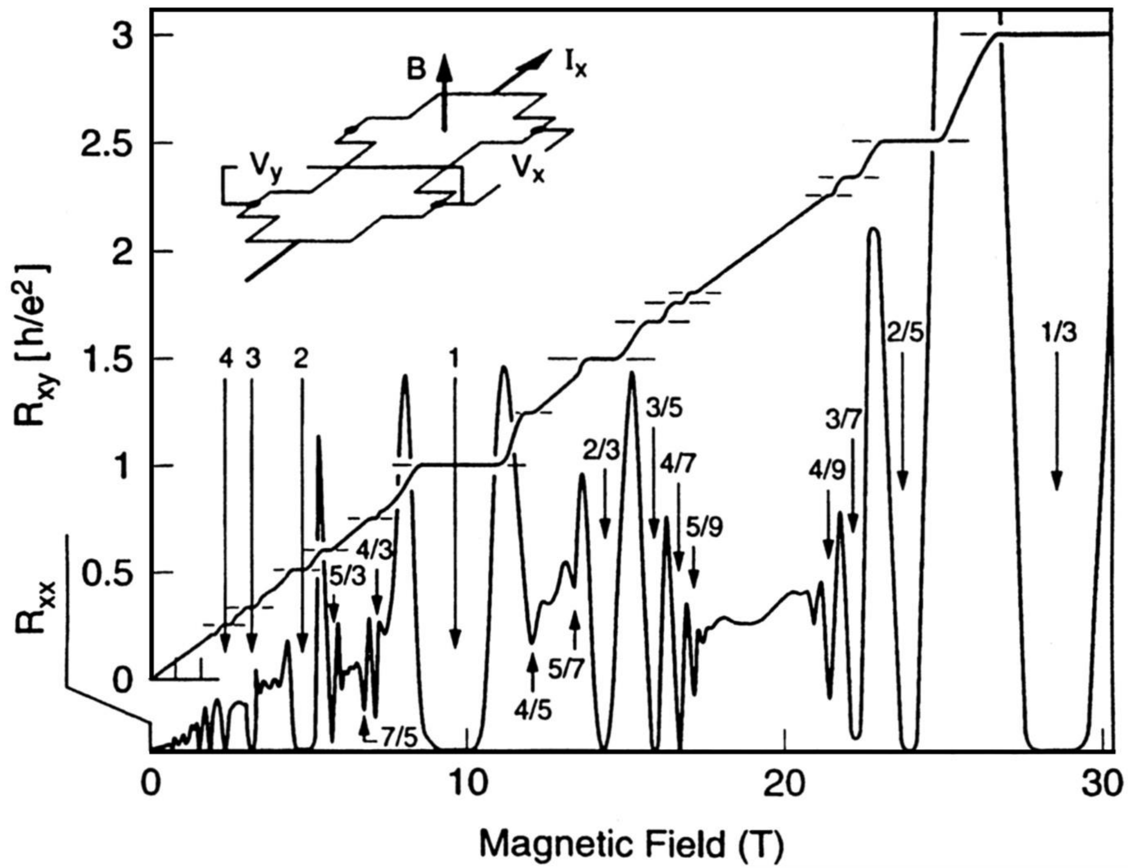

Laughlin


Störmer

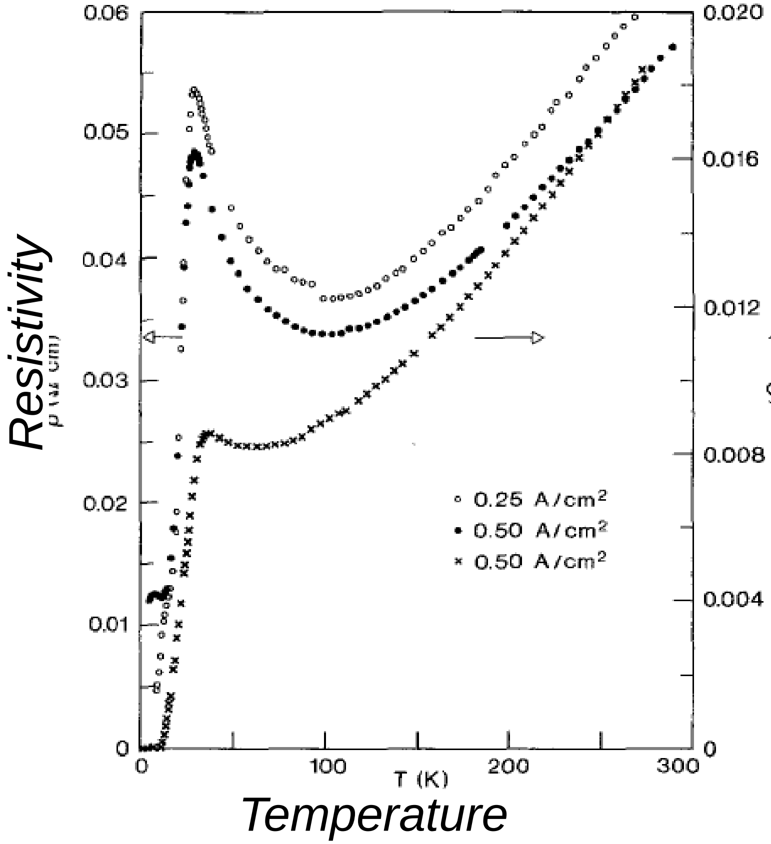

Tsui

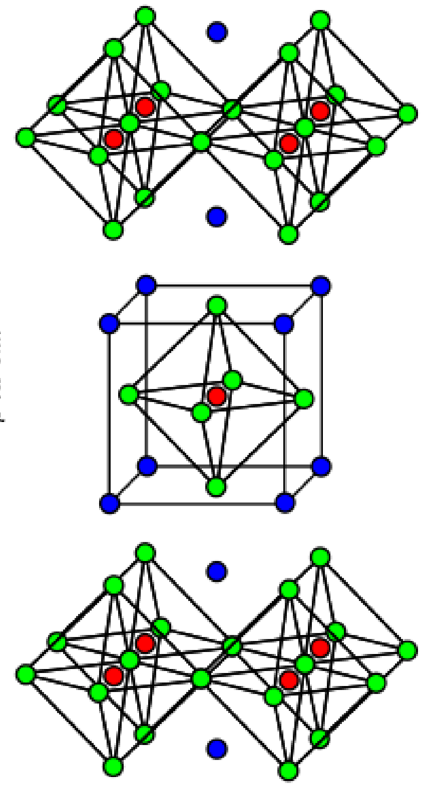


Electrons carry charge!

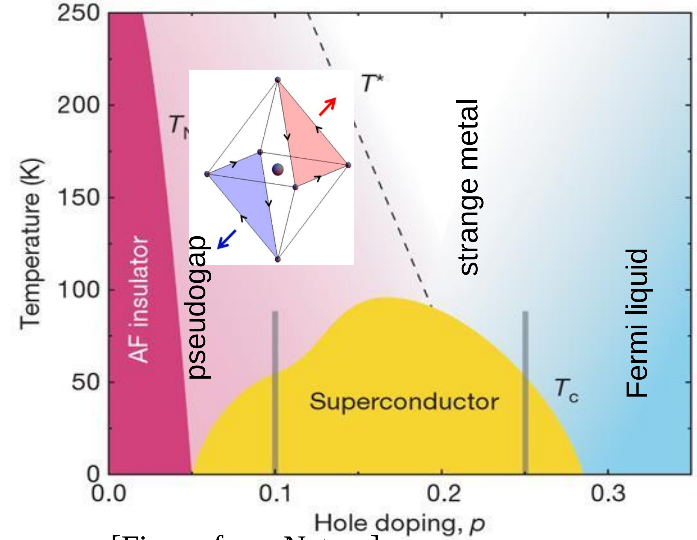


Superconductors

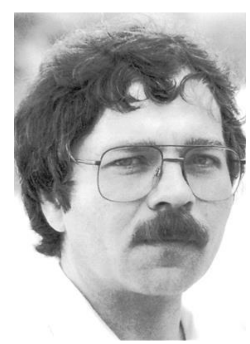




-cuprate phase diagram



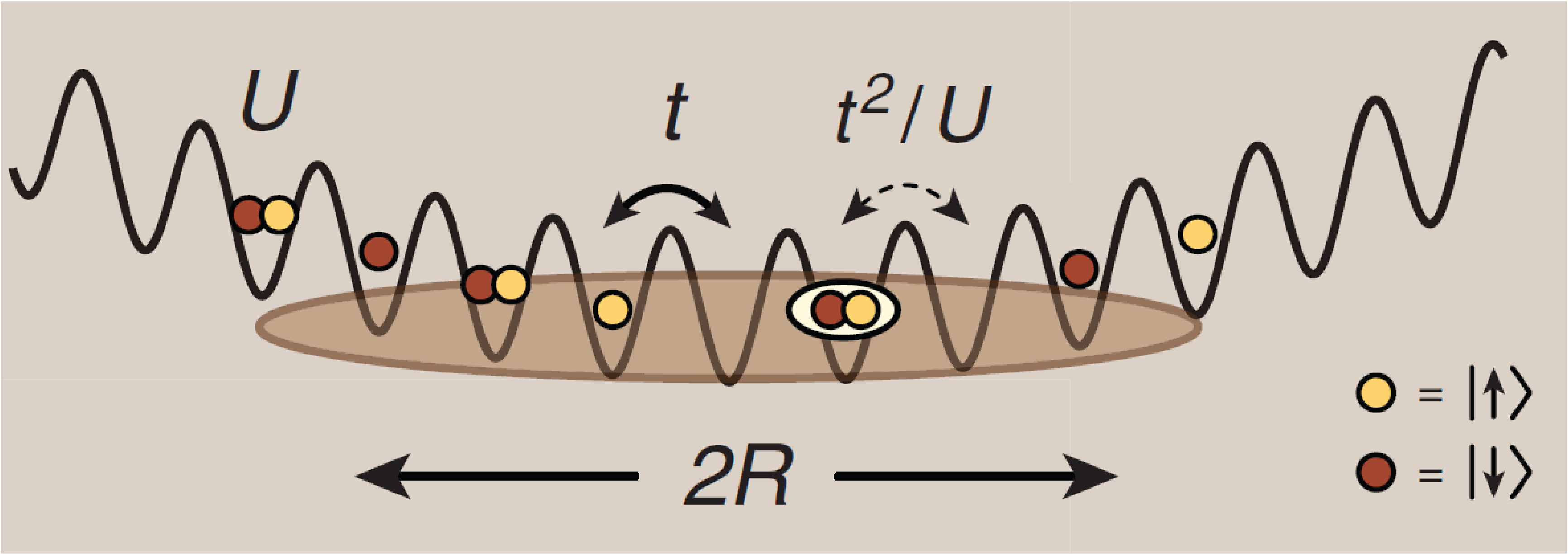
[Figure from Nature]

