Interacting surface states of topological insulators

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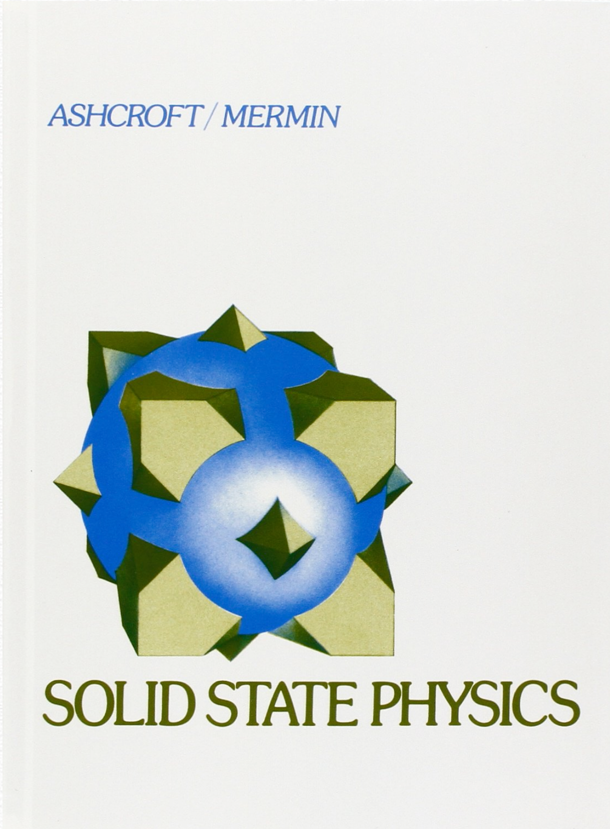
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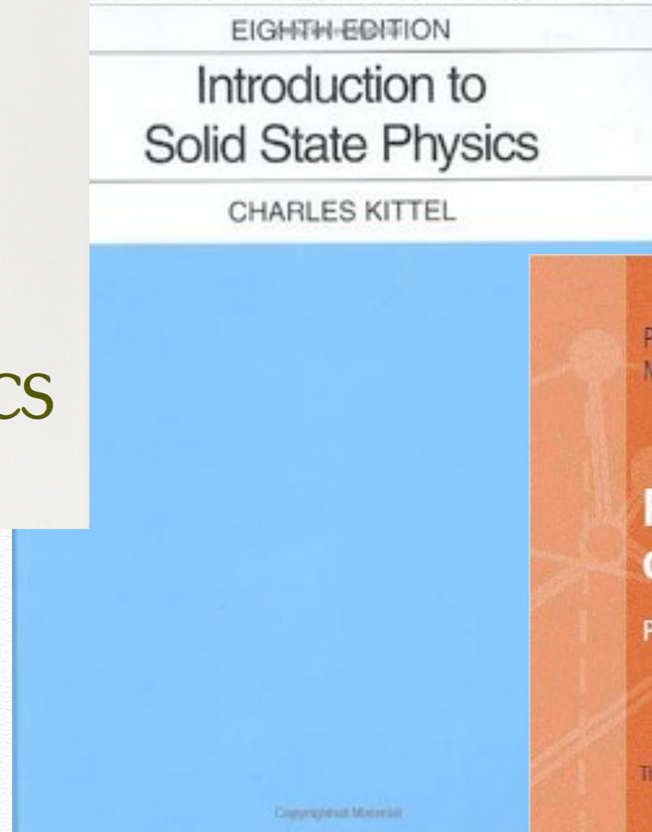
SOLID STATE PHYSICS


EIGHTH EDITION
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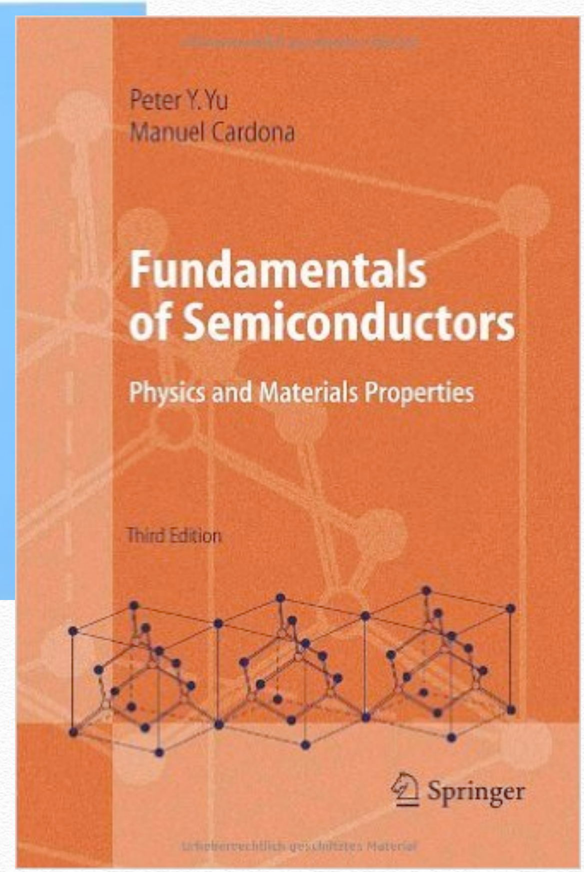
CHARLES KITTEL


Peter Y. Yu
Manuel Cardona
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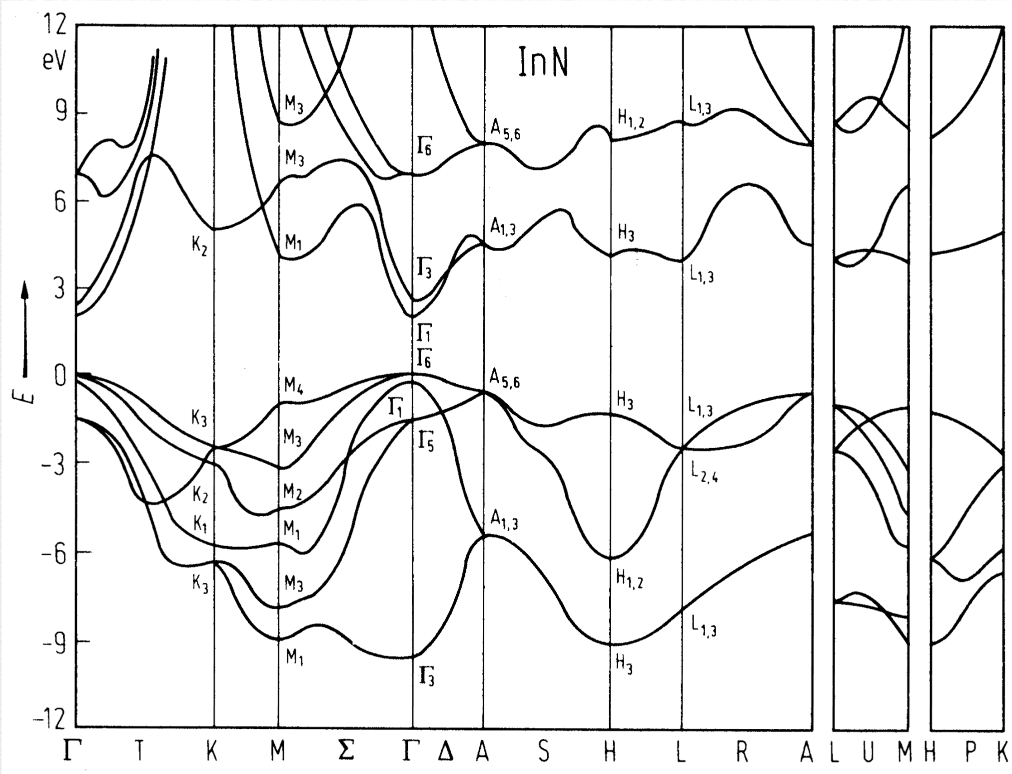
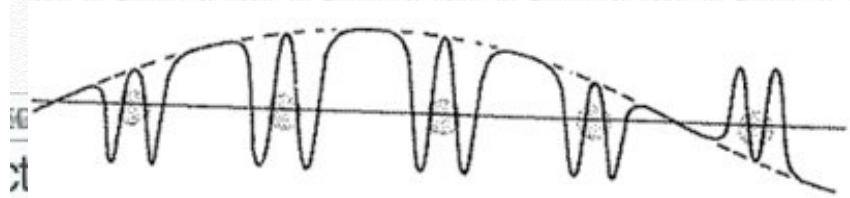
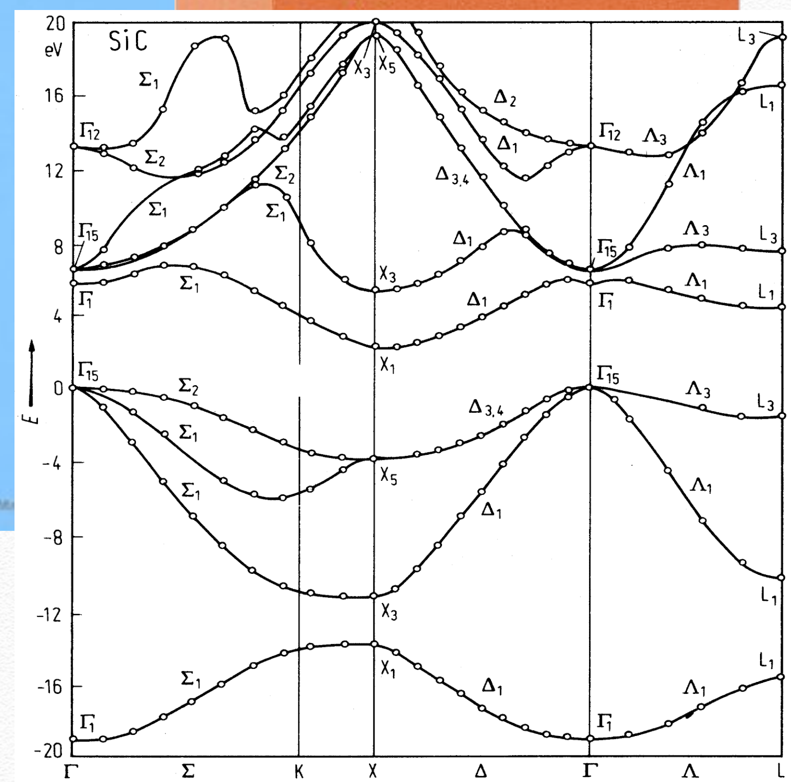
Physics and Materials Properties
Third Edition
Springer



