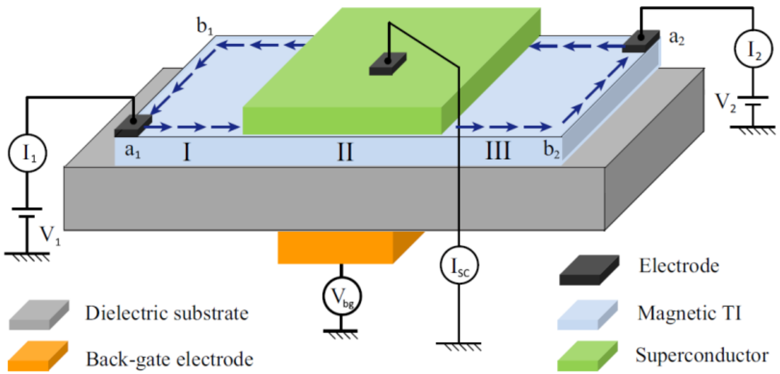
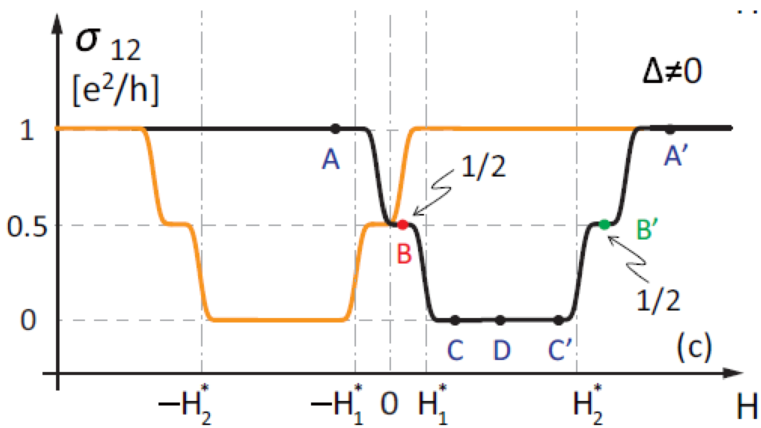
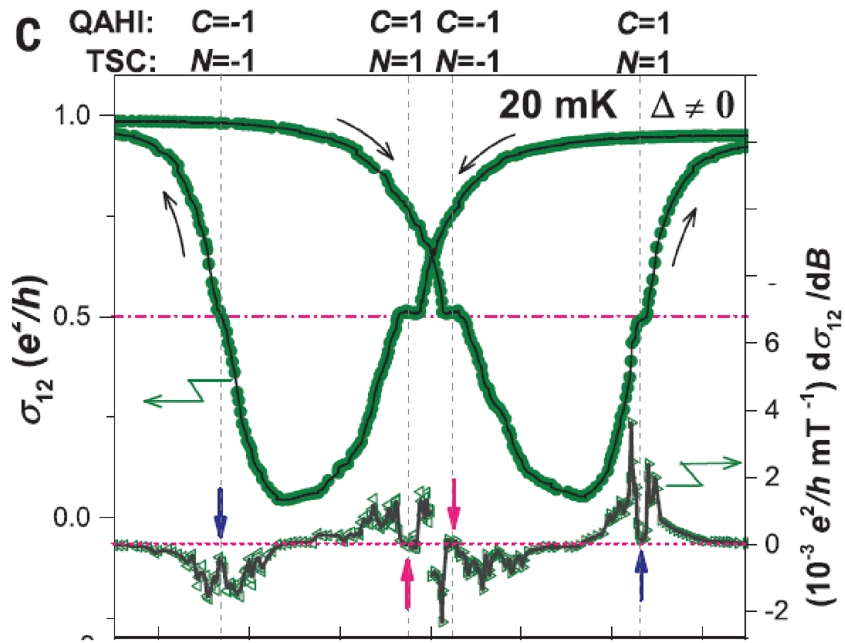
Chiral Majorana fermion from quantum anomalous Hall plateau transition





Phys. Rev. B, 2015



Science, 2017

王靖
复旦大学物理系
wjingphys@fudan.edu.cn

Acknowledgements

Stanford

Biao Lian

Quan Zhou

Xiao-Liang Qi

Shou-Cheng Zhang

Tsinghua

Yang Feng

Xiao Feng

Ke He

Yayu Wang

Qi-Kun Xue

UCLA

Xufeng Kou

Qinglin He

Kang L Wang

应用表面物理国家重点实验室

STATE KEY LABORATORY OF SURFACE PHYSICS

References

[1] J. Wang, B. Lian, S. C. Zhang, Phys. Rev. B 89, 085106 (2014)

[2] J. Wang, B. Lian, S. C. Zhang, Phys. Scr. T164, 014003 (2015)

[3] J. Wang, Q. Zhou, B. Lian, S. C. Zhang, Phys. Rev. B 92, 064520 (2015)

[4] X. Kou, L. Pan, J. Wang *et al*, Nat. Commun. 6, 8474 (2015)

[5] Y. Feng, X. Feng, Y. Ou *et al*, Phys. Rev. Lett. 115, 126801 (2015)

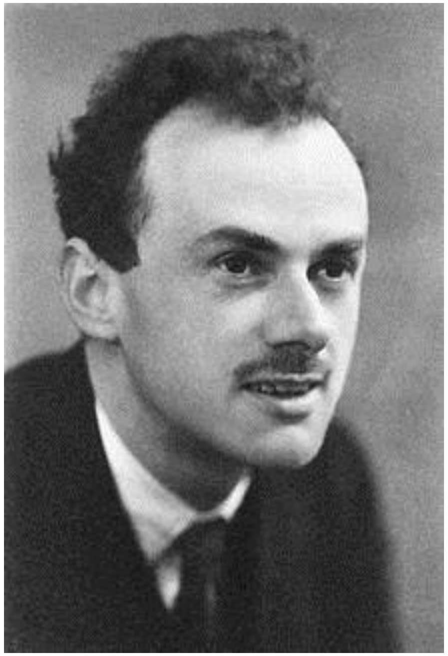
[7] B. Lian, J. Wang, S. C. Zhang, Phys. Rev. B 93, 161401 (2016)

[8] J. Wang, Phys. Rev. B 94, 214520 (2016)

[9] Q. L. He *et al*, Science 357, 294 (2017)

[10] B. Lian, J. Wang, X. Sun, A. Vaezi, S. C. Zhang, arXiv: 1709.05558 (2017)

Dirac equation and the anti-particle



In 1928, Dirac unified Einstein's special theory of relativity with quantum mechanics, and introduced Dirac equation

$$i\hbar\gamma^\mu\partial_\mu\psi - mc\psi = 0$$

where γ^μ are Dirac's anticommuting *Gamma matrices*.

Dirac equation gives negative energy solutions, which led Dirac to predict the existence of anti-particle.

$$E = \pm\sqrt{p^2 + m^2}$$

In 1932, the positron, the anti-particle of the electron was discovery by CD Anderson in cosmic rays.

Majorana and his fermion

In 1937, Ettore Majorana asked the question: can fermions be their own antiparticles?

The ***Dirac equation*** is known to describe charged fermions:

$$i\hbar\gamma^\mu\partial_\mu\psi - mc\psi = 0$$

where γ^μ are Dirac's anticommuting *Gamma matrices*.

Ettore Majorana

Majorana claimed if all γ^μ are selected imaginary, on can ψ make real, describing a charge neutral, spin ½ fermion, obeying majorana equation

$$\gamma^0 = \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} i\sigma^3 & 0 \\ 0 & i\sigma^3 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} 0 & -\sigma^2 \\ \sigma^2 & 0 \end{pmatrix},$$

$$\gamma^3 = \begin{pmatrix} -i\sigma^1 & 0 \\ 0 & -i\sigma^1 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix},$$

Gamma matrices in Majorana equation.

Properties of the Majorana fermion

Neutrino could be a Majorana fermion, with Majorana mass term.

Majorana fermion is essential for supersymmetry.

Chiral Majorana fermion could exist in 1+1 and 9+1 dimensions, both essential for the superstring theory. With only fermion-number-parity conservation.

Majorana fermion could arise as quasi-particles of topological states of quantum matter.

Majorana fermion could be used for topological quantum computing.

Search for hypothetical particles/wave

Higgs boson, gravitational wave

Majorana fermion

Magnetic monopole

Axion

Dark matter particle

Particle = anti-particle

A Majorana fermion is its own anti-particle

$$1 = -1$$

In other words:

$$2 = 0$$

In the sense of modular (clock) arithmetic like 11+14=25=1 in clock counting

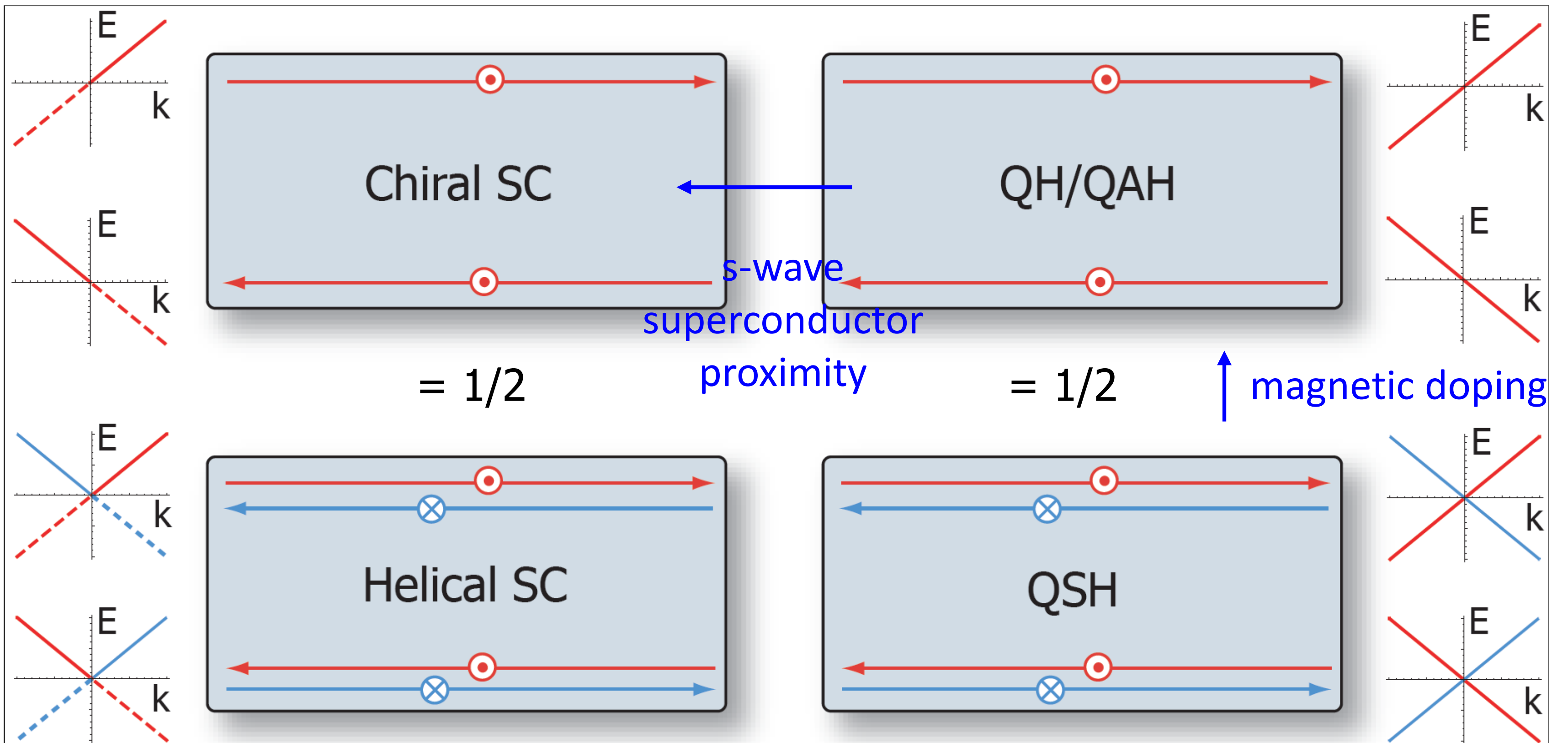
In particle physics, Majorana neutrino violates lepton number conservation by 2

In condensed matter physics, Majorana fermion are associated with superconductors, which also violates particle number conservation by 2.