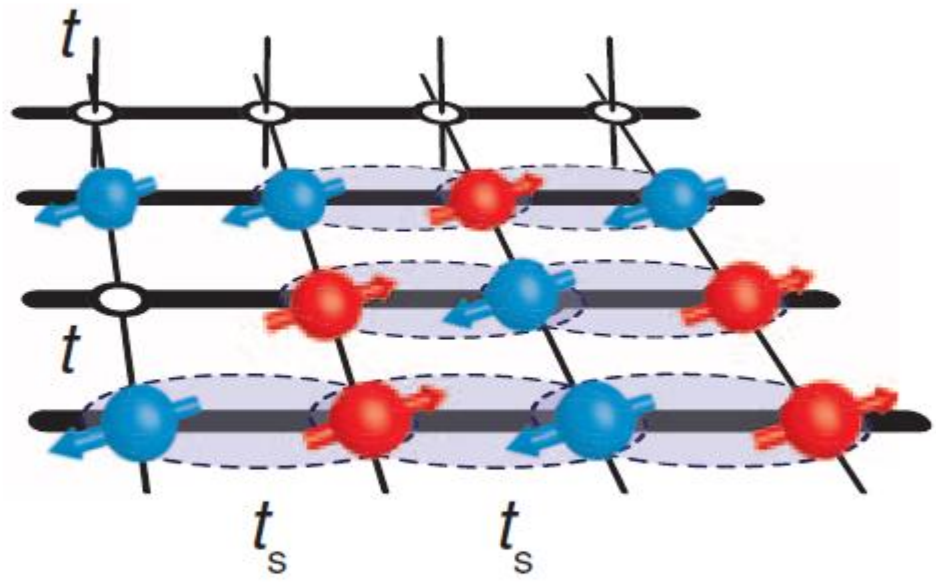


Bednorz

Müller

The atomic Fermi-Hubbard model

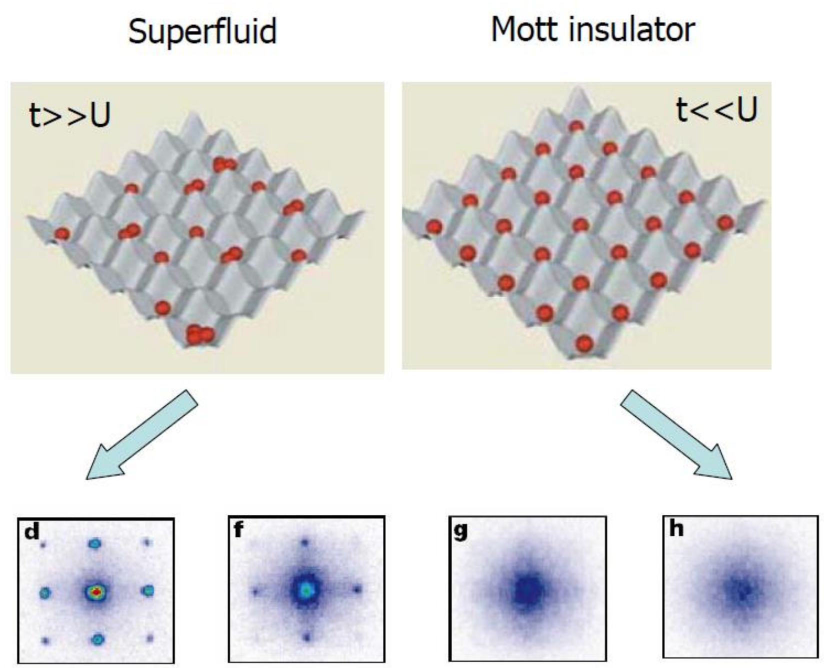




U Schneider, I. Bloch et al., Science (2008)
D. Grief, T. Esslinger, Science (2013)
R. A. Hart, R. Hulet et al., Nature (2014)

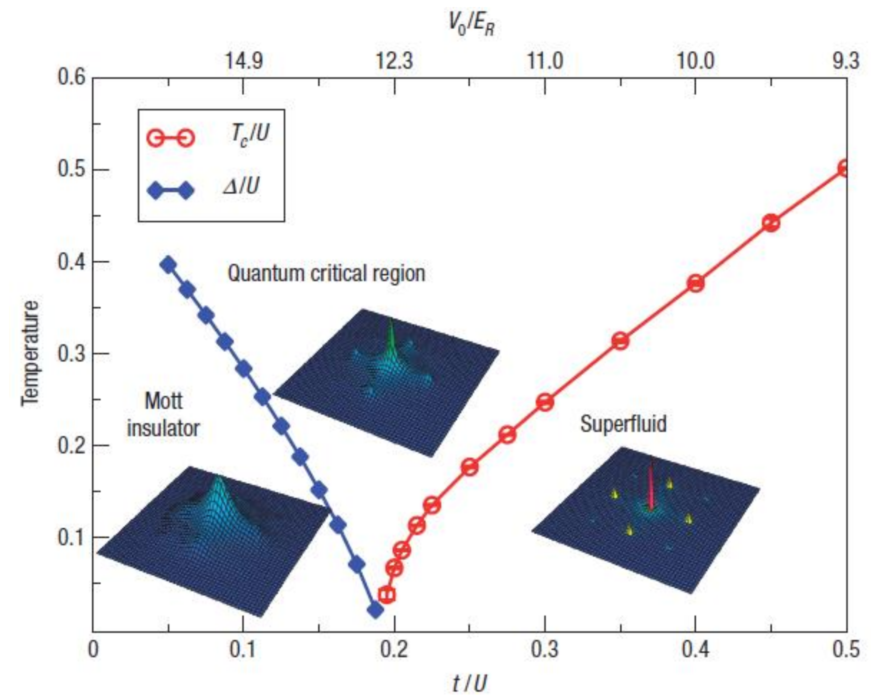
- ✓ Fermi Hubbard model realized
- ✓ Mott state achieved
- ✓ Short range anti-ferromagnetic correlations
- ✓ Non-Equilibrium, Many-body localization

The atomic Bose-Hubbard model



^{87}Rb

M. Greiner et al., Nature (2002)



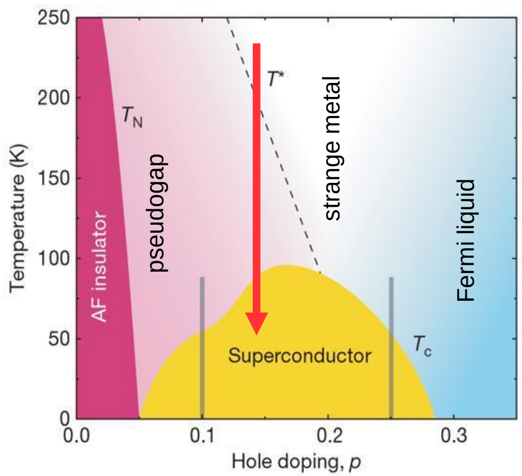
Y. Kato, Qi Zhou et al., Nat Phys (2008)

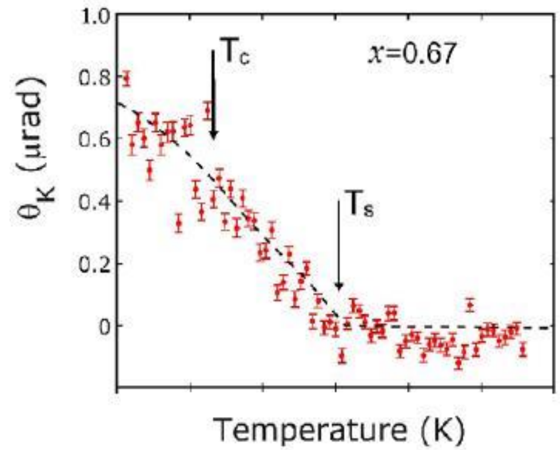
- Bose-Hubbard model, criticality and phase diagram [M.P.A. Fisher et al., PRB(1989)]
- Theoretical proposal: optical lattice realization [D. Jaksch, C. Bruder, J. I. Cirac, C. W. Gardiner, and P. Zoller, PRL (1998)]
- Experimental demonstration [M. Greiner et al., Nature (2002), ...]
- Single-site addressing [J.F. Sherson, I. Bloch, S. Kurn et al., Nature (2010), W.S. Bakr, M. Greiner et al., Nature (2009)], allowing quench dynamics, quantum walk, entanglement measurement, …

Loop currents in cuprates

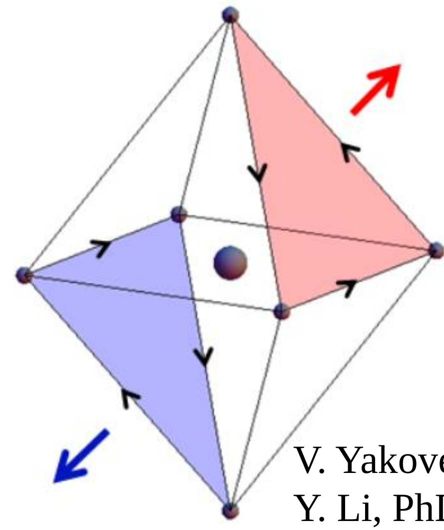
More is different

-kerr effect



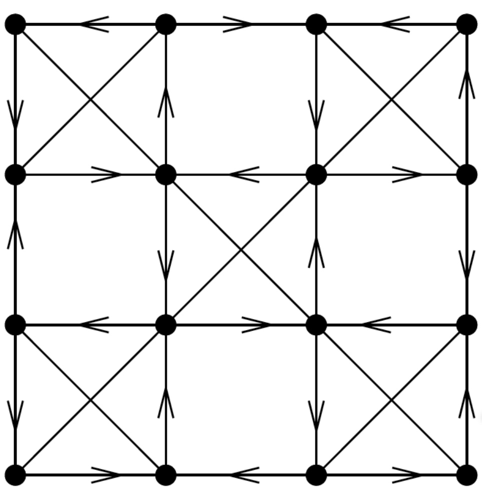


J. Xia et al., PRL (2008)



V. Yakovenko, arXiv (2014)
Y. Li, PhD thesis (2010)

Emergent/effective gauge fields



-D-density waves
C. Nayak, PRB (2000);
S. Chakravarty et al., PRBs (2011)
R. Laughlin et al., PRB (2014)
S. Sachdev et al.,, PRBs (2014)
…

$\langle c_{\mathbf{r}}^\dagger c_{\mathbf{r}'} \rangle$ is complex

-Effective magnetic fields

$$V_2 c_{\mathbf{r}}^\dagger c_{\mathbf{r}'}^\dagger c_{\mathbf{r}'} c_{\mathbf{r}} = V_2 \left[-\langle c_{\mathbf{r}}^\dagger c_{\mathbf{r}'} \rangle c_{\mathbf{r}'}^\dagger c_{\mathbf{r}} + H.c. \right] + \ldots$$

$$= \lambda e^{iA_{\mathbf{rr}'}} c_{\mathbf{r}'}^\dagger c_{\mathbf{r}}$$

Effective complex tunneling from spontaneous current order. This mechanism does not require particles to carry charge!