$$\left[-\frac{\hbar}{2m}\nabla^2 + U(\vec{r})\right]\psi = E\psi$$





$$\psi_{\vec{k}}(\vec{r}) = u_{\vec{k}}(\vec{r})e^{i\vec{k}.\vec{r}}$$

What about interactions?

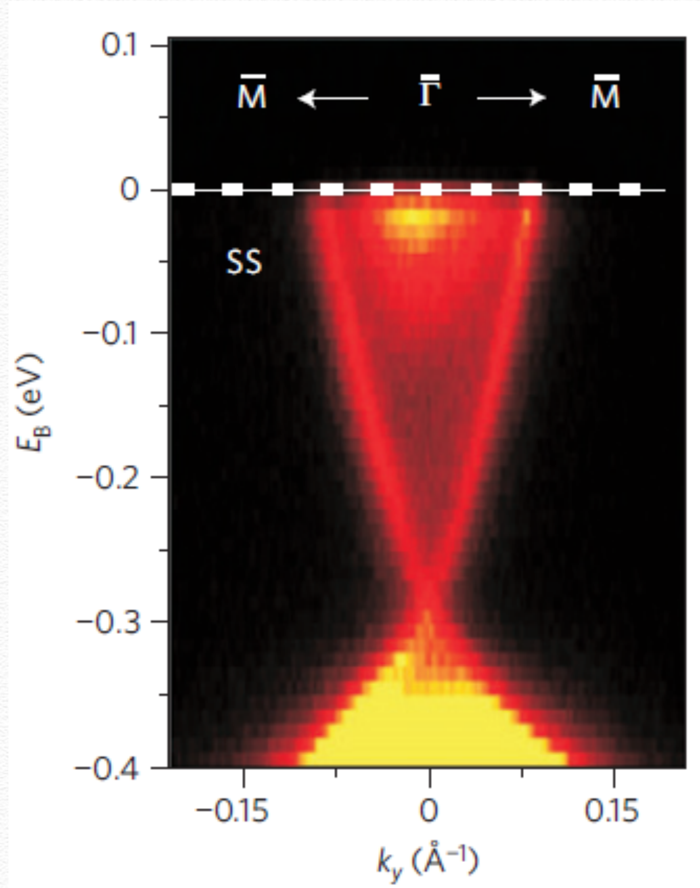
- Problems of interacting electrons are hard...
- A lot depends on microscopics: chemistry, lattices...

Topological phases

- Topological phases of matter are nice, because their long-wavelength properties are universal
- Bulk: quantized response, emergent gauge and/or matter d.o.f.
- Surface: robust gapless d.o.f.
- Bulk is gapped, focus on effect of interactions on surface

3D topological insulators

- Surface state = 2D Dirac fermion
- Goal: universal (materials-independent) description of surface state interactions & instabilities



Bi_2Se_3

(Xia et al., Nat. Phys. 2009)

Collaborators

R. Lundgren
(UT Austin)

W. Witczak-Krempa
(Harvard)

Outline

- Weak correlations: Landau theory of helical Fermi liquids

R. Lundgren and JM, PRL **115**, 066401 (2015)

- Strong correlations: Universal conductivity at semimetal-superconductor QCP

W. Witczak-Krempa and JM, arXiv:1510.06397

Landau Fermi liquid theory

- Fundamental paradigm of many-body physics (Landau 1956; Abrikosov, Khalatnikov 1957)

- Adiabatic continuity between energy levels of free & interacting systems: QP with momentum **k**, spin **σ**, distribution function $n_{\mathbf{k}\sigma}$

Landau Fermi liquid theory

- Landau functional: energy of many-body excited state (configuration of QPs) relative to GS

$$\delta E[\delta n] = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} \delta n_{\mathbf{k}\sigma} + \frac{1}{2V} \sum_{\mathbf{k},\mathbf{k}',\sigma,\sigma'} f_{\sigma\sigma'}(\mathbf{k},\mathbf{k}') \delta n_{\mathbf{k}\sigma} \delta n_{\mathbf{k}'\sigma'}$$

$\delta n_{\mathbf{k}\sigma} = n_{\mathbf{k}\sigma} - n_{\mathbf{k}\sigma}^0$

Landau parameters

- Most general symmetry-allowed short-range interaction: TRS, spatial SO(3) rotations, spin SU(2) rotations

$$f_{\sigma\sigma'}(\mathbf{k},\mathbf{k}') = f_{\sigma\sigma'}(\mathbf{k}_F,\mathbf{k}'_F) = f^s(\theta) + \sigma\sigma' f^a(\theta)$$

$$f_l^{s,a} = (2l+1)\int_0^\pi \frac{d\Omega}{4\pi} f^{s,a}(\theta) P_l(\cos\theta)$$

$$F_l^{s,a} = 2N^*(0) f_l^{s,a}$$

- Interactions between QPs near the FS: Landau parameters F_l^s, F_l^a

Landau parameters

- (Finite) renormalization of physical properties due to interactions

effective mass	$\frac{m^*}{m} = 1 + \frac{1}{3}F_1^s$	Galilean invariance
specific heat (cv = γT)	$\frac{\gamma}{\gamma_0} = \frac{m^*}{m}$	
compressibility	$\frac{\kappa}{\kappa_0} = \frac{m^*}{m}\frac{1}{1+F_0^s}$	
spin susceptibility	$\frac{\chi}{\chi_0} = \frac{m^*}{m}\frac{1}{1+F_0^a}$	

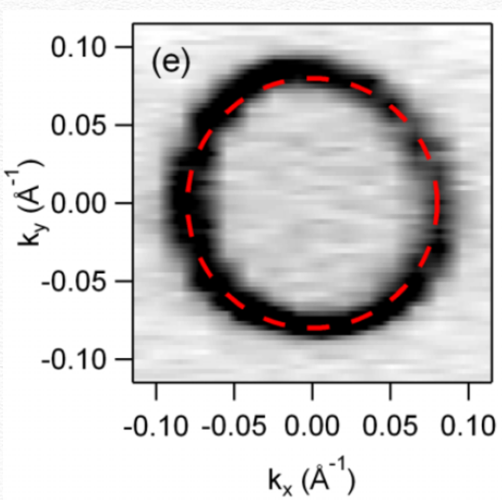
A theory of helical Fermi liquids?

- Phenomenological Landau theory for the 3D TI surface state?
- Qualitative differences from ordinary FL theory due to SOC

Symmetries of the helical FL

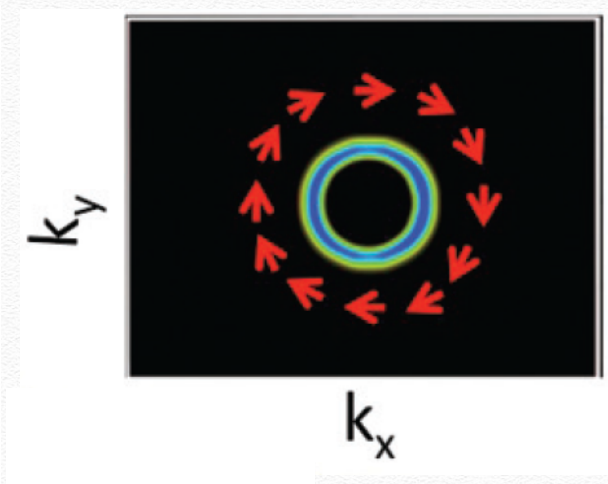
- TRS = protecting symmetry of 3D TI
- Rotation symmetry: focus on materials with (almost) perfectly circular FS