Topological insulators and superconductors in 2D

Full pairing gap in the bulk, gapless Majorana edge and surface states

Chiral Majorana fermions = 1/2 Chiral fermions

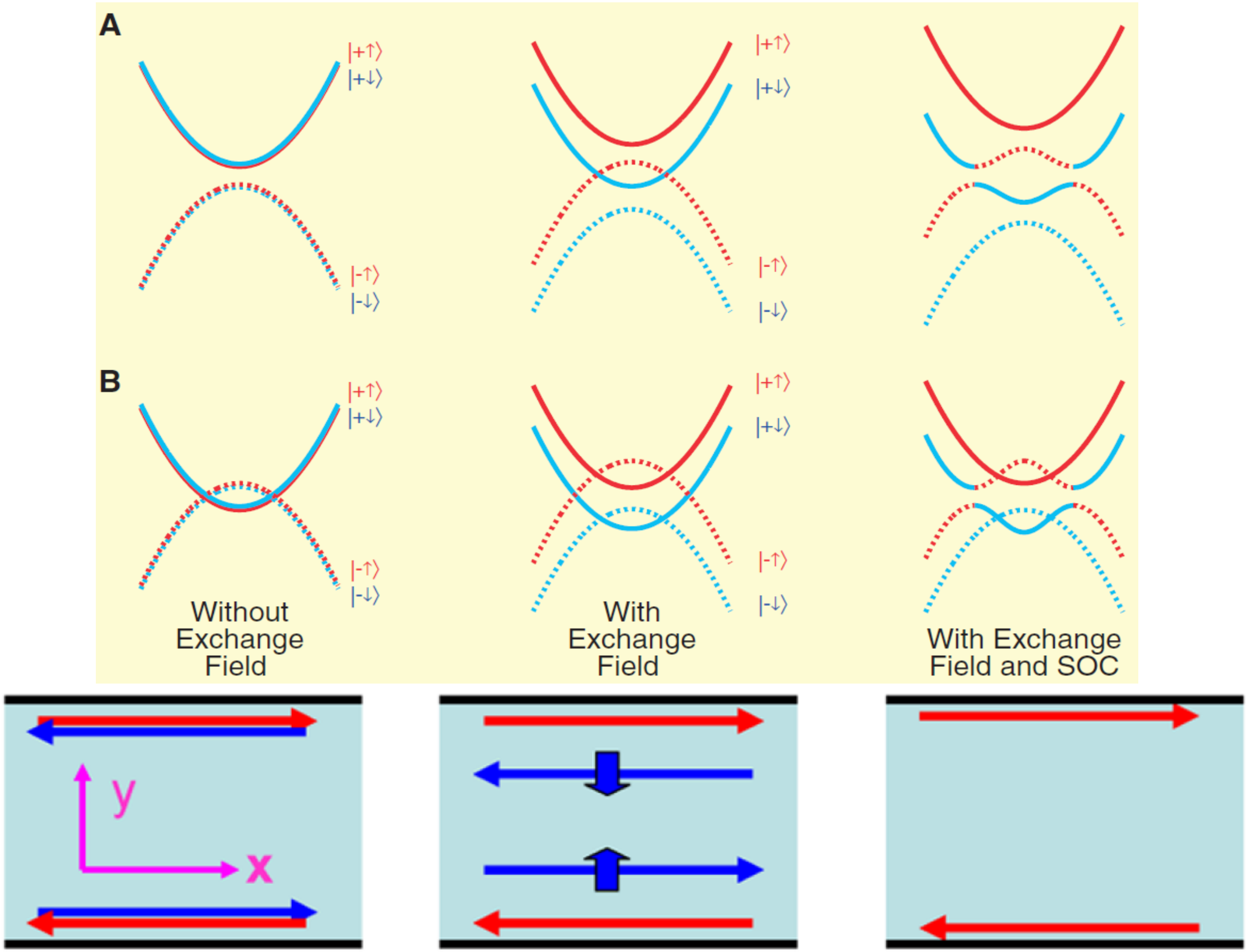


massless Majorana fermions massless Dirac fermions

Qi, Hughes, Raghu and Zhang, PRL, 2009

Basic mechanism of the QAH effect

- Key point to get *Quantum* Anomalous Hall effect: spin polarized *band inversion*

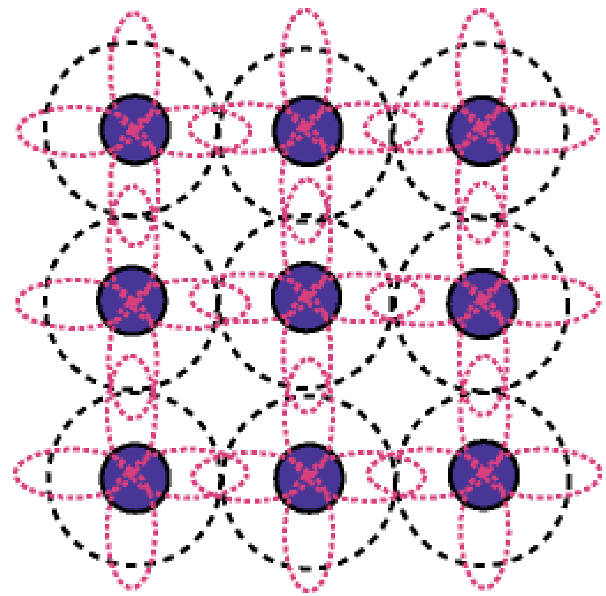


Qi, Wu & Zhang, PRB **74**, 085308 (2006): general theory
Liu et al, PRL, **101**, 146802 (2008): HgMnTe
Yu et al, Science **329**, 61, 2010 : $(BiCr)_2Te_3$

The model of the 2D topological insulator (BHZ, Science 2006)

Square lattice with 4-orbitals per site:

$$\left|s,\uparrow\right\rangle, \left|s,\downarrow\right\rangle, \left|(p_x+ip_y,\uparrow\right\rangle, \left|-(p_x-ip_y),\downarrow\right\rangle$$



Nearest neighbor hopping integrals. Mixing matrix elements between the s and the p states must be odd in k.

$$H_{eff}(k_x,k_y)=\begin{pmatrix} h(k) & 0 \\ 0 & h^*(-k) \end{pmatrix}$$

$$h(k)=\begin{pmatrix} m(k) & A(\sin k_x - i\sin k_y) \\ A(\sin k_x + i\sin k_y) & -m(k) \end{pmatrix} \equiv d_a(k)\tau^a$$

$$\Rightarrow \begin{pmatrix} m+Bk^2 & A(k_x-ik_y) \\ A(k_x+ik_y) & -m-Bk^2 \end{pmatrix} \qquad \begin{matrix} m/B<0, & \text{Edge state} \\ m/B>0, & \text{No edge state} \end{matrix}$$

Similar to relativistic Dirac equation in 2+1 dimensions, with a mass term tunable by the sample thickness d!

QAH model: FM exchange field

$$H_{eff}(k_x,k_y)=\begin{pmatrix} h(k) & 0 \\ 0 & h^*(-k) \end{pmatrix} \qquad h_{exchange}(k)=\begin{pmatrix} \Delta & 0 & 0 & 0 \\ 0 & -\Delta & 0 & 0 \\ 0 & 0 & -\Delta & 0 \\ 0 & 0 & 0 & \Delta \end{pmatrix}$$

$$h(k)\Rightarrow\begin{pmatrix} m+\Delta+Bk^2 & A(k_x-ik_y) \\ A(k_x+ik_y) & -m-\Delta-Bk^2 \end{pmatrix}$$

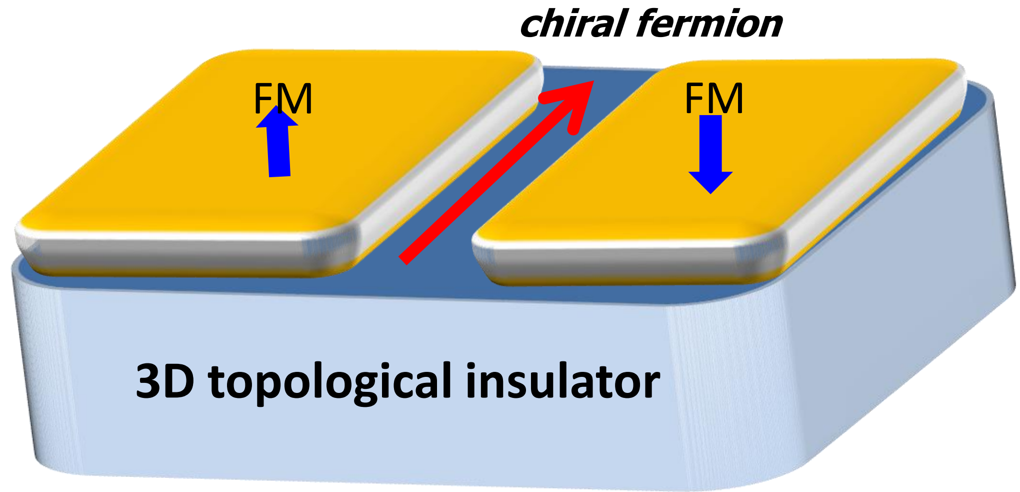
$$h^*(-k)\Rightarrow\begin{pmatrix} m-\Delta+Bk^2 & A(-k_x-ik_y) \\ A(-k_x+ik_y) & -m+\Delta-Bk^2 \end{pmatrix}$$

