# *Quantum Impurities*

## *A Challenge for Quantum Simulation with Ultracold Atoms*

**Richard Schmidt**

*- Harvard University / ITAMP -*

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Theoretical Physics Seminar
Tsinghua University, Institute for Advanced Study
Beijing, 10/29/14

**Collaborators**

**Harvard/ITAMP:** David Benjamin, Eugene Demler, Mikhail Lukin, Hossein Sadeghpour

**Munich:** Steffen P. Rath, Wilhelm Zwerger

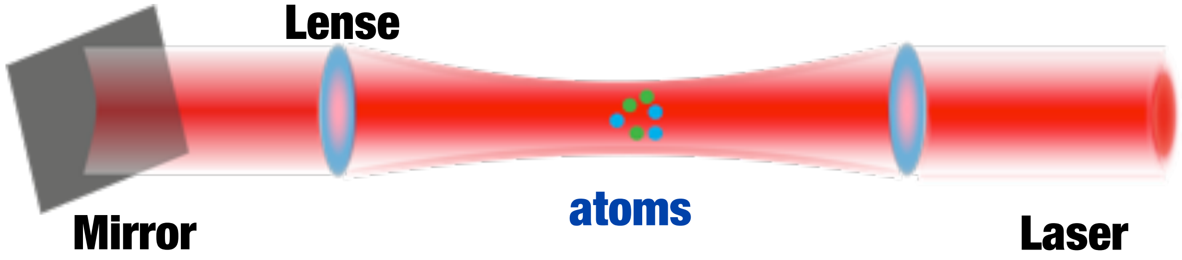
**Vienna:** Mikhail Lemeshko

**Harvard University**

# ultracold quantum gases

## atoms trapped by laser in harmonic confinement

➡ cooled to very low temperatures $\sim 100\,\mathrm{nK}$



## condensed matter system with well controlled *Hamiltonian*

e.g.

$$H = \sum_{\mathbf{p}} \epsilon_{\mathbf{p}} \hat{c}^{\dagger}_{\mathbf{p}} \hat{c}_{\mathbf{p}} + \underbrace{g}_{\longrightarrow \text{low scattering energies: } \textit{contact} \text{ interactions}} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} \hat{c}^{\dagger}_{\mathbf{k}'-\mathbf{q}} c^{\dagger}_{\mathbf{k}+\mathbf{q}} \hat{c}_{\mathbf{k}} \hat{c}_{\mathbf{k}'}$$

low scattering energies: *contact* interactions

➡ interaction strength tunable

➡ experimental accessible:

- density
- transport coefficients (viscosity, spin diffusion...)
- responses, correlations

**quantum simulator of Hamiltonians**
**&**
**testground for few and many-body theories**

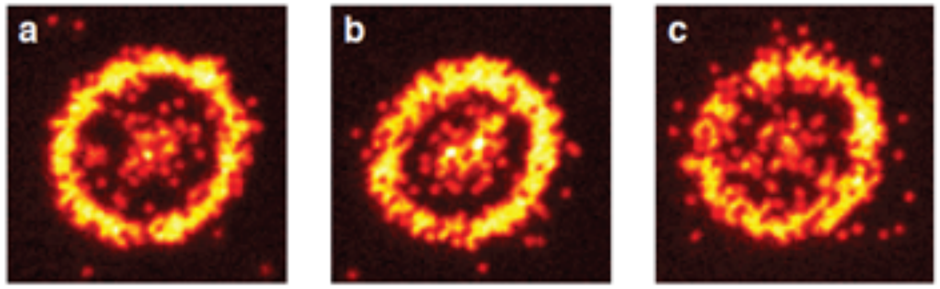
# ultracold quantum gases

## cold atoms as quantum simulators

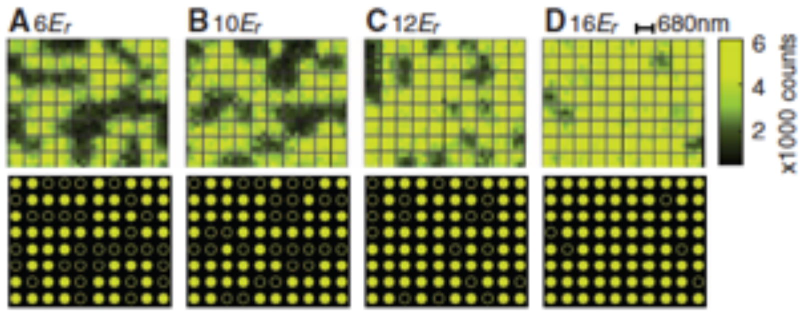
*optical lattices / single-site detection:*

- Mott-Insulator to Superfluid transition

**MOTT-SUPERFLUID TRANSITION**

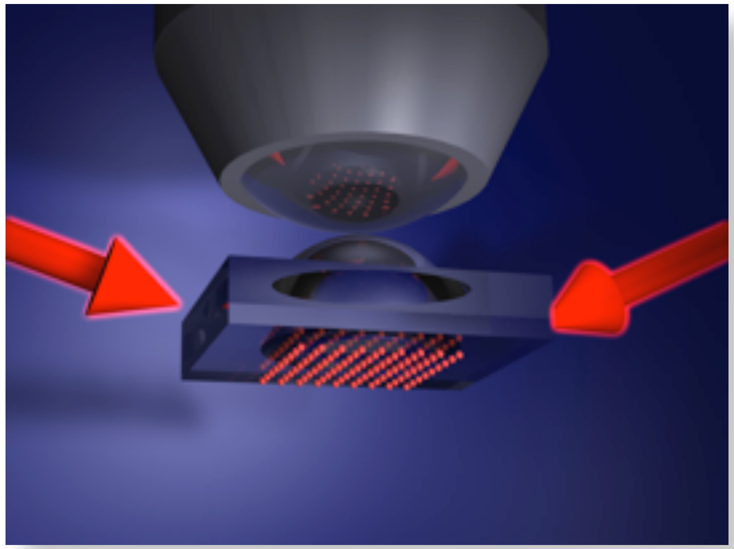


SHERSON ET AL., NATURE 467 (2010)



BAKR ET AL. SCIENCE 329 (2010)

**QUANTUM GAS MICROSCOPE**



BAKR ET AL. SCIENCE 329 (2010)
IMAGE: CUAWEB.MIT.EDU - GREINER GROUP

# ultracold quantum gases

## cold atoms as quantum simulators

*optical lattices / single-site detection:*

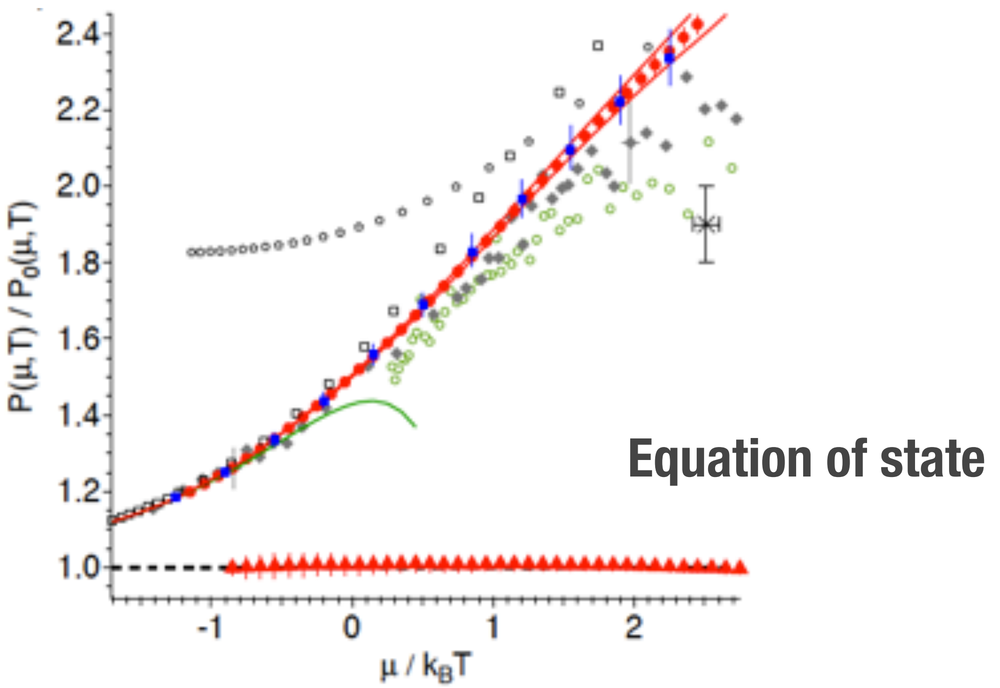
- Mott-Insulator to Superfluid transition

*Feshbach resonances:*

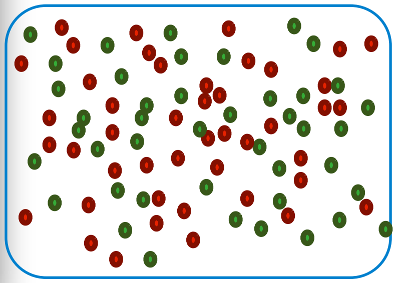
- unitary Fermi gas / BEC-BCS crossover
- ...

### UNITARY FERMI GAS

ZWIERLEIN (MIT)



Van Houcke et al. Nature Phys. 8 (2012)



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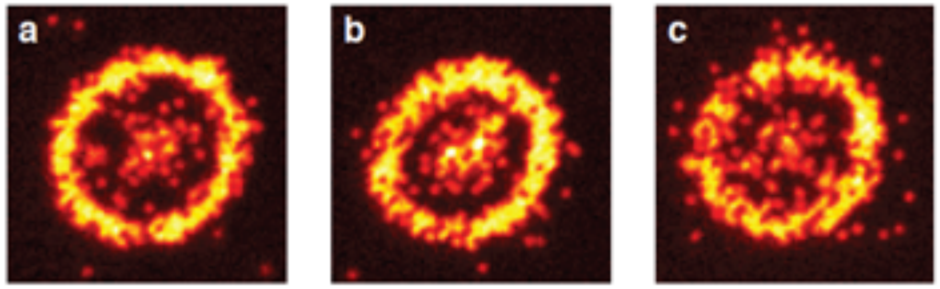

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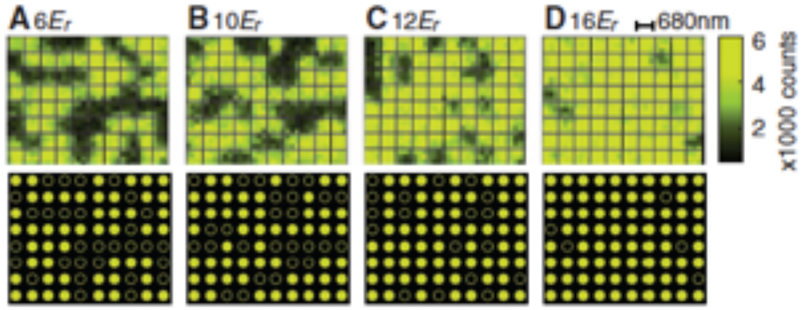

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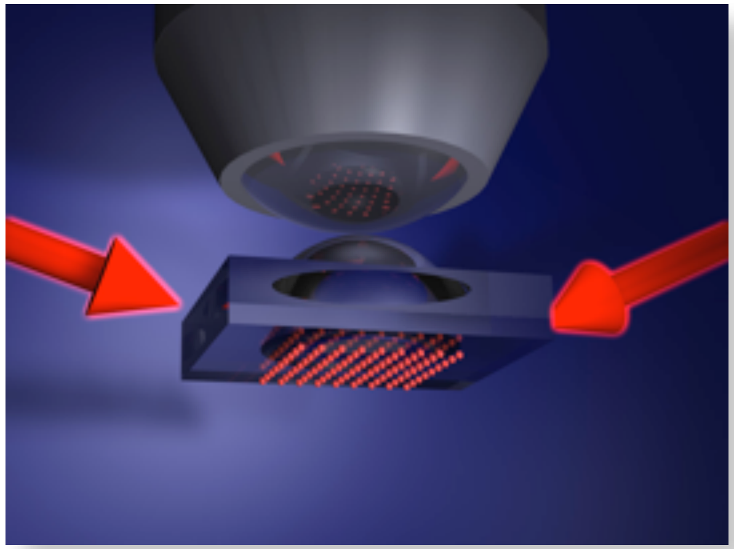
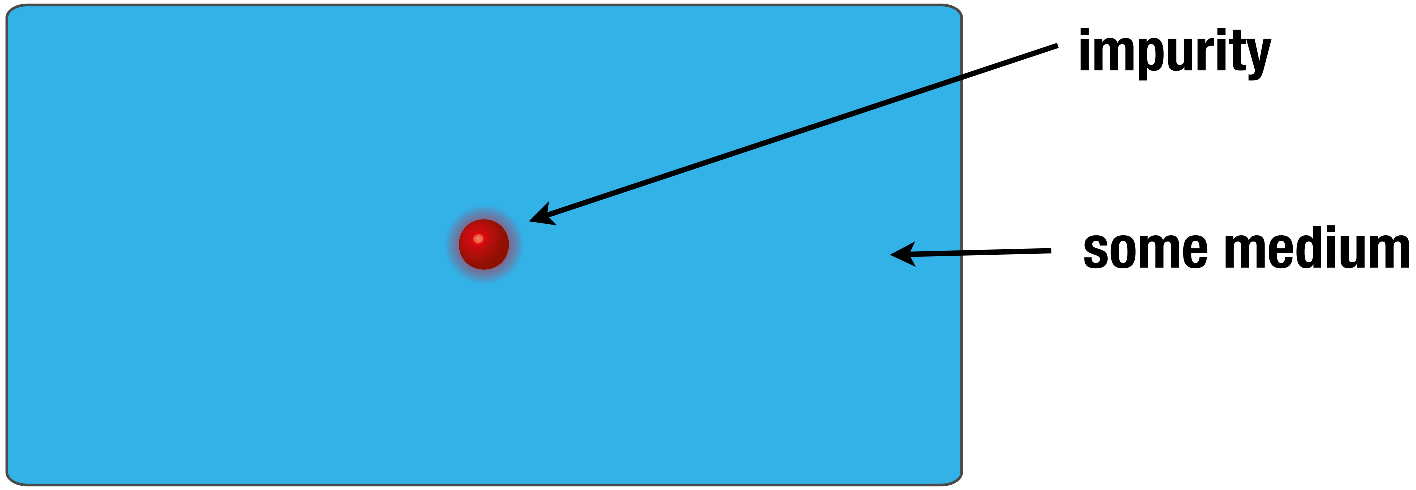


BAKR ET AL. SCIENCE 329 (2010)

IMAGE: CUAWEB.MIT.EDU - GREINER GROUP

# This talk: Impurity physics

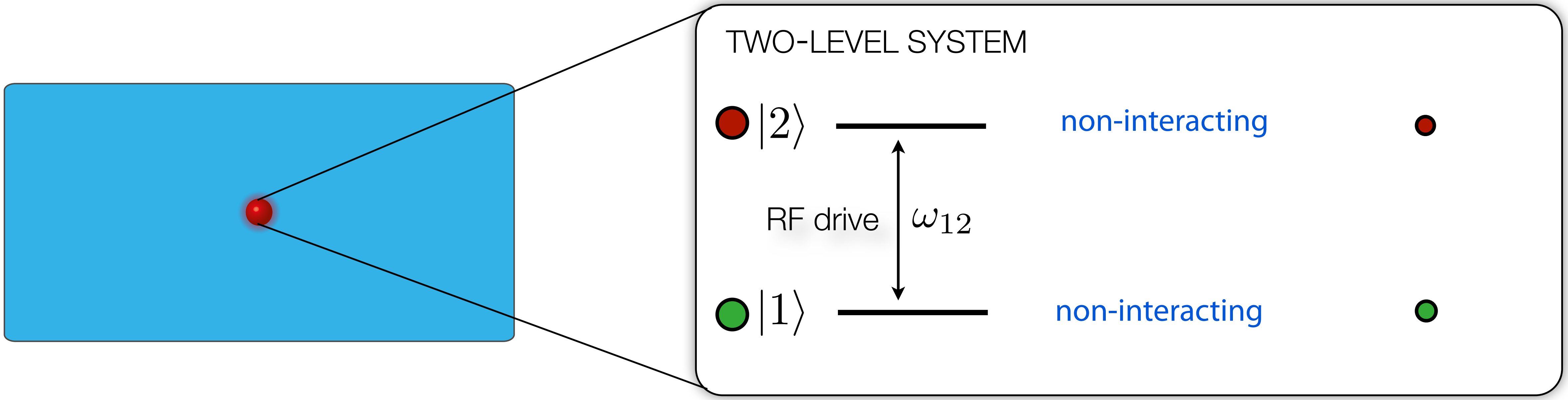
**Quantum impurities** ⟷ **Paradigm of condensed matter physics**



- appear in many flavors in condensed matter physics
- relatively simple system from many-body perspective: allow to advance theory in ‘controlled way’
- system on the verge from few- to many-body physics

# This talk: Impurity physics

- impurity physics appears also naturally in: quantum optics, quantum dots, NV centers, atomic clocks, cavity QED...