Bi_2Se_3



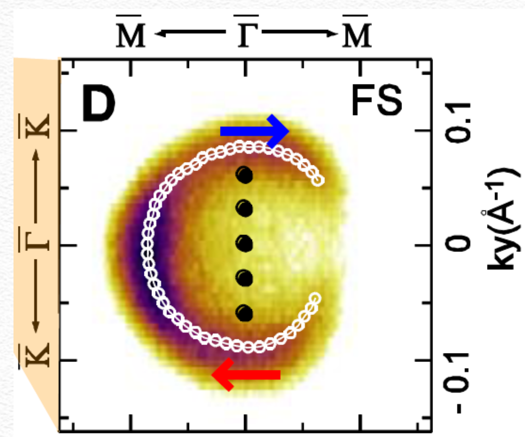
Pan et al., PRL 2011

Bi_2Te_2Se



Neupane et al., PRB 2013

$TlBiSe_2$



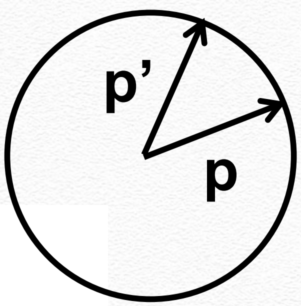
Kuroda et al., PRB 2015

Landau functional

- SOC: QP distribution function is 2x2 matrix

$$\delta n_{\boldsymbol{p}}^{\alpha\beta} \equiv n_{\boldsymbol{p}}^{\alpha\beta} - n_{\boldsymbol{p}}^{(0)\alpha\beta}$$

- Landau functional



$$\delta E[\delta n_{\boldsymbol{p}}] = \int đp \, h_{\alpha\beta}(\boldsymbol{p}) \delta n_{\boldsymbol{p}}^{\alpha\beta} + \frac{1}{2} \int đp \, đp' \, V_{\alpha\beta;\gamma\delta}(\hat{\boldsymbol{p}}, \hat{\boldsymbol{p}}') \delta n_{\boldsymbol{p}}^{\alpha\beta} \delta n_{\boldsymbol{p}'}^{\gamma\delta}$$

$$h(\boldsymbol{p}) = v_F \hat{\boldsymbol{z}} \cdot (\boldsymbol{\sigma} \times \boldsymbol{p})$$

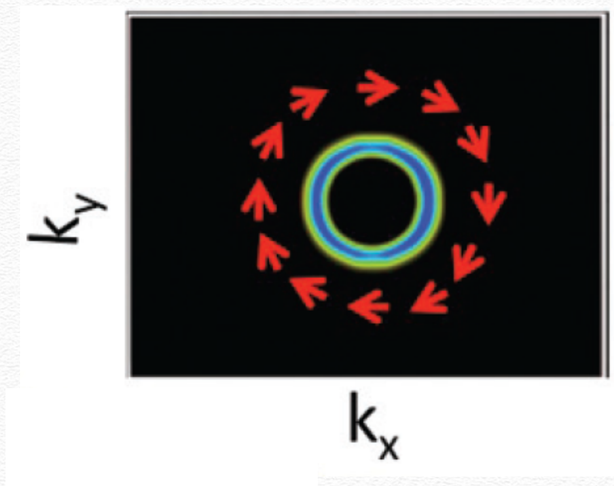
Spin & charge densities

$$\delta\rho_{\boldsymbol{p}} = \sigma^0_{\alpha\beta}\delta n^{\alpha\beta}_{\boldsymbol{p}} = \delta_{\alpha\beta}\delta n^{\alpha\beta}_{\boldsymbol{p}}$$

$$\delta s^i_{\boldsymbol{p}} = \frac{1}{2}\sigma^i_{\alpha\beta}\delta n^{\alpha\beta}_{\boldsymbol{p}}$$

Spin-orbit rotation symmetry

- L_z and S_z not good quantum numbers, only $J_z=L_z+S_z$ is



- Determine most general interaction invariant under J_z rotations and TRS

Allowed interactions

- Charge-charge: identical to spinless 2D FL theory
- Spin-spin: XXZ, Dzialoshinski-Moriya, and "compass model"
- Direct spin-charge interaction allowed by SOC

Landau parameters

- 10 Landau parameters (per angular momentum):

$$f_l^{cc}$$

$$f_l^{ss,1}, \ldots, f_l^{ss,5}$$

$$f_l^{sc,1}, \ldots, f_l^{sc,4}$$

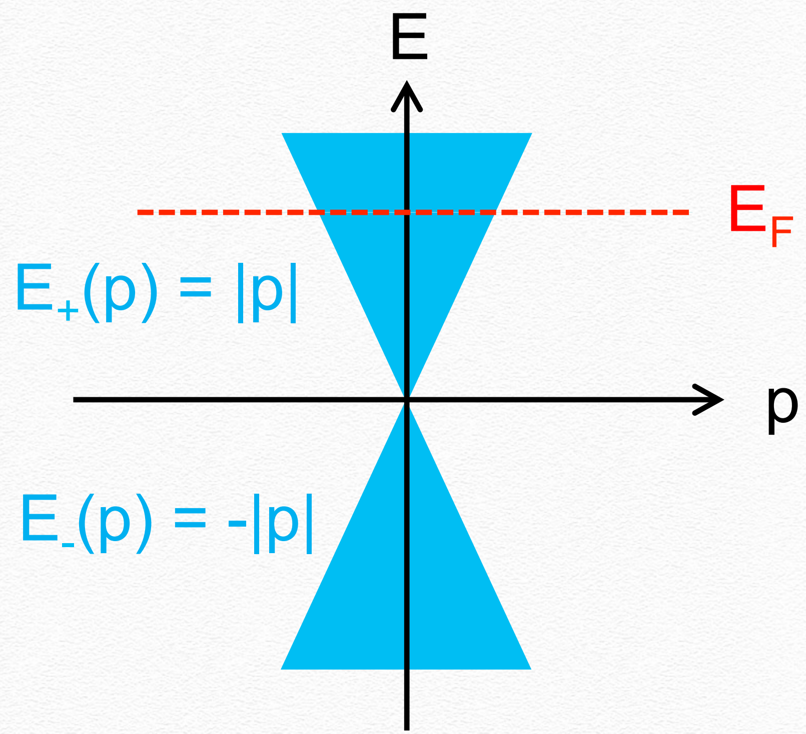
- Compared to 2 for standard FL theory

Projected Fermi liquid theory

- FL theory: only keep d.o.f. near FS

$$c_{\boldsymbol{p}\uparrow} = \frac{ie^{-i\theta_{\boldsymbol{p}}}}{\sqrt{2}}(\psi_{\boldsymbol{p}+} + \psi_{\boldsymbol{p}-})$$

$$c_{\boldsymbol{p}\downarrow} = \frac{1}{\sqrt{2}}(\psi_{\boldsymbol{p}+} - \psi_{\boldsymbol{p}-})$$



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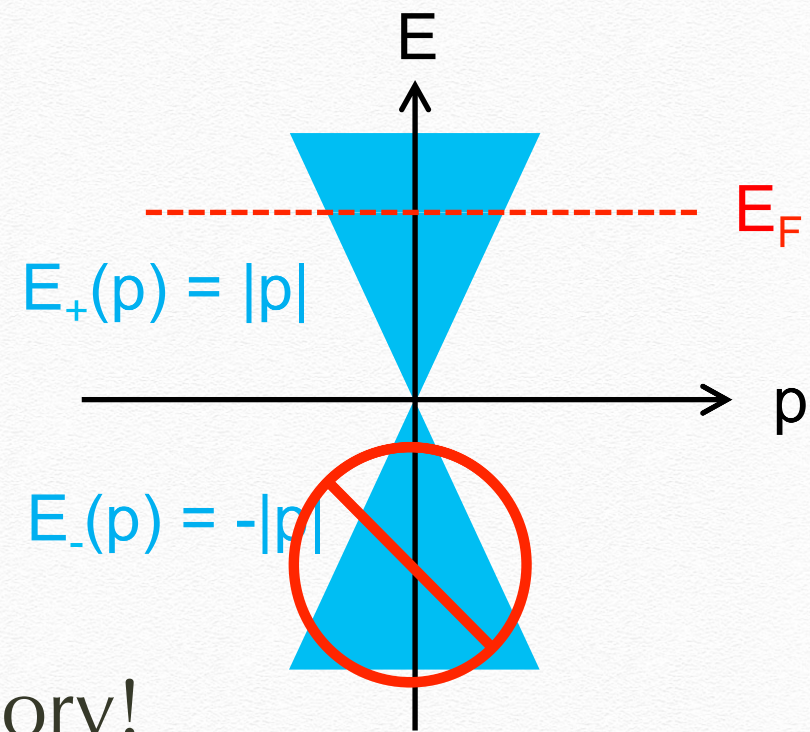


- effectively spinless theory!