Outline

- Experimental signatures of loop currents in optical lattices

- Chiral spin condensate, spontaneous spin loop currents and emergent spin Hall effect in double-valley lattices
  [XL, S. Natu, A. Paramekanti, S. Das Sarma, Nat Commun (2014)]

- Chiral density waves and emergent Weyl fermions with Rydberg-dressed atoms
  [XL, S. Das Sarma, Nat Commun (2015)]

Spontaneous Loop Currents of atoms in optical lattices?

-Current in a lattice model

$$J_{\mathbf{r}'\to\mathbf{r}} = -it_{\mathbf{r}\mathbf{r}'}\psi^\dagger_{\mathbf{r}}\psi_{\mathbf{r}'} + h.c.$$

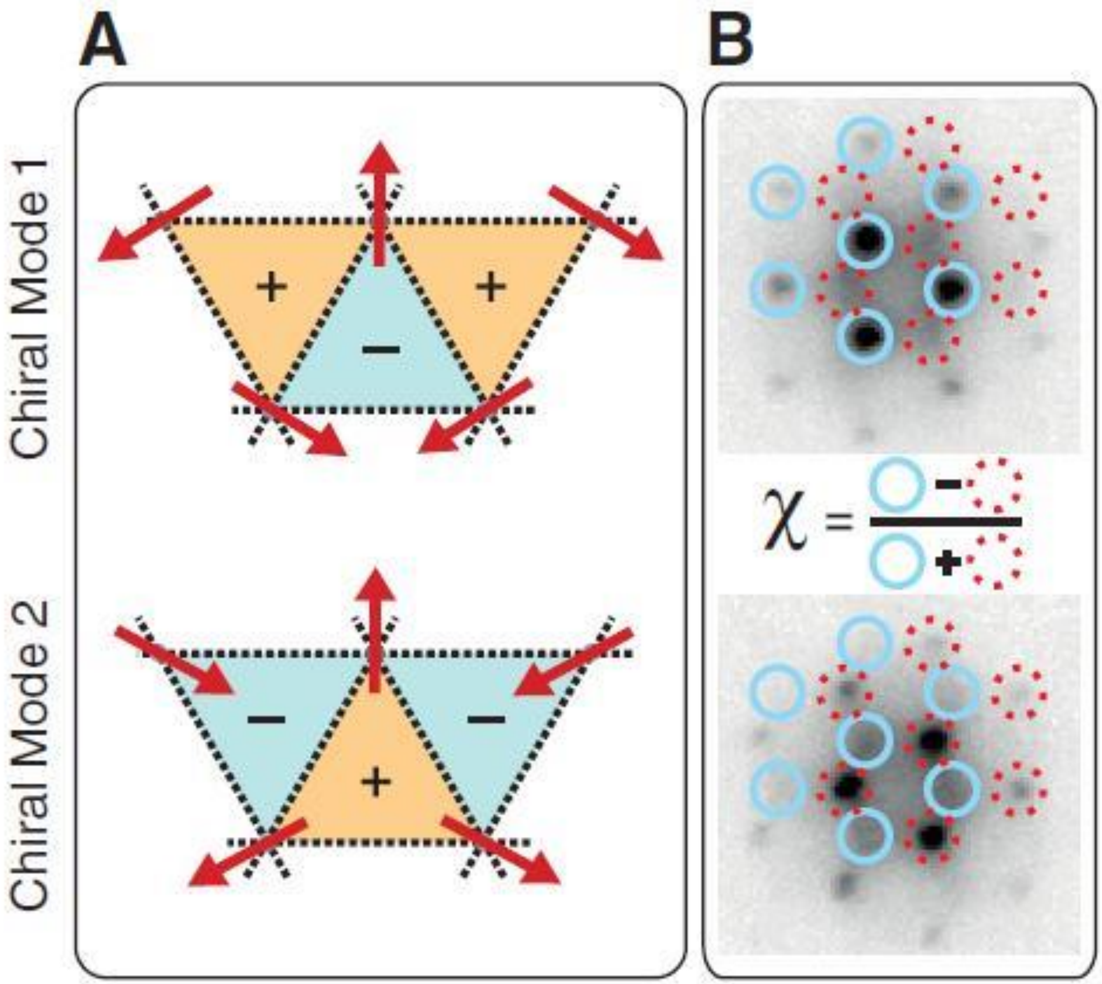
[defined from U(1) symmetry and charge conservation law]

For Bose-Einstein condensates, current means phase modulations in condensate wave functions.

$$\langle\psi_{\mathbf{r}}\rangle = \sqrt{n_{\mathbf{r}}}e^{i\theta_{\mathbf{r}}}$$

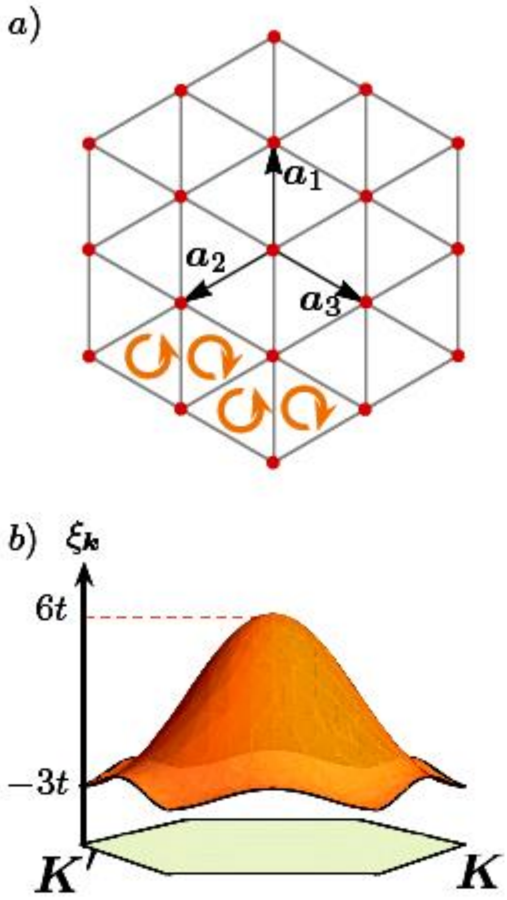
$$\langle J_{\mathbf{r}'\to\mathbf{r}}\rangle = \sqrt{n_{\mathbf{r}}n_{\mathbf{r}'}}t_{\mathbf{r}\mathbf{r}'}\sin(\theta_{\mathbf{r}'} - \theta_{\mathbf{r}})$$

Pi-flux triangular lattice



J. Struck, K. Sengstock et al., Science (2010)

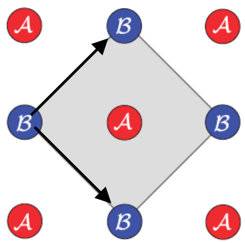
$$\langle \psi_{\mathbf{r}} \rangle = \sqrt{n_s} e^{i\mathbf{K}\cdot\mathbf{r}}$$
$$\mathbf{K} = (\pm\tfrac{2\pi}{3}, 0)$$

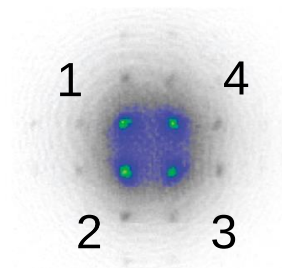


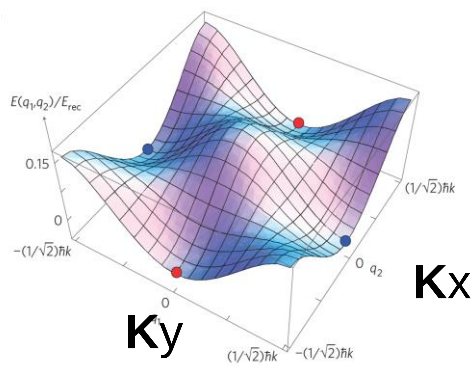
M. P. Zaletel, et al., PRB (2013)

Measurement is simple when the band minima are not time-reversal invariant points.

2nd band of Checkerboard lattice

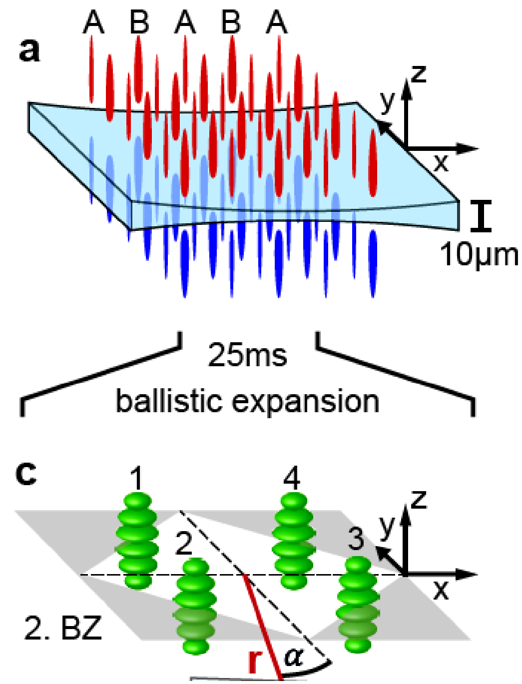


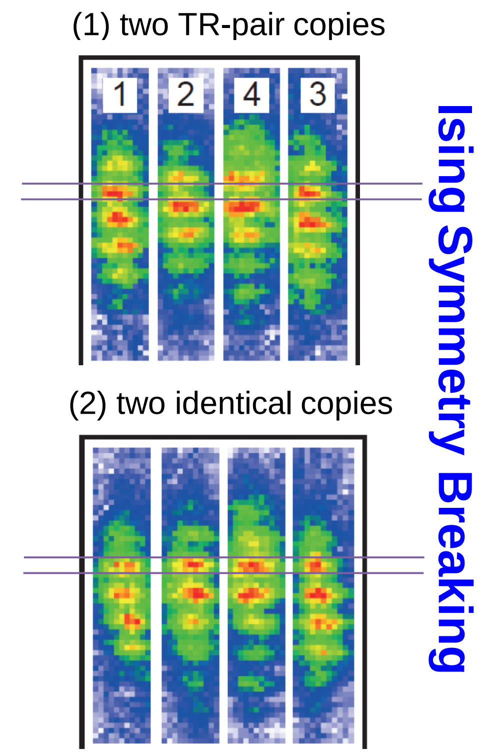




Band minima at time-reversal invariant points!