TWO-LEVEL SYSTEM

no interactions: ▸ coherent Rabi oscillations

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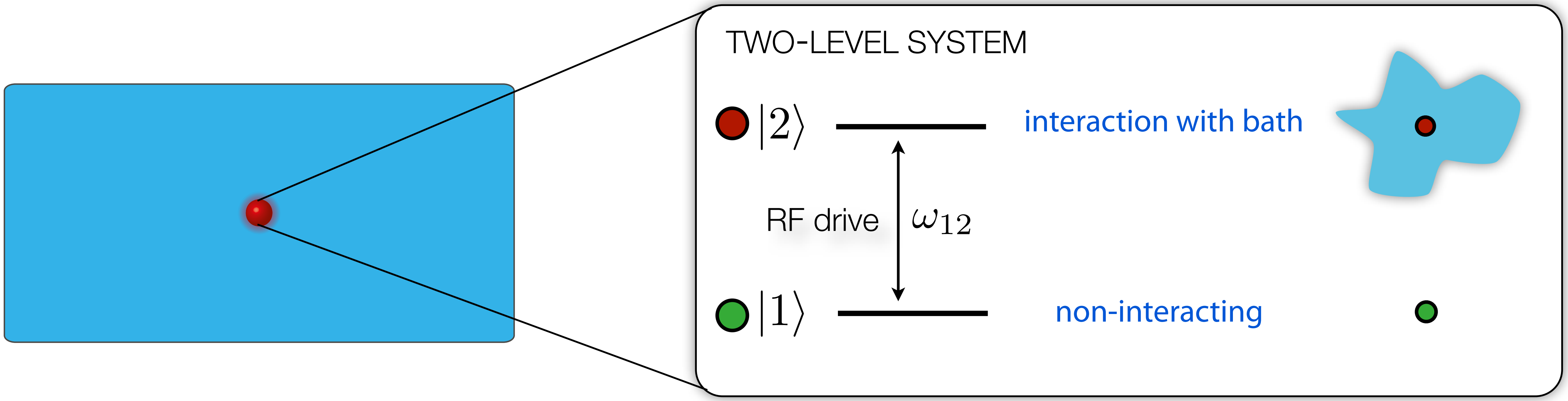
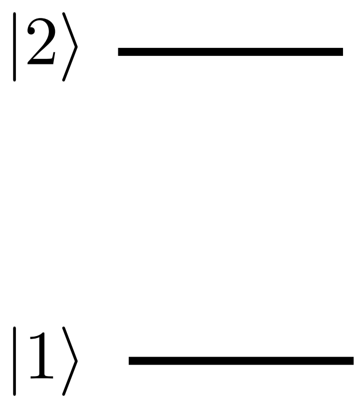


no interactions: ‣ coherent Rabi oscillations

finite interactions: ‣ decoherence of Rabi oscillations



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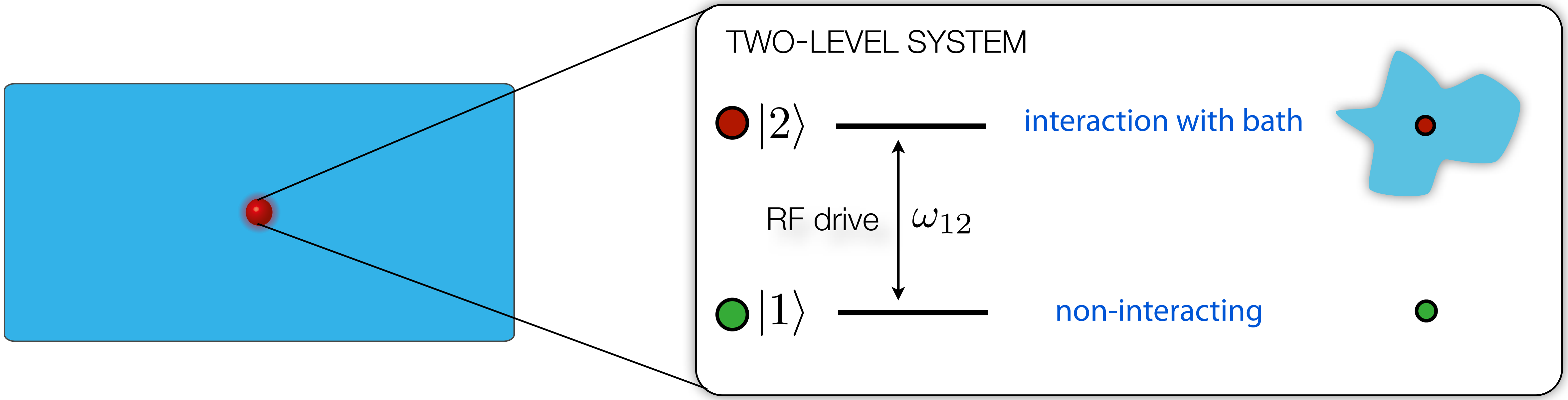
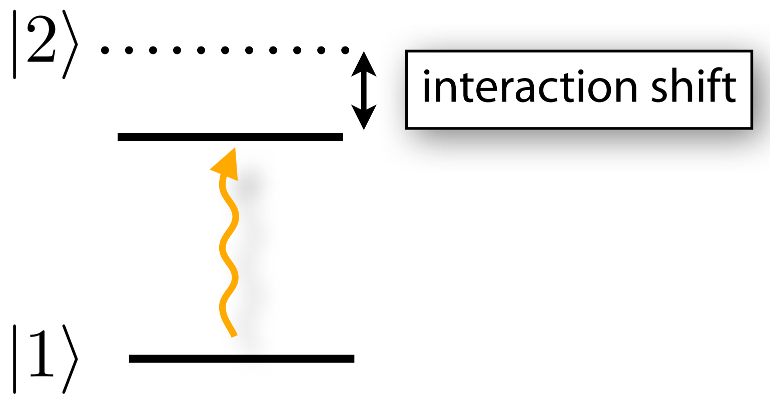


no interactions:
- coherent Rabi oscillations

finite interactions:
- decoherence of Rabi oscillations
- level shifts
- line broadening



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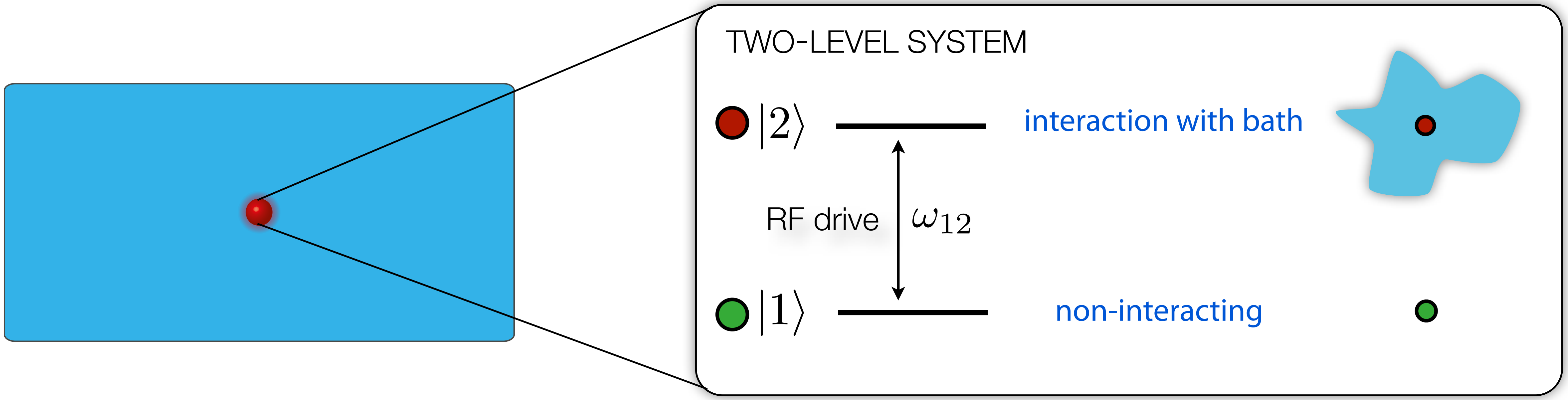

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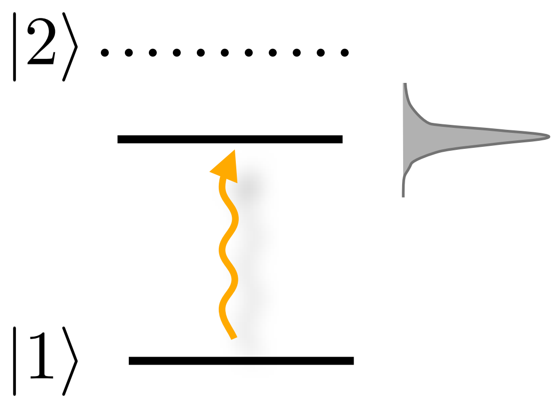

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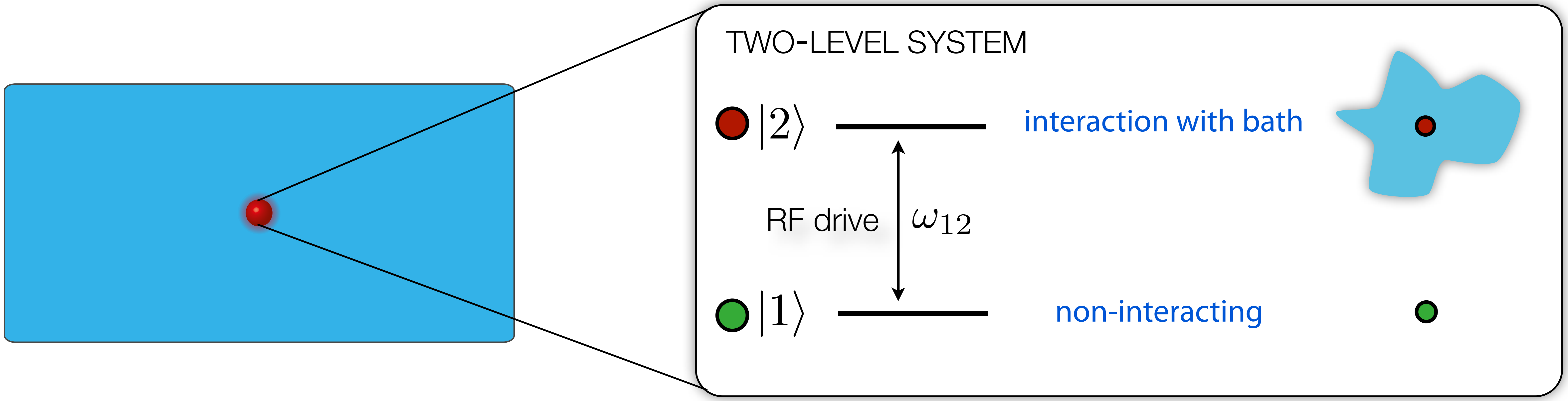

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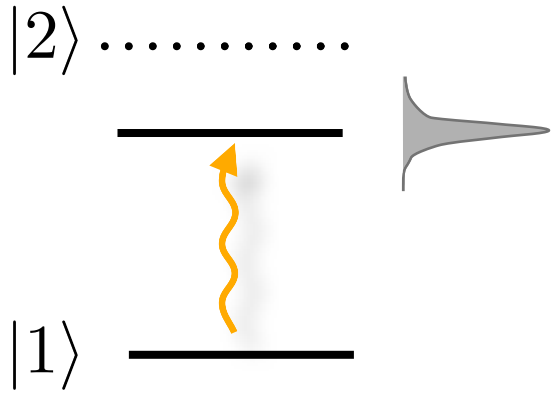


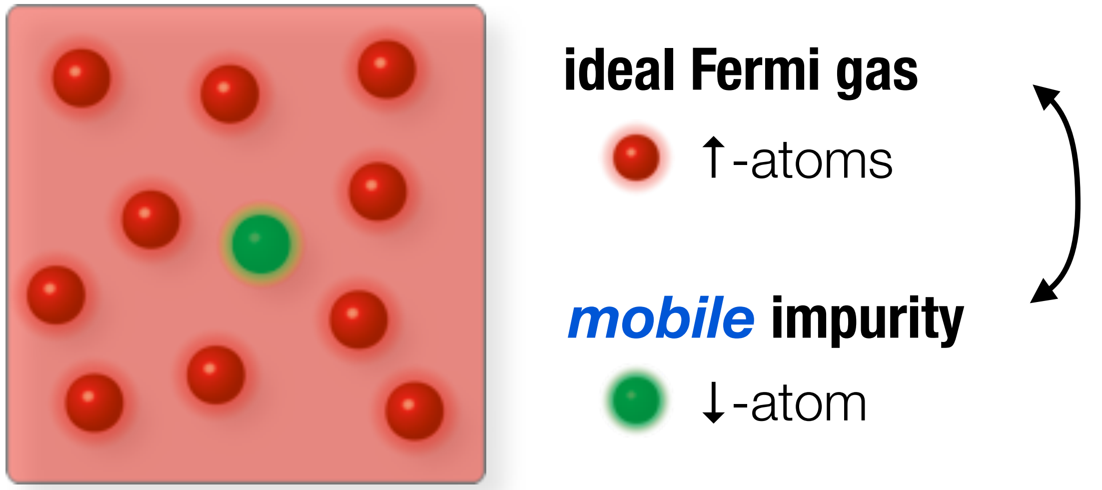
**Theoretical challenge:** Calculate properties of impurity (strongly) coupled to environment

# Impurity experiments with ultracold atoms

**Mainly studied experimentally:**

Theory Repulsive Branch Physics: Cui, Zhai, PRA 81 (2010), RS, Enss, PRA 83 (2011), Massignan, Bruun, EPJD 65 (2011)
Review: Massignan, Zaccanti, Bruun Rep. Prog. Phys. 77, 034401 (2014)

**Fermi polaron**
*[in continuum]*



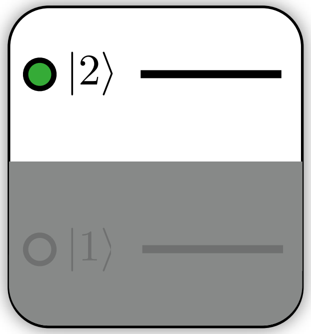
**ideal Fermi gas**

● ↑-atoms

***mobile* impurity**

● ↓-atom

strongly coupled



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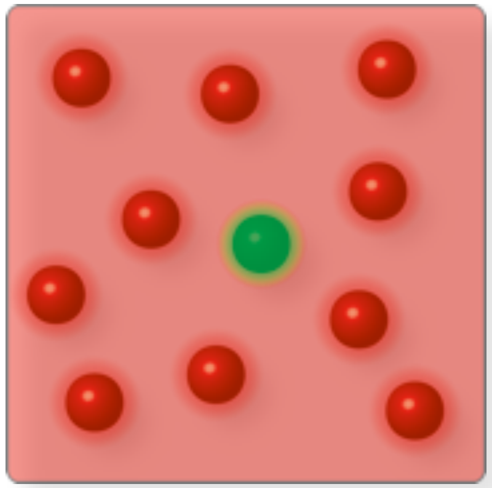

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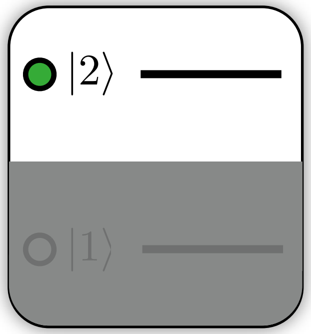

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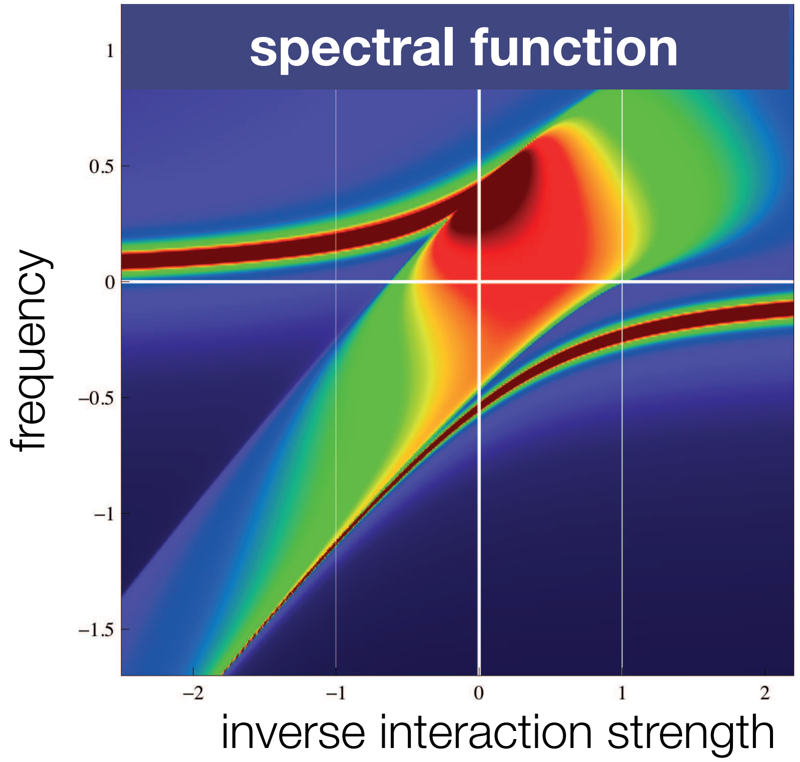

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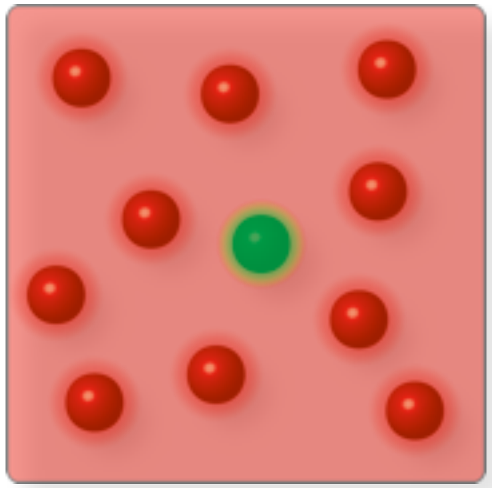

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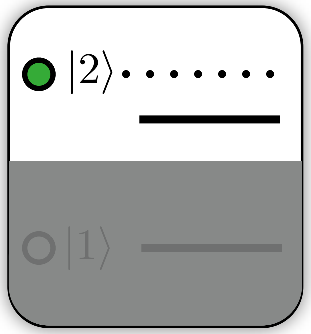

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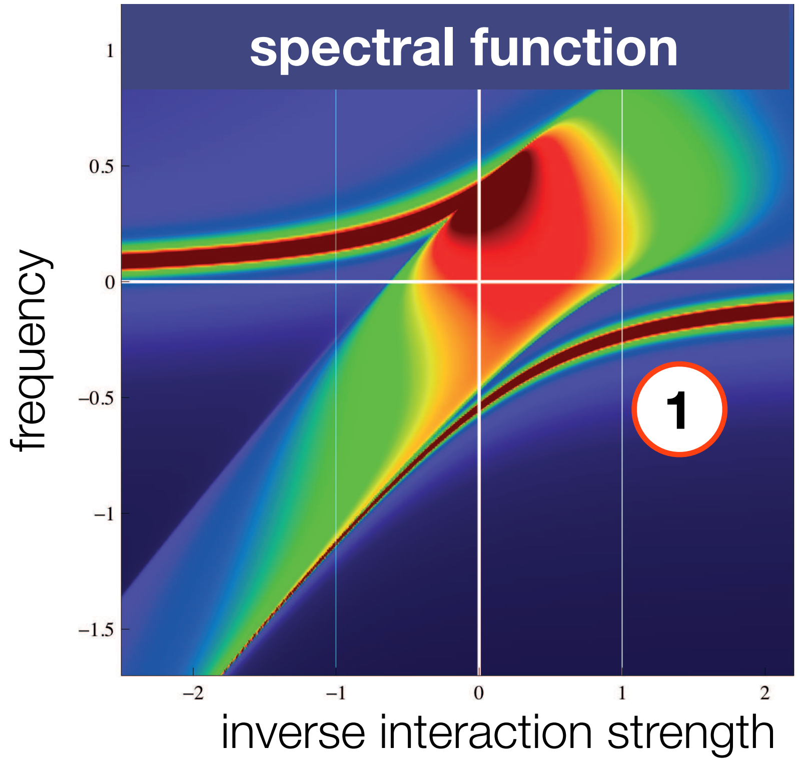

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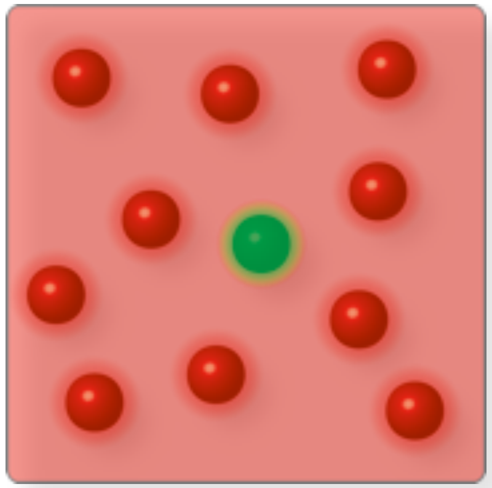

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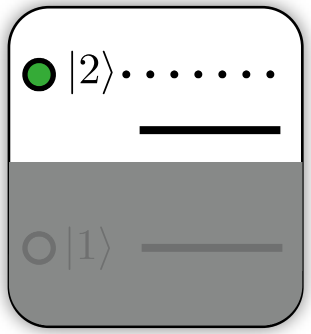