Projected Fermi liquid theory

- Projected Landau parameters

$$\bar{f}_l = f_l^{cc} - f_l^{sc,3} - \frac{1}{4} f_l^{ss,5} + \frac{1}{8}\left(f_{l-1}^{ss,1} - f_{l-1}^{ss,3} + f_{l+1}^{ss,1} + f_{l+1}^{ss,3}\right)$$

- Projection to helical FS can effectively raise/lower angular momentum of the interaction (cf. Fu, Kane, PRL 2008)

$$\bar{f}_1 = f_1^{cc} - f_1^{sc,3} - \frac{1}{4} f_1^{ss,5} + \frac{1}{8}\left(f_0^{ss,1} - f_0^{ss,3} + f_2^{ss,1} + f_2^{ss,3}\right)$$

p-wave

s-wave

Physical properties

$$\frac{v_F^0}{v_F} = 1 + \bar{F}_1$$

but no Galilean invariance!

$$\frac{\gamma}{\gamma_0} = \left(\frac{v_F^0}{v_F}\right)^2$$

$$\frac{\kappa}{\kappa_0} = \left(\frac{v_F^0}{v_F}\right)^2 \frac{1}{1 + \bar{F}_0}$$

Pomeranchuk instabilities

- Instabilities towards spontaneous distortions of the FS (Pomeranchuk, JETP 1958)

$$p_F(\theta) - p_F = \sum_{l=-\infty}^{\infty} A_l e^{il\theta}$$

$$\delta \bar{E}[\delta \bar{n}_{\boldsymbol{p}}] = \frac{\epsilon_F}{2\pi\hbar^2} \sum_{l=0}^{\infty} (1 + \bar{F}_l)|A_l|^2$$

- Stability of FS requires $\bar{F}_l > -1$

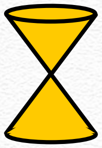
Pomeranchuk instabilities

- l=0: phase separation

$$\frac{\kappa}{\kappa_0} = \left(\frac{v_F^0}{v_F}\right)^2 \frac{1}{1+\bar{F}_0}$$

- l=1: in-plane magnetic order (Xu, PRB 2010)

$$\chi_{xx} = \frac{1}{8} g^2 \mu_B^2 \rho(\epsilon_F) \frac{1}{1+\bar{F}_1}$$



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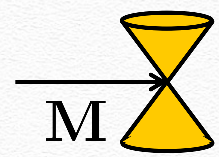


Pomeranchuk instabilities

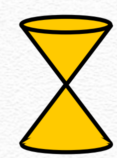
- l=2: nematic instability

$$\cos 2\theta_{\boldsymbol{p}\boldsymbol{p}'}\delta\bar{n}_{\mathbf{p}}\delta\bar{n}_{\mathbf{p}'} = \frac{1}{2}\,\mathrm{Tr}\,\bar{Q}(\boldsymbol{p})\bar{Q}(\boldsymbol{p}')$$

$$\bar{Q}_{ij}(\boldsymbol{p}) = (2\hat{p}_i\hat{p}_j - \delta_{ij})\delta\bar{n}_{\mathbf{p}}$$

- Unprojected theory: quadrupolar "spin-orbital" order parameter (Park, Chung, JM, PRB 2015; Fu, PRL 2015)

$$Q_{ij}(\boldsymbol{p}) = \hat{p}_i\delta s^j_{\mathbf{p}} + \hat{p}_j\delta s^i_{\mathbf{p}} - \delta_{ij}\hat{\boldsymbol{p}}\cdot\delta\boldsymbol{s}_{\mathbf{p}}$$



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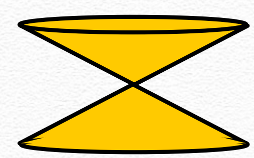


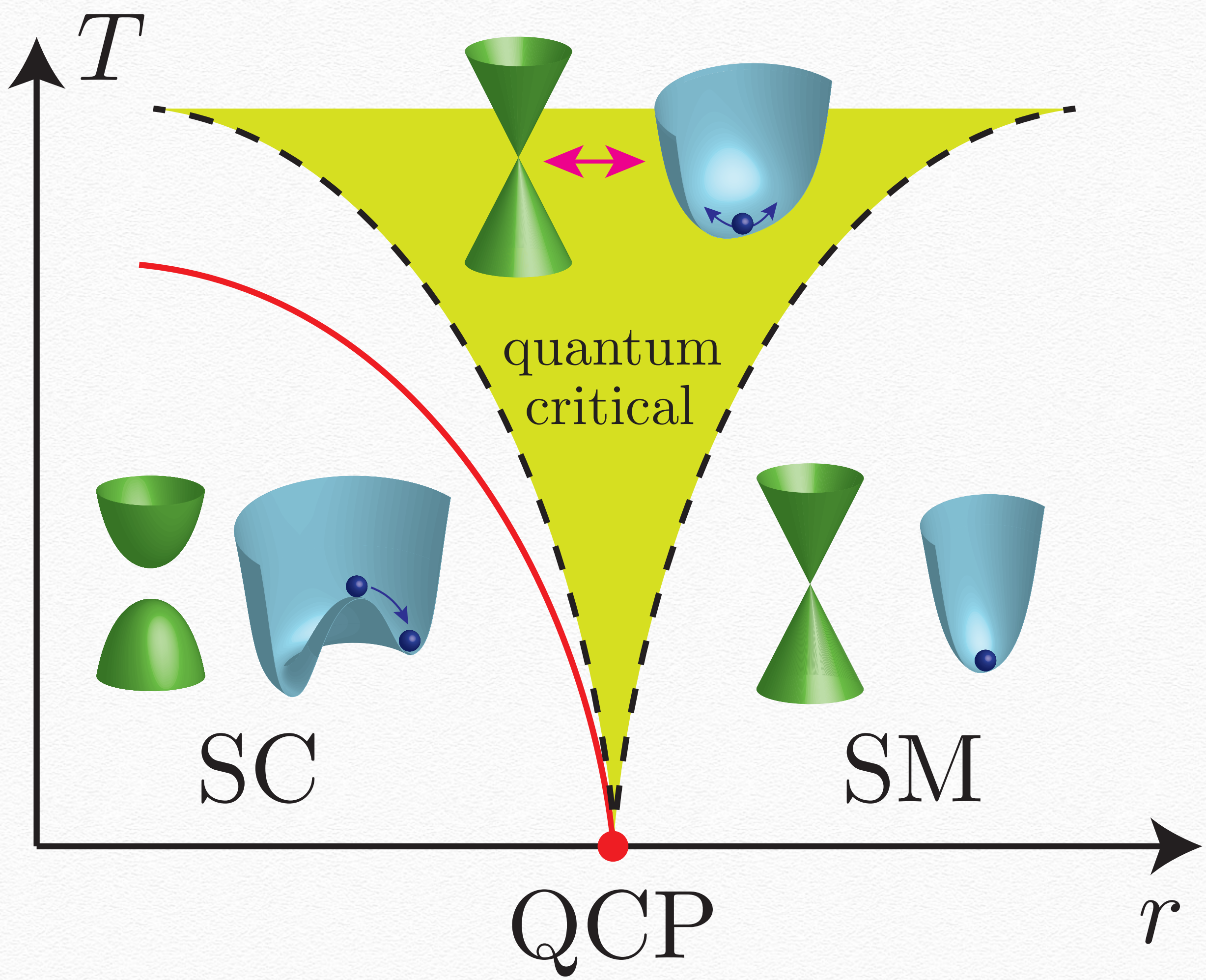
- Strong correlations: Universal conductivity at semimetal-superconductor QCP

W. Witczak-Krempa and JM, arXiv:1510.06397

SC instability of TI surface state

- FL theory: instabilities in particle-hole channel
- Consider pairing instability of Dirac surface state at $\mu = 0$
- Vanishing DOS: finite threshold attraction strength -> QCP

SC instability of TI surface state



SUSY QCP

- QCP has emergent N=2 SUSY! (Grover, Sheng, Vishwanath, Science 2014; Ponte, Lee, NJP 2014)

- Strongly coupled (2+1)D CFT: N=2 Wess-Zumino model

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial_\mu\psi + \frac{1}{2}|\partial_\mu\phi|^2 + \frac{r}{2}|\phi|^2 + \frac{\lambda}{4!}|\phi|^4 + h(\phi^*\psi^T i\gamma_2\psi + \text{c.c.})$$

- Finite $h^2 \propto \lambda$ at the QCP: universality class neither Gaussian nor 3D XY

SUSY QCP

- SUSY fixes exact anomalous dimensions of ψ, ϕ

$$\eta_\phi = \eta_\psi = \frac{1}{3}$$

- Correlation length exponent not fixed by SUSY

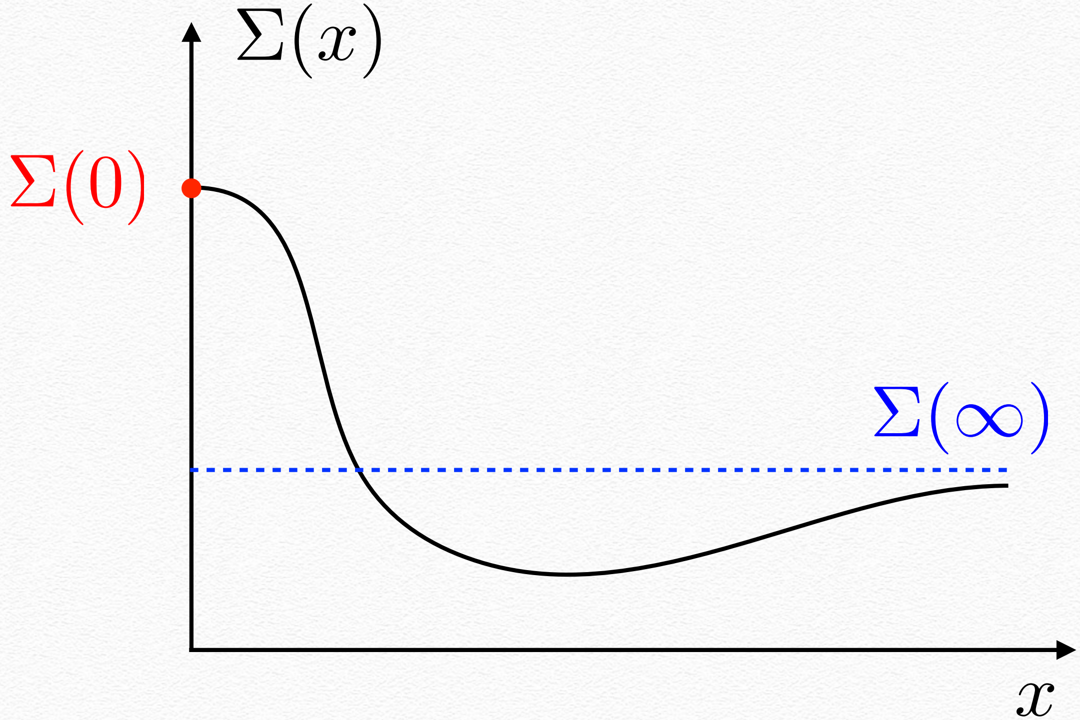
$$\nu = \frac{1}{2} + \frac{\epsilon}{4} + \mathcal{O}(\epsilon^2) \approx 0.75$$ 1-loop RG (Thomas, 2005)

SUSY QCP

- Can SUSY tell us anything else?