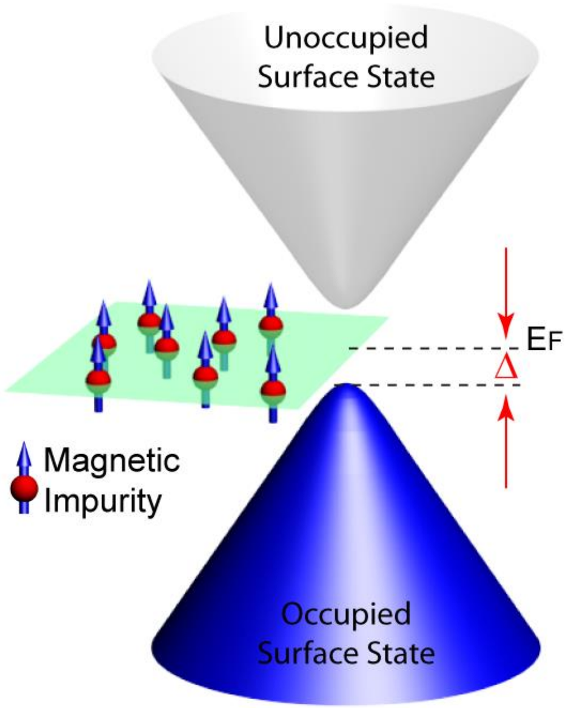
$|m|>|\Delta|$, $(m+\Delta)/B$ the same sign $(m-\Delta)/B$

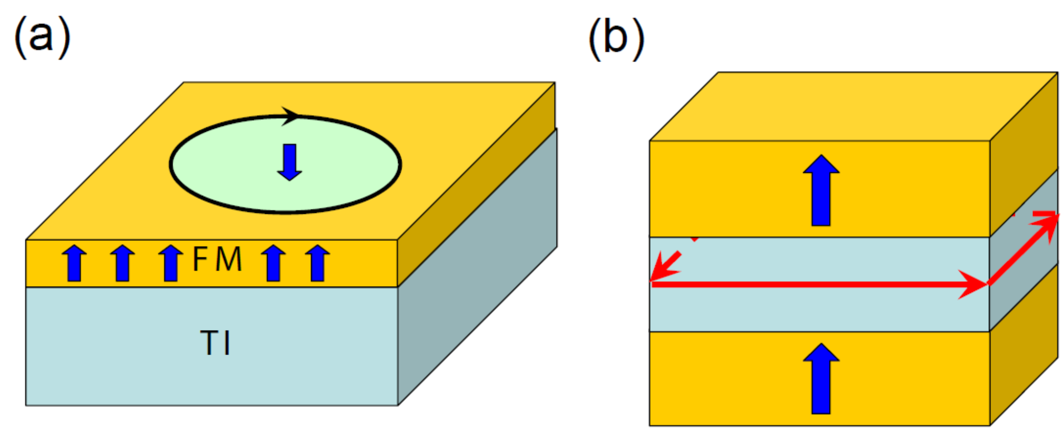
$|m|<|\Delta|$, $(m+\Delta)/B$ the opposite sign $(m-\Delta)/B$

Gapped Dirac fermions on the surface, chiral fermions on the domain wall

- 2D system
- Ferromagnetic
- Topological
- Insulating







QAH can be realized in ferromagnetic TI (Qi, Hughes, Zhang, PRB 2008)

Phase diagram: QAH in a magnetic TI thin film

Key message: strong FM ordering

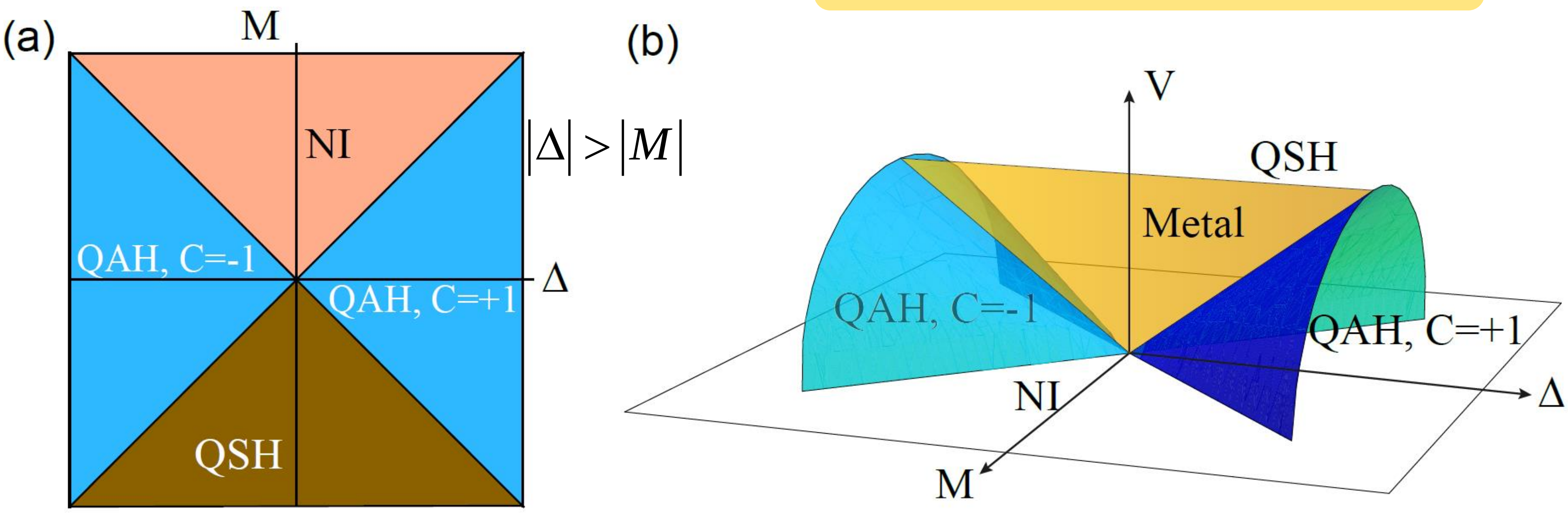
$$\mathcal{H}_{\mathrm{surf}}(k_x,k_y)+\mathcal{H}_{\mathrm{Zeeman}}(k_x,k_y)$$

$$=\begin{pmatrix} 0 & iv_F k_- & m(k) & 0 \\ -iv_F k_+ & 0 & 0 & m(k) \\ m(k) & 0 & 0 & -iv_F k_- \\ 0 & m(k) & iv_F k_+ & 0 \end{pmatrix} + \begin{pmatrix} \Delta & 0 & 0 & 0 \\ 0 & -\Delta & 0 & 0 \\ 0 & 0 & \Delta & 0 \\ 0 & 0 & 0 & -\Delta \end{pmatrix}$$

$$|t\uparrow\rangle \quad |t\downarrow\rangle \quad |b\uparrow\rangle \quad |b\downarrow\rangle$$

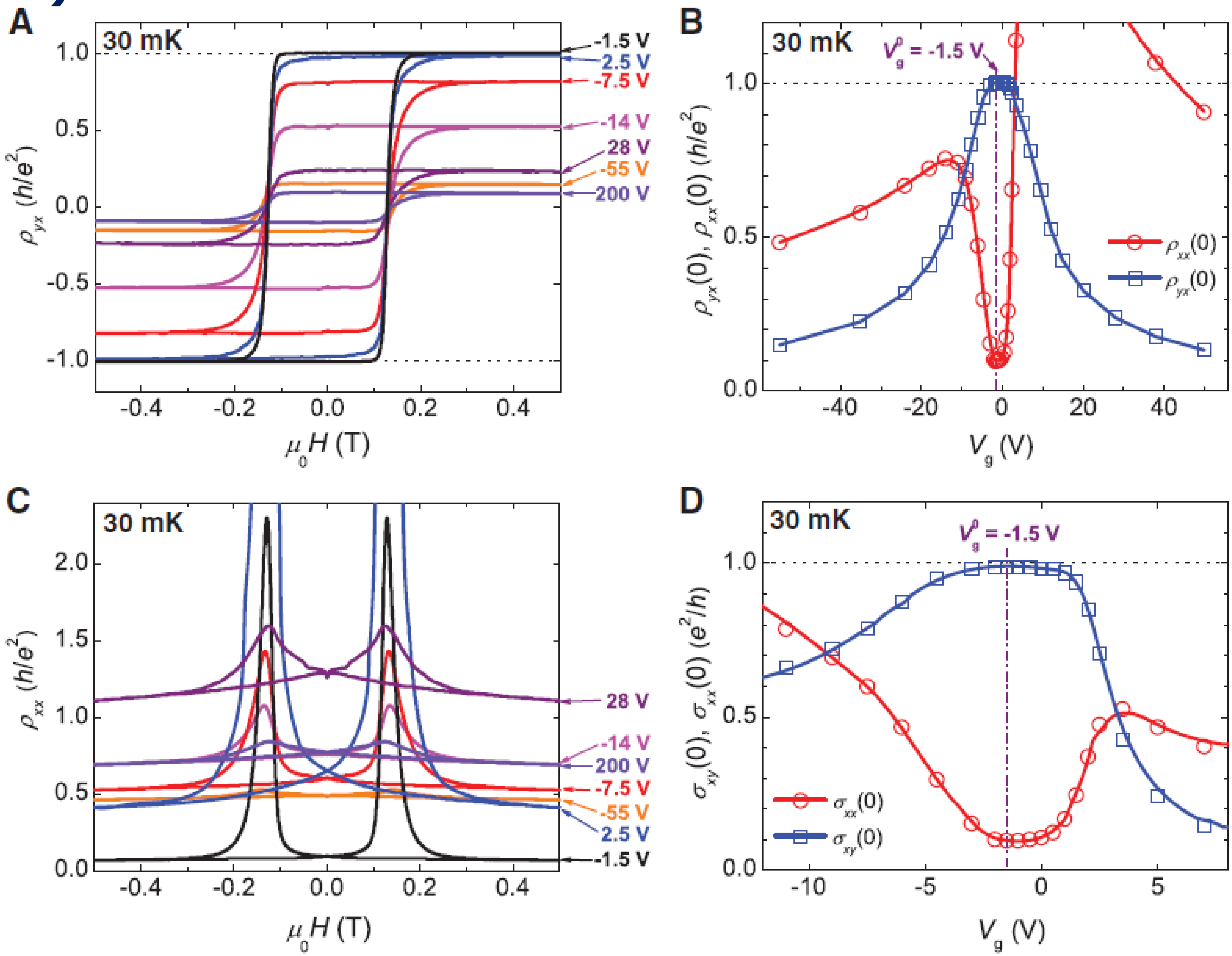
$$\mathcal{H}_{\mathrm{inv}} = \begin{pmatrix} V & 0 & 0 & 0 \\ 0 & V & 0 & 0 \\ 0 & 0 & -V & 0 \\ 0 & 0 & 0 & -V \end{pmatrix}$$