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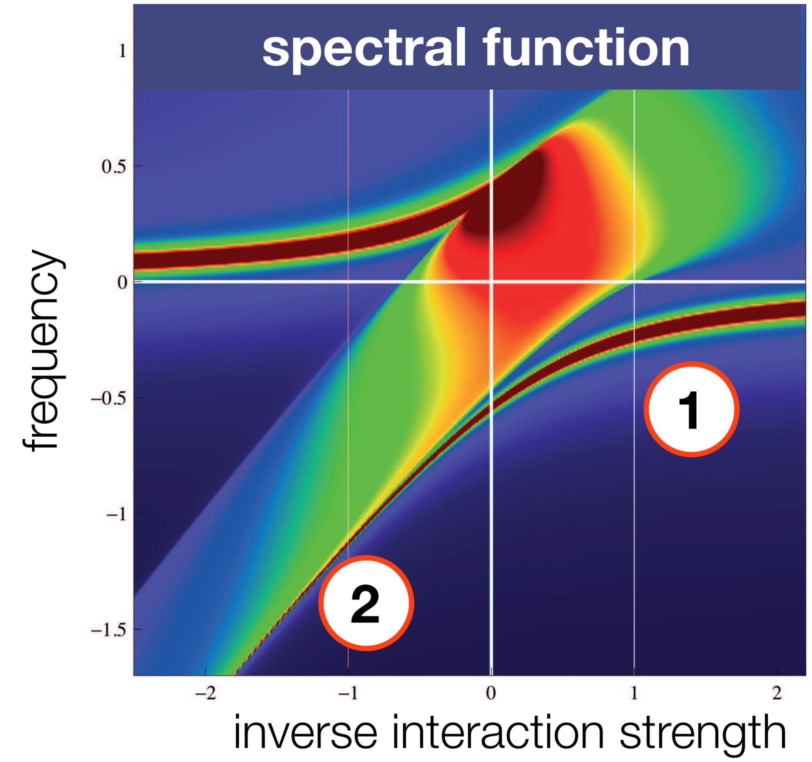

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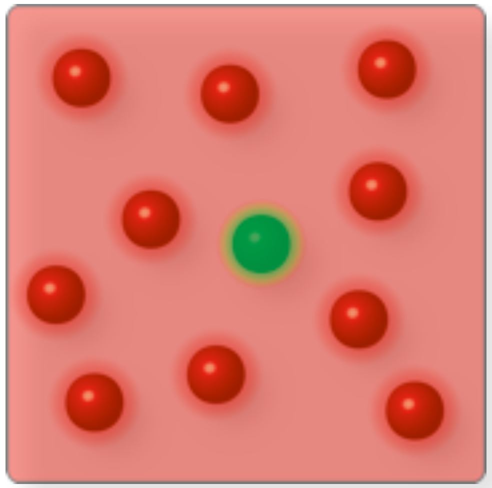

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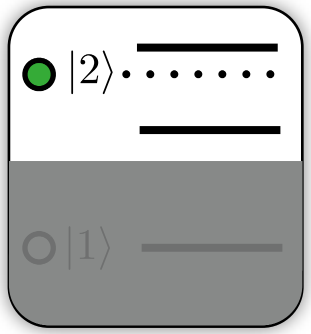

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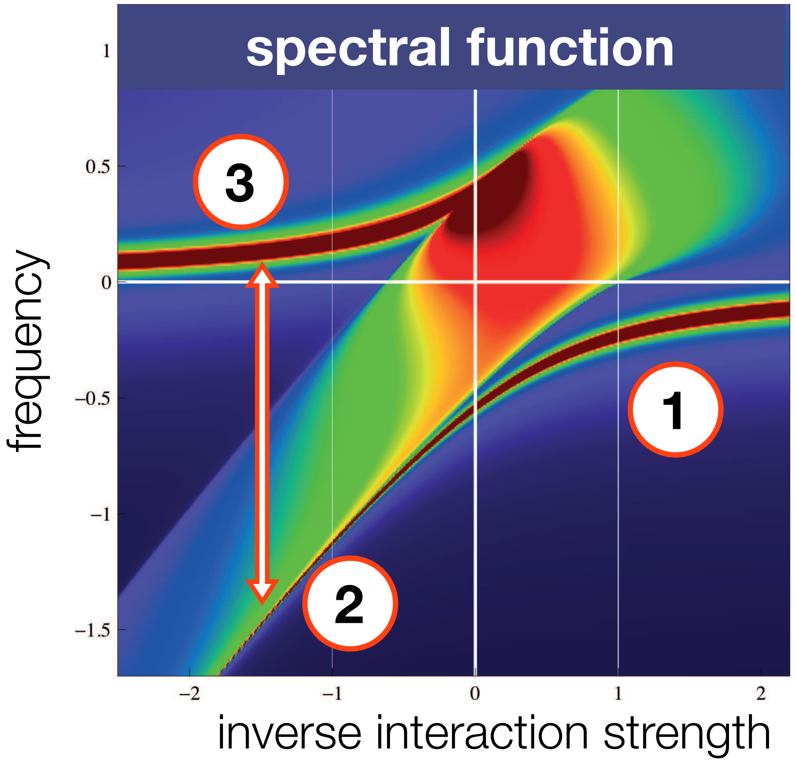
  - energy shift of coherent level

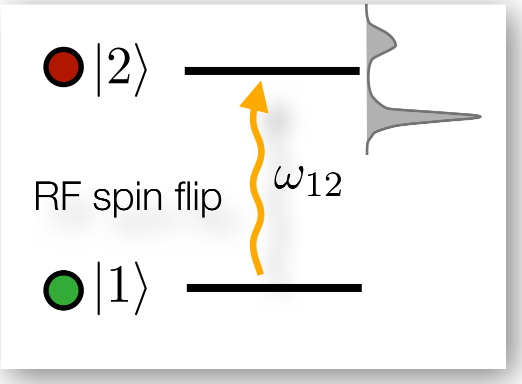
② regime of strong attractive interactions

③ *emergent* effective repulsive interactions
  - single state has split into two branches
  - repulsive branch: ferromagnetic transition CF. WORK BY TIN-LUN HO, AND MASSIGNAN, YU, BRUUN, PRL 110 (2013)

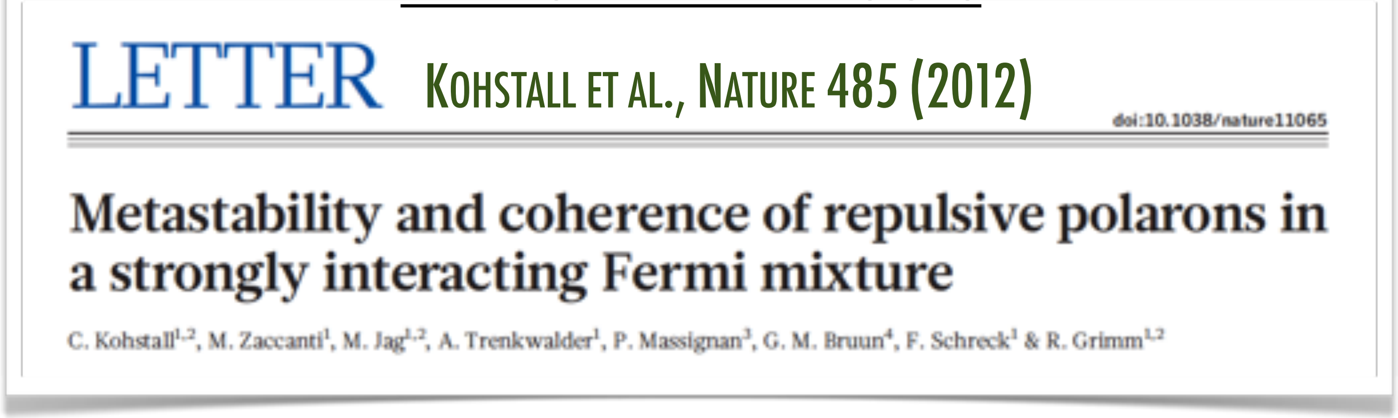


# Experimental observation

**our proposal: inverse radio-frequency [rf] spectroscopy** RS, Enss, PRA 83 (2011)



## THREE SPATIAL DIMENSIONS



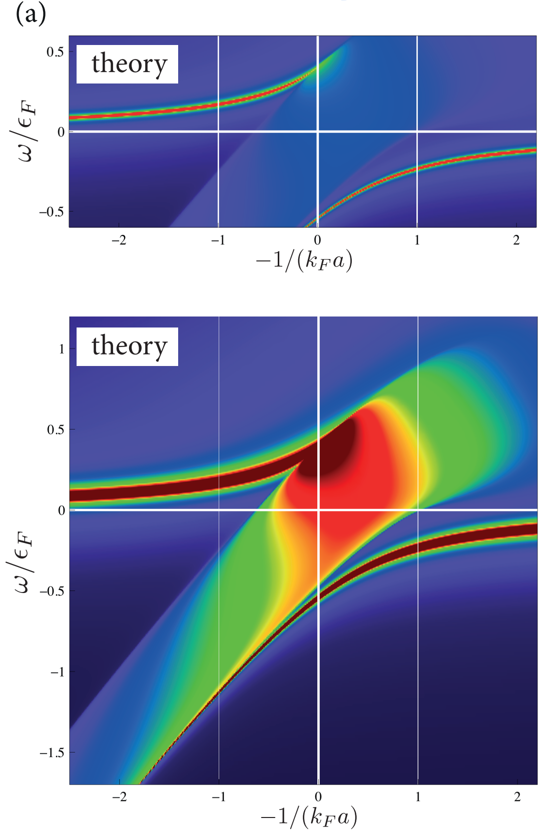
LETTER Kohstall et al., Nature 485 (2012)

doi:10.1038/nature11065

**Metastability and coherence of repulsive polarons in a strongly interacting Fermi mixture**

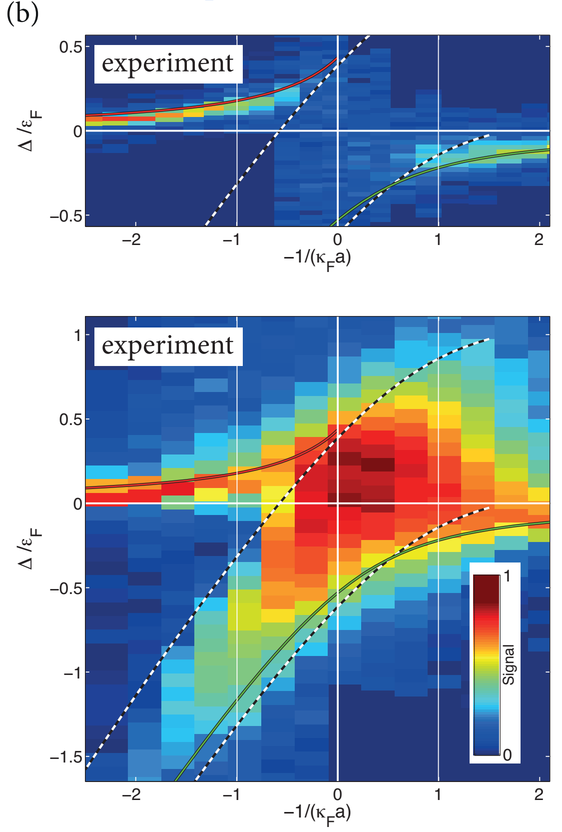
C. Kohstall[1,2], M. Zaccanti[1], M. Jag[1,2], A. Trenkwalder[1], P. Massignan[3], G. M. Bruun[4], F. Schreck[1] & R. Grimm[1,2]

**theory**



RS, Enss, PRA 83 (2011)
Massignan, Bruun, EPJD 65 (2011)
RS, PhD Thesis (2013)

**experiment**



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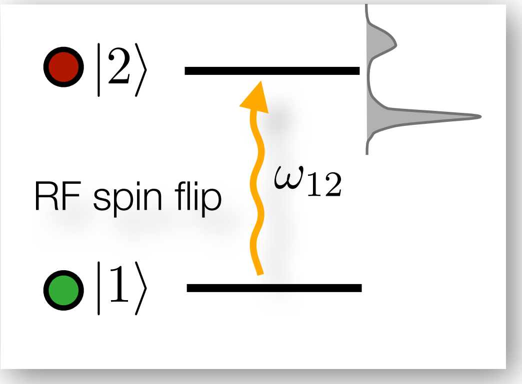

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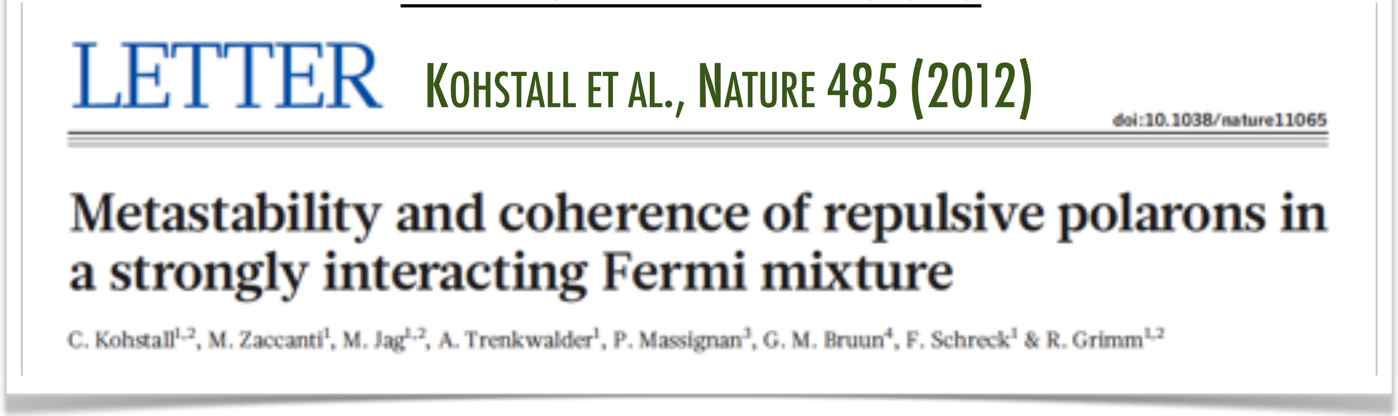

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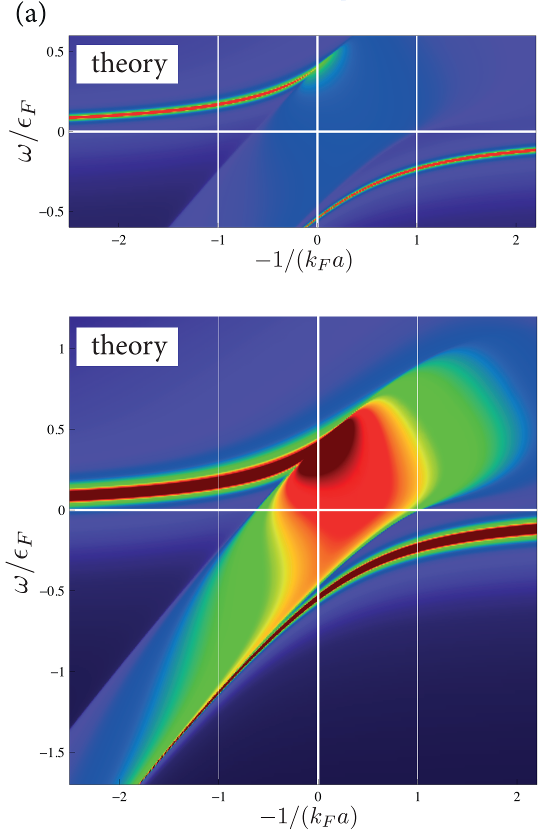

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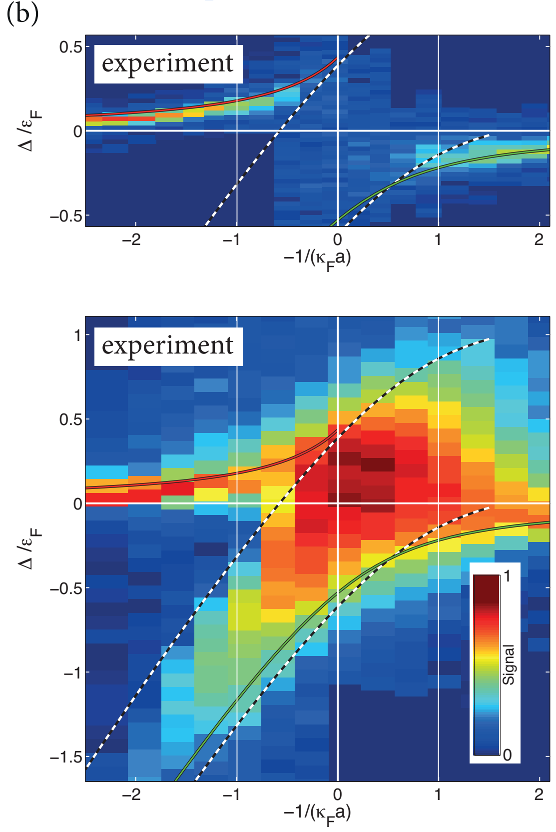


Kohstall et al., Nature 485 (2012)

**TWO SPATIAL DIMENSIONS**

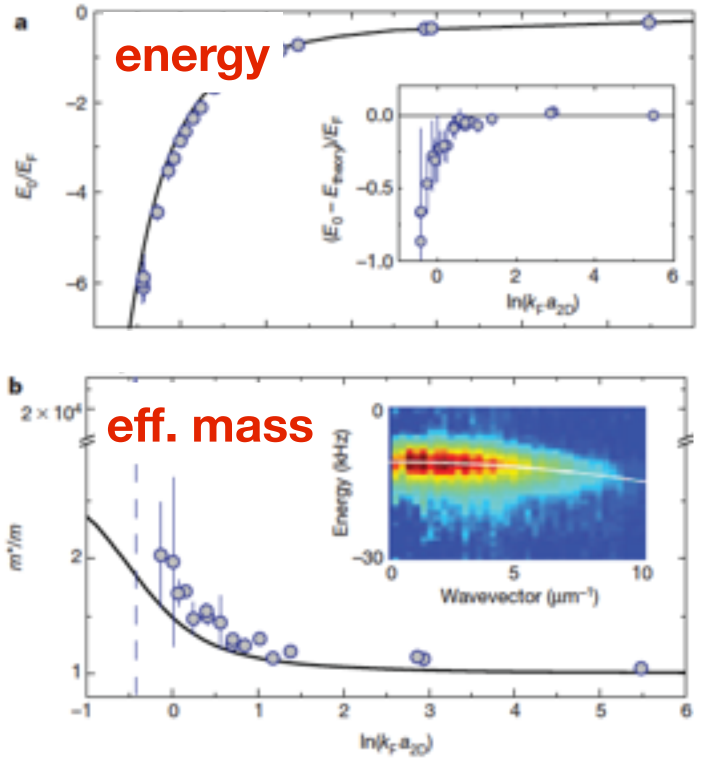
LETTER

31 MAY 2012 | VOL 485 | NATURE | 619

doi:10.1038/nature11151

Attractive and repulsive Fermi polarons in two dimensions

Marco Koschorreck[1]*, Daniel Pertot[1]*, Enrico Vogt[1], Bernd Fröhlich[1], Michael Feld[1] & Michael Köhl[1]



inverse interaction strength

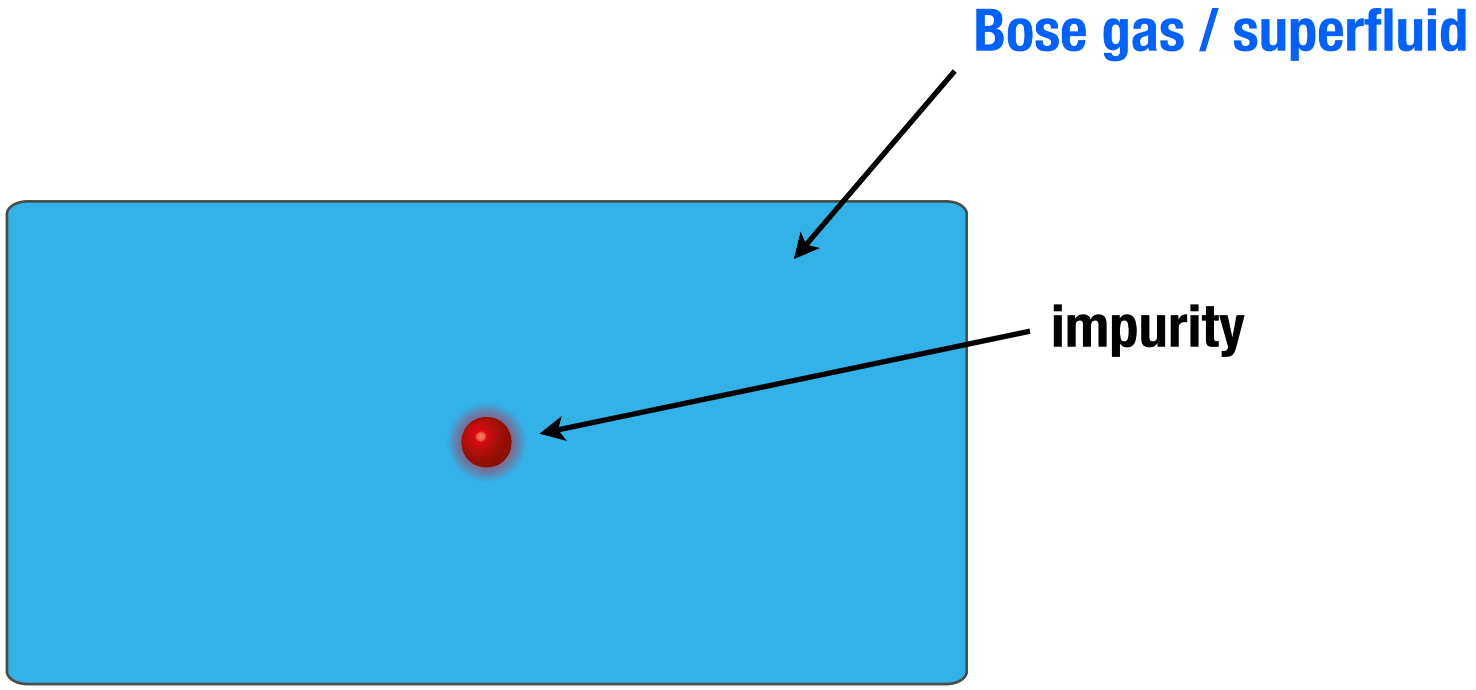
○ **experiment**

Koschorreck et al., Nature 485 (2012)