Optical conductivity

$$\sigma(\omega, T) = \frac{e^2}{\hbar}\left(\frac{k_B T}{\hbar c}\right)^{(d-2)/z} \Sigma\left(\frac{\hbar\omega}{k_B T}\right)$$



Damle & Sachdev, PRB 1997

Optical conductivity: (2+1)D

$$\sigma(\omega, T) = \frac{e^2}{\hbar} \Sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

- T=0 optical conductivity = universal constant

$$\sigma(\omega, 0) = \frac{e^2}{\hbar} \Sigma(\infty) = \frac{e^2}{\hbar} \sigma_\infty$$

- σ_∞ related to T=0 JJ correlation function (Kubo)

PRL **101,** 196405 (2008) PHYSICAL REVIEW LETTERS week ending 7 NOVEMBER 2008

Measurement of the Optical Conductivity of Graphene

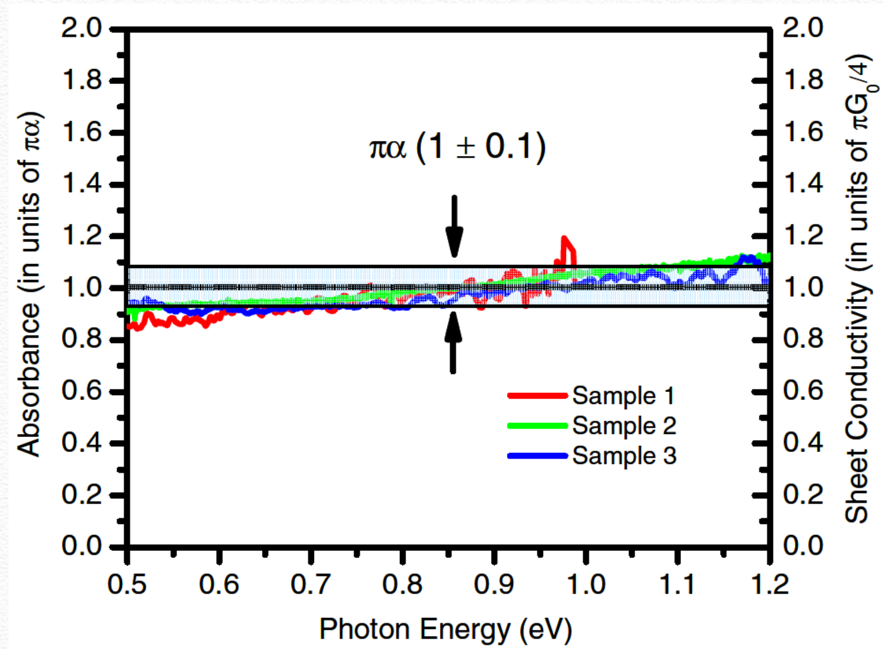
Kin Fai Mak,[1] Matthew Y. Sfeir,[2] Yang Wu,[1] Chun Hung Lui,[1] James A. Misewich,[2] and Tony F. Heinz[1,*]
[1] *Departments of Physics and Electrical Engineering, Columbia University, 538 West 120th Street, New York, New York 10027, USA*
[2] *Brookhaven National Laboratory, Upton, New York 11973, USA*
(Received 28 June 2008; published 7 November 2008)

Optical reflectivity and transmission measurements over photon energies between 0.2 and 1.2 eV were performed on single-crystal graphene samples on a SiO_2 substrate. For photon energies above 0.5 eV, graphene yielded a spectrally flat optical absorbance of $(2.3 \pm 0.2)\%$. This result is in agreement with a constant absorbance of $\pi\alpha$, or a sheet conductivity of $\pi e^2/2h$, predicted within a model of noninteracting massless Dirac fermions.

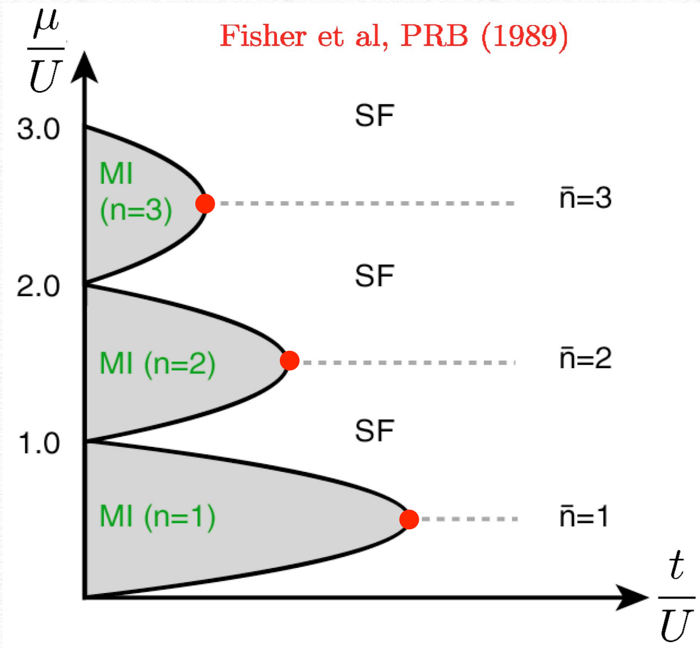
$$\sigma_\infty = 1/4$$

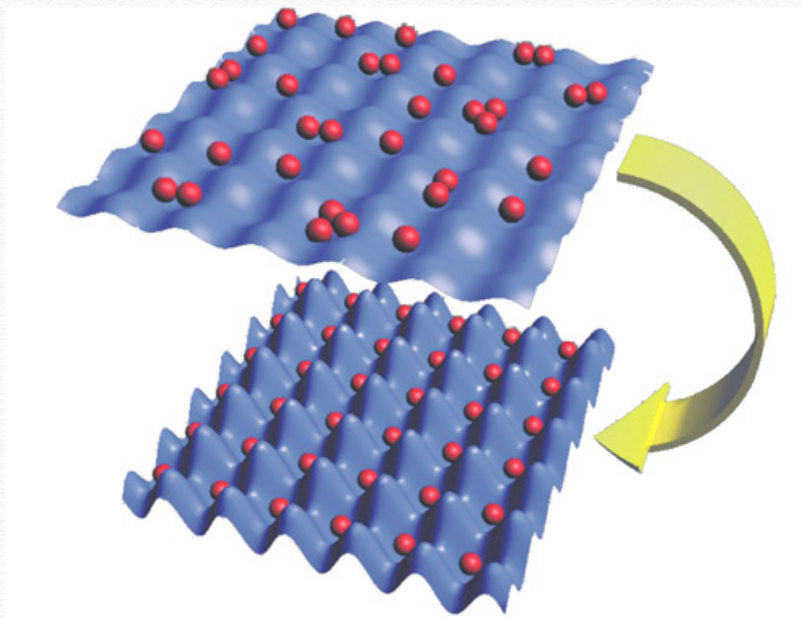
- Graphene = free Dirac CFT

$$\frac{\hbar\omega}{k_B T} \sim \frac{1\ \text{eV}}{300\ \text{K}} \sim 39 = \infty$$