G. Wirth, A. Hemmerich et al., Nat Phys (2011)



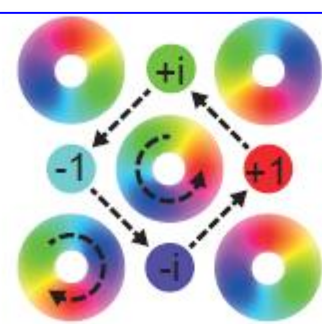


T. Kock, A. Hemmerich et al., PRL (2015)

-Condensate wave function

$$\langle \psi_{\mathbf{r}} \rangle = \sqrt{n_s}\left(e^{i\mathbf{K}_x\cdot\mathbf{r}} \pm i e^{i\mathbf{K}_y\cdot\mathbf{r}}\right)$$

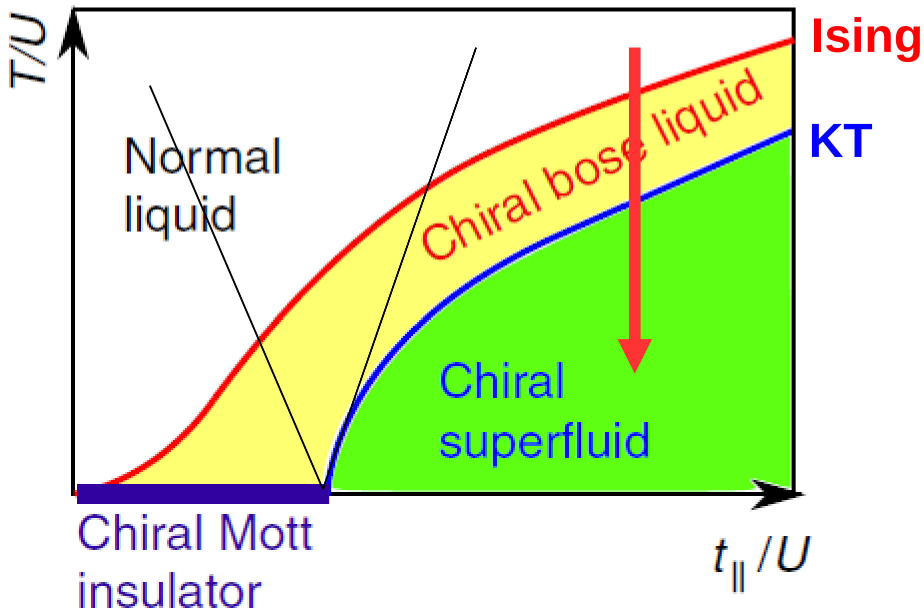
$\mathbf{K}_x = (\pi, 0)$ $\quad\quad$ $\mathbf{K}_y = (0, \pi)$

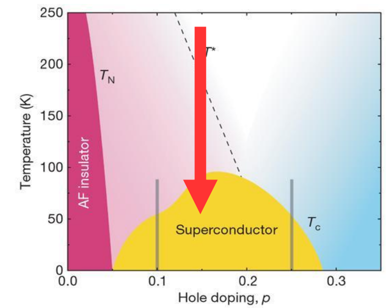


Theory work:
A. Isacsson and S. Girvin, PRA (2005)
W. V. Liu, C. Wu, PRA (2006);
A. B. Kuklov, PRL (2006)
XL, Z.-X. Zhang, W.V. Liu, PRL (2012)
…

Finite temperature phase transition

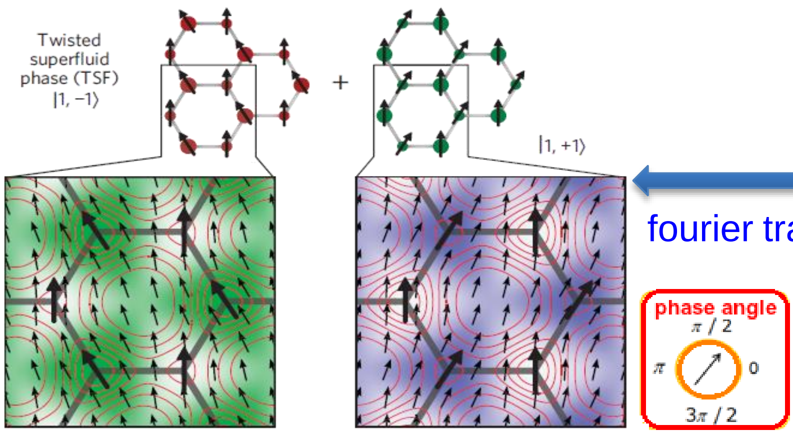
[XL et al., Nat Commun 5:3205 (2014)]



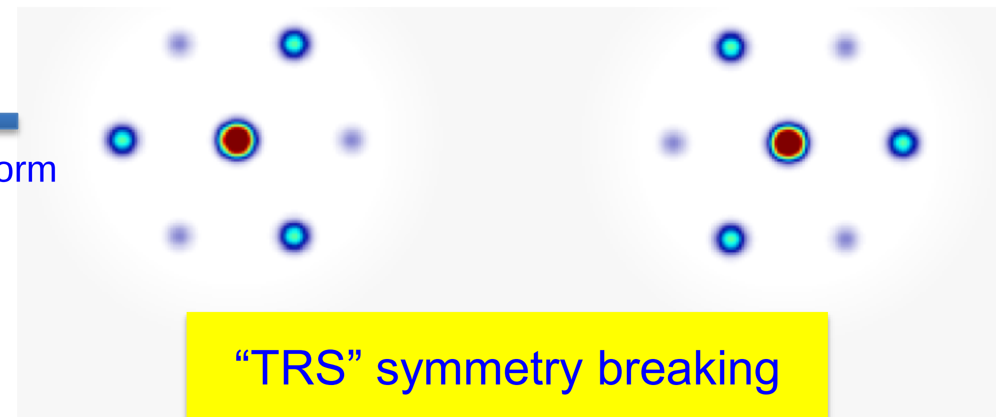


Zero temperature phase diagram by QMC: F. Hebert, Z. Cai, et al., PRB (2013)