— **theory**

RS, Enss, Pietila, Demler, PRA 85, 021602 (2012)

# Our recent work: The Bose Polaron

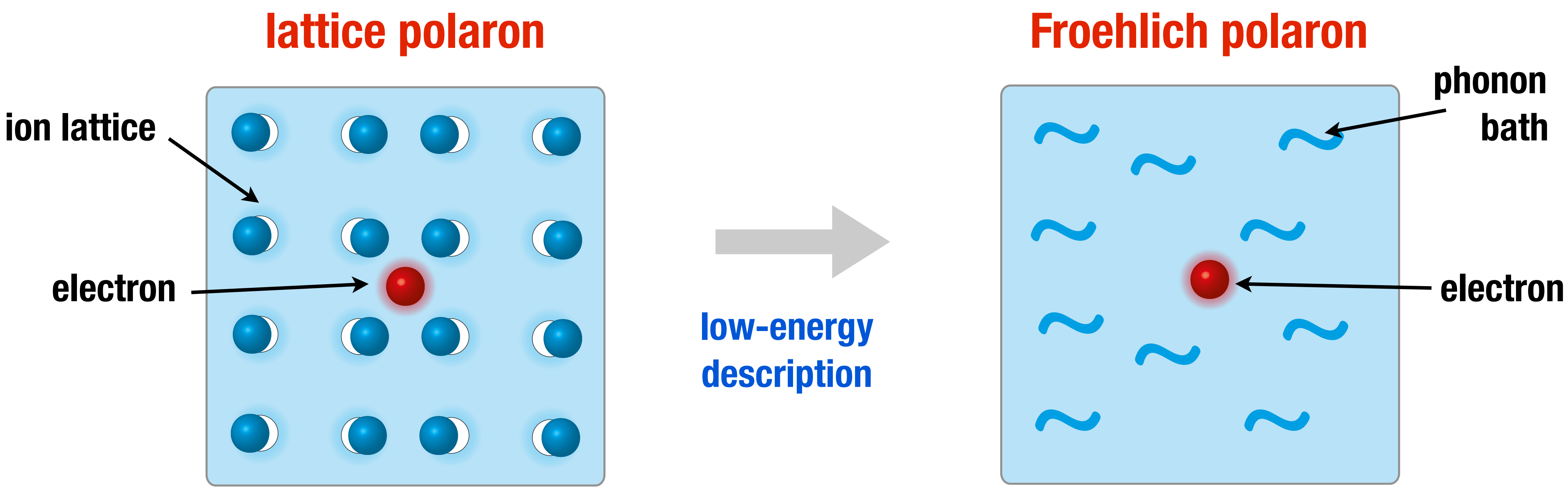


**what happens if medium is Bose gas?**

$\Longrightarrow$ **Bose polaron**

# A cond-mat motivation: The Froehlich polaron

## a paradigm condensed matter model:



### Fröhlich Hamiltonian

Fröhlich, Adv. Phys. 3, 325 (1954)

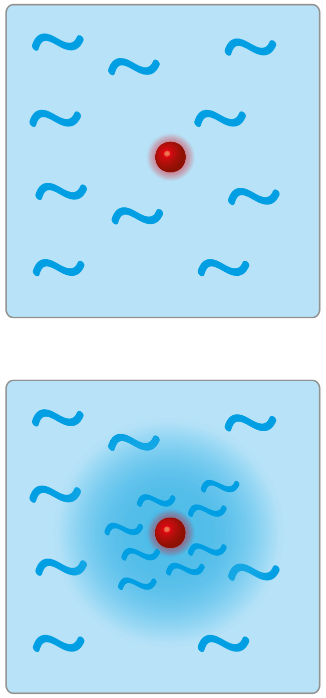
$$\hat{H} = \underbrace{\sum_{\mathbf{p}} \omega_p \hat{b}^\dagger_{\mathbf{p}} \hat{b}_{\mathbf{p}}}_{\text{phonons}} + \underbrace{\sum_{\mathbf{p}} \epsilon_{\mathbf{p}} \hat{c}^\dagger_{\mathbf{p}} \hat{c}_{\mathbf{p}}}_{\text{impurity}} + \underbrace{\sum_{\mathbf{q},\mathbf{p}} \alpha_q \, \hat{c}^\dagger_{\mathbf{p}+\mathbf{q}} \hat{c}_{\mathbf{p}} (\hat{b}^\dagger_{-\mathbf{q}} + \hat{b}_{\mathbf{q}})}_{\text{impurity-phonon interaction}}$$

# The Fröhlich polaron

## Fröhlich Hamiltonian FRÖHLICH, ADV. PHYS. 3, 325 (1954)

$$\hat{H} = \sum_{\mathbf{p}} \omega_p \hat{b}^\dagger_{\mathbf{p}} \hat{b}_{\mathbf{p}} + \sum_{\mathbf{p}} \epsilon_{\mathbf{p}} \hat{c}^\dagger_{\mathbf{p}} \hat{c}_{\mathbf{p}} + \sum_{\mathbf{q},\mathbf{p}} \alpha_q \, \hat{c}^\dagger_{\mathbf{p}+\mathbf{q}} \hat{c}_{\mathbf{p}} (\hat{b}^\dagger_{-\mathbf{q}} + \hat{b}_{\mathbf{q}})$$

$$\hookrightarrow \alpha > 0$$



- impurity dressed by *phonon cloud* becomes the '**Fröhlich polaron**'
- enhanced effective mass, renormalized energy SEE E.G. MILLER ET AL. PHYS. REV. 127 ('62)

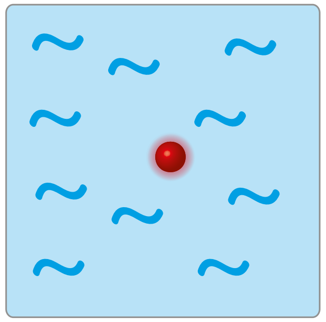
# The Fröhlich polaron

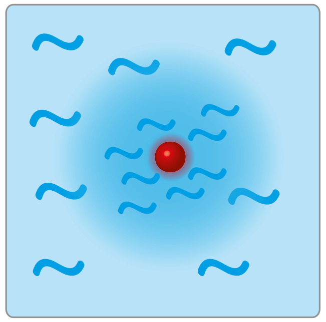
## Fröhlich Hamiltonian

Fröhlich, Adv. Phys. 3, 325 (1954)

$$\hat{H} = \sum_{\mathbf{p}} \omega_p \hat{b}^\dagger_{\mathbf{p}}\hat{b}_{\mathbf{p}} + \sum_{\mathbf{p}} \epsilon_{\mathbf{p}}\hat{c}^\dagger_{\mathbf{p}}\hat{c}_{\mathbf{p}} + \sum_{\mathbf{q},\mathbf{p}} \alpha_q \, \hat{c}^\dagger_{\mathbf{p}+\mathbf{q}}\hat{c}_{\mathbf{p}}(\hat{b}^\dagger_{-\mathbf{q}} + \hat{b}_{\mathbf{q}})$$

$$\hookrightarrow \alpha > 0$$



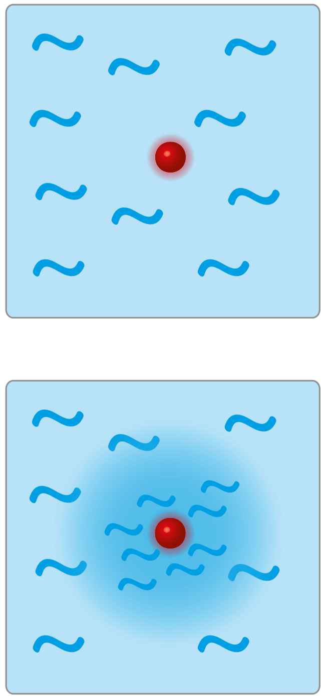


- impurity dressed by *phonon cloud* becomes the '**Fröhlich polaron**'
- enhanced effective mass, renormalized energy See e.g. Miller et al. Phys. Rev. 127 ('62)

*perturbation theory:* $m^* = \frac{m}{1-\alpha/6} \xrightarrow[\substack{\alpha>0\\ \text{large}}]{} \infty$ → **self-localization?**

# The Fröhlich polaron

## **Fröhlich Hamiltonian** Fröhlich, Adv. Phys. 3, 325 (1954)



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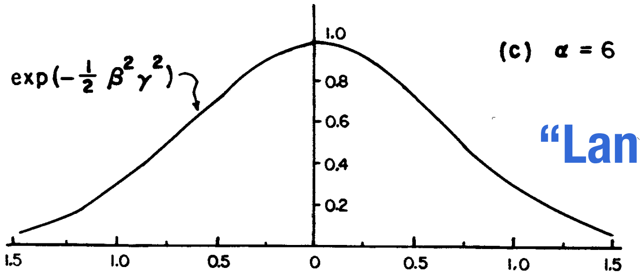
large

**self-localization?**

- strong interactions: variational wave function

Landau, Pekar, JETP 18 (1948); Feynman, Cohen, Phys. Rev. 102 (1956)



**"Landau Pekar polaron"**

  - describes localized particle
  - yields energy smaller than pert. theory at strong coupling, further evidence of self-localization

# The Fröhlich polaron

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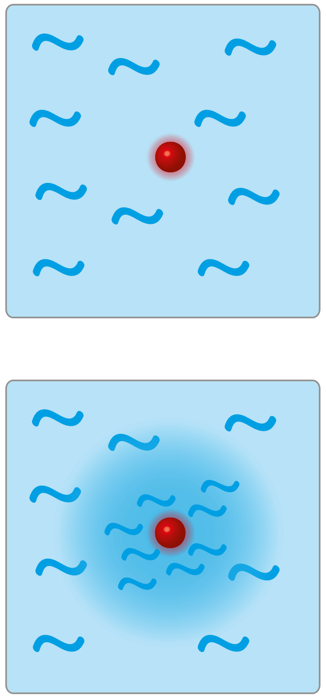

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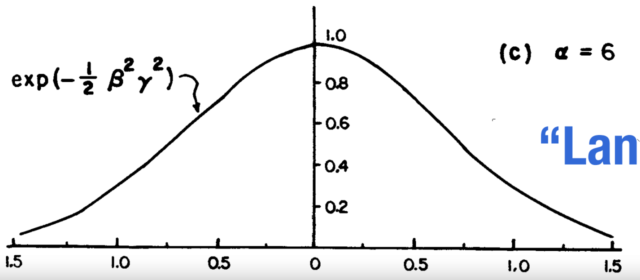


**“Landau Pekar polaron”**

**superfluid + strong interactions:**

**Realizable with ultracold atoms!** **Let’s do quantum simulation!**

# The Bose polaron with ultracold atoms

## impurity in Bose gas: Bose polaron

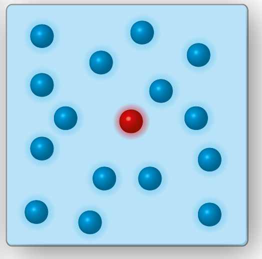
e.g. take strongly imbalanced mixture of ultracold atoms CF. GENERAL CASE: B. LIU, J.HU INT. J. MOD. PHYS, 26 (2012)

$$S = \int d^4x \underline{\varphi^*(x)} \left(\partial_\tau - \frac{1}{2m_B}\nabla^2 - \mu_B\right) \underline{\varphi(x)} + \frac{g_B}{2}\underline{[\varphi(x)^*\varphi(x)]^2}$$

$$+ \underline{\psi^*(x)} \left(\partial_\tau - \frac{1}{2m_I}\nabla^2 - \mu_I\right) \underline{\psi(x)} + g_{IB}\underline{\psi(x)^*\psi(x)}\,\underline{\varphi(x)^*\varphi(x)}$$

— **bosons**

— **impurity**



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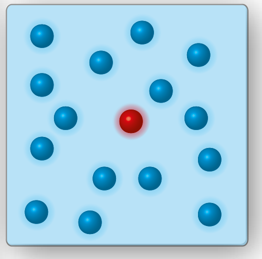
— **bosons**

— **impurity**



weakly interacting BEC: Bogoliubov approximation for BEC

$$\varphi(\mathbf{x},t) = \underset{\substack{\nearrow \\ \textbf{mean-field}}}{\sqrt{n_B}} + \underset{\substack{\nwarrow \\ \textbf{fluctuations}}}{\phi(\mathbf{x},t)}$$

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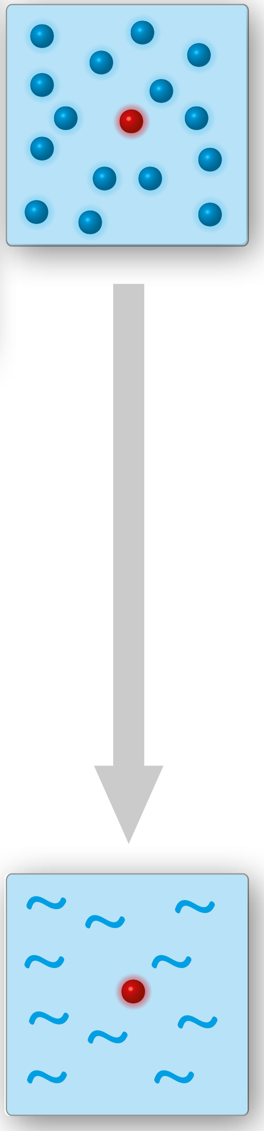
— **bosons**

— **impurity**



weakly interacting BEC: Bogoliubov approximation for BEC

$$\varphi(\mathbf{x},t) = \sqrt{n_B} + \phi(\mathbf{x},t)$$

**mean-field** **fluctuations**

### at weak coupling: **Fröhlich Hamiltonian**

$$S_{\text{eff}} = S_{\text{kin}}^{\text{Bos}} + S_{\text{kin}}^{\text{Imp}} + \underbrace{g_{IB}\int_x n_B(\mathbf{r})\,|\psi(\mathbf{r})|^2}_{\text{MF energy shift}} + g_{IB}\int \Big\{ \underbrace{\sqrt{n_B}\,\psi^*_{\mathbf{k}+\mathbf{q}}\psi_{\mathbf{k}}\,(\phi_{\mathbf{q}} + \phi^*_{-\mathbf{q}})}_{\text{"Fröhlich term"}}$$

ELECTRON IN BEC: PFAU GROUP [STUTTGART] - BALEWSKI ET AL., NATURE 502 (2013)

# Bose poloron at strong coupling

$$S_{\text{eff}} = S^{\text{Bos}}_{\text{kin}} + S^{\text{Imp}}_{\text{kin}} + g_{IB} \int \left\{ \underbrace{\sqrt{n_B}\psi^*_x\psi_x(\phi_x + \phi^*_x) + n_B\psi^*_x\psi_x}_{\text{"Fröhlich terms"}} \right.$$

$$g_{IB} = \frac{2\pi\hbar^2}{m_r} a_{IB}$$

**strong *effective* phonon-impurity interaction wanted**

Heiselberg et al., PRL 85 (2000)
Cucchientti, Timmermans, PRL 96 (2006)
Kalas, Blume, PRA 73 (2006)
Wang, PRL 96 (2006)
Enss, Zwerger, EPJB 68 (2009)
Tempere, Oberthaler et al., PRB 80 (2009)

Casteels et al., PRA 83,84,86 (2011)
Casteels, Cauteren, Tempere,Devreese, Laser Phys. 21 (2011)
Casteels, Tempere, Devreese, PRA 84 (2011)
Casteels, Tempere, Devreese, PRA 86 (2012)
Dasenbrook, Komnik, PRB 87 (2013)
Blinova, Boshier, Timmermans, PRA 88 (2013)
Shashi, Grusdt, Abanin, Demler, PRA 89, 053617 (2014)
Grusdt, Shchadilova, Rubtsov, Demler, 1410.2203 (2014)
...