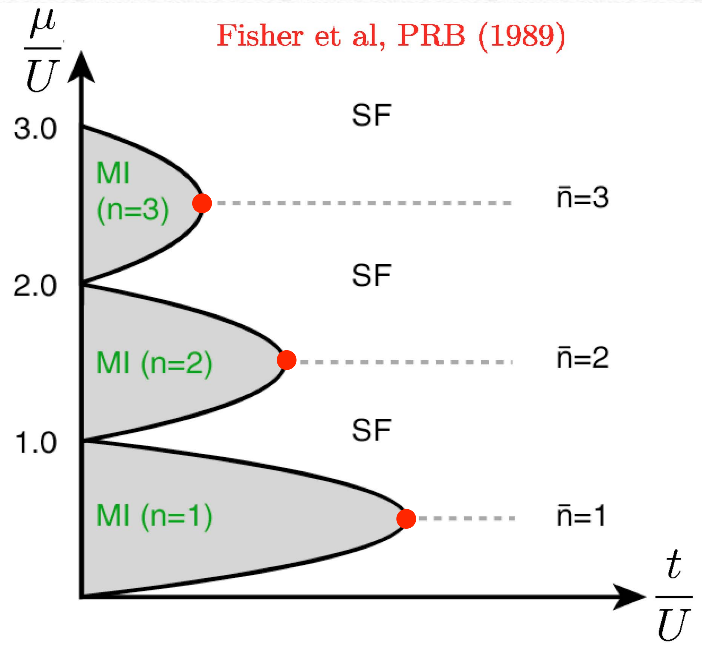
Boson superfluid-insulator QCP

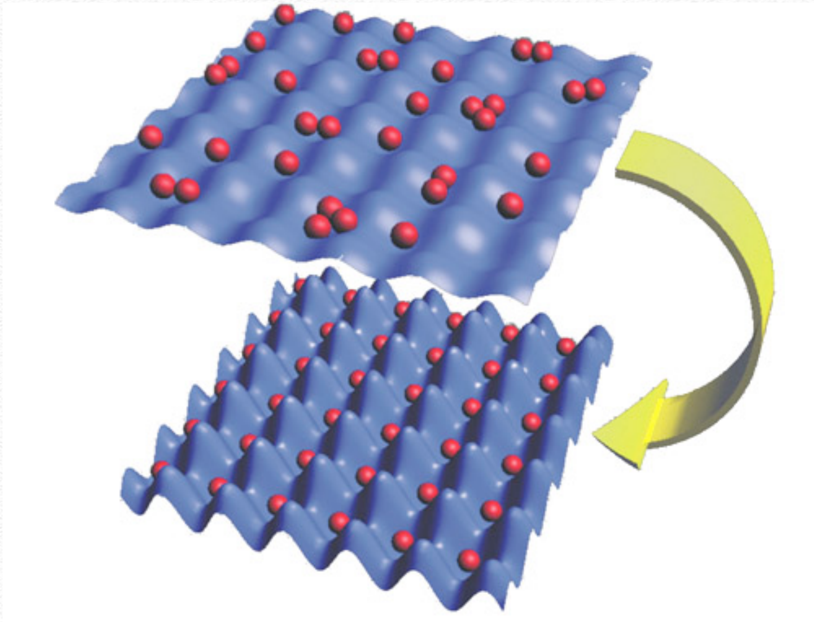




- Universal conductivity σ_∞ : no exact result, long history (Fisher, Grinstein, Girvin, PRL 1990; Cha et al., PRB 1991; Fazio & Zappalà, PRB 1996; Šmakov & Sørensen, PRL 2005; ...)

Boson superfluid-insulator QCP





- QMC + holography + conformal bootstrap (Katz et al., PRB 2014; Gazit et al., PRB 2013, PRL 2014; Chen et al., PRL 2014; Witczak-Krempa et al., Nat. Phys. 2014; Kos et al., arXiv 2015)

$$\sigma_\infty \simeq 0.226$$

Kubo for CFTs

- Ground-state JJ correlation function, constrained by conformal symmetry (Osborn & Petkou, Ann. Phys. 1994)

$$\langle J_\mu(x) J_\nu(0) \rangle = C_J \frac{I_{\mu\nu}(x)}{|x|^4}$$

$$\sigma_\infty = \frac{\pi^2}{2} C_J$$

- Can C_J be computed at our SUSY QCP?

N=2 SCFTs in (2+1)D

- U(1) current and stress tensor are related by SUSY

$$\mathcal{J}_\mu = J_\mu - (\theta\gamma^\nu\bar{\theta})2T_{\nu\mu} + \dots$$

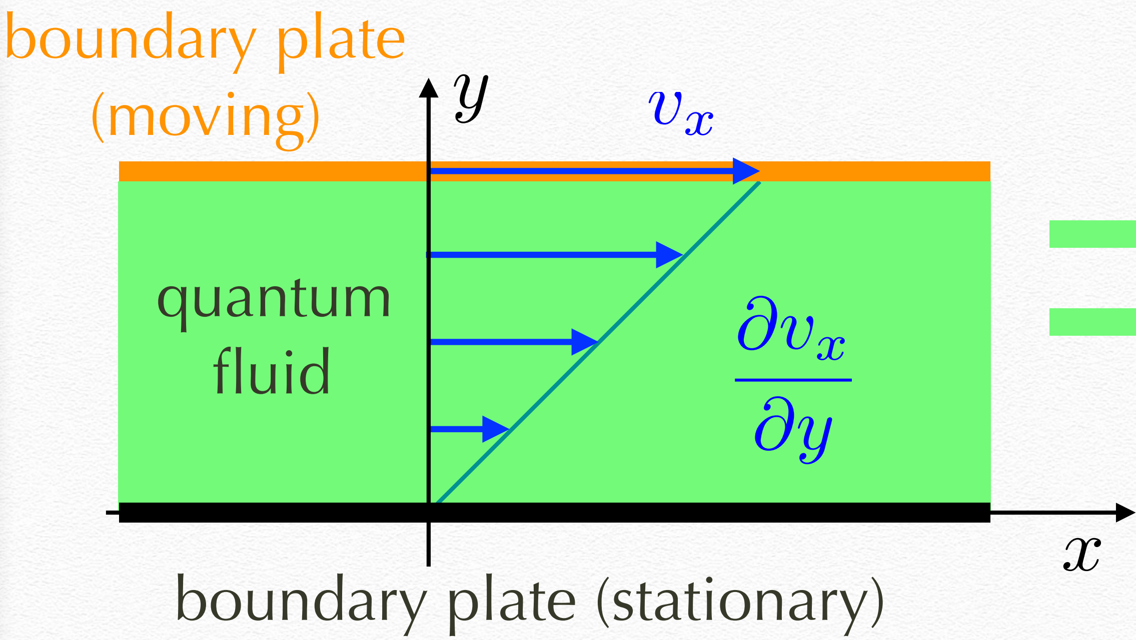
- <JJ> and <TT> are related by SUSY

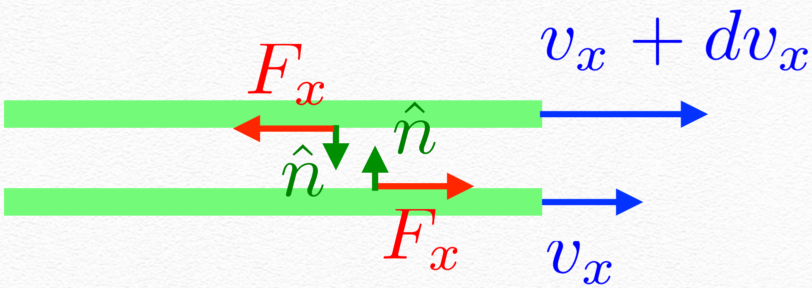
$$\langle J_\mu(x)J_\nu(0)\rangle = C_J\frac{I_{\mu\nu}(x)}{|x|^4}$$

$$\langle T_{\mu\nu}(x)T_{\rho\sigma}(0)\rangle = C_T\frac{I_{\mu\nu,\rho\sigma}(x)}{|x|^6}$$