Spinor BEC in a Hexagonal lattice

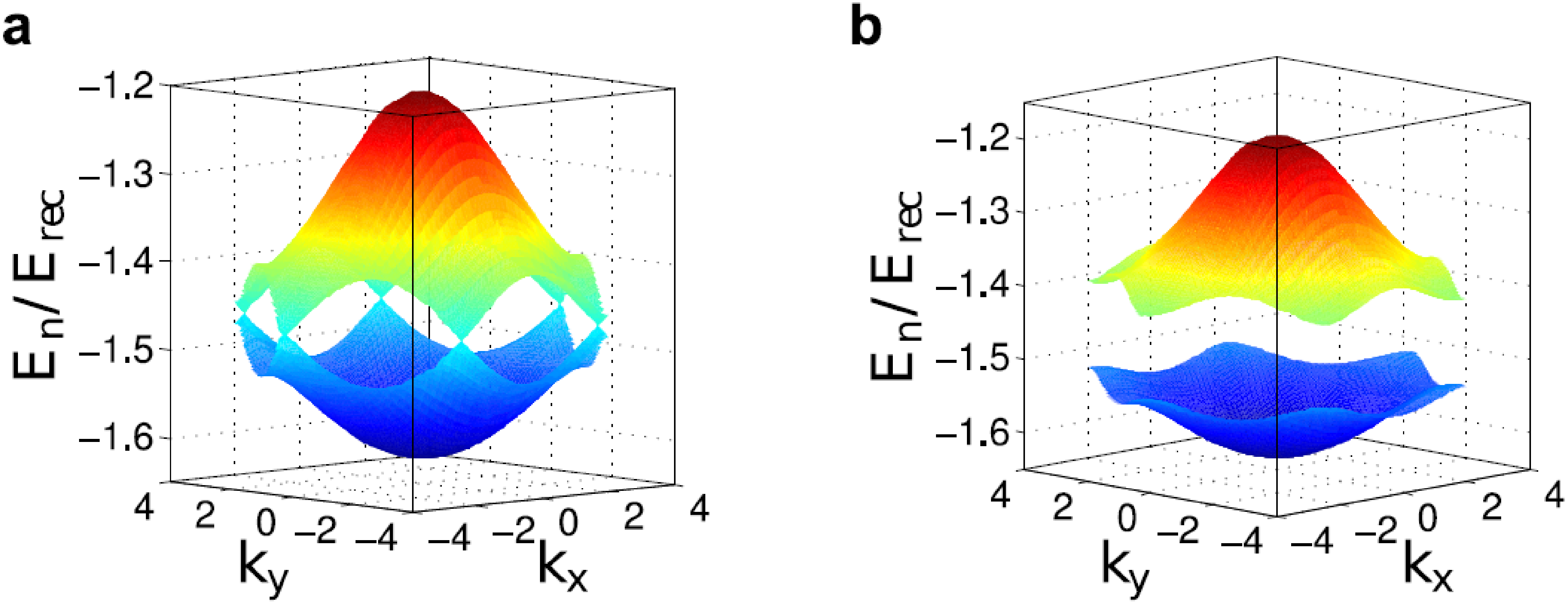


fourier transform



P. Soltan-Panahi K. Sengstock et al., Nat Phys 8, 71-75 (2012)

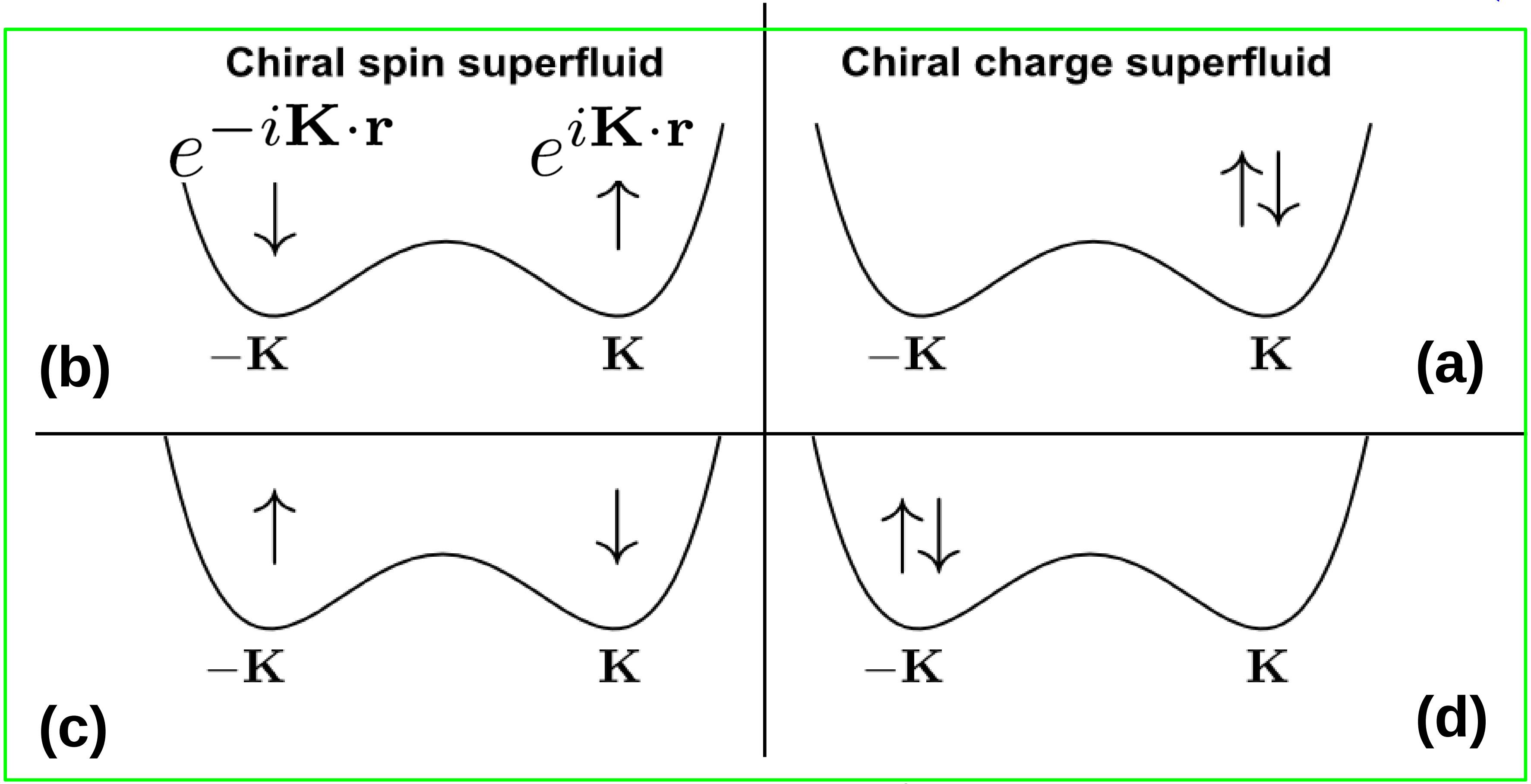
Bandstructure of the hexagonal lattice



Question: What if some particles are left in the massive Dirac valleys of the 2nd band.

Spinor Bosons in a double-valley band

[XL, S. Natu, A. Paramekanti, S. Das Sarma, Nat Commun (2014)]



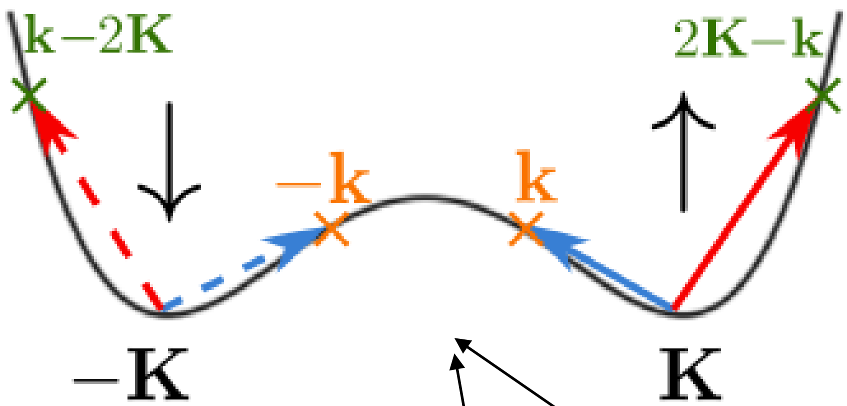
$$\varphi_{\downarrow\mathbf{r}} \to \varphi^*_{\downarrow\mathbf{r}}$$

$$E[\varphi_{\uparrow\mathbf{r}}, \varphi^*_{\downarrow\mathbf{r}}] = E[\varphi_{\uparrow\mathbf{r}}, \varphi_{\downarrow\mathbf{r}}]$$

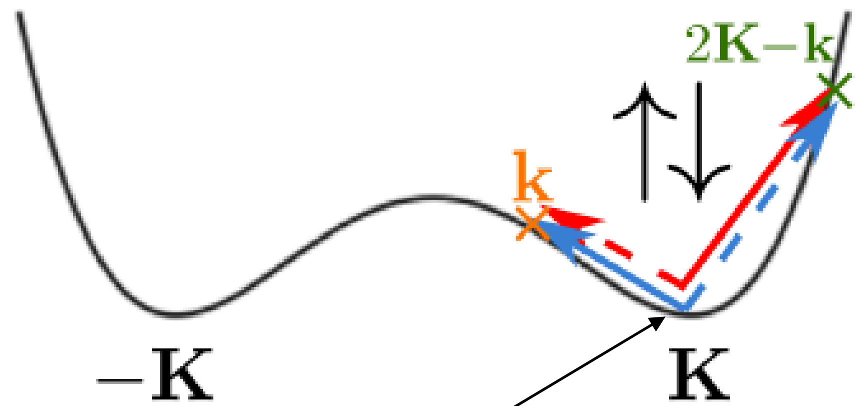
Second order perturbation theory

[XL, S. Natu, A. Paramekanti, S. Das Sarma, Nat Commun (2014)]

Chiral spin superfluid



Chiral charge superfluid



$$\Delta E^{(2)}/N_s = -\int \frac{d^d\mathbf{k}}{(2\pi)^d}\rho_\uparrow\rho_\downarrow \left\{ \frac{|U_{\uparrow\downarrow}(\mathbf{k}-\mathbf{K})|^2}{\epsilon(\mathbf{k})+\epsilon(\mathbf{Q}-\mathbf{k})} - \frac{1}{2}\frac{|U_{\uparrow\downarrow}(\mathbf{k}-\mathbf{K})|^2}{\epsilon(\mathbf{k})+\epsilon(-\mathbf{k})} - \frac{1}{2}\frac{|U_{\uparrow\downarrow}(\mathbf{K}-\mathbf{k})|^2}{\epsilon(\mathbf{Q}-\mathbf{k})+\epsilon(\mathbf{k}-\mathbf{Q})} \right\},\ \mathbf{Q}=2\mathbf{K}$$

$$\Delta E^{(2)} = E^{(2)}_{\chi_c} - E^{(2)}_{\chi_s}$$

TRS: $T\phi_\sigma(\mathbf{k})T^{-1} = \phi_\sigma(-\mathbf{k})$

an anti-unitary transformation

Logarithmic divergence and renormalized theory

[XL, S. Natu, A. Paramekanti, S. Das Sarma, Nat Commun (2014)]

In two dimensions, the bare perturbative result has a logarithmic divergence

$\int d^2\mathbf{k}\,\frac{1}{\mathbf{k}^2}$ ⟶ infrared log divergence

-renormalized theory

$$\Delta E^{(2)}/N_s = -\tfrac{1}{2}\rho_\uparrow\rho_\downarrow \int_{\mathbf{k}} g^2(\mathbf{k})$$ ⟶ effective scattering among quasi-particles

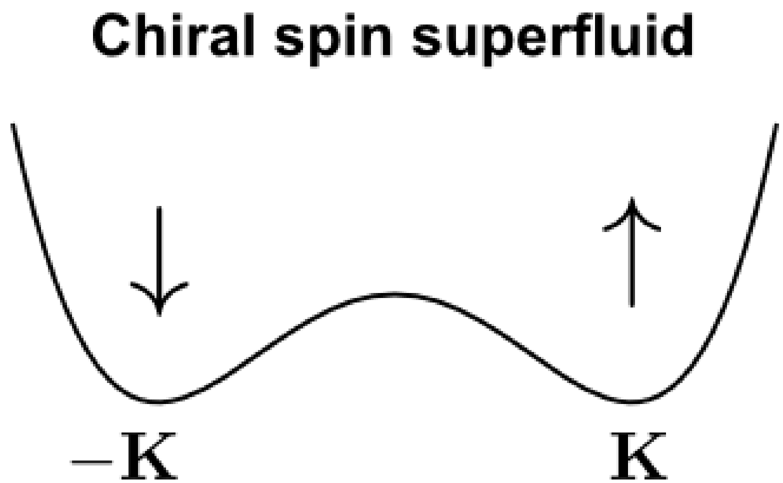
$$\times \left\{ \frac{2}{\varepsilon_\uparrow(\mathbf{k},\mathbf{Q}-\mathbf{k})+\varepsilon_\downarrow(\mathbf{k},\mathbf{Q}-\mathbf{k})} \right.$$ ⟶ Bogoliubov spectra

$$- \frac{1}{\varepsilon_\uparrow(\mathbf{k},\mathbf{Q}-\mathbf{k})+\varepsilon_\downarrow(-\mathbf{Q}+\mathbf{k},-\mathbf{k})+\Delta\epsilon(\mathbf{k},\mathbf{Q}-\mathbf{k})-\Delta\epsilon(-\mathbf{Q}+\mathbf{k},-\mathbf{k})}$$

$$\left. - \frac{1}{\varepsilon_\downarrow(-\mathbf{Q}+\mathbf{k},-\mathbf{k})+\varepsilon_\uparrow(\mathbf{k},\mathbf{Q}-\mathbf{k})+\Delta\epsilon(-\mathbf{Q}+\mathbf{k},-\mathbf{k})-\Delta\epsilon(\mathbf{k},\mathbf{Q}-\mathbf{k})} \right\}$$

Universal Chiral spin superfluid

[XL, S. Natu, A. Paramekanti, S. Das Sarma, Nat Commun (2014)]