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"Fröhlich terms"

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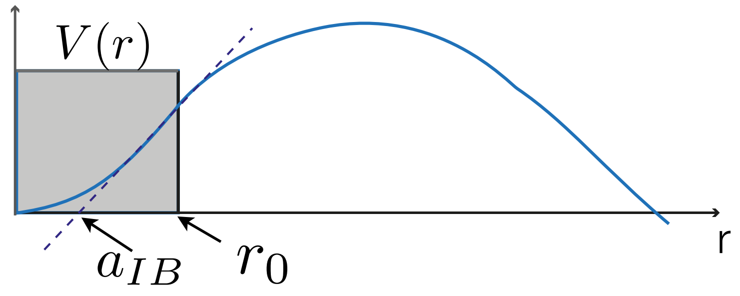
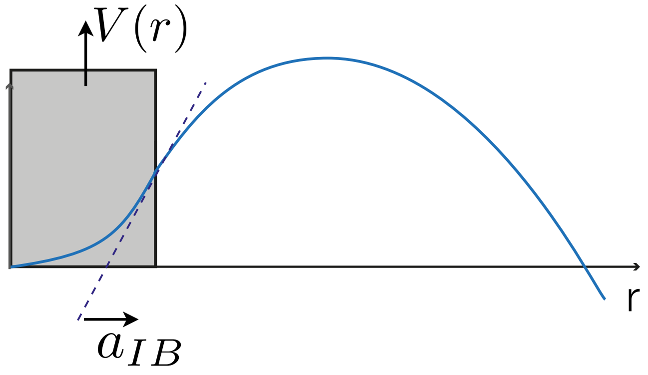
**How to achieve large, positive scattering length $a_{IB}$?**

1. way: large microscopic repulsion





**not sufficient:**

for infinite repulsion

$$a_{\rm max} = r_0$$

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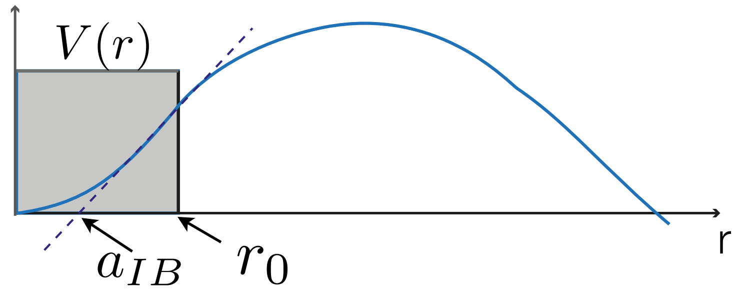

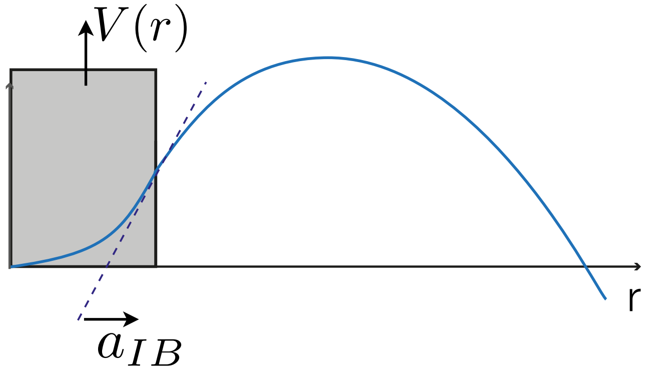

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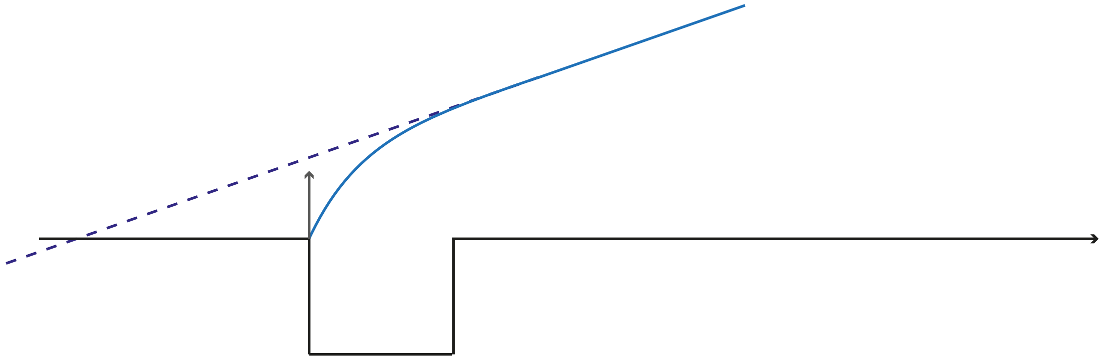

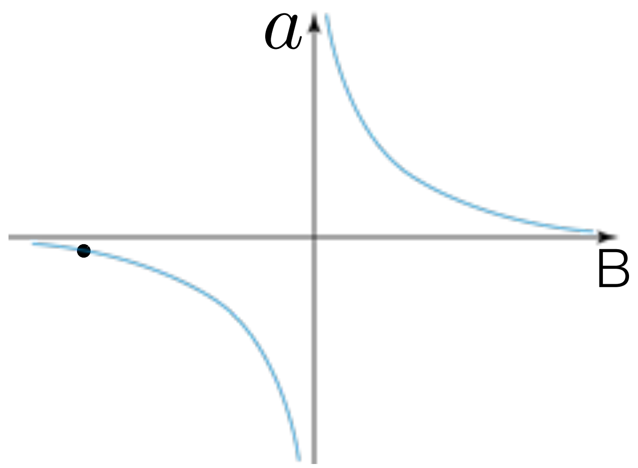

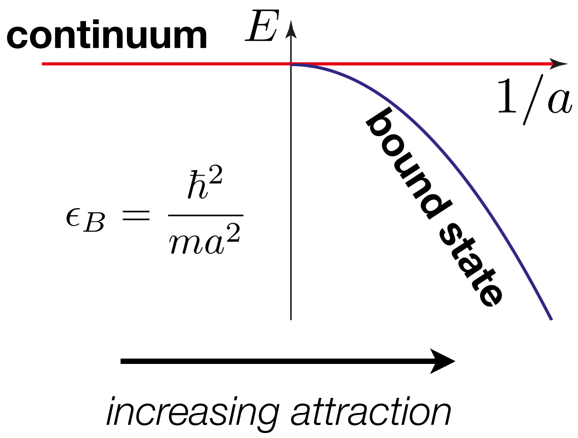

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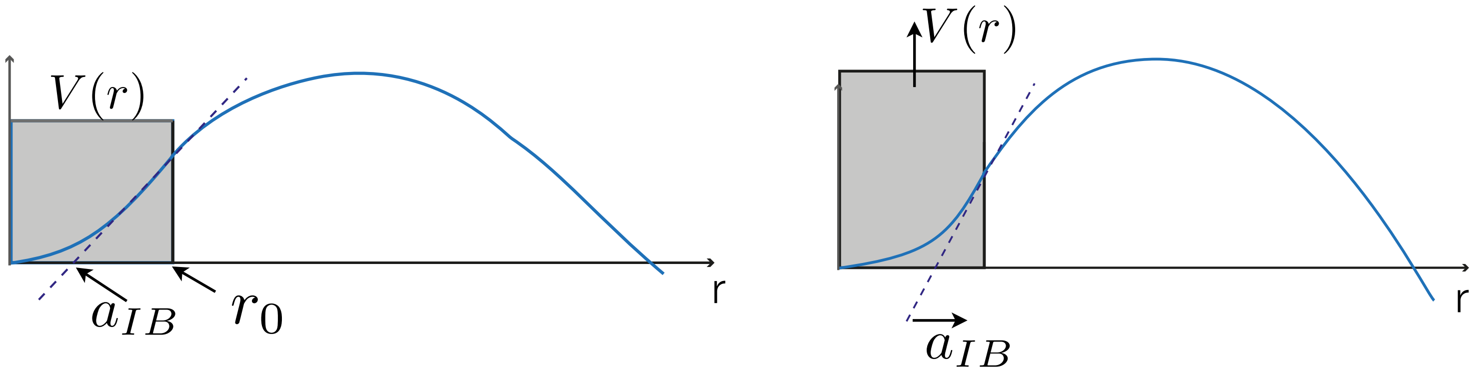

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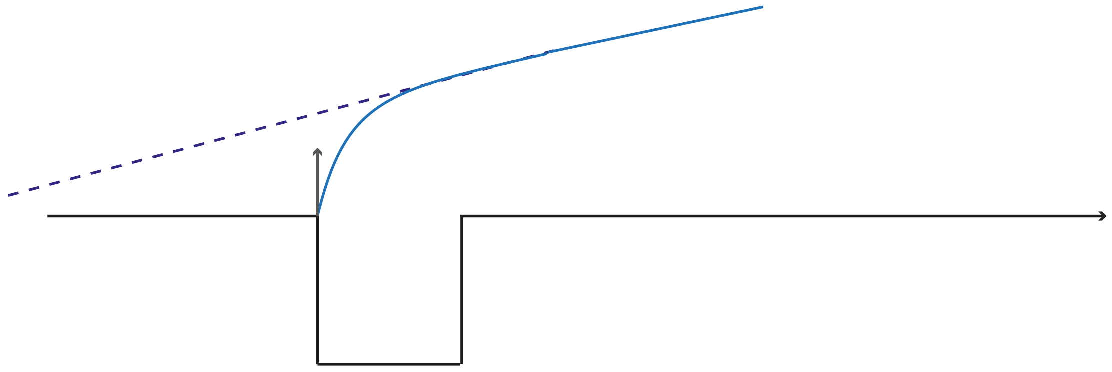

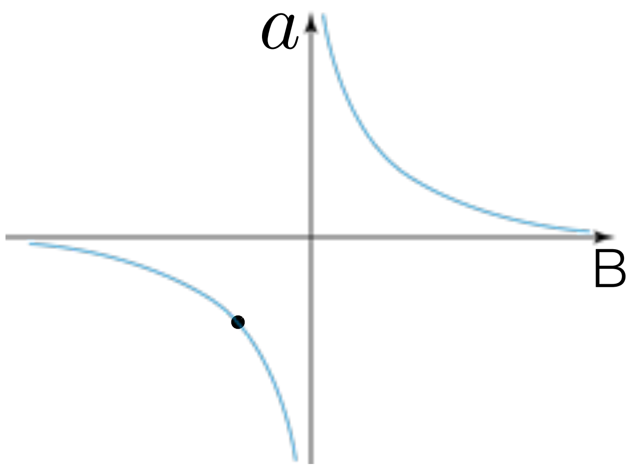

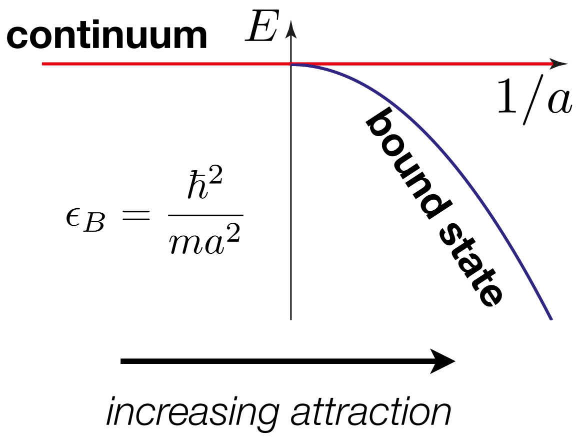

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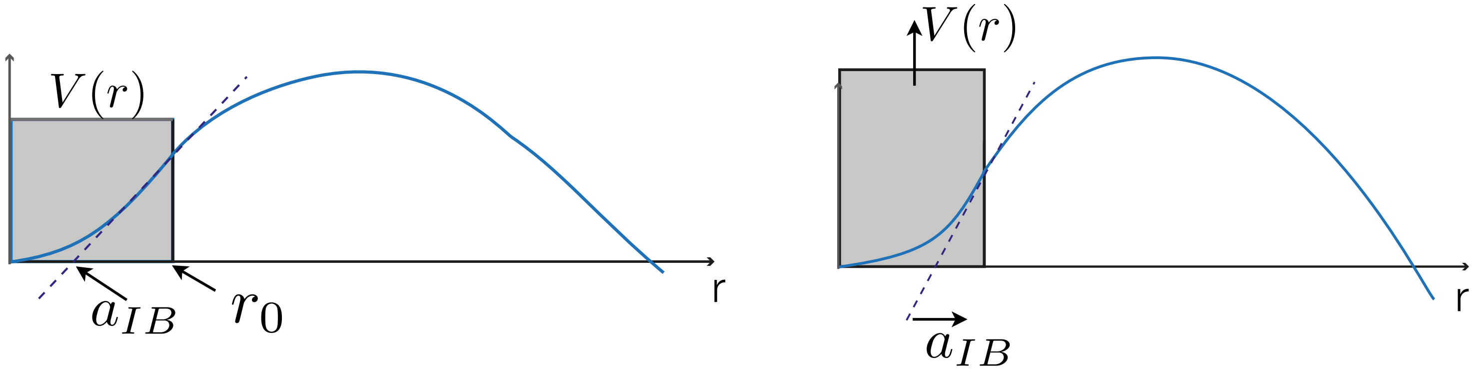

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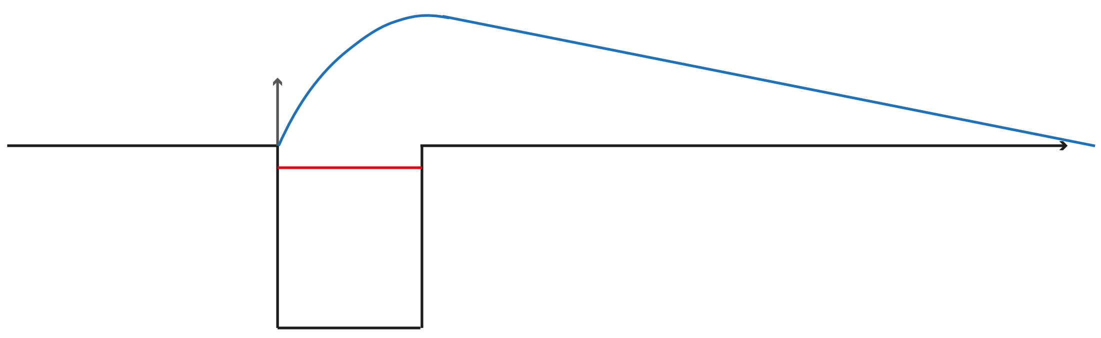

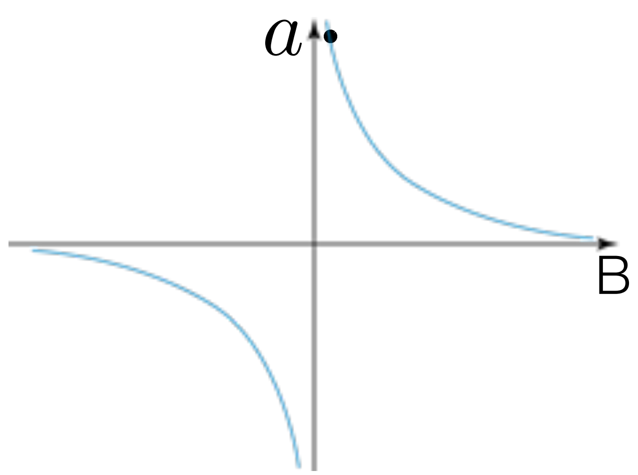

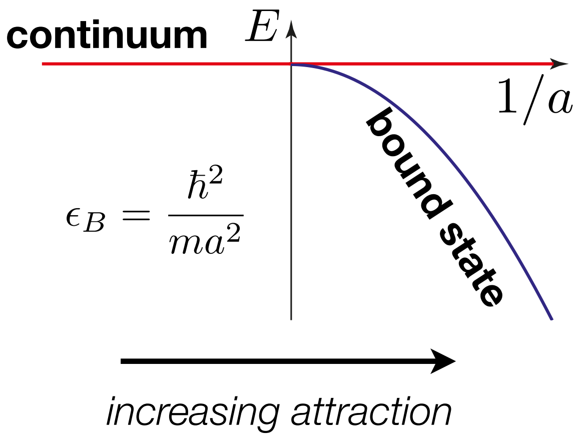

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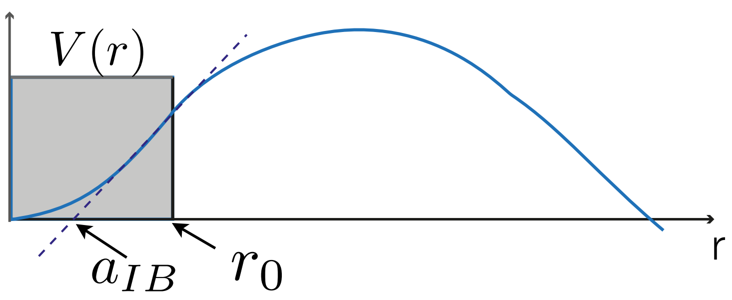

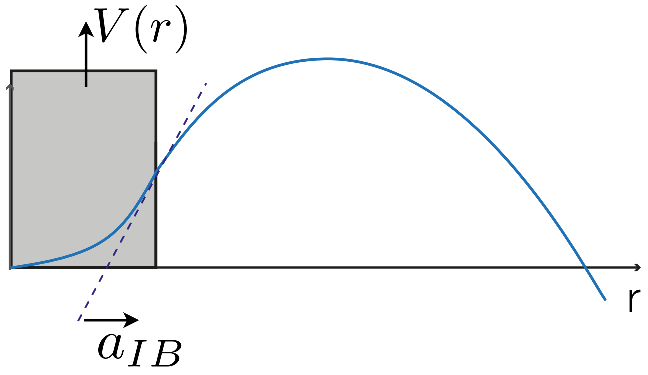

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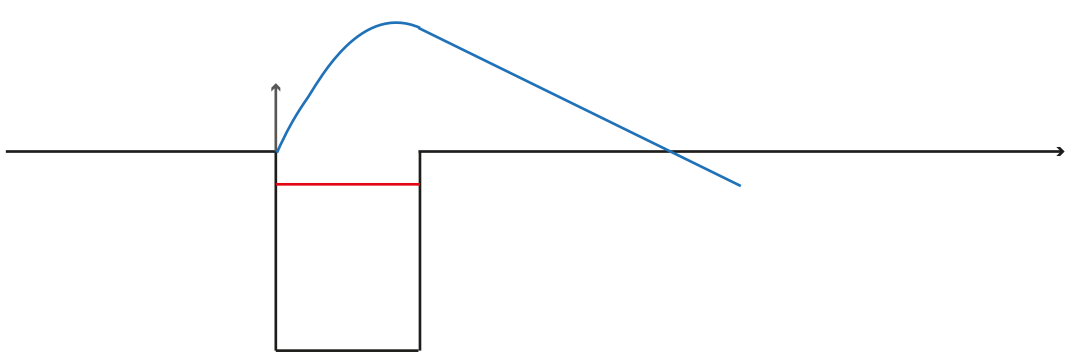

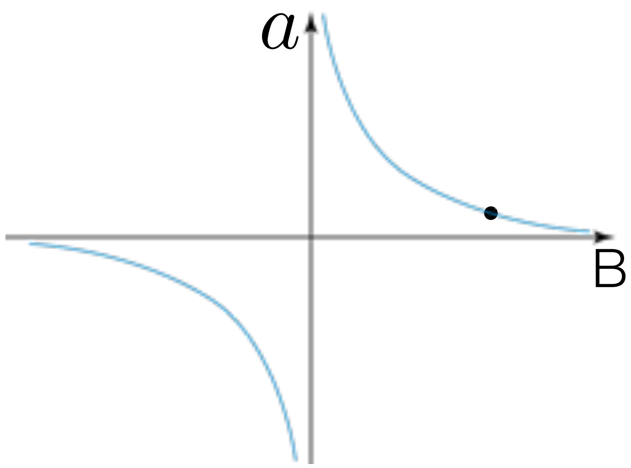

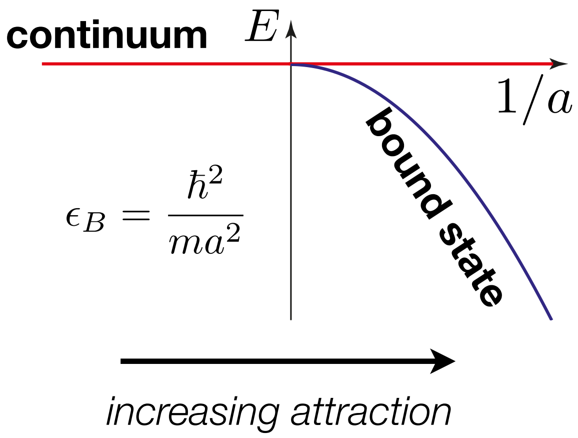

arbitrarily large $a_{IB}$ possible

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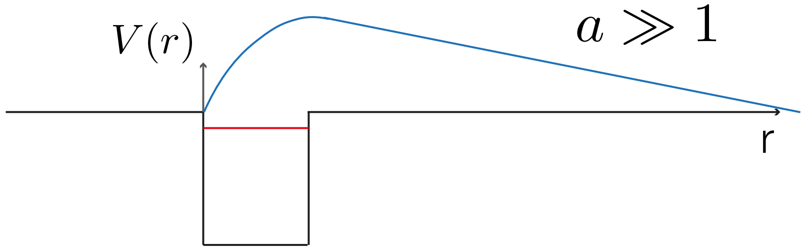
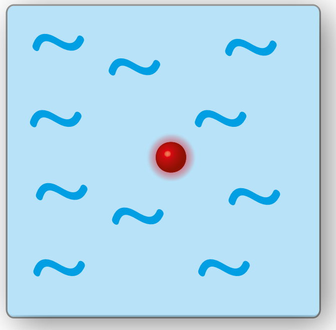
"Fröhlich terms"

**strong *effective* phonon-impurity interaction comes at a prize**

1. microscopic attraction needed





$$g_{IB} \neq \frac{2\pi\hbar^2}{m_r} a_{IB}$$ 'mean-field replacement' invalid!