Condition for QAH: $\Delta^2 > M^2 + V^2$



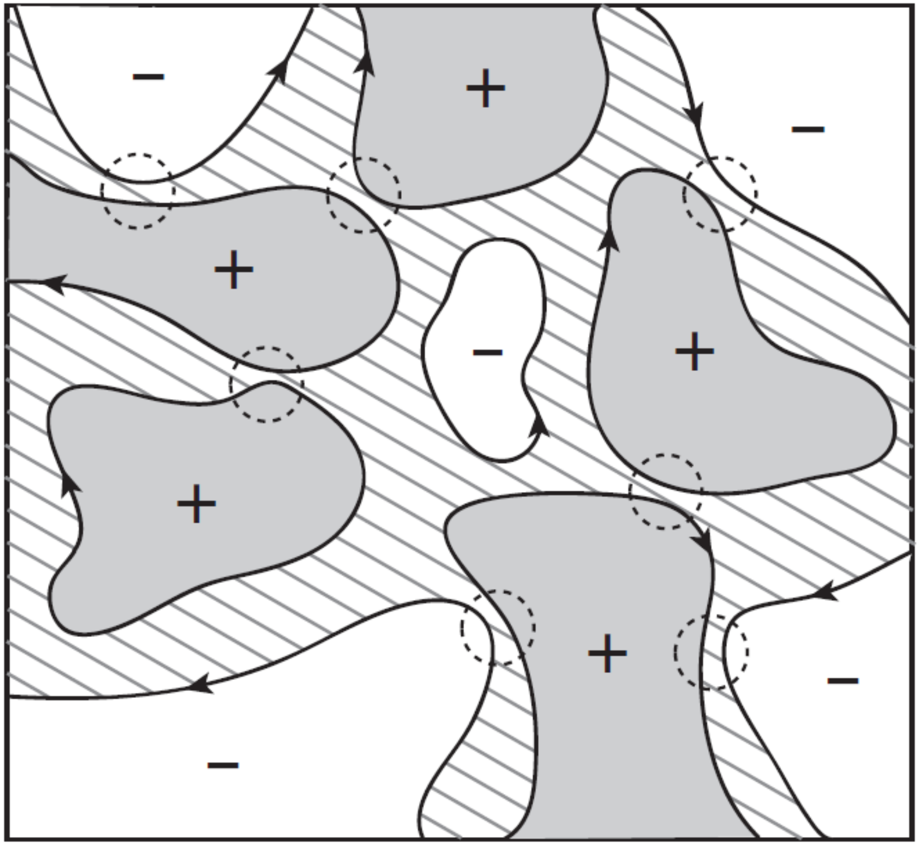
J. Wang, B. Lian, S. C. Zhang, Phys. Scr. T164, 014003 (2015)

Experimental observation of the QAHE in Cr-BiSbTe3 (Tsinghua 2013, RIKEN, UCLA, Stanford, MIT, Princeton and PSU…)



QAH plateau transition: Chalker-Cottington model and effectively tune magnetic exchange coupling

Magnetic TI: Two copy of Dirac model with opposite Chern number



Network of chiral edge states at random magnetic domain walls

$$\mathcal{H}_0(k_x,k_y) = \begin{pmatrix} \mathcal{H}_+(k) & 0 \\ 0 & \mathcal{H}_-(k) \end{pmatrix},$$

$$\mathcal{H}_\pm(k) = k_y\tau_1 \mp k_x\tau_2 + (m(k) \pm \Delta)\tau_3,$$

$$C = \begin{cases} \Delta/|\Delta|, & \text{for } |\Delta| > |m_0| \\ 0, & \text{for } |\Delta| < |m_0| \end{cases}.$$

Three kinds of disorder potential:

$$\mathcal{H}_A = A_x(x,y)\tau_2 \otimes \sigma_3 - A_y(x,y)\tau_1 \otimes 1,$$

$$\mathcal{H}_\Delta = \Delta(x,y)\tau_3 \otimes \sigma_3,$$

$$\mathcal{H}_V = V(x,y),$$

J. Wang *et al*, Phys. Rev. B **89**, 085106 (2014)

Critical behavior in QAH plateau transitions
Theoretical Prediction

Two copy of Dirac model with opposite Chern number

1. At coercivity field, and at low enough temperature,

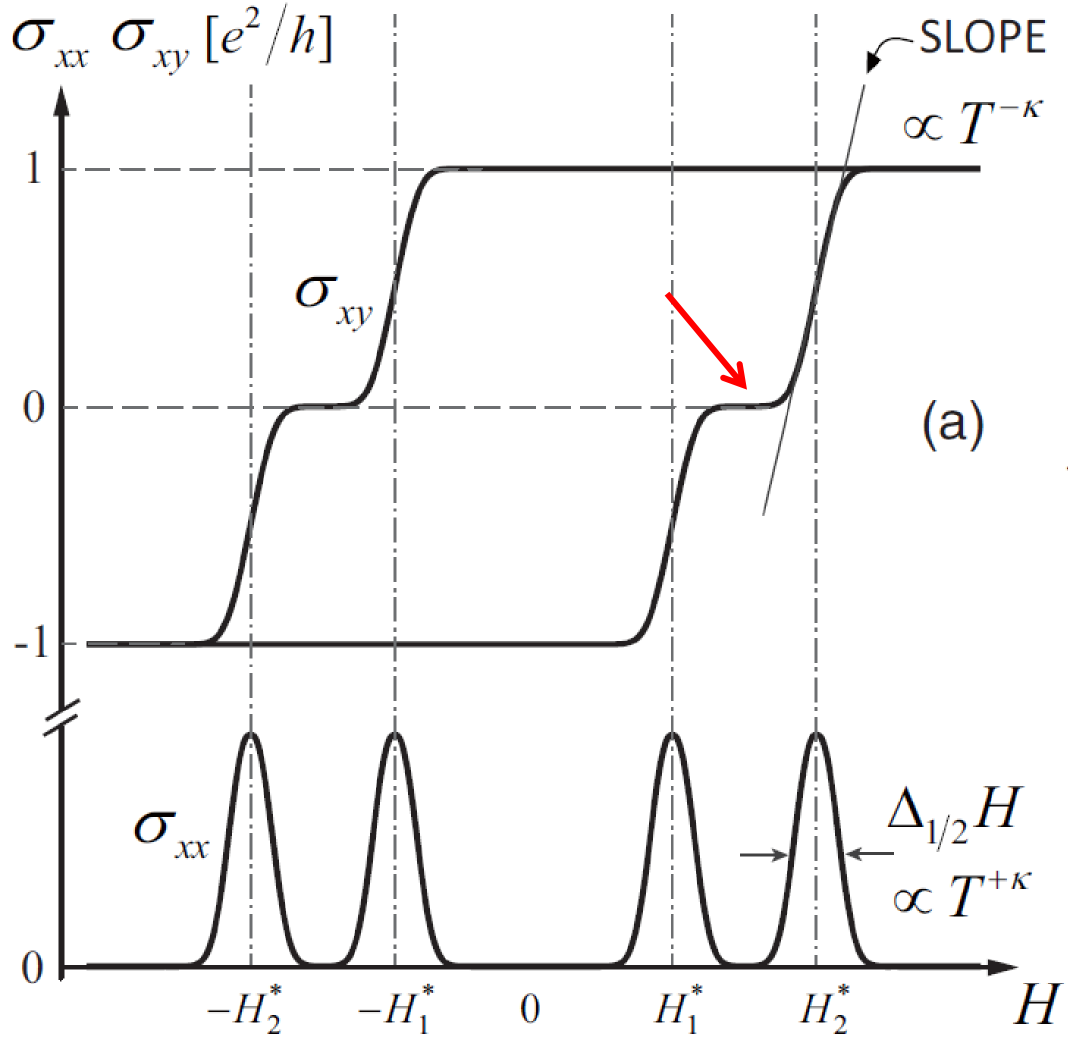
σ_{xy} plateaus at 0!

$$(\partial\sigma_{xy}/\partial H)_{\max} \propto T^{-\kappa}$$
$$\Delta_{1/2}H \propto T^{\kappa}$$

$\kappa \simeq 3/14 = 0.214$

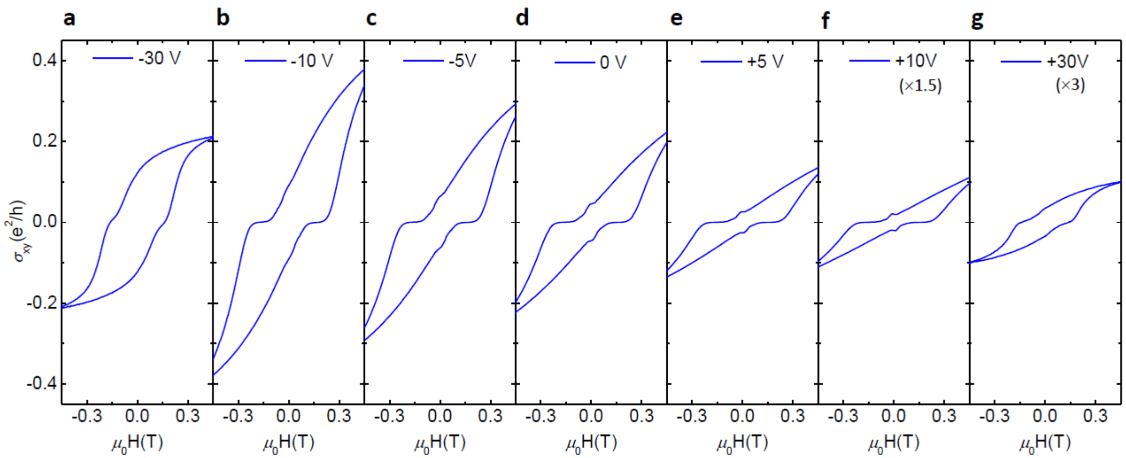
$\kappa = p/2\nu$ $\quad p \simeq 1$ 2D dirty limit

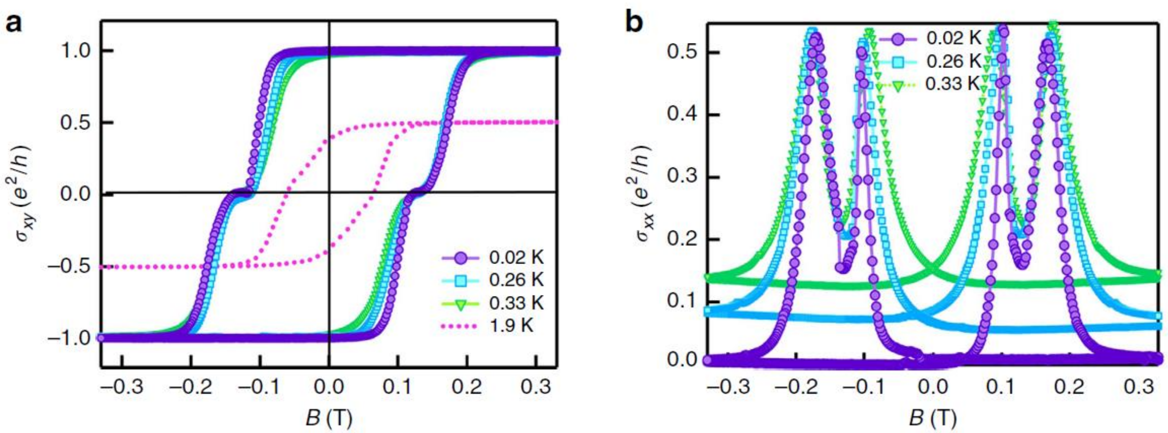
$L_{\text{in}}(T) \propto T^{-p/2}$ as $T \to 0$