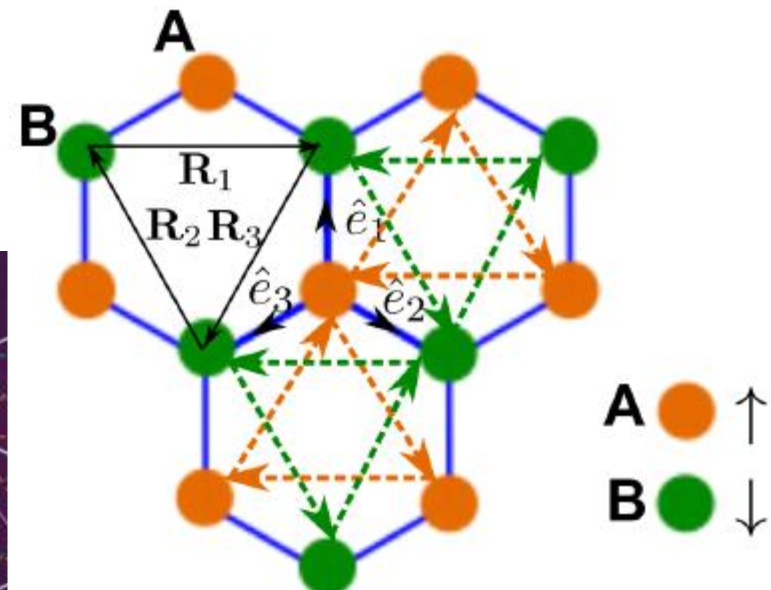
double-valley bandstructure

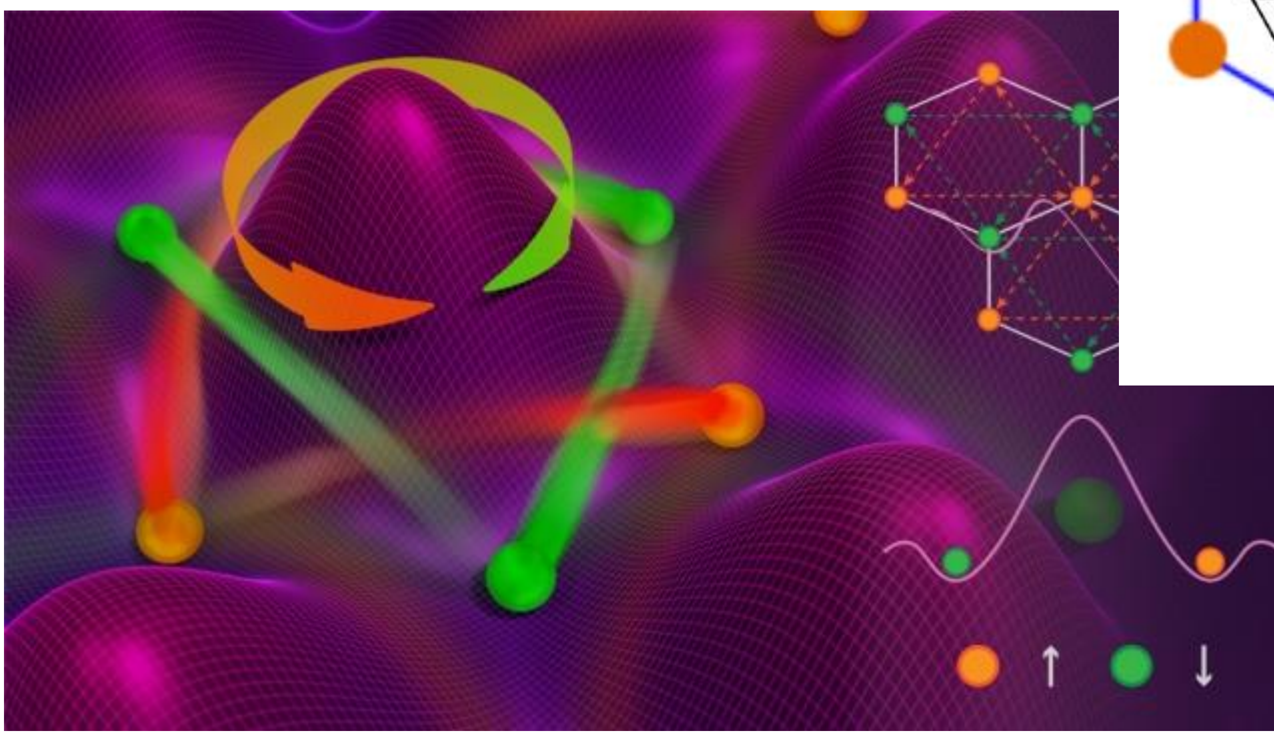
$$\int d^d\mathbf{k}\,\mathbf{k}\,[n_\uparrow(\mathbf{k}) - n_\downarrow(\mathbf{k})] \neq 0$$

With two component bosons loaded into a double-valley band, quantum fluctuations universally select the chiral spin superfluid through a quantum order by disorder mechanism.

Spin-Loop Current

[XL, S. Natu, A. Paramekanti, S. Das Sarma, Nat Commun (2014)]

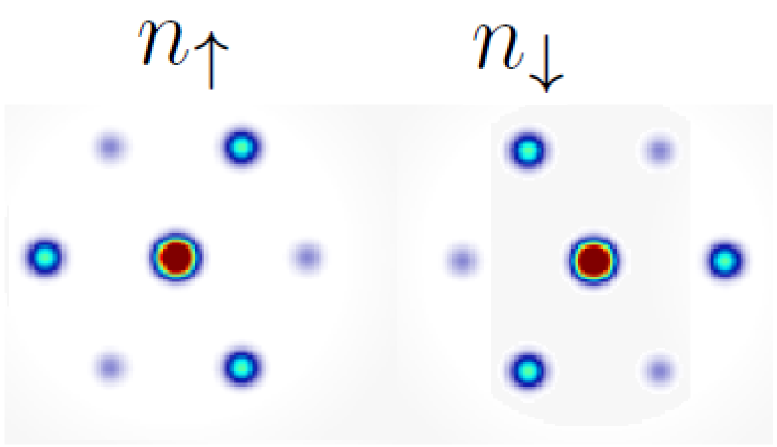




[Figure from JQI website]

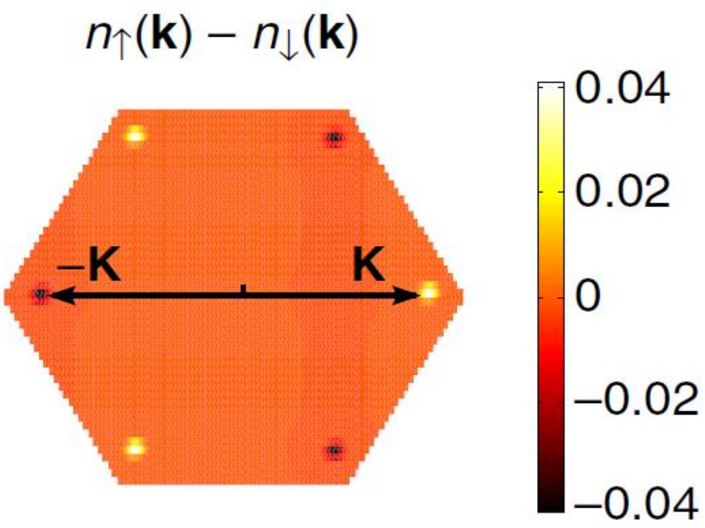
Experimental signatures

-experimental data



P. Soltan-Panahi et al., Nat Phys 8, 71-75 (2012)

-our theory prediction



XL, et al., Nat Commun 5:5174 (2014)

Spontaneous spin Hall effect

[XL, S. Natu, A. Paramekanti, S. Das Sarma, Nat Commun (2014)]

-Berry curvature

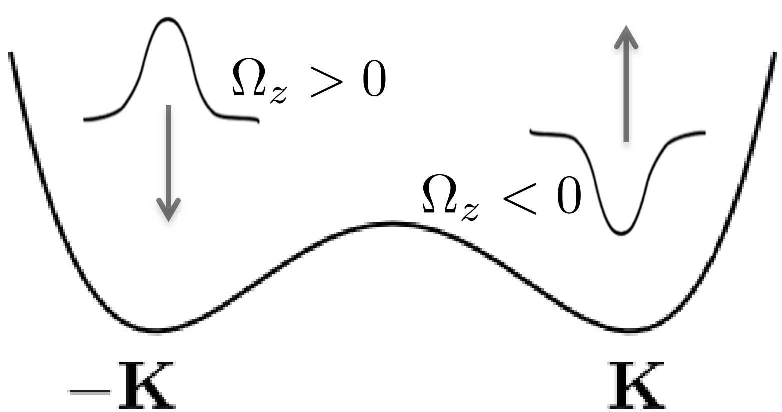
[Xiao, Chang, Niu, RMP (2010)]

$$\mathbf{\Omega}(\mathbf{k}) = \nabla_{\mathbf{k}} \times \langle u(\mathbf{k})|i\nabla_{\mathbf{k}}|u(\mathbf{k})\rangle$$

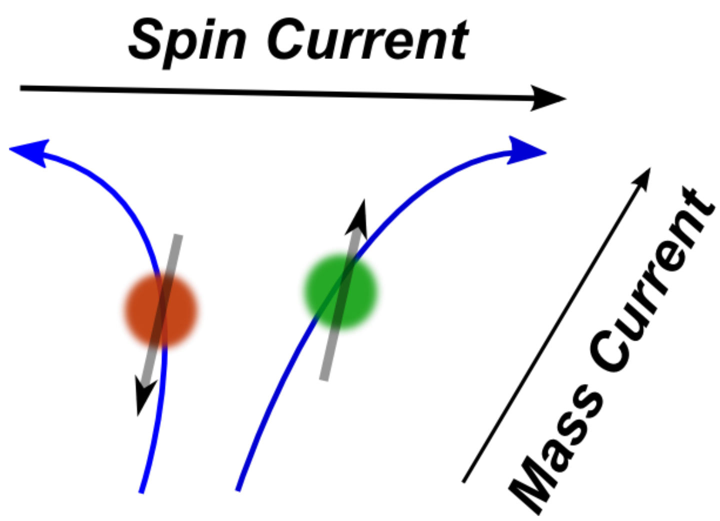
$$\mathbf{\Omega}(\mathbf{K}) = \mathbf{\Omega}(-\mathbf{K})$$ Inversion symmetry

$$\mathbf{\Omega}(\mathbf{K}) = -\mathbf{\Omega}(-\mathbf{K})$$ Time-reversal symmetry

Chiral spin superfluid



Absence of Inversion

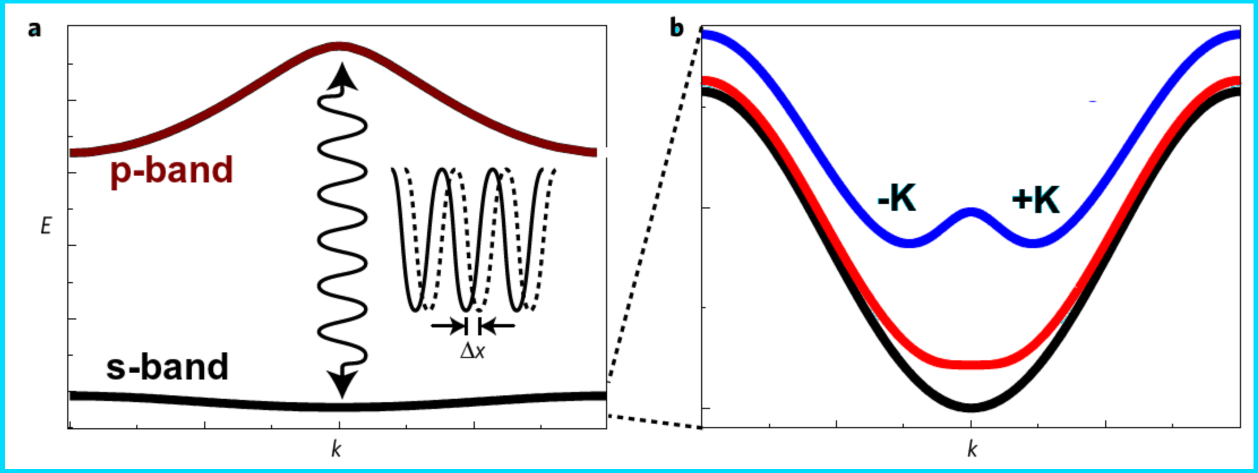


This spin Hall response vanishes above some transition temperature!