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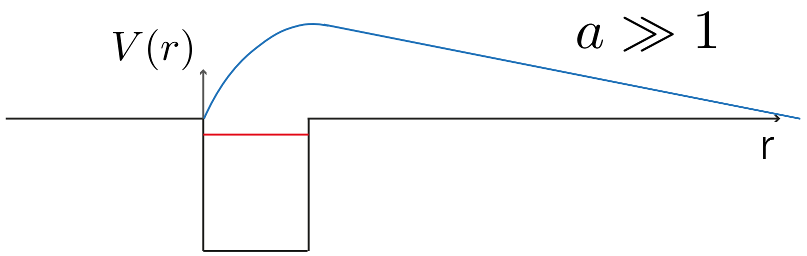
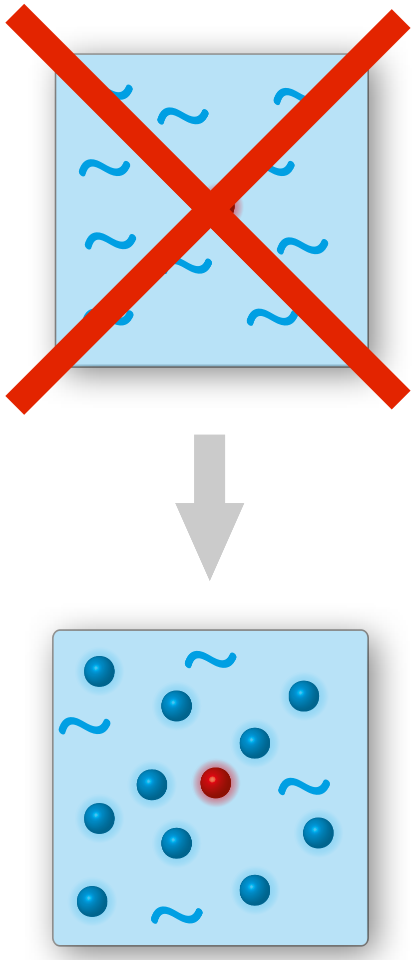


$$g_{IB} \neq \frac{2\pi\hbar^2}{m_r} a_{IB}$$ ‘mean-field replacement’ invalid!

2. pairing fluctuations become relevant

→ MF approach & Fröhlich Hamiltonian becomes invalid



**in RG language:** *Froehlich*: weak coupling RG fixed point
*cold atoms at Feshbach resonance:* strong coupling RG fixed point

SIMILAR RG ANALYSIS FOR BOSE-FERMI MIXTURE: B. LIU, J.HU INT. J. MOD. PHYS, 26 (2012)

# Our work: Bose polaron from a truly attractive model

## simple quantum field-theory approach

$$S = \int d^4x \underline{\varphi^*(x)} \left(\partial_\tau - \frac{1}{2m_B}\nabla^2 - \mu_B\right)\varphi(x) + \underline{\psi^*(x)}\left(\partial_\tau - \frac{1}{2m_I}\nabla^2 - \mu_I\right)\psi(x)$$

$$+\frac{g_B}{2}\underline{[\varphi^*(x)\varphi(x)]^2} + \tilde{g}_{IB}\underline{\psi^*(x)\psi(x)}\,\underline{\varphi^*(x)\varphi(x)}$$

**bosons**

**impurity**

**attractive interaction**

RATH, RS, PRA 88 (2013)

assume homogeneous, weakly interacting BEC

$$\varphi(\mathbf{x},t) = \rho_0^{1/2} + \phi(\mathbf{x},t)$$

**mean-field** **fluctuations**

Bogoliubov approximation for bosons: keep all terms up to quadratic in $\phi$ , $\phi^*$

$$S_{\text{eff}} = \int_{\omega,\boldsymbol{p}} \left\{ \frac{1}{2}\begin{pmatrix}\phi_p^* \\ \phi_{-p}\end{pmatrix} \begin{pmatrix} -\left[G_\phi^{(0)}(-p)\right]^{-1} & g_{\phi\phi}\rho_0 \\ g_{\phi\phi}\rho_0 & -\left[G_\phi^{(0)}(p)\right]^{-1}\end{pmatrix} \begin{pmatrix}\phi_p \\ \phi_{-p}^*\end{pmatrix} + \psi_p^*\left(-i\omega + \frac{\boldsymbol{p}^2}{2m_\psi} - \mu_\psi\right)\psi_p \right\}$$

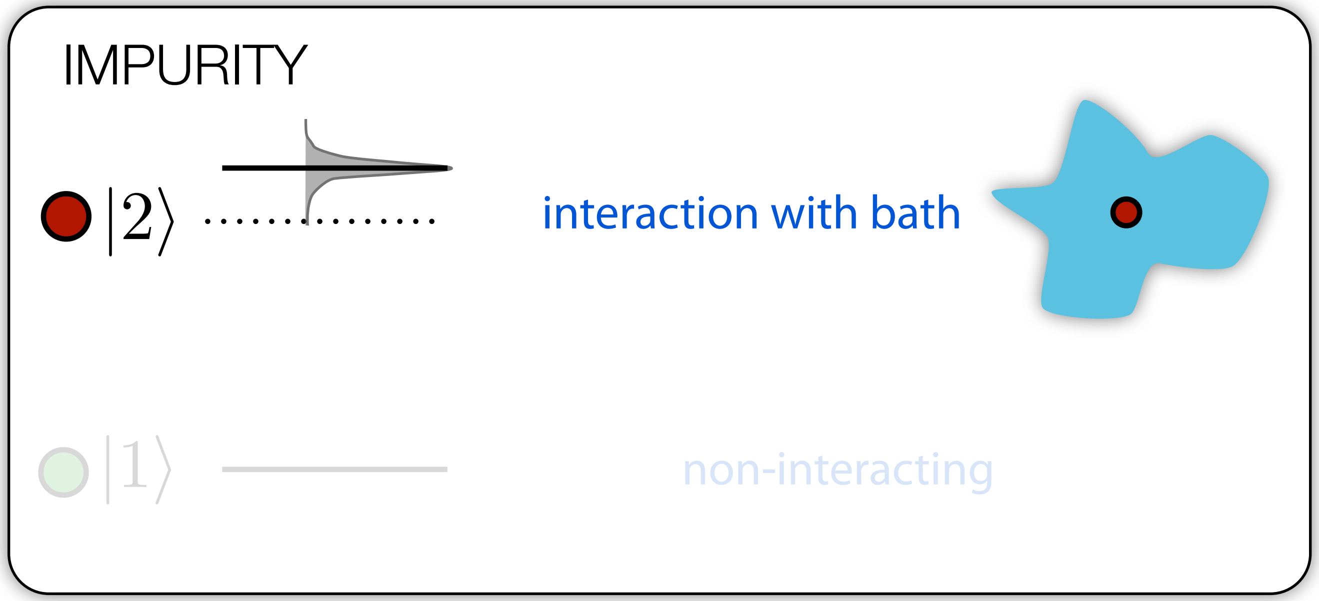
$$+\tilde{g}_{\phi\psi}\int_x \left[\psi_x^*\psi_x\phi_x^*\phi_x + \sqrt{\rho_0}\psi_x^*\psi_x(\phi_x + \phi_x^*) + \rho_0\psi_x^*\psi_x\right]$$

Unlike previous approaches, we keep **pairing fluctuations**

"Fröhlich terms"

# The Question

## What is the spectrum of the model?



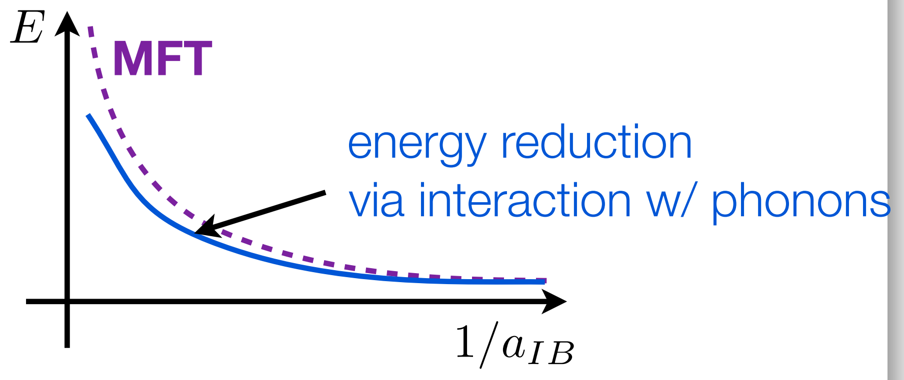
IMPURITY

$|2\rangle$ interaction with bath

$|1\rangle$ non-interacting

**condensed matter/**
**pure Froehlich model**

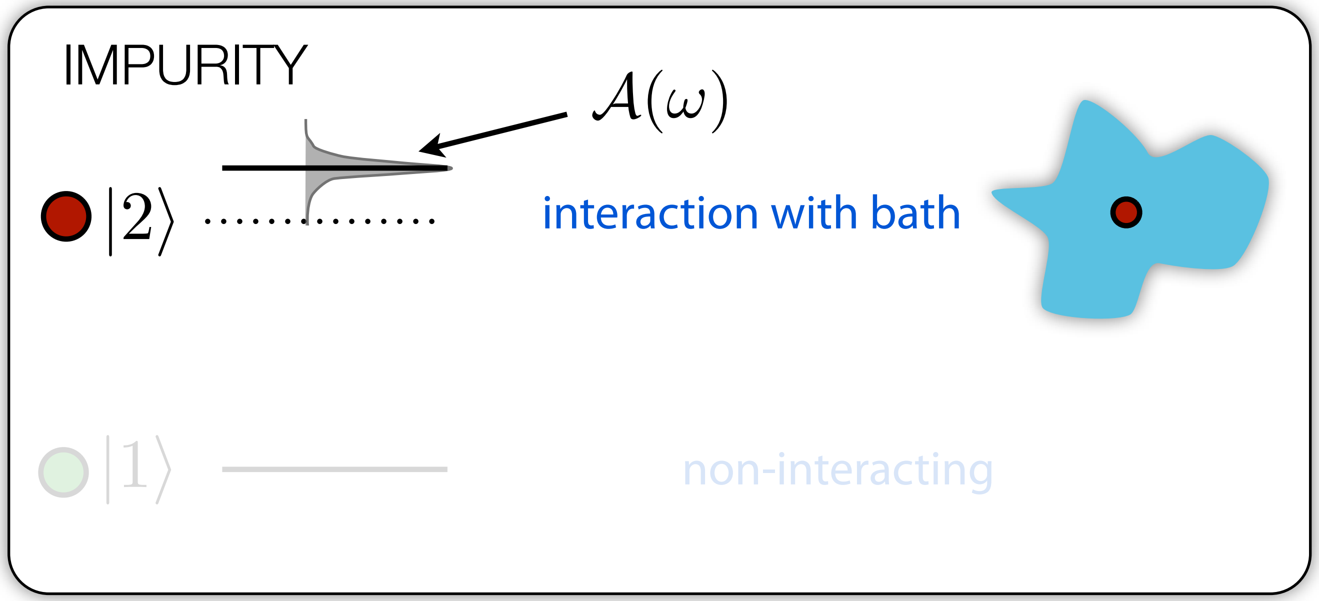
predicts sharp quasi-particle excitation



**do cold atoms forget underlying microscopic physics?**

# The Question

## What is the spectrum of the model?



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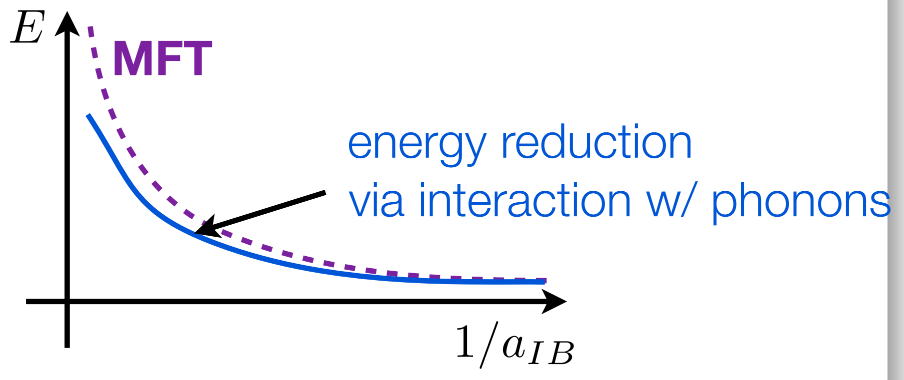


**do cold atoms forget underlying microscopic physics?**

## Quantity to address this question in quantum field theory:

**Spectral function** (gives access to radio-frequency response etc...)

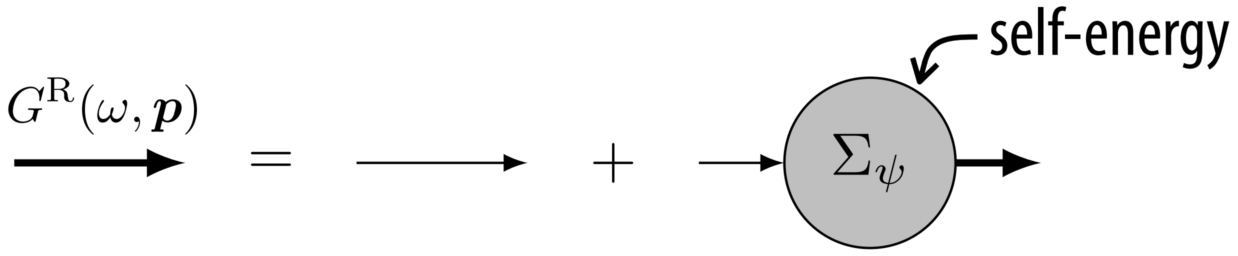
$$\mathcal{A}(\omega, \boldsymbol{p}) = -2\,\mathrm{Im}\,G^{\mathrm{R}}(\omega, \boldsymbol{p})$$

# T-matrix approximation

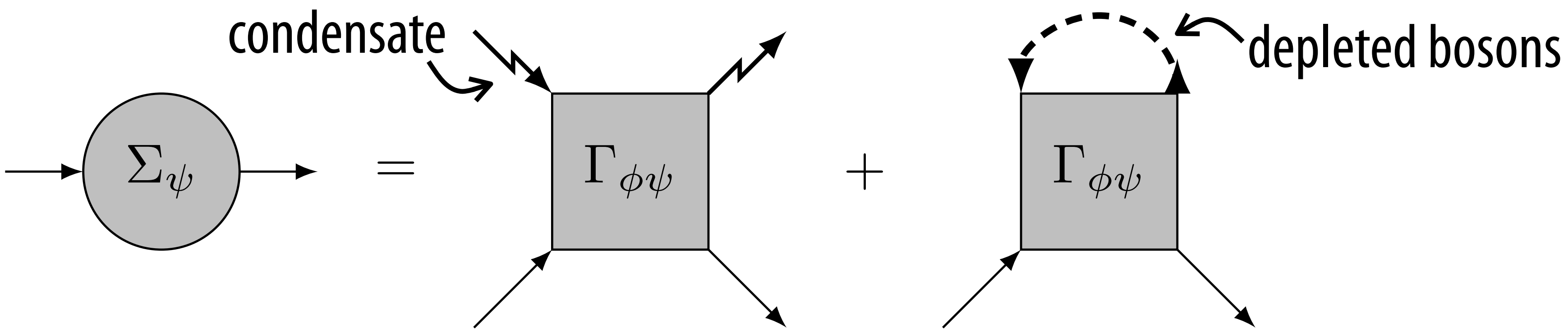
See also for Fermions: **RS, Enss, PRA 83 (2011)**

**impurity spectral function** $A_{\mathrm{pol}}(\omega, \boldsymbol{p}) = -2\,\mathrm{Im}\,G^{\mathrm{R}}(\omega, \boldsymbol{p})$ **from Dyson equation**

**full Green's function:**



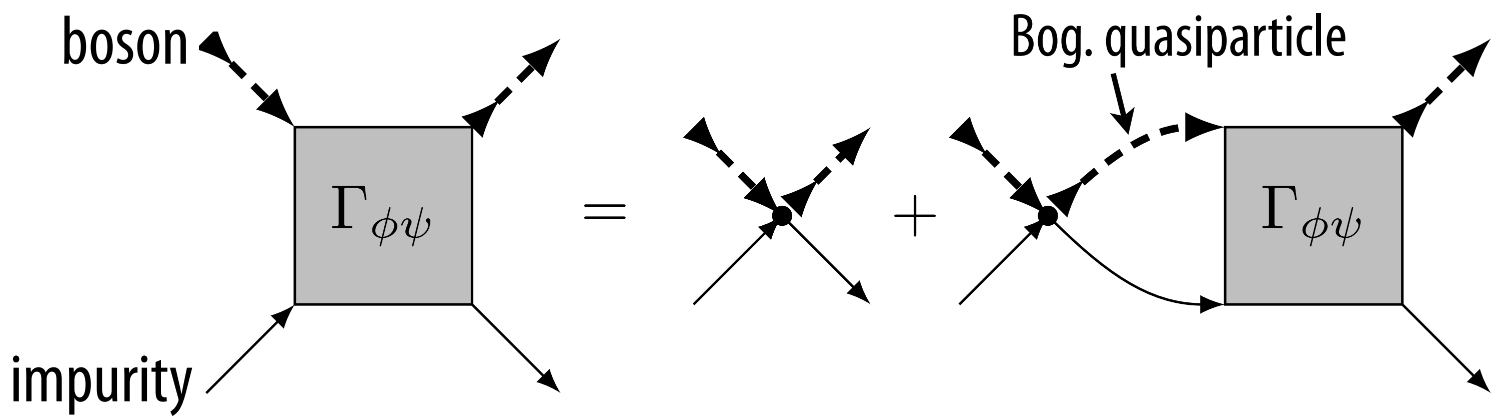
**self-energy:**



**prerequisite:** **recover exact two-body solution** [unlike previous works]

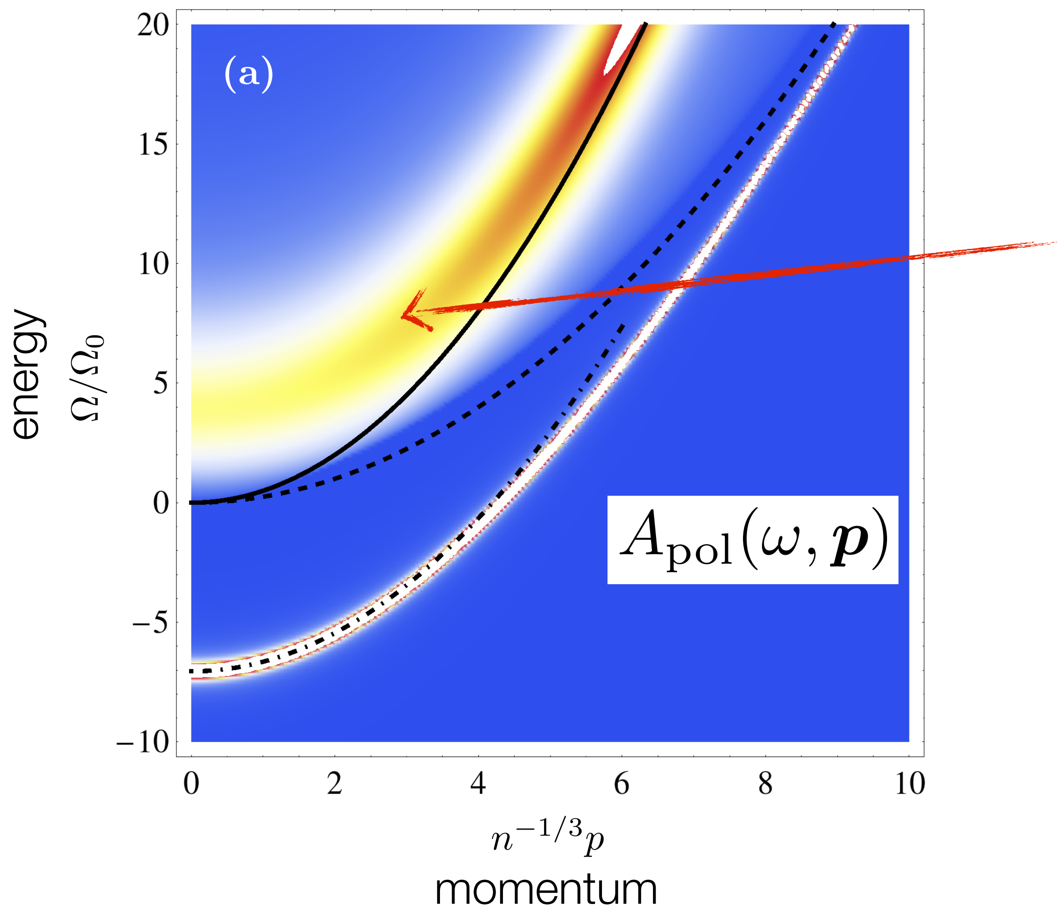
*resummed perturbation theory*

**T-matrix equation:**



# Result for momentum resolved spectral function

$n^{1/3}a_{IB}^{-1} = 1$



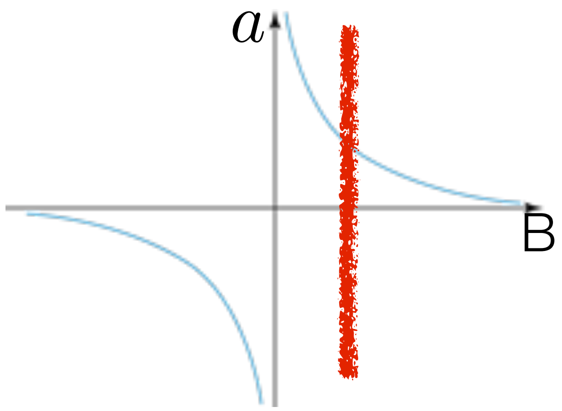
RATH, RS, PRA 88 (2013)