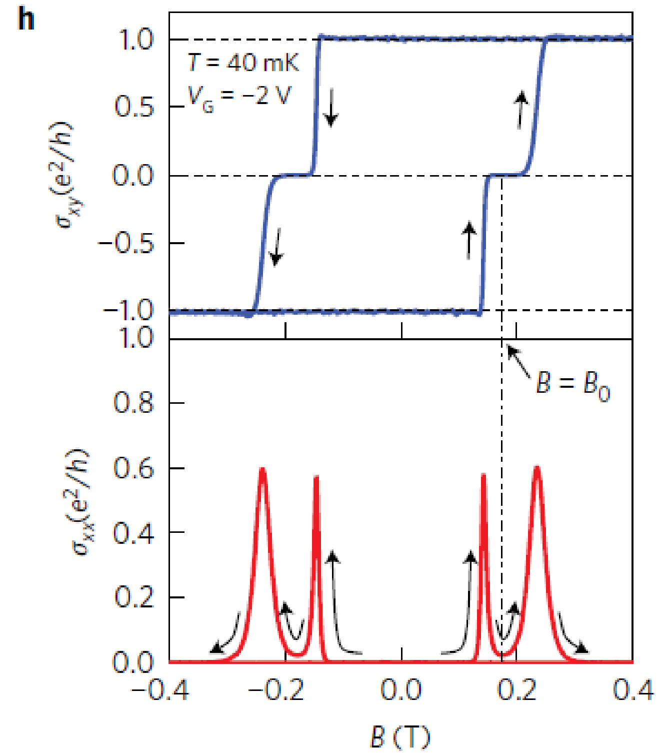


J. Wang, B. Lian, SCZ, PRB **89**, 085106 (2014)

Observation of zero Hall plateau state (Tsinghua, UCLA, 2015, RIKEN 2017)

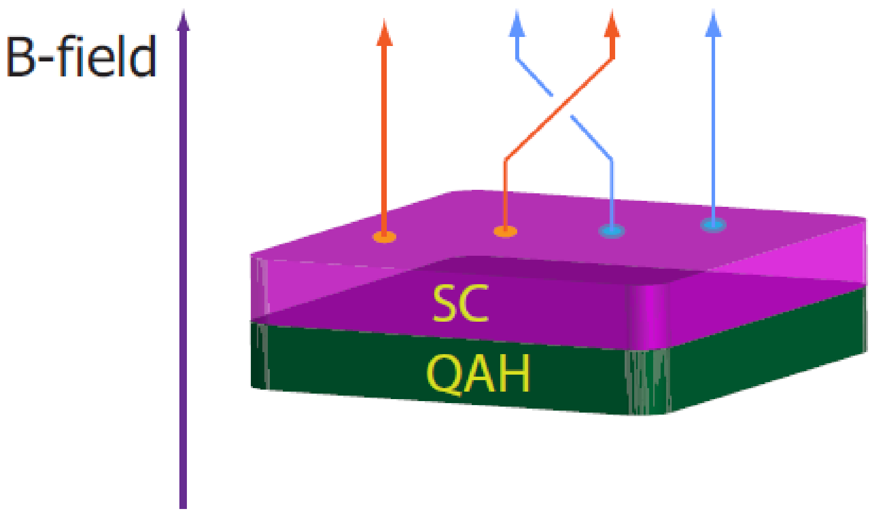


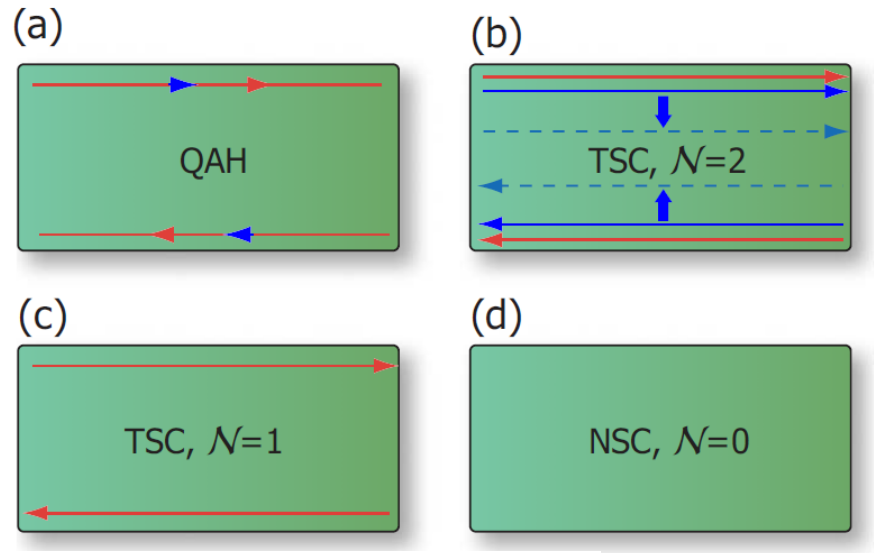




Chiral topological superconductivity in 2D

1 chiral fermion=2 identical majorana fermion



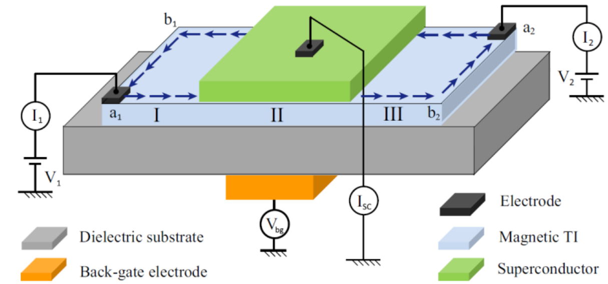


$$H_{\rm BdG} = \frac{1}{2}\sum_{\mathbf{p}} \Psi_{\mathbf{p}}^{\dagger} \begin{pmatrix} h_{\rm QAH}(\mathbf{p}) - \mu & i\Delta\sigma^{y} \\ -i\Delta^{*}\sigma^{y} & -h_{\rm QAH}^{*}(-\mathbf{p}) + \mu \end{pmatrix} \Psi_{\mathbf{p}}$$

$$h_{\rm QAH}(\mathbf{p}) = \begin{pmatrix} m(p) & A(p_x - ip_y) \\ A(p_x + ip_y) & -m(p) \end{pmatrix}$$

X L Qi *et al*, PRB **82**, 184516 (2010)

Model for superconductor proximity coupled QAH effect in magnetic TI



$$H_{\mathrm{BdG}} = \begin{pmatrix} H_0(\mathbf{k}) - \mu & \Delta_{\mathbf{k}} \\ \Delta_{\mathbf{k}}^{\dagger} & -H_0^*(-\mathbf{k}) + \mu \end{pmatrix},$$

$$\Delta_{\mathbf{k}} = \begin{pmatrix} i\,\Delta_1\sigma_y & 0 \\ 0 & i\,\Delta_2\sigma_y \end{pmatrix}. \qquad H_0(\mathbf{k}) = k_y\sigma_x\widetilde{\tau}_z - k_x\sigma_y\widetilde{\tau}_z + m(k)\widetilde{\tau}_x + \lambda\sigma_z,$$

In a simple case for $\mu = 0$ and $\Delta_1 = -\Delta_2 = \Delta$

$$H_{\mathrm{BdG}} = \begin{pmatrix} H_+(\mathbf{k}) & 0 \\ 0 & H_-(\mathbf{k}) \end{pmatrix},$$

$$H_\pm(\mathbf{k}) = k_y\sigma_x \mp k_x\sigma_y\varsigma_z + [m(k) \pm \lambda]\sigma_z\varsigma_z \mp \Delta\sigma_y\varsigma_y$$

The topological properties of H_+

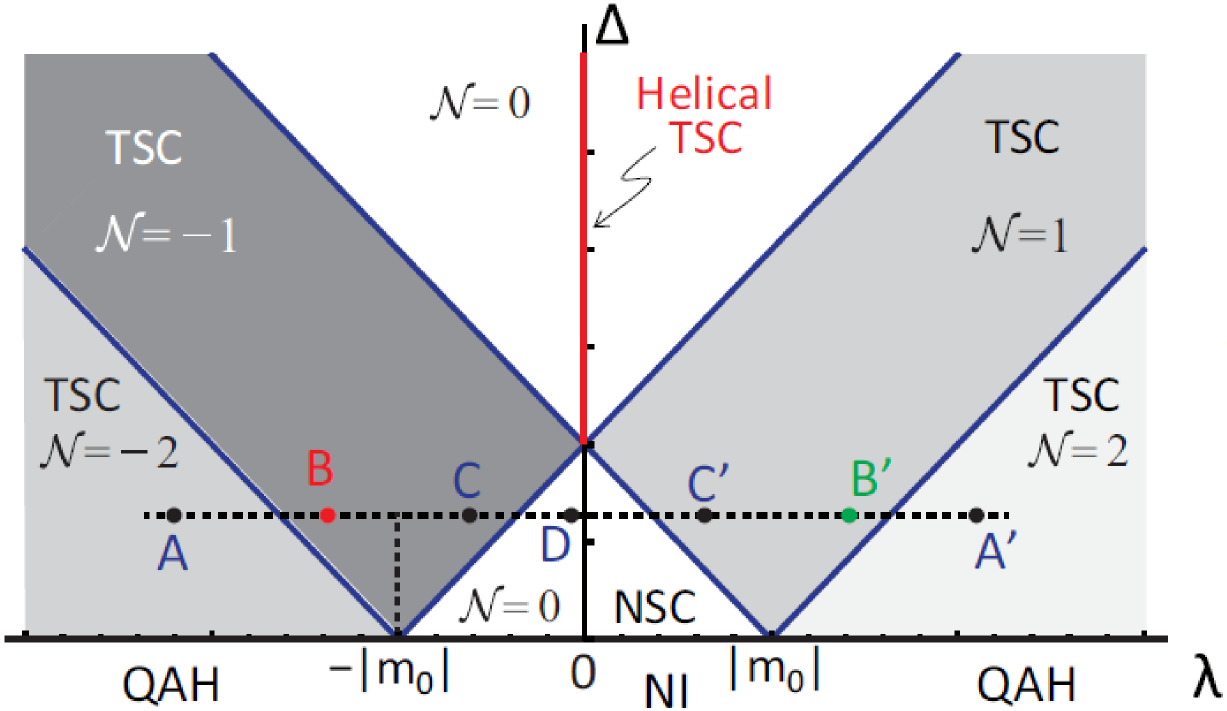
$$H_+(\mathbf{k}) = \begin{pmatrix} h_+(\mathbf{k}) & 0 \\ 0 & -h_-^*(-\mathbf{k}) \end{pmatrix},$$

$$h_\pm(\mathbf{k}) = k_y\sigma_x - k_x\sigma_y + \boxed{[m(k) + \lambda \pm |\Delta|]\sigma_z}$$

characterize a px+/-ipy superconductor, depending on mass term

Jing Wang *et al*, PRB **92**, 064520 (2015)