Relevance to other double-valley bands

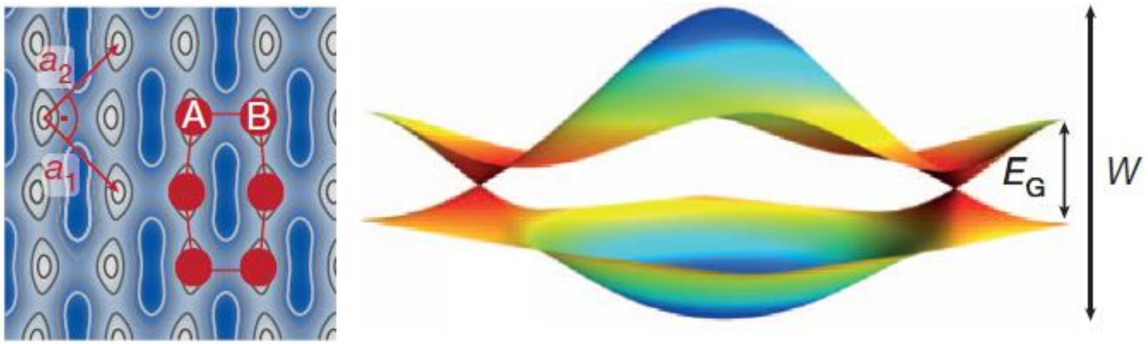
Chin group (Chicago)

[C. Parker et al., Nat Phys (2013)]



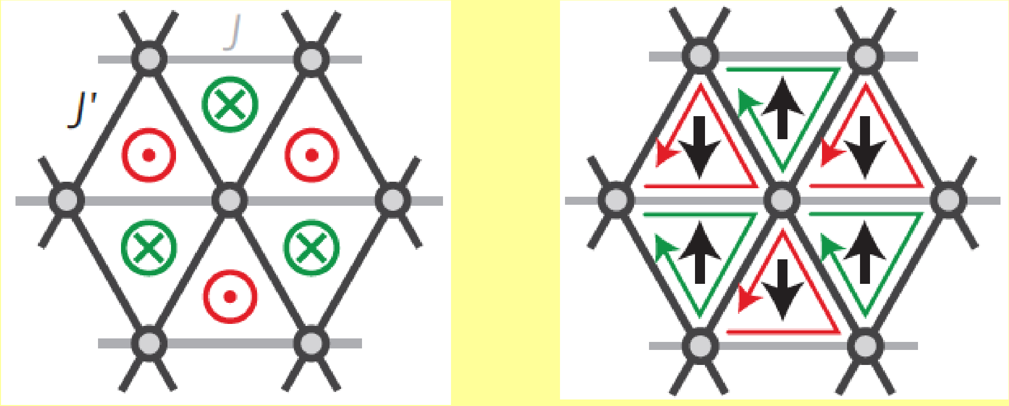
Esslinger group (ETH)

[L. Tarruell et al., Nature (2012)]



Sengstock group (U Hamburg)

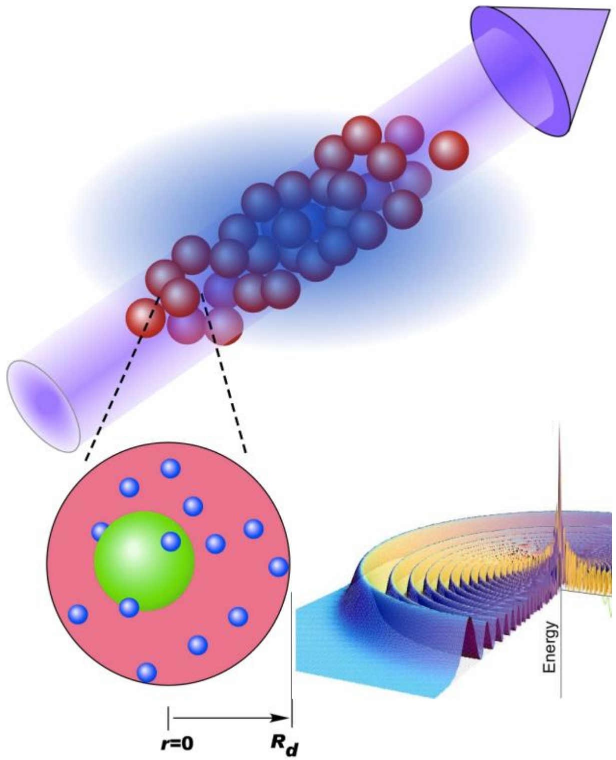
[J. Struck et al., Nature Physics (2013)]

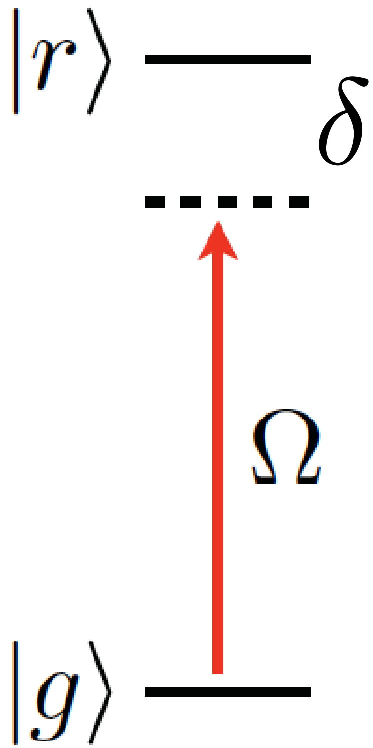


Outline

- ✓ Experimental signatures of loop currents optical lattices
- ✓ Chiral spin condensate and spin loop currents in double-valley lattices
  [XL, S. Natu, A. Paramekanti, S. Das Sarma, Nat Commun (2014)]
- ❑ Chiral density waves and emergent Weyl fermions with Rydberg-dressed atoms
  [XL, S. Das Sarma, Nat Commun (2015)]

Rydberg state and off-resonant dressing





$$|\text{Atom}\rangle = |g\rangle + \frac{\Omega}{\delta}|r\rangle$$

Spontaneous emission: $\Gamma \propto \left(\frac{\Omega}{\delta}\right)^2$

$$V(\mathbf{x}) = \frac{V_6}{1 + (|\mathbf{x}|/x_c)^6}$$

Interaction strength: $V_6 \propto \frac{\Omega^4}{\delta^3}$

- ✧ New approach of controlling atomic interactions [N. Henkel et al., PRL (2010); G. Pupillo et al., PRL (2010); A. Dauphin et al., PRA (2012); XL et al., Nat Commun (2015)];
- ✧ Quantum simulations of spin ice and lattice gauge theory [P. Zoller et al., PRL/PRX (2014)];
- ✧ Experimental achievements: Quantum Computing [Saffman et al., RMP (2010)]; Non-linear Photonics [Lukin et al., Nature (2013)]; Dynamical Crystallization [Bloch et al., Nature (2012), Science (2015)]; Off-resonant Dressing [Balewski et al., NJP (2014)]; …

Non-local interaction and Density wave instability

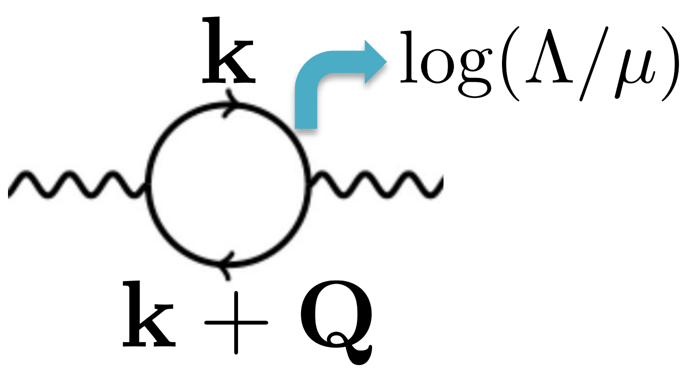
[XL, S. Das Sarma, Nat Commun (2015)]

-Rydberg-dressed fermions on a cubic lattice

$$\epsilon(\mathbf{k}) = -2t\,(\cos k_x + \cos k_y + \cos k_z)$$

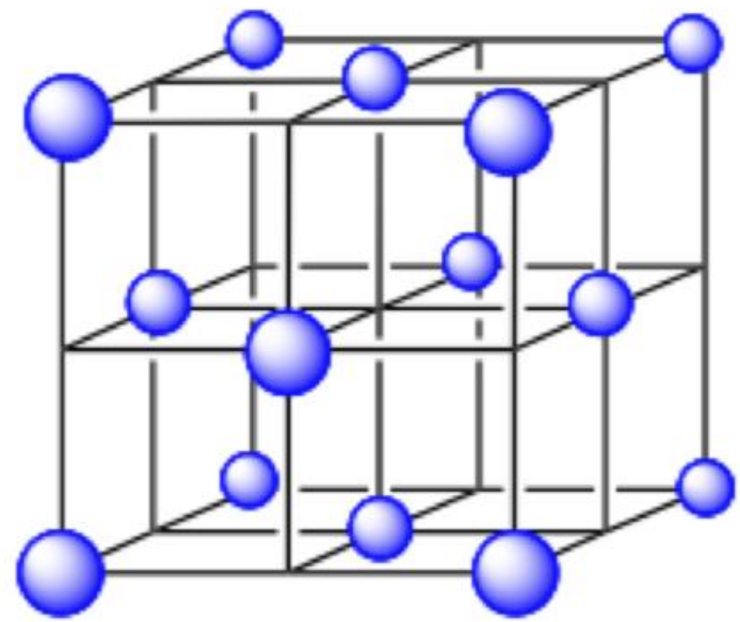
$$\epsilon(\mathbf{k}) = -\epsilon(\mathbf{k}+\mathbf{Q}) \qquad \mathbf{Q} = (\pi, \pi, \pi)$$

Fermi surface nesting at half filling



-trivial density wave with short-range interaction

$$\rho(\mathbf{k}) = \langle \psi^\dagger(\mathbf{k}+\mathbf{Q})\psi(\mathbf{k})\rangle \sim \textit{const}$$



Unconventional density waves

[XL, S. Das Sarma, Nat Commun (2015)]

Momentum dependence of $\rho(\mathbf{k}) = \langle \psi^\dagger(\mathbf{k}+\mathbf{Q})\psi(\mathbf{k})\rangle$

Table 1 | Symmetry classification of three-dimensional density wave orders. The classification is according to irreducible representation of the symmetry group $O_h \times \mathcal{T}$.