$$\frac{C_J}{C_T} = \frac{5}{3}$$

Shear viscosity



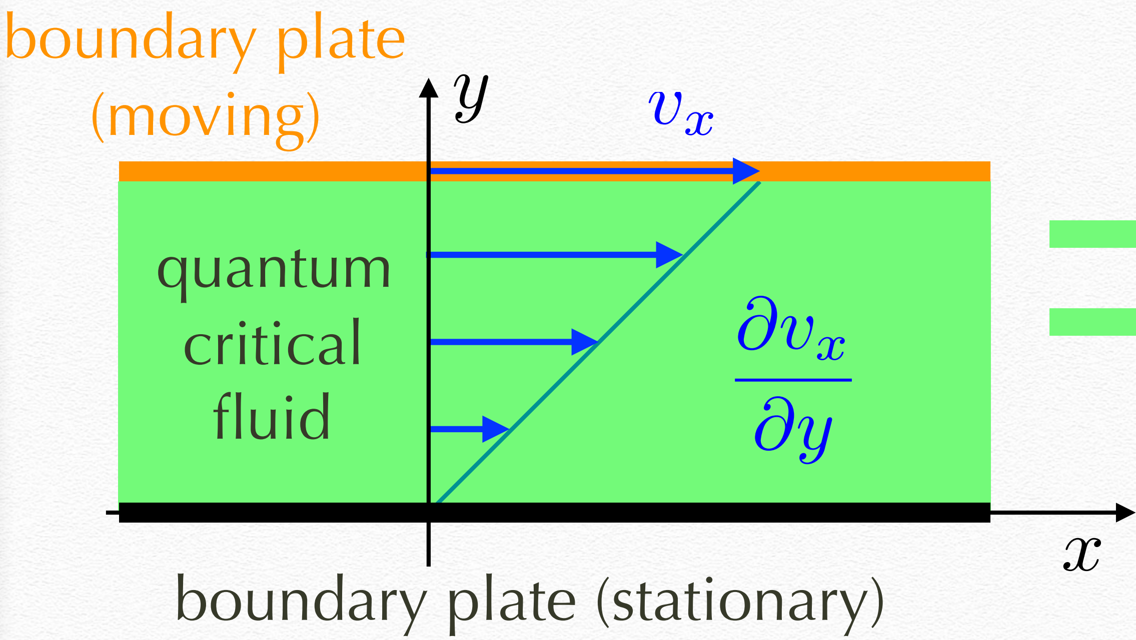


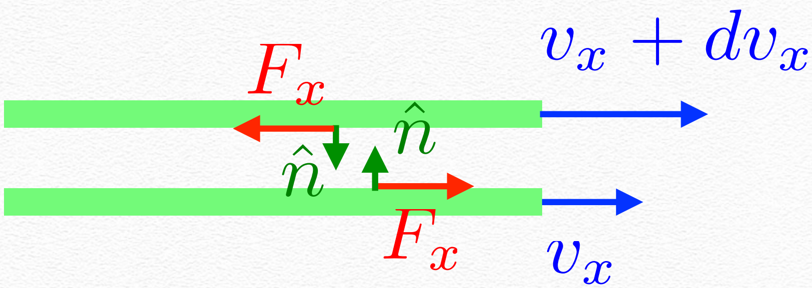
$$T_{xy} = \frac{F_x}{L}$$

shear stress

$$T_{xy} = \eta \frac{\partial v_x}{\partial y} = \eta \delta \dot{g}_{xy}$$

Shear viscosity





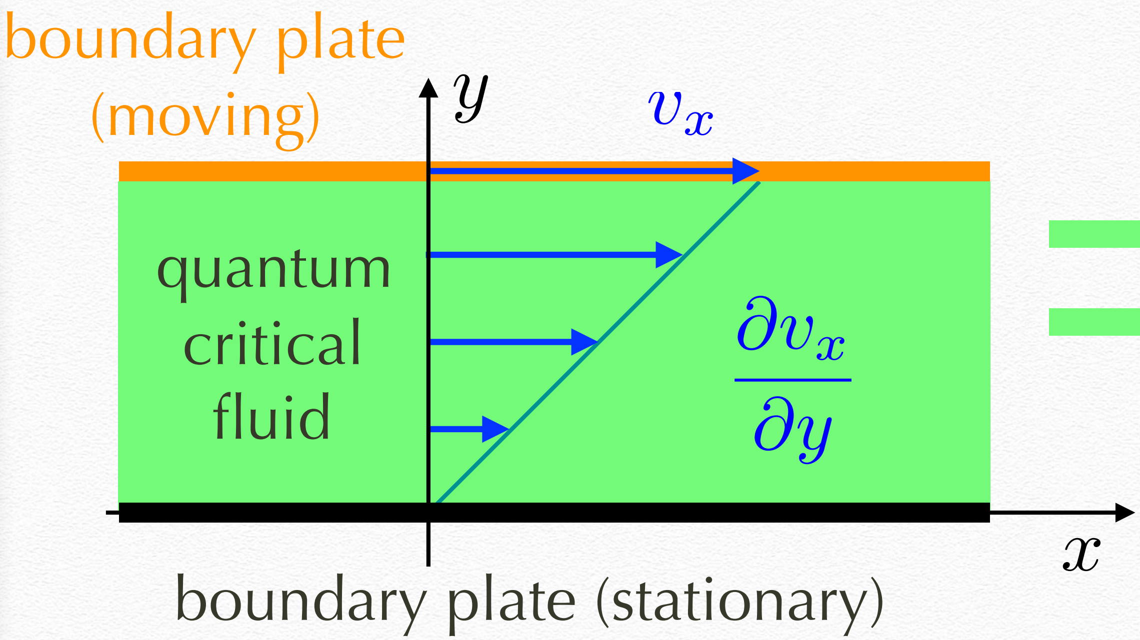
$$T_{xy} = \frac{F_x}{L}$$

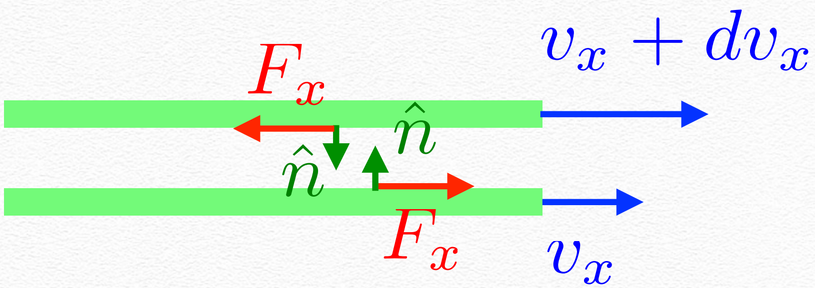
shear stress

$$\eta(i\omega_n) = \frac{1}{\omega_n} \langle T_{xy}(\omega_n) T_{xy}(-\omega_n) \rangle_T = \eta_\infty \omega_n^2 + \dots$$

(dynamical) shear viscosity

Shear viscosity





$$T_{xy} = \frac{F_x}{L}$$

shear stress

$$\eta(i\omega_n) = \frac{1}{\omega_n} \langle T_{xy}(\omega_n) T_{xy}(-\omega_n) \rangle_T = \eta_\infty \omega_n^2 + \ldots$$

(dynamical) shear viscosity $\eta_\infty = \frac{\pi^2}{48} C_T$

Conductivity vs viscosity

$$\frac{\sigma_\infty}{\eta_\infty} = 40$$

- Exact universal ratio at the QCP: consequence of SUSY

Exact universal conductivity

- C_T can be calculated exactly for the N=2 WZ model by localization on the squashed 3-sphere (Closset et al., JHEP 2013; Nishioka & Yonekura, JHEP 2013)

$$\sigma_\infty = \frac{5(16\pi - 9\sqrt{3})}{243\pi} \simeq 0.2271$$

Exact universal conductivity

- C_T can be calculated exactly for the N=2 WZ model by localization on the squashed 3-sphere (Closset et al., JHEP 2013; Nishioka & Yonekura, JHEP 2013)

$$\sigma_\infty = \frac{5(16\pi - 9\sqrt{3})}{243\pi} \simeq 0.2271$$

- Exact result for T=0 conductivity (and shear viscosity) of "realistic" strongly coupled quantum fluid in (2+1)D

Exact universal conductivity

	Dirac SM-SC	Gaussian	SC-Ins. (Cooper pairs)
σ_∞	$\frac{5(16\pi - 9\sqrt{3})}{243\pi} \approx 0.227$	$\frac{5}{16} = 0.3125$	0.226

- Reduced conductivity = increase scattering due to interactions

Finite temperature?

$$\frac{\sigma(\omega)}{e^2/\hbar} = \sigma_\infty + b_{|\phi|^2}\left(\frac{iT}{\omega}\right)^{3-1/\nu} + b_T\left(\frac{iT}{\omega}\right)^3 + \dots$$

identity

$|\phi|^2$

stress tensor

Katz et al., PRB 2014

Finite temperature?

$$\frac{\sigma(\omega)}{e^2/\hbar} = \sigma_\infty + b_{|\phi|^2}\left(\frac{iT}{\omega}\right)^{3-1/\nu} + b_T\left(\frac{iT}{\omega}\right)^3 + \dots$$

- Can't say much about $b_{|\phi|^2}$: probably nonzero

Finite temperature?

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- Combine conformal invariance + Ward identities (Osborn & Petkou, Ann. Phys. 1994), and SUSY (Buchbinder, Kuzenko, Samsonov, JHEP 2015):

$$\boxed{b_T = 0}$$

for all (2+1)D QCPs with N=2 SUSY!

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for all (2+1)D QCPs with N=2 SUSY!

- Exact result for finite-T, dynamical response of strongly coupled quantum fluid in (2+1)D

What about the real world?

What about the real world?

ARTICLE

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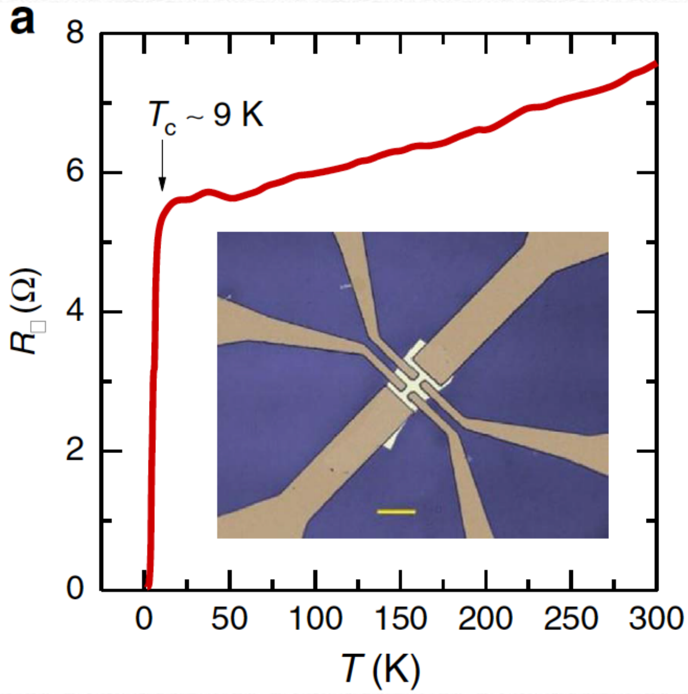
Emergent surface superconductivity in the topological insulator Sb_2Te_3