**unlike condmat: two coherent quasi-particle excitations!**

**almost "standard" repulsive polaron** $a > 0$

- at positive energy
- enhanced effective mass
- finite lifetime!
- largely reduced quasi-particle weight

**Feshbach resonance**



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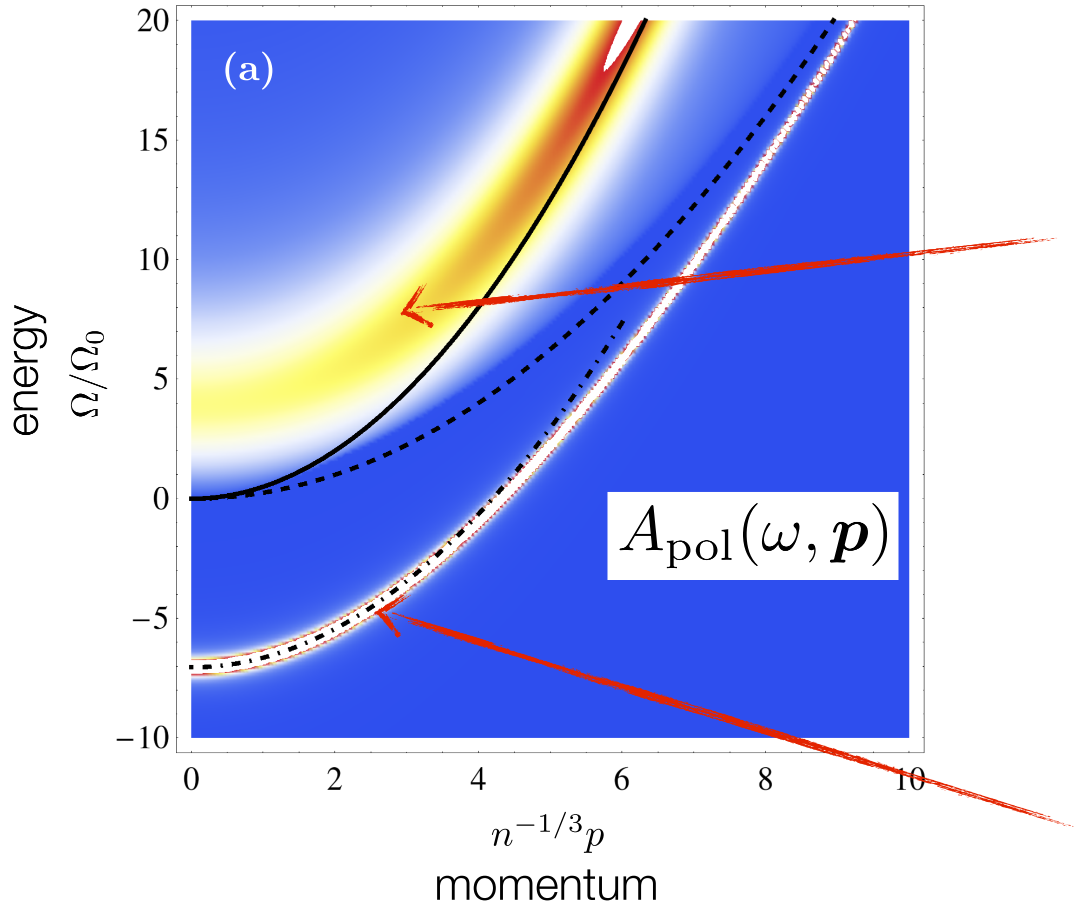

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- actual ground state at negative energies!
- cannot be found in Fröhlich approaches
- interacts attractively with BEC
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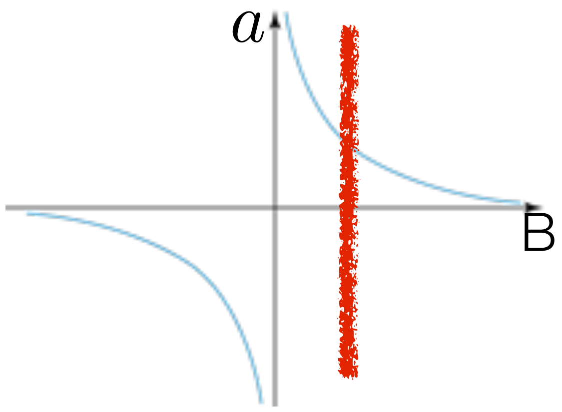

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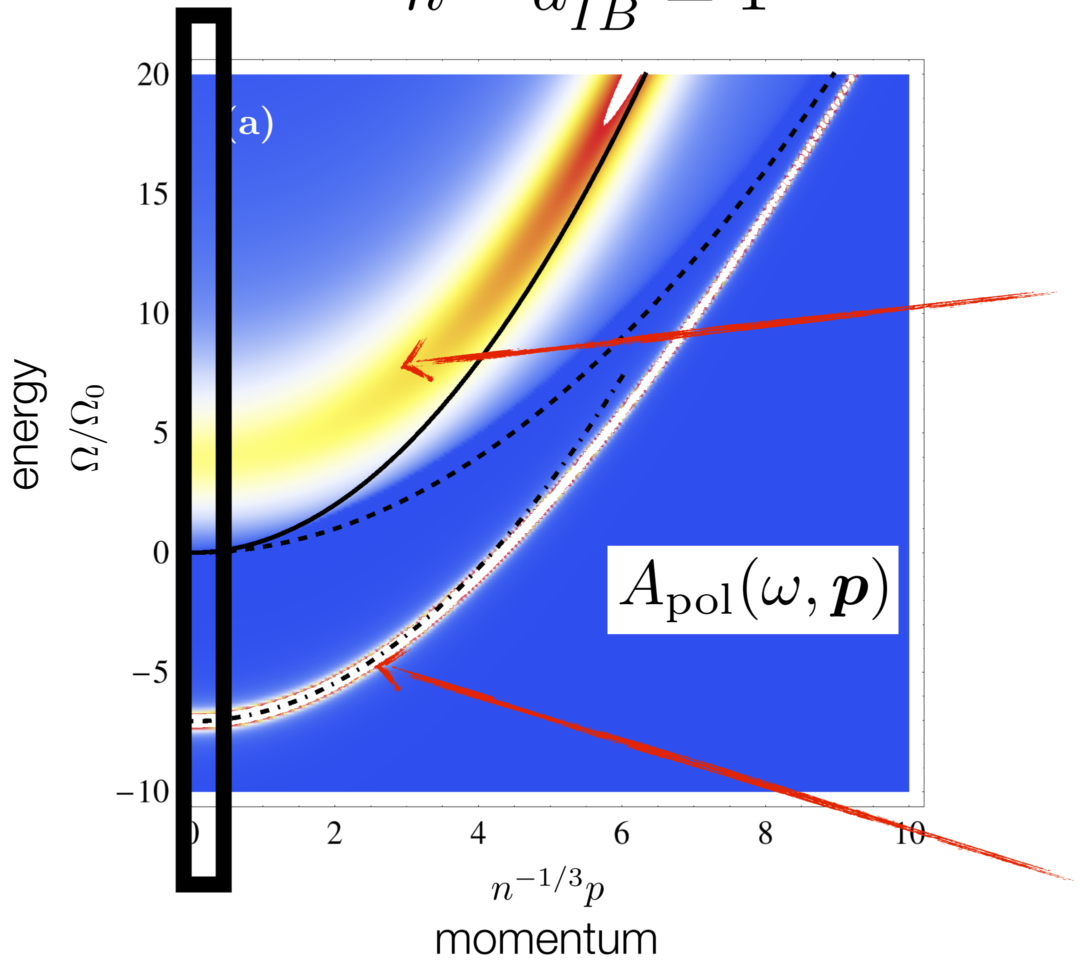

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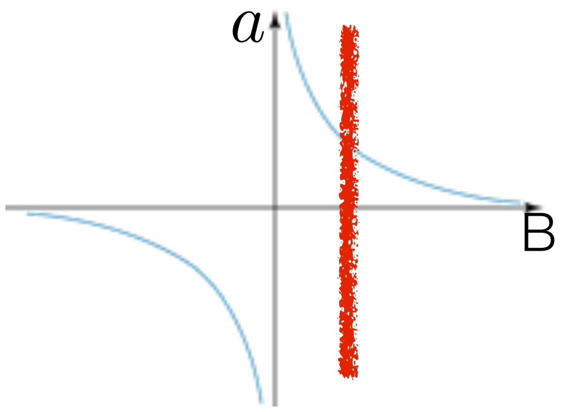
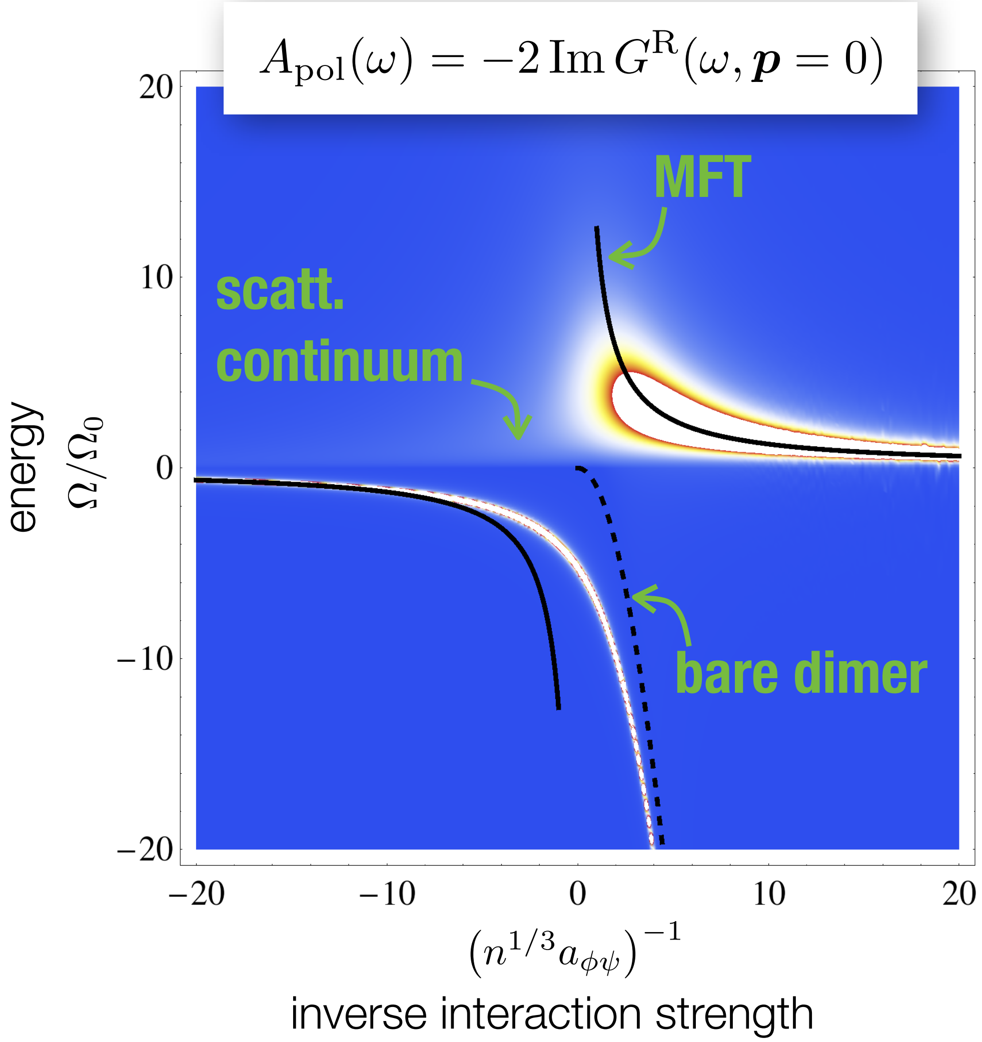
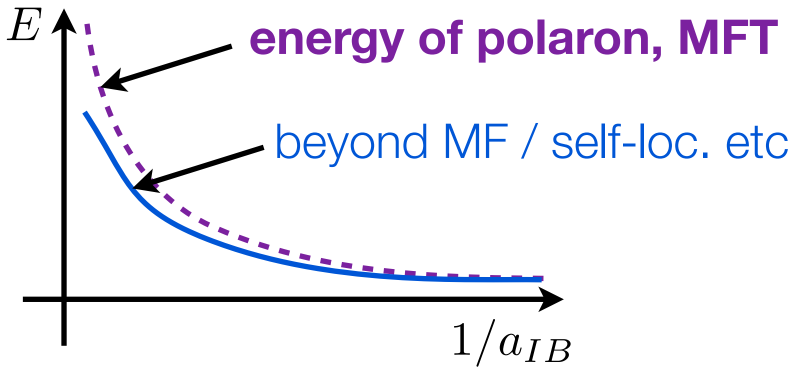


# Energy spectrum for impurity at rest



**previous “quantum simulation proposals”**



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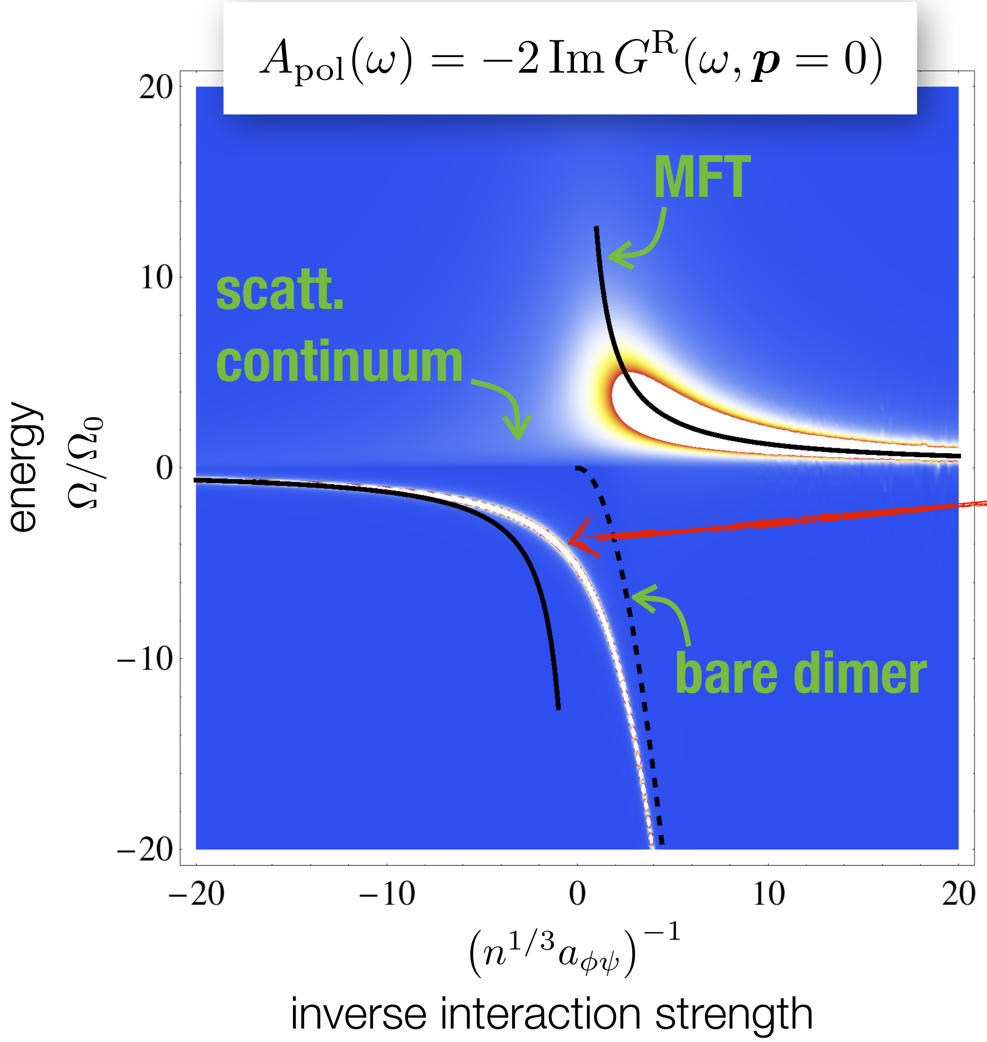
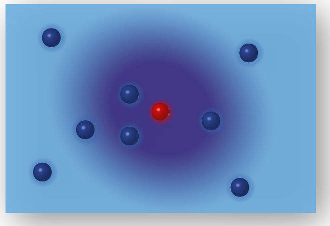


$$A_{\text{pol}}(\omega) = -2\,\text{Im}\,G^{\text{R}}(\omega, \boldsymbol{p} = 0)$$

**attractive polaron**



- stable ground state at all scattering lengths

OBSERVED AT WEAK COUPLING Pfau Group [Stuttgart]: Balewski et al., Nature 502 (2013)

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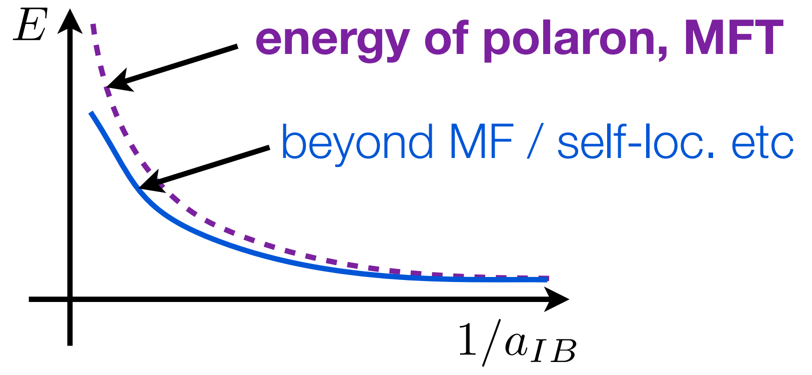