Simple criteria: chiral TSC emerges at the QAH plateau transition



Phase boundary:

$$\Delta \pm (m_0 \pm \lambda) = 0$$

Usually

$\lambda \gg \Delta, m_0$

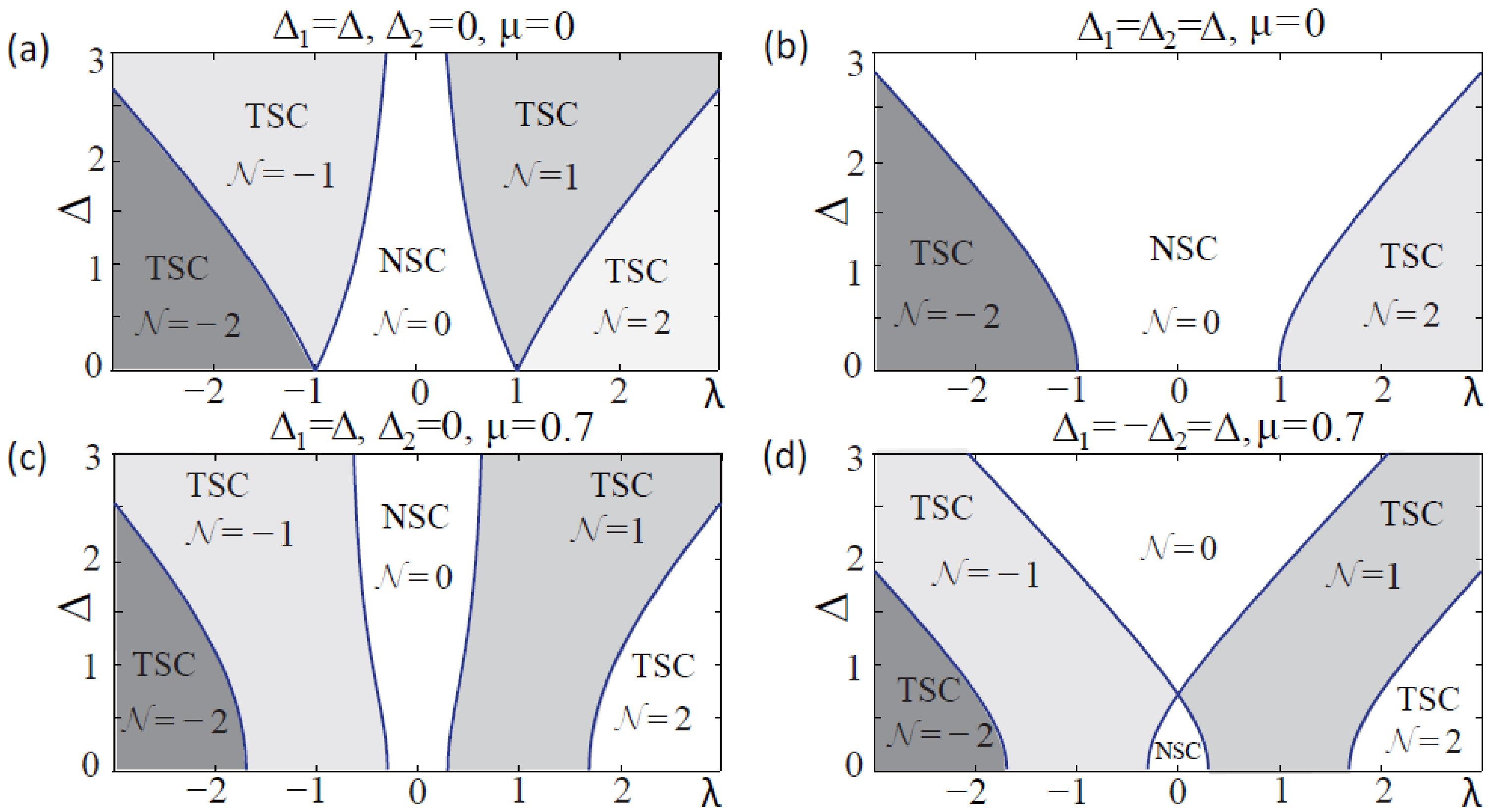
Competetion between Δ m_0 λ

$|m_0| < |\lambda|$ ⟶ QAH effect

$|\lambda \pm m_0| < |\Delta|$ ⟶ Chiral TSC

Phase diagram and realization of TSC in magnetic TI

1. Finite chemical potential.
2. Top and bottom surface better have coupling, otherwise fine tuning of chemical potential into gap is needed.
3. SC proximity only to one surface. (top and bottom have different SC pairing order).

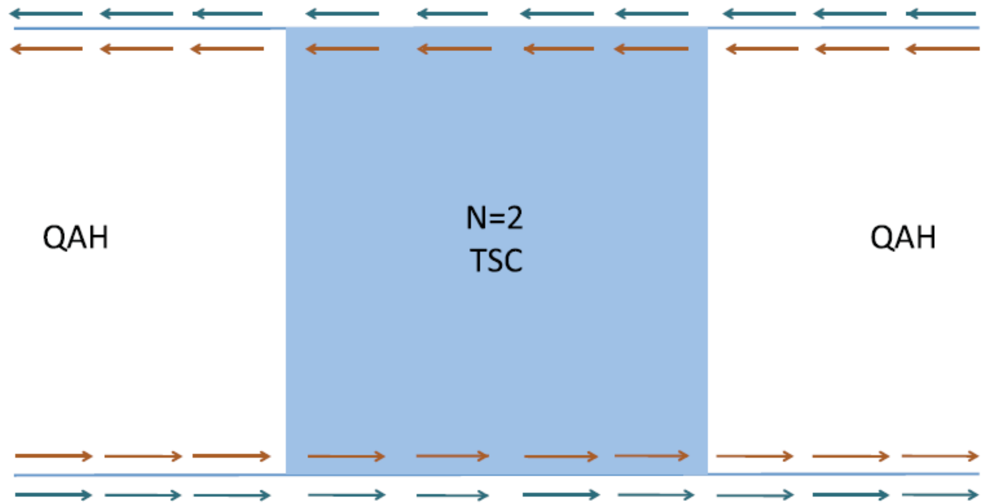


J. Wang *et al*, PRB 92, 064520 (2015)

Smoking gun of chiral edge Majorana fermion: transport signature

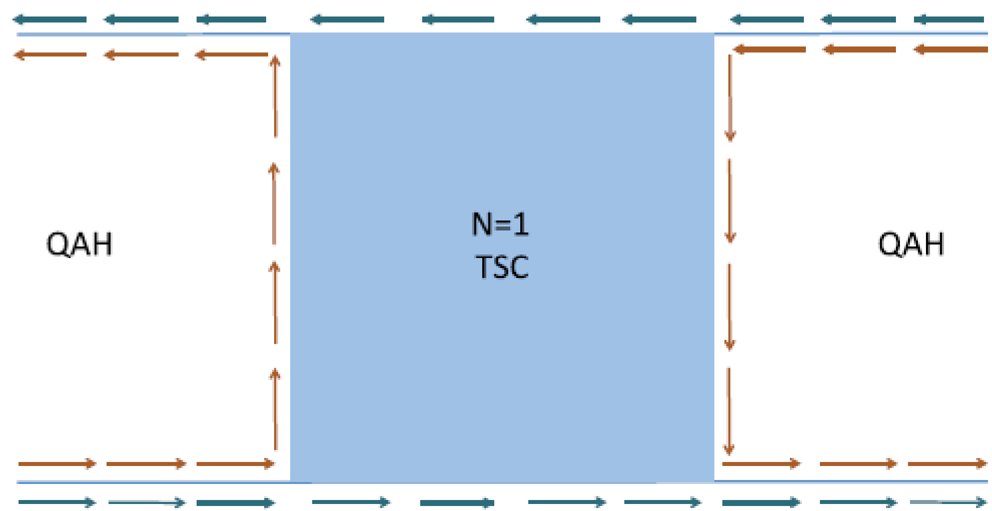
1. QAH and N=2 TSC interface

- No backscattering
- Edge current perfectly transmitted



2. QAH and N=1 TSC

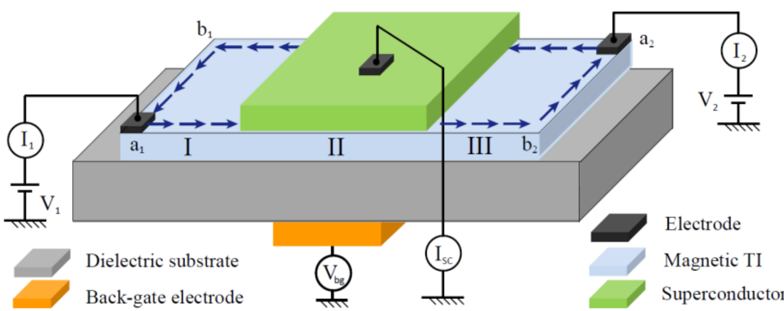
- One chiral majorana complete backscattering
- The other chiral majorana perfectly transmitted