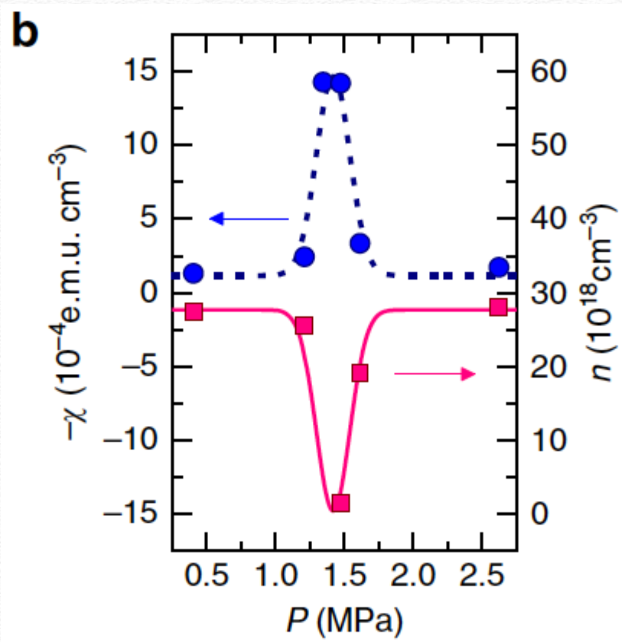
Lukas Zhao[1], Haiming Deng[1], Inna Korzhovska[1], Milan Begliarbekov[1], Zhiyi Chen[1], Erick Andrade[2], Ethan Rosenthal[2], Abhay Pasupathy[2], Vadim Oganesyan[3,4] & Lia Krusin-Elbaum[1,4]

Surface SC in Sb_2Te_3?

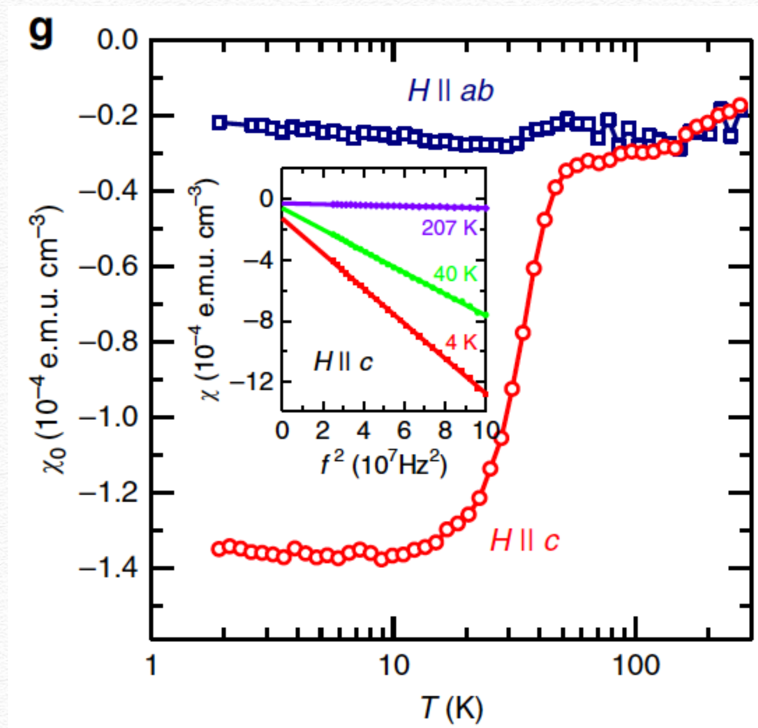


- Resistive transition at $T_c = 8.6$ K

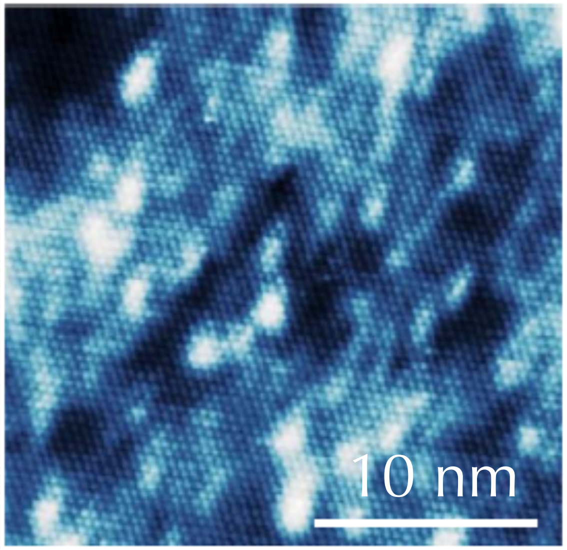
Surface SC in Sb_2Te_3?

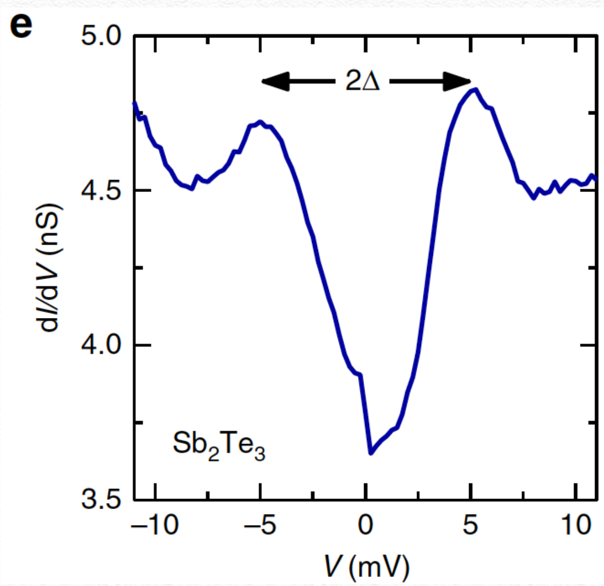


- Resistive transition at T_c = 8.6 K
- Anisotropic (2D) diamagnetic screening below T ~ 50 K (~2% of Meissner value)



Surface SC in Sb_2Te_3?





- Resistive transition at T_c = 8.6 K
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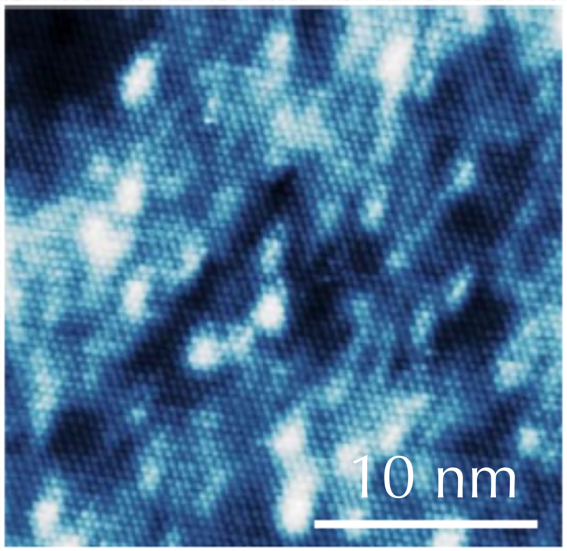
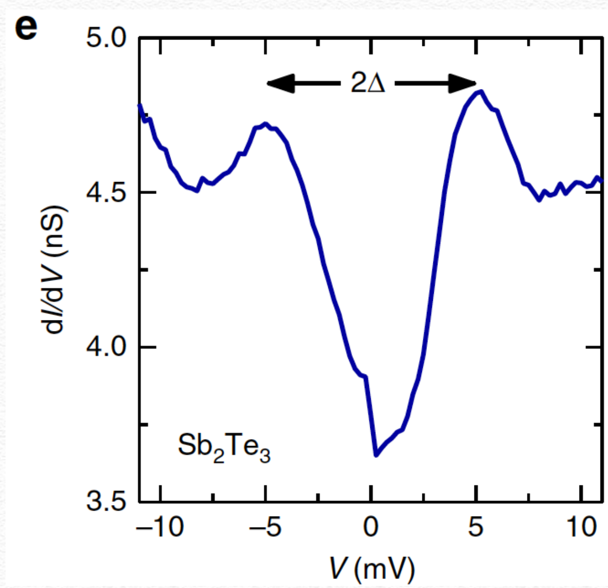
- Inhomogeneous STM maps, largest local "gap" (~ 20 meV) consistent with local BCS T_c ~ 60 K

Surface SC in Sb_2Te_3?





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- Anisotropic (2D) diamagnetic screening below T ~ 50 K (~2% of Meissner value)
- Inhomogeneous STM maps, largest local "gap" (~ 20 meV) consistent with local BCS T_c ~ 60 K
- Inhomogeneous BCS pairing in local Dirac "puddles" at T ~ 50-60 K, onset of global phase coherence at T = 8.6 K? (Nandkishore, JM, Huse, Sondhi, PRB 2013)

Surface SC in Sb_2Te_3?

- Far from ideal system... but cleaner materials may lead to desired physics

Summary

- Weakly correlated surface state can be described in a materials-independent way by a effectively spinless, phenomenological "projected" Landau Fermi liquid theory
- SUSY allows us to calculate exactly dynamical response properties (e.g. optical conductivity) at zero and finite temperature for the strongly coupled SM-SC surface QCP in (2+1)D