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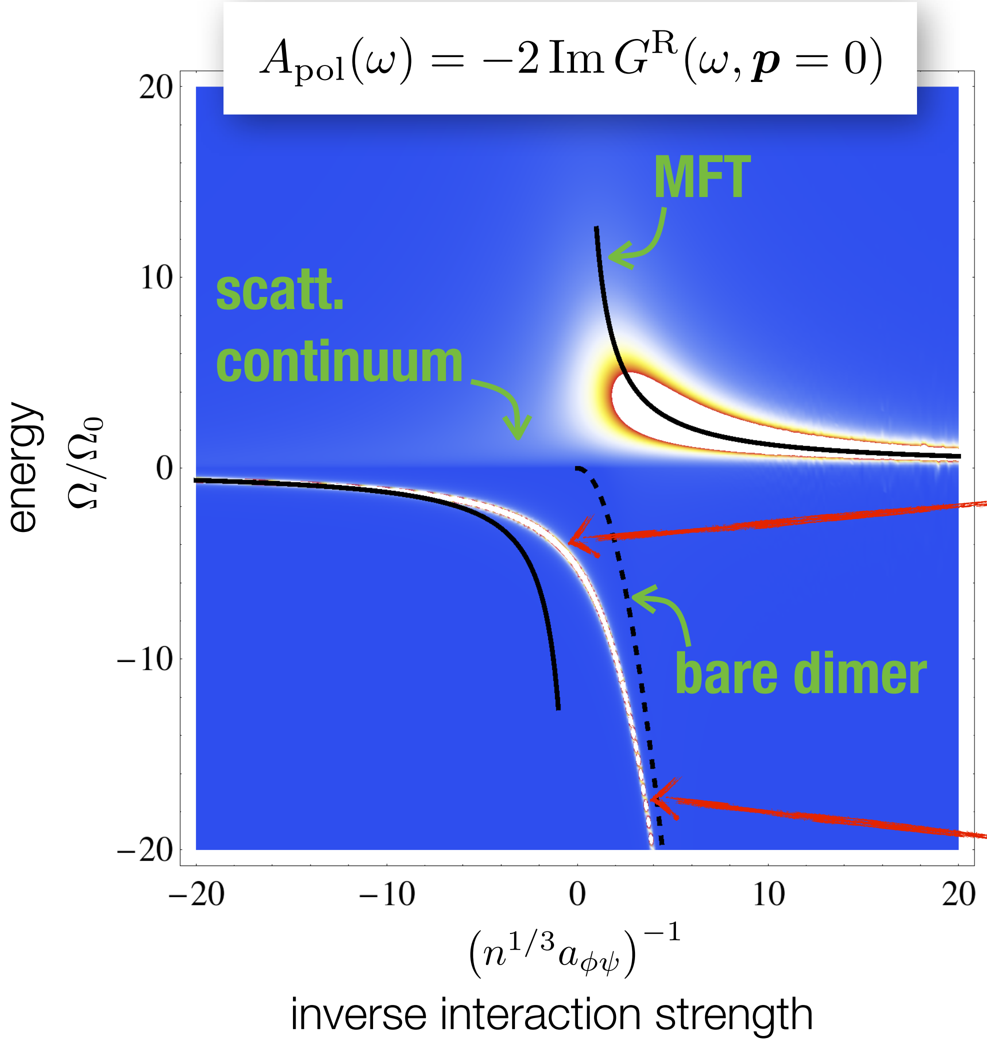

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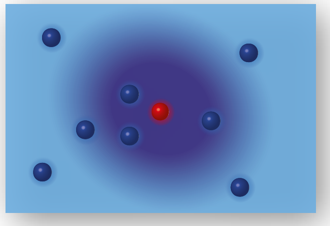

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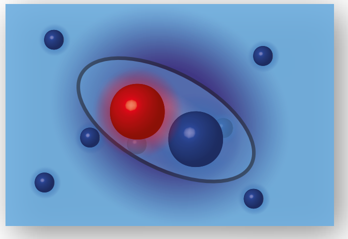
smooth **crossover to molecular state** - hybridization

*- different from transition for Fermi polaron -* Rath, RS, PRA 88 (2013)

DISCUSSED IN CONTEXT OF B/F MIXTURES BY
Marchetti, ... Parish, PRB 78 (2008)



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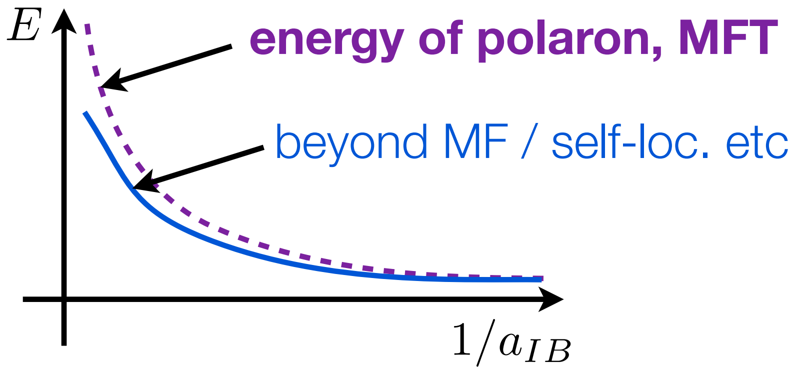

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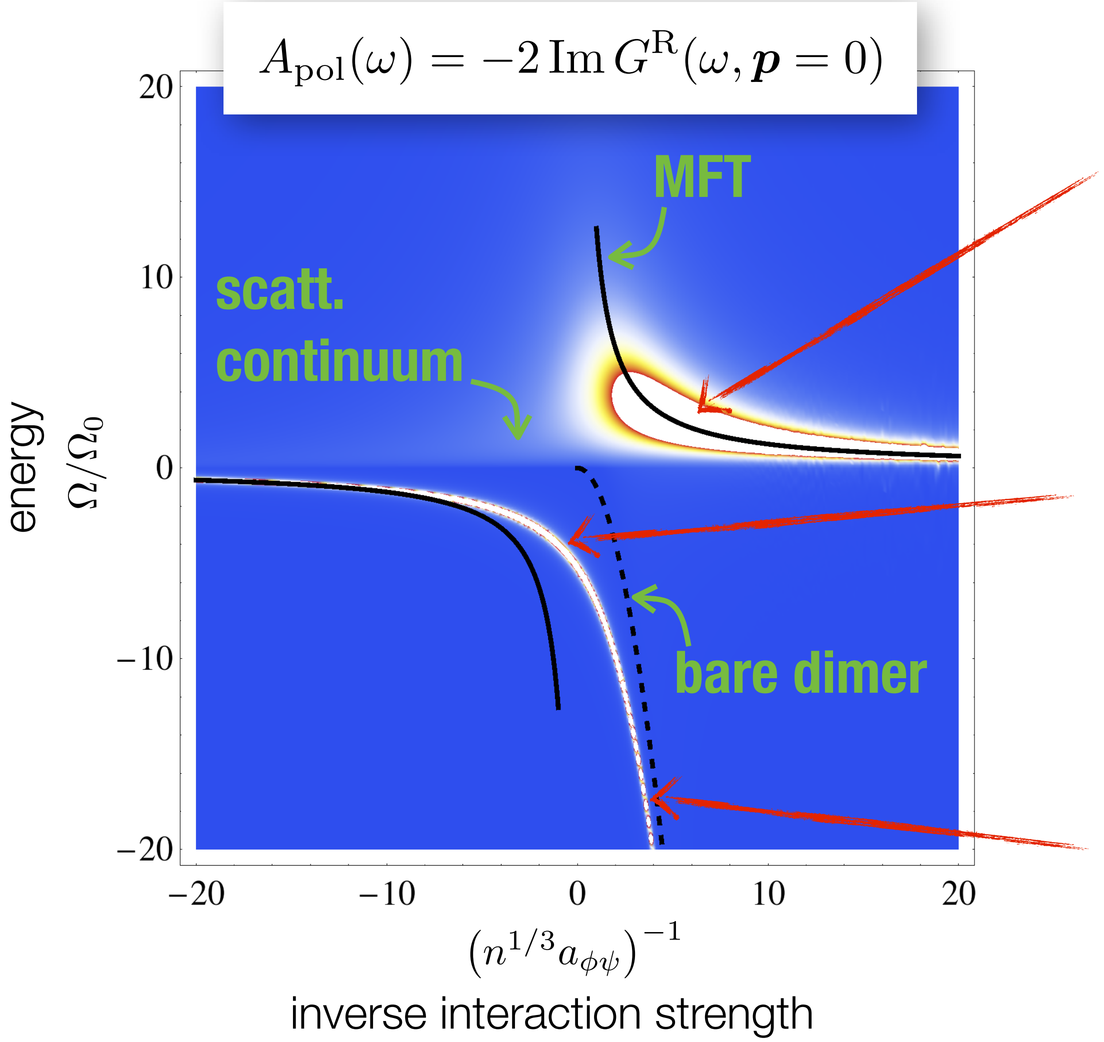
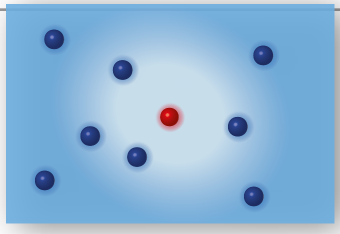
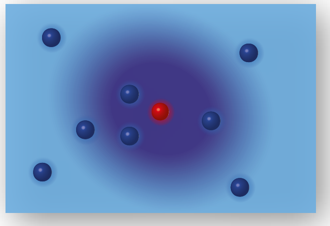




**repulsive polaron**

- extremely unstable in strong-coupling regime! *[molecule formation]*
- self-localization challenging to observe



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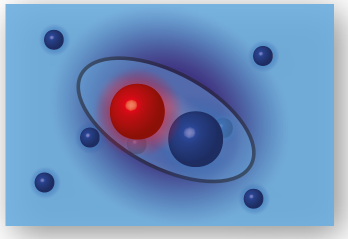
OBSERVED AT WEAK COUPLING Pfau Group [Stuttgart]: Balewski et al., Nature 502 (2013)



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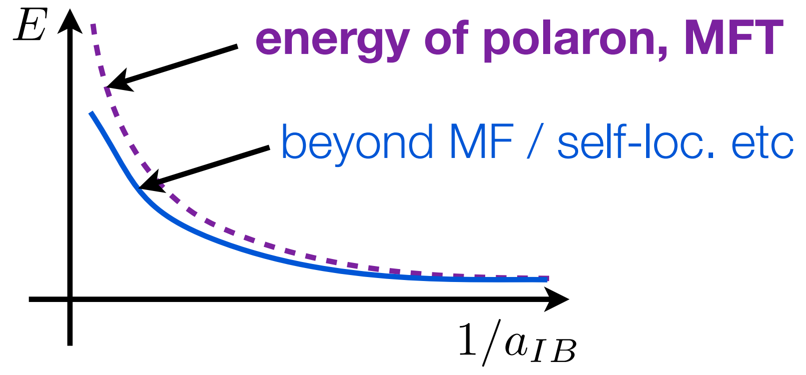
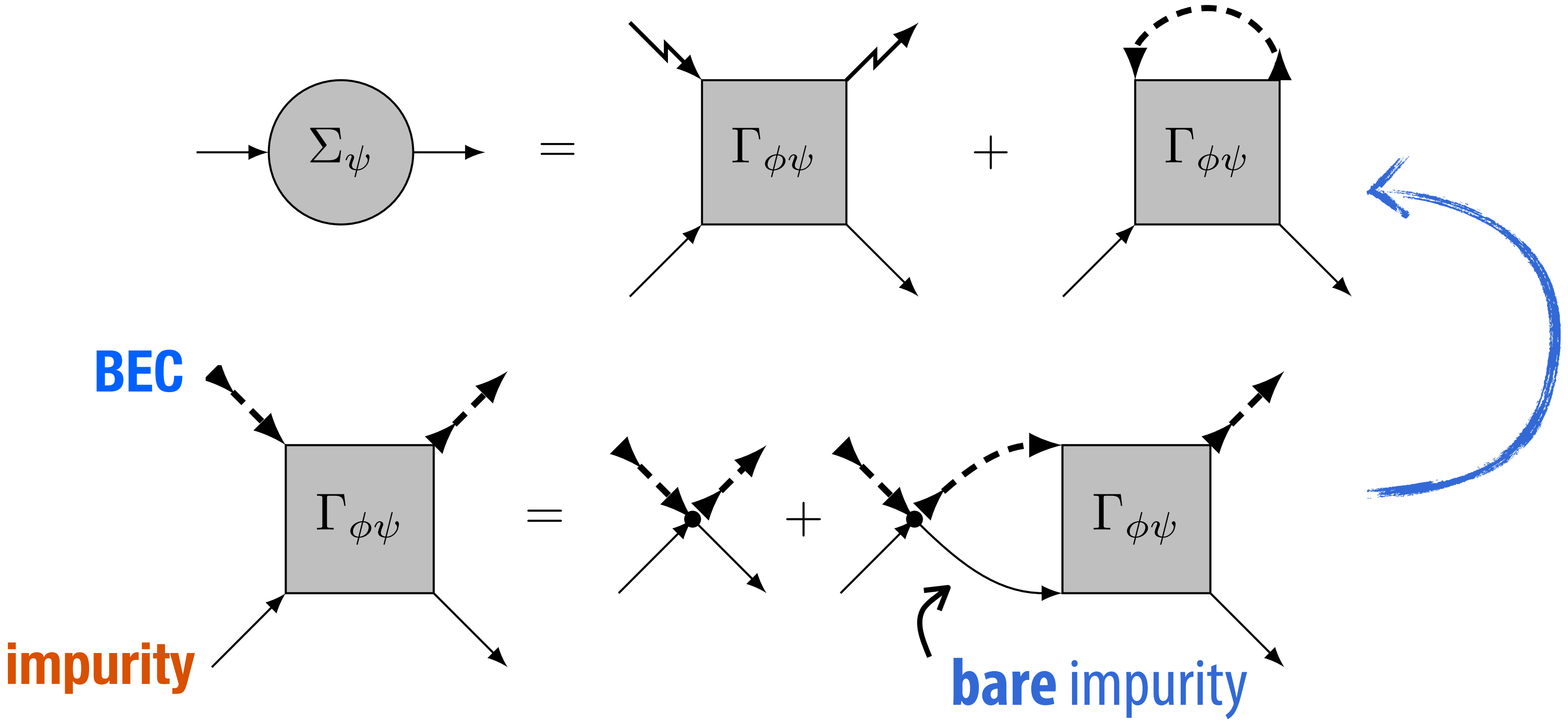


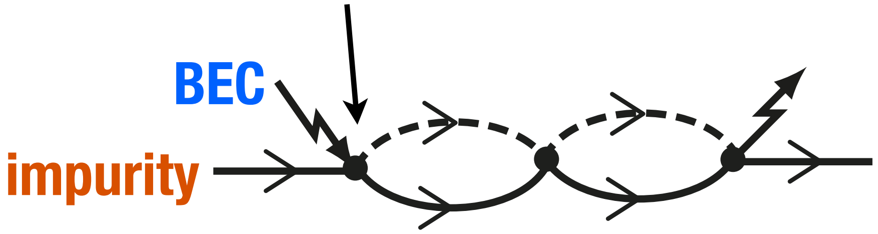
# Self-consistent T-matrix

## So far: Non-selfconsistent T-matrix approach



**Non-self-consistent T-matrix**

- single boson taken out of condensate



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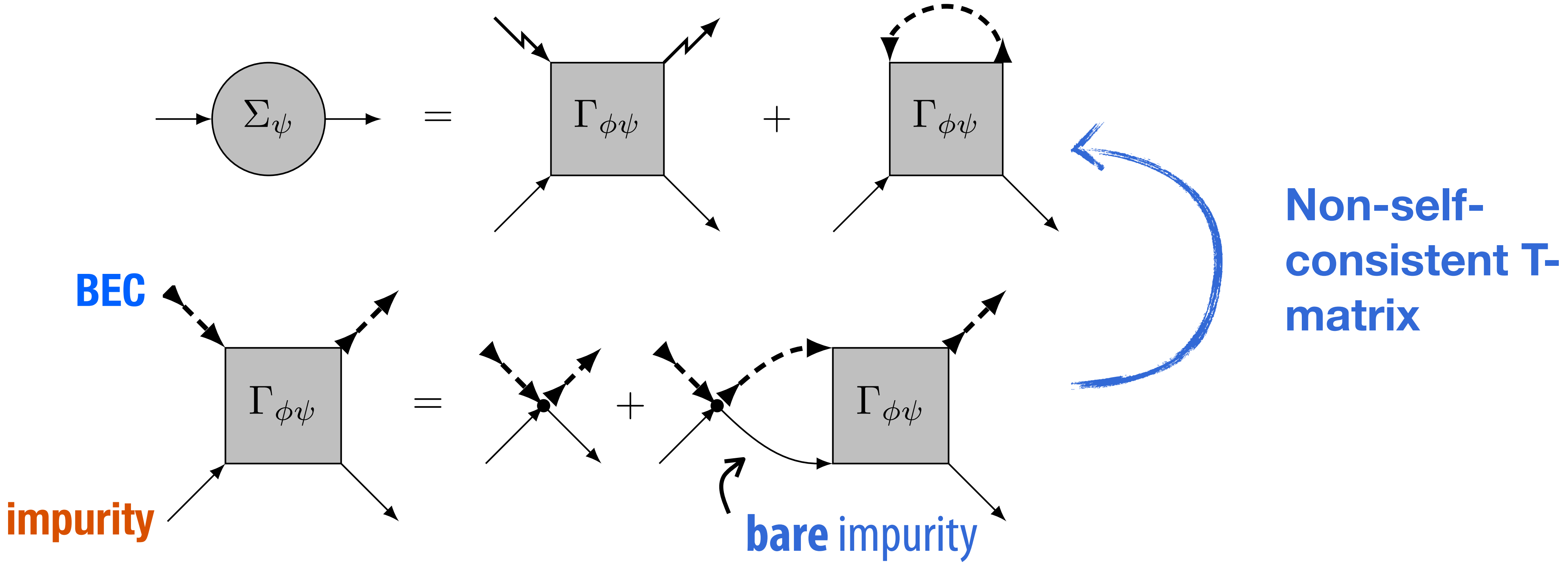

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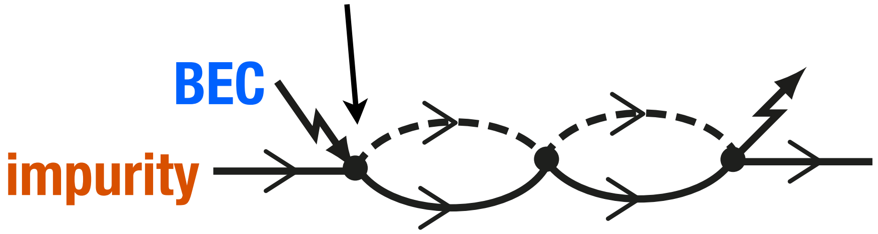


- **equivalent to simple variational wave function**

$$|\psi_0\rangle = \sqrt{Z}\hat{c}^\dagger_{\mathbf{0}}|\text{BEC}\rangle + \sum_{\mathbf{k}} \mathcal{A}(\mathbf{k})\hat{c}^\dagger_{-\mathbf{k}}\hat{b}^\dagger_{\mathbf{k}}|\text{BEC}\rangle$$

impurity