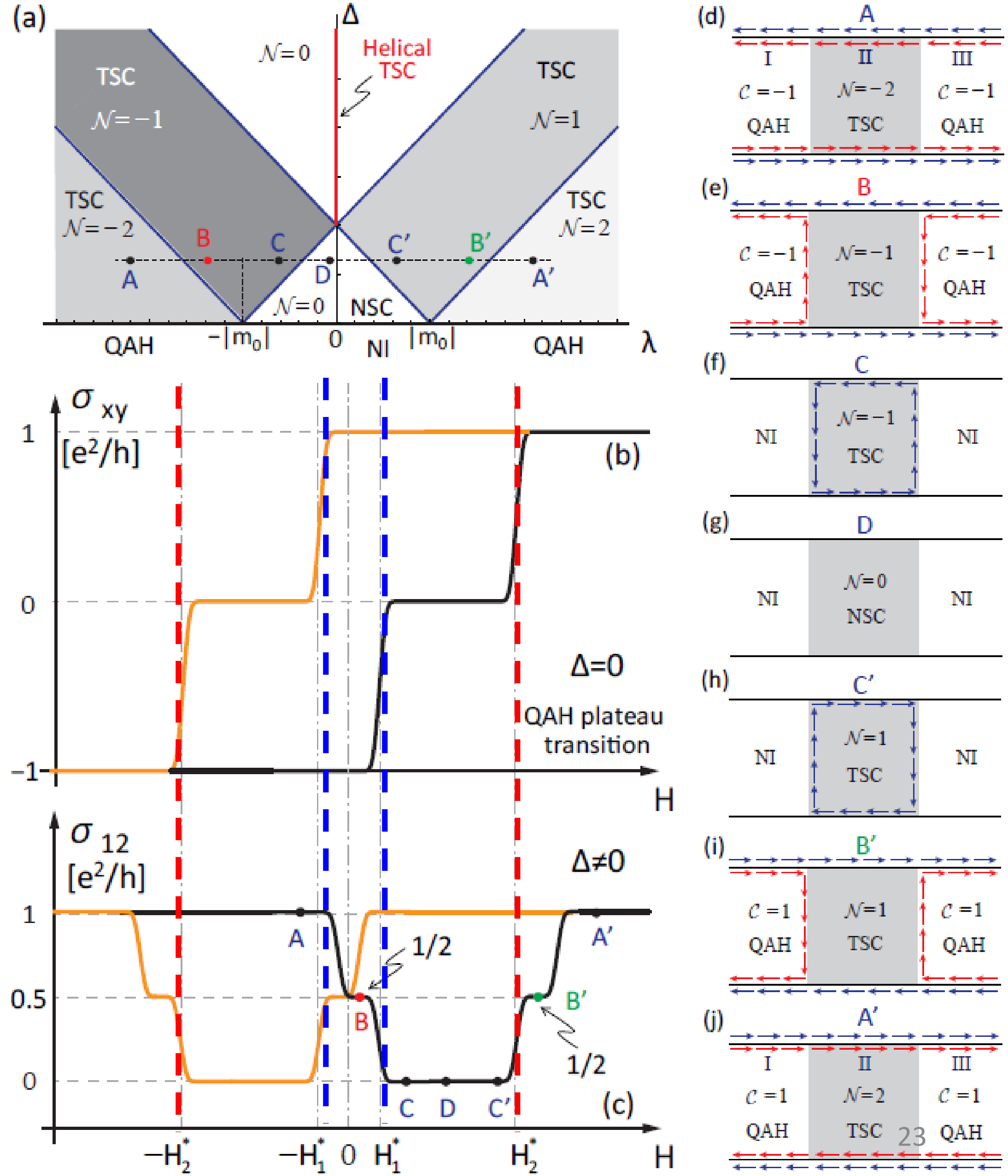


S. B. Chung *et al*, PRB 83, 100512 (2011), J. Wang *et al*, PRB 92, 064520 (2015)

Smoking gun: transport signature half-integer conductance plateau at the coercive field

½ conductance plateau for N=1 TSC and chiral Majorana edge state



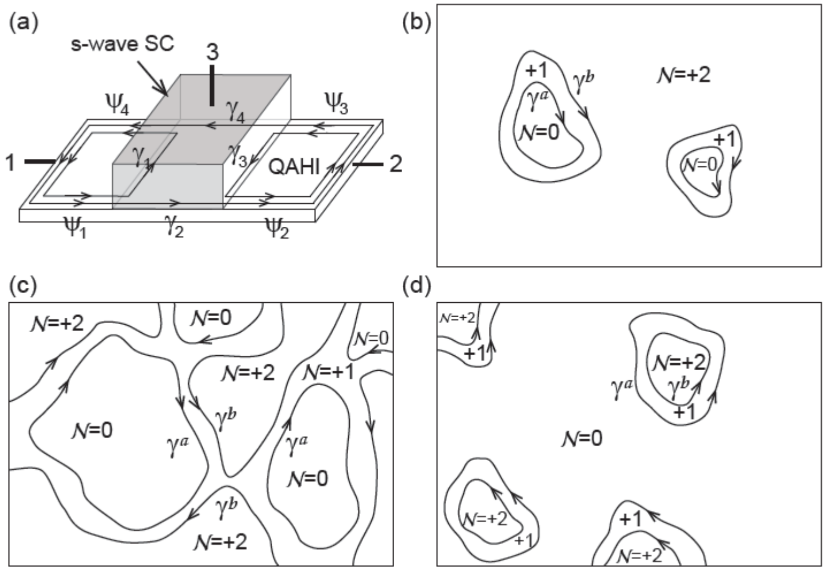


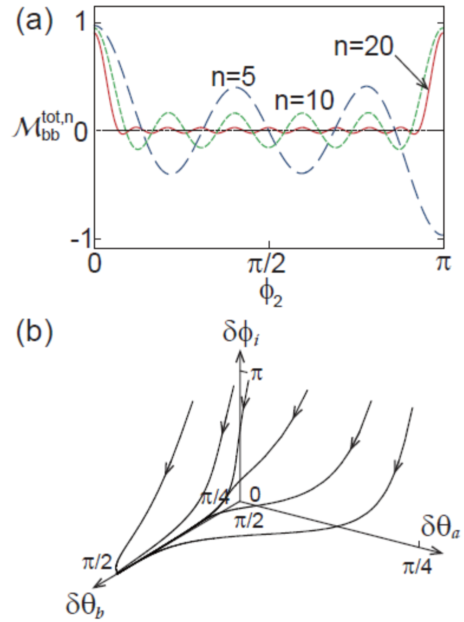
$$V_1 = -V_2 = \frac{V}{2} \qquad V_{SC} = 0$$

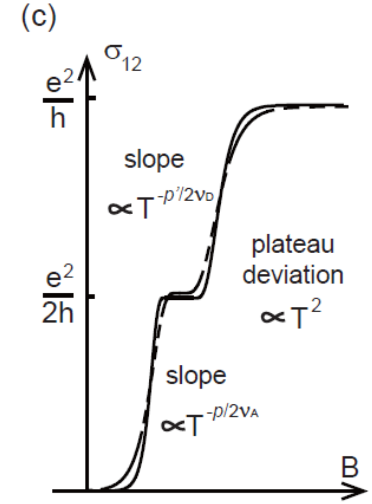
$$I_1 = -I_2 = \frac{e^2}{2h} V$$

J Wang *et al*, PRB 92, 064520 (2015)

Stability of Chiral TSC against disorder



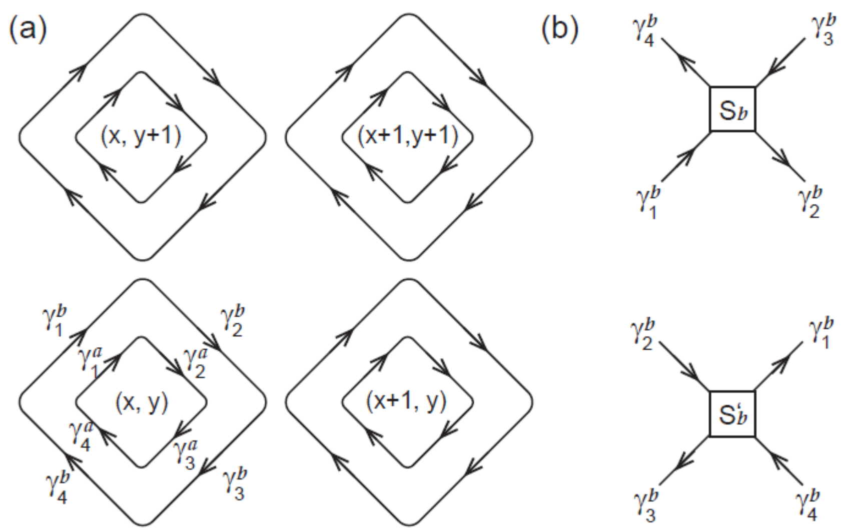




Quantum percolation theory in D class with particle hole symmetry

Critical scaling: 0→1/2, A class

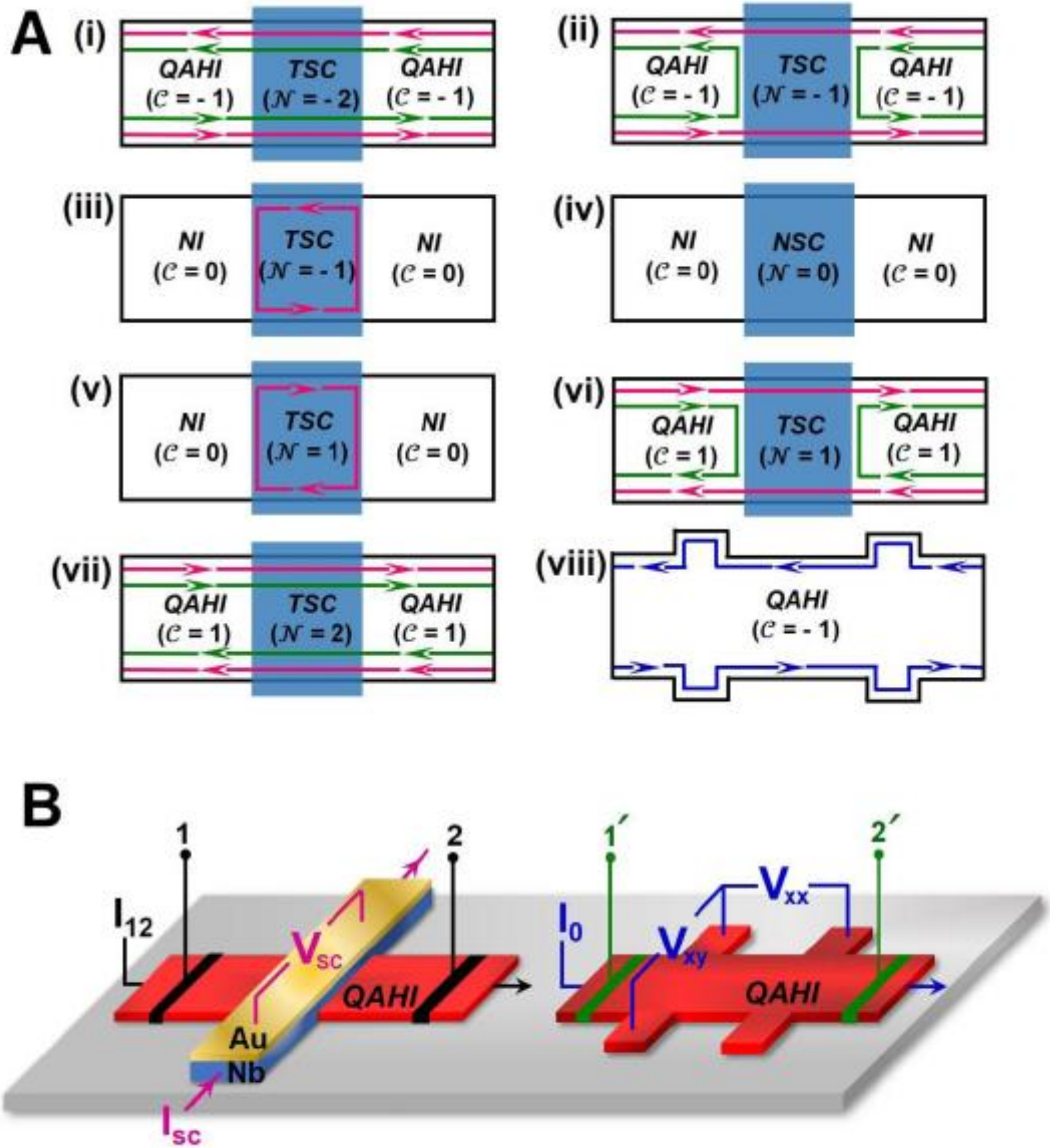
1/2 →1, D class



B. Lian, J. Wang, X. Sun, A. Vaezi,

S. C. Zhang, arXiv: 1709.05558 (2017)

Experimental signature of chiral Majorana edge state
(UCLA & Stanford, 2017) Science 357, 294 (2017)



6 quintuple layers

$(Cr_{0.12}Bi_{0.26}Sb_{0.62})_2Te_3$

2 mm × 1 mm

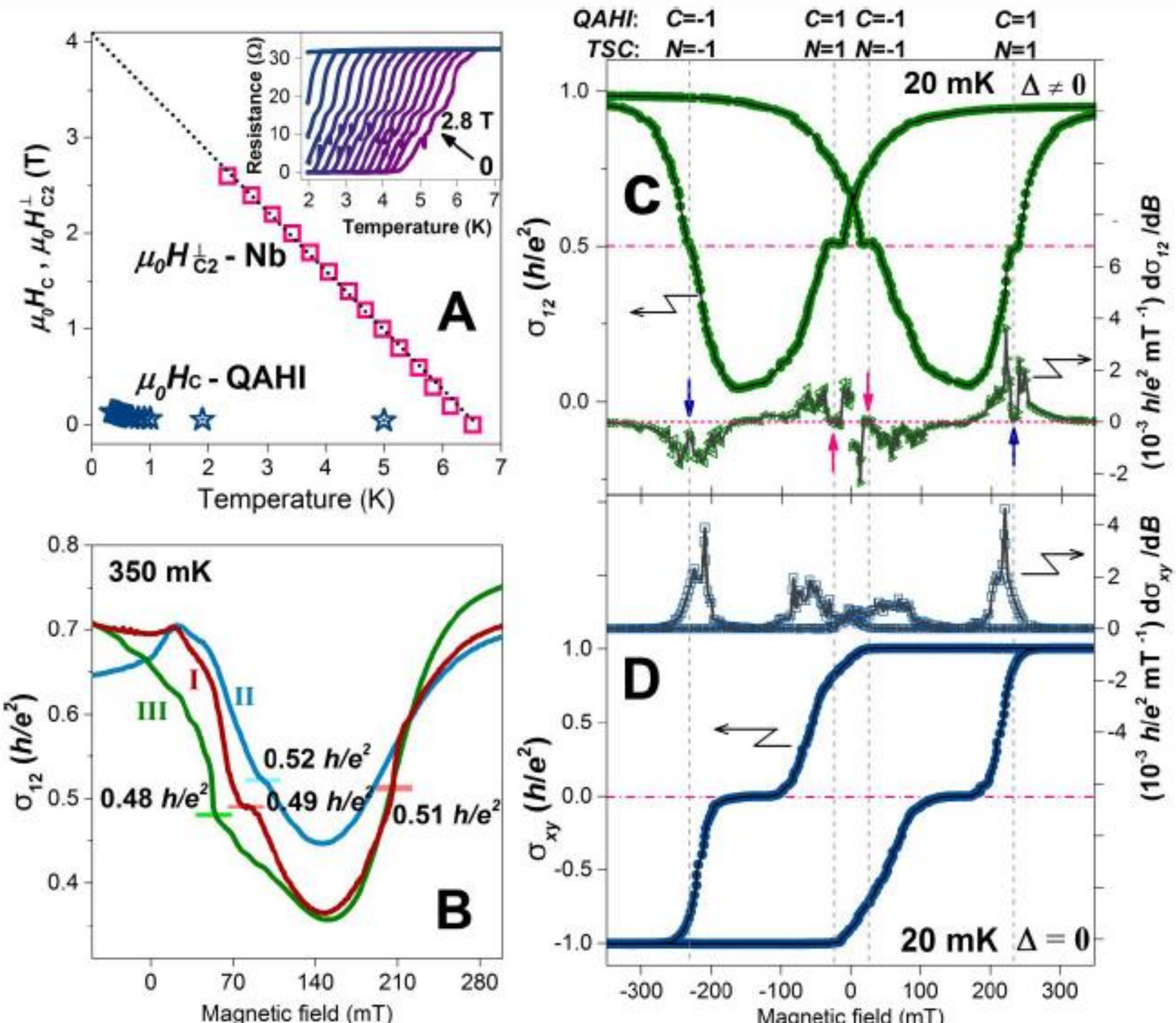
GaAs (111)B substrate

200 nm layer of Nb

8 mm × 0.6 mm

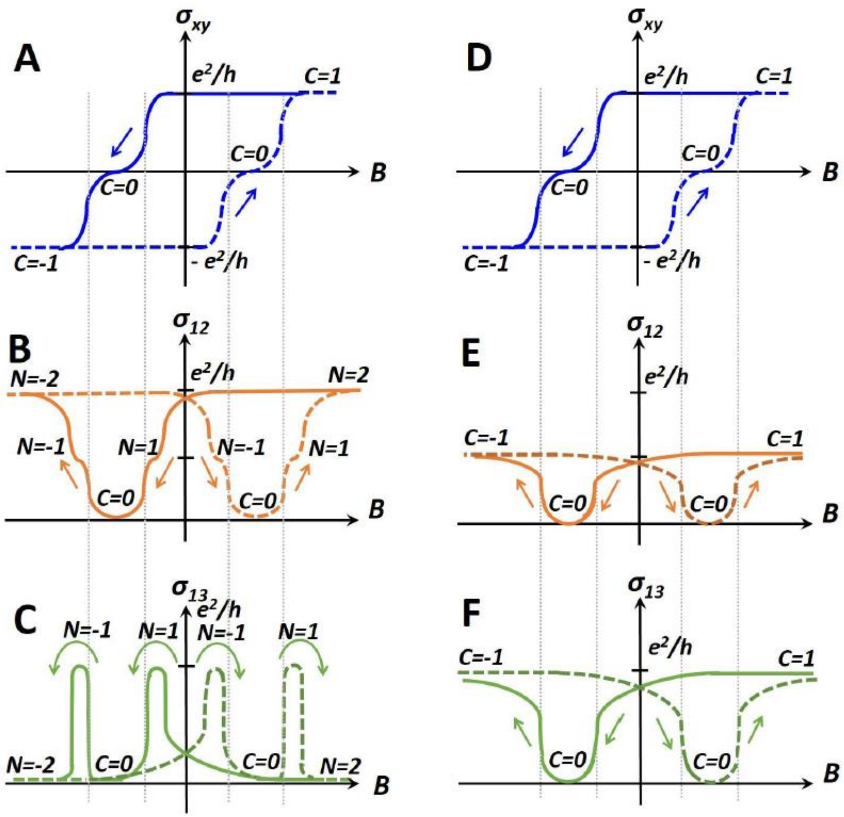
Experimental signature of chiral Majorana edge state

(UCLA & Stanford, 2017) Science 357, 294 (2017)

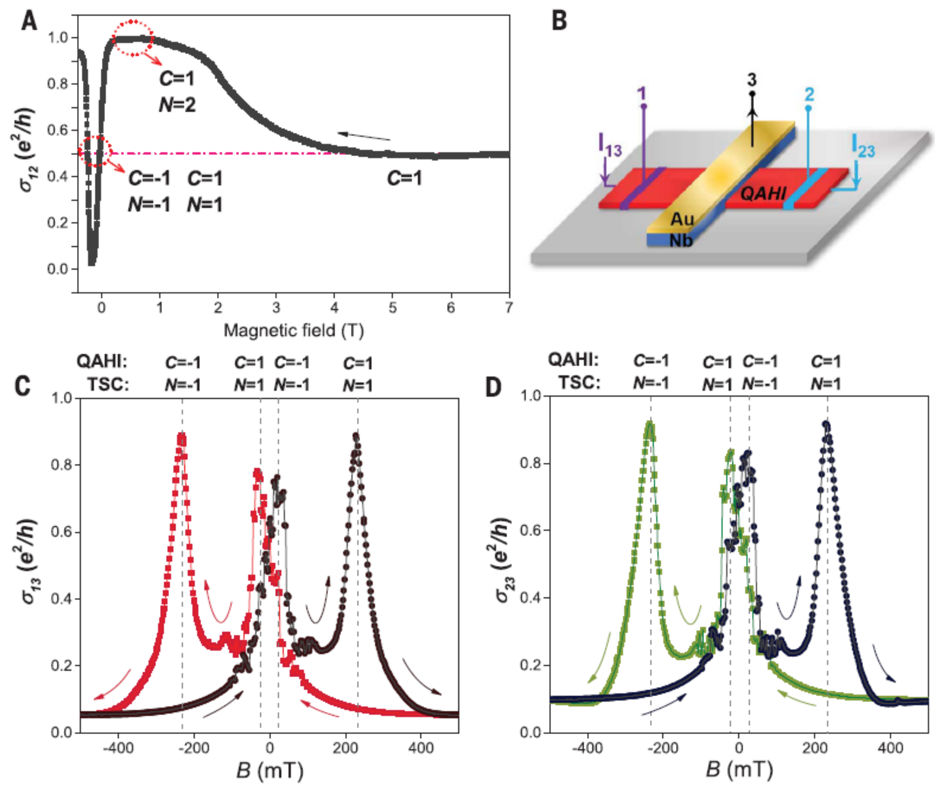


σ_{13} measurement instead of σ_{12}

THEORY



EXPERIMENT



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Journal Club for Condensed Matter Physics JCCM_July_2017_01
https://www.condmatjclub.org

Mobilizing Majorana fermions

Chiral Majorana fermion modes in a quantum anomalous Hall insulator-superconductor structure

Authors: Qing Lin He, Lei Pan, Alexander L. Stern, Edward C. Burks, Xiaoyu Che, Gen Yin, Jing Wang, Biao Lian, Quan Zhou, Eun Sang Choi, Koichi Murata, Xufeng Kou, Zhijie Chen, Tianxiao Nie, Qiming Shao, Yabin Fan, Shou-Cheng Zhang, Kai Liu, Jing Xia, and Kang L. Wang

Science **357**, 294-299 (2017); arXiv:1606.05712

Recommended with a Commentary by Jason Alicea, Caltech

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Journal Club for Condensed Matter Physics JCCM_September_2017_01
http://www.condmatjclub.org

Angels & Demons: Majorana & Dirac fermions in a quantum Hall edge channel

1. **A mechanism of $e^2/2h$ conductance plateau without 1D chiral Majorana fermions**
   Authors: Wenjie Ji and Xiao-Gang Wen
   arXiv:1708.06214
2. **Disorder-induced half-integer quantized conductance plateau in quantum anomalous Hall insulator–superconductor structures**
   Authors: Yingyi Huang, F. Setiawan, and Jay D. Sau
   arXiv:1708.06752

Recommended with a Commentary by Carlo Beenakker, Instituut-Lorentz, Leiden University

Quantum phase transition of chiral Majorana fermion in the presence of disorder, B. Lian, J. Wang, X. Sun, A. Vaezi, S. C. Zhang, arXiv: 1709.05558 (2017)

Summary

1. Chiral topological superconductor can be realized from superconductor proximity coupled quantum anomalous Hall state.
2. Tunable parameters, such as magnetism and hybridization gap make magnetic topological insulator a good platform for chiral TSC.
3. Experimental observation of the ½ plateau as a compelling evidence of chiral Majorana fermion.
4. Hybrid topological materials host a lot of interesting topological phenomena.

Thank you for your attention!