A_{1g}^+	A_{1g}^-	A_{1u}^+	A_{1u}^-
$1, \cos k_x \cos k_y + \cos k_y \cos k_z + \cos k_z \cos k_x$,	$i(\cos k_x + \cos k_y + \cos k_z)$	—	—
A_{2g}^+	A_{2g}^-	A_{2u}^+	A_{2u}^-
—	—	—	—
E_g^+	E_g^-	E_u^+	E_u^-
$\begin{cases} \cos k_z(\cos k_x - \cos k_y) \\ 2\cos k_x \cos k_y - \cos k_z(\cos k_x + \cos k_y) \end{cases}$	$\begin{cases} i(\cos k_x - \cos k_y) \\ i(2\cos k_z - \cos k_x - \cos k_y) \end{cases}$	—	—
T_{1g}^+	T_{1g}^-	T_{1u}^+	T_{1u}^-
—	—	$\begin{cases} \sin k_x(\cos k_y + \cos k_z) \\ \sin k_y(\cos k_z + \cos k_x) \\ \sin k_z(\cos k_x + \cos k_y) \end{cases}$	$\begin{cases} i\sin k_x \\ i\sin k_y \\ i\sin k_z \end{cases}$
T_{2g}^+	T_{2g}^-	T_{2u}^+	T_{2u}^-
$\begin{cases} \sin k_x \sin k_y \\ \sin k_y \sin k_z \\ \sin k_z \sin k_x \end{cases}$	—	$\begin{cases} \sin k_x(\cos k_y - \cos k_z) \\ \sin k_y(\cos k_z - \cos k_x) \\ \sin k_z(\cos k_x - \cos k_y) \end{cases}$	—

TRS even | **TRS odd (Loop Current)** | **TRS odd (Loop Current)** | **TRS even**

* $J_{\mathbf{r}' \to \mathbf{r}} = -i t_{\mathbf{r}\mathbf{r}'} \psi_{\mathbf{r}}^\dagger \psi_{\mathbf{r}'} + h.c.$

Topological density waves (3D Quantum Hall)

[XL, S. Das Sarma, Nat Commun (2015)]

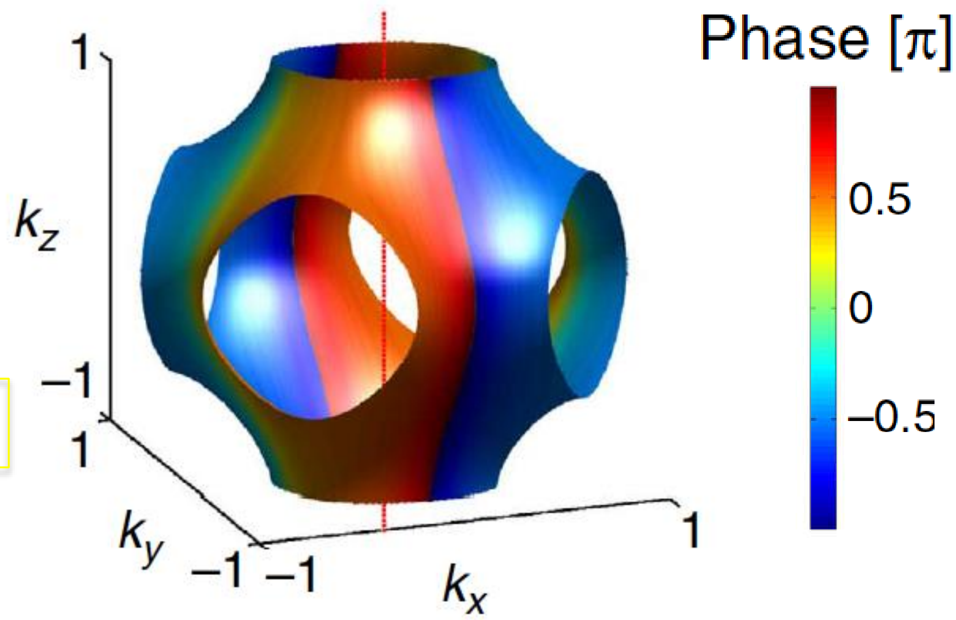
-quasi-particle Hamiltonian

$$H_{\mathrm{BdG}}(\mathbf{k}) = \begin{bmatrix} \epsilon_{\mathbf{k}} & \Delta_{\mathbf{k}} \\ \Delta^*_{\mathbf{k}} & -\epsilon_{\mathbf{k}} \end{bmatrix}$$

$\epsilon_{\mathbf{k}} \to \mathbf{k}$, $-\epsilon_{\mathbf{k}} \to \mathbf{k}+\mathbf{Q}$

$$\Delta_{\mathbf{k}} \approx 2\Delta_{T^+_{2g}} \sin k_x \sin k_y + i\Delta_{E^-_g}(\cos k_x - \cos k_y)$$

(Loop Current in real space)



Topological density waves (3D Quantum Hall + Weyl)

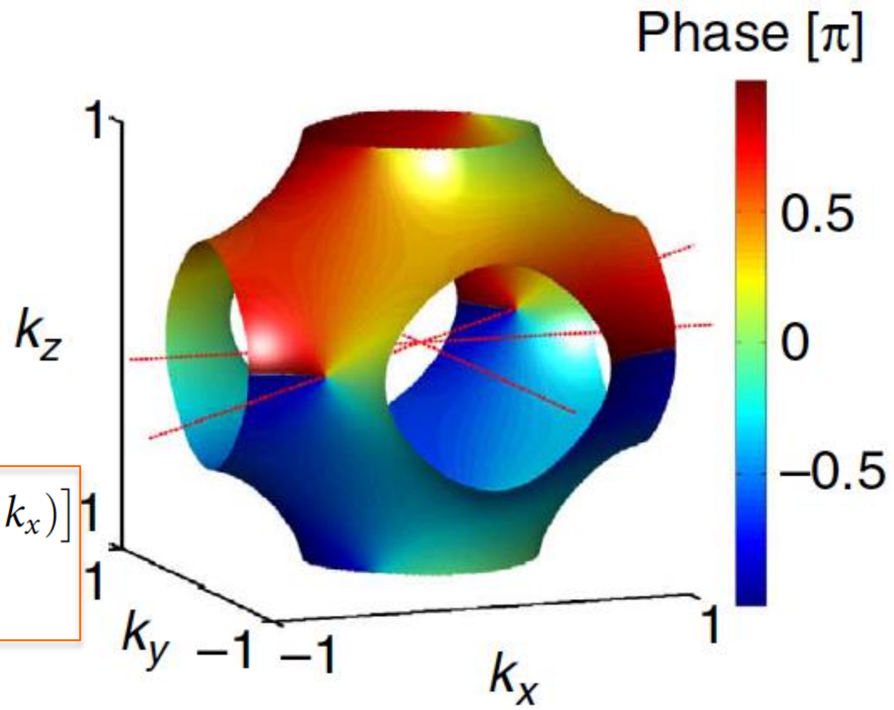
[XL, S. Das Sarma, Nat Commun (2015)]

-quasi-particle Hamiltonian

$$H_{\mathrm{BdG}}(\mathbf{k}) = \begin{bmatrix} \epsilon_{\mathbf{k}} & \Delta_{\mathbf{k}} \\ \Delta^*_{\mathbf{k}} & -\epsilon_{\mathbf{k}} \end{bmatrix}$$

$$\Delta_{\mathbf{k}} = \Delta_{\mathrm{T}^+_{2u}}\left[\sin k_x(\cos k_y - \cos k_z) + \sin k_y(\cos k_z - \cos k_x)\right] + i\sqrt{2}\Delta_{\mathrm{T}^-_{1u}} \sin k_z.$$

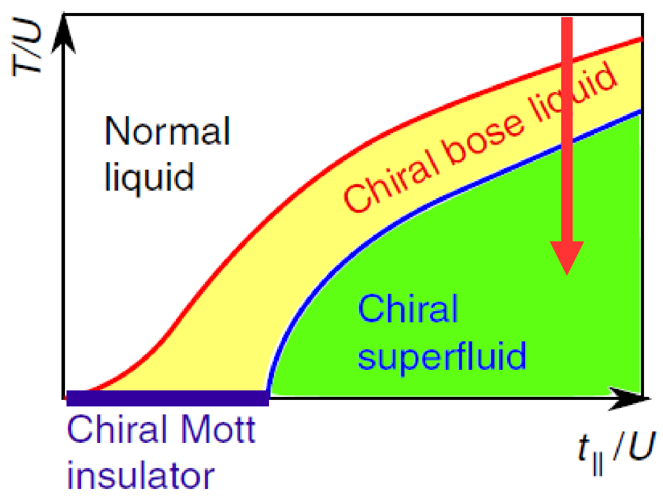
(Loop Current in real space)

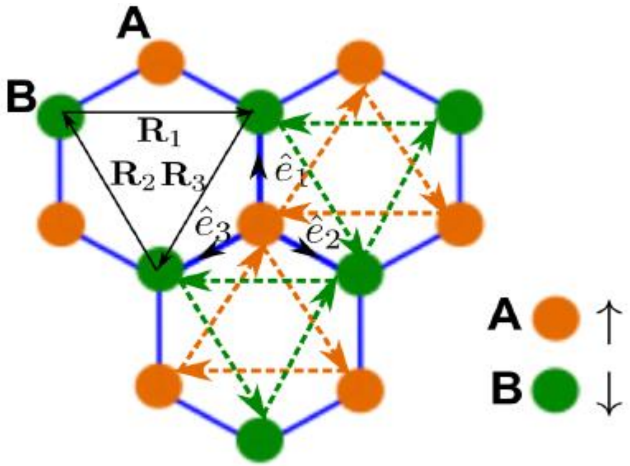


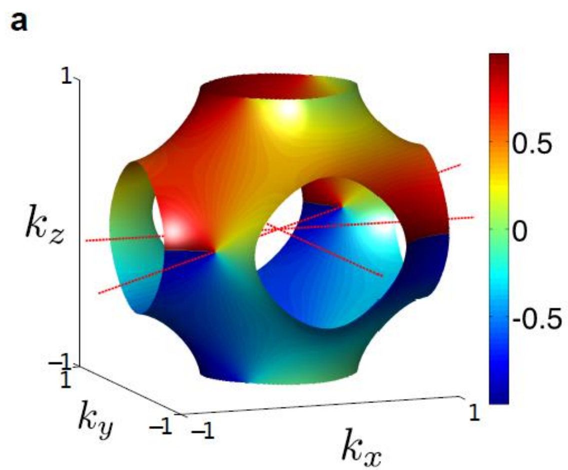
Spontaneous loop currents provide effective gauge fields and give rise to topological states

Summary

- ✓ Experimental evidence of spontaneous loop currents and time-reversal symmetry breaking in double-valley optical lattices
- ✓ Chiral spin condensation and spin loop currents as generic phenomena for spinor bosons in double-valley lattices
- ✓ Topological properties of chiral density waves with Rydberg dressed fermions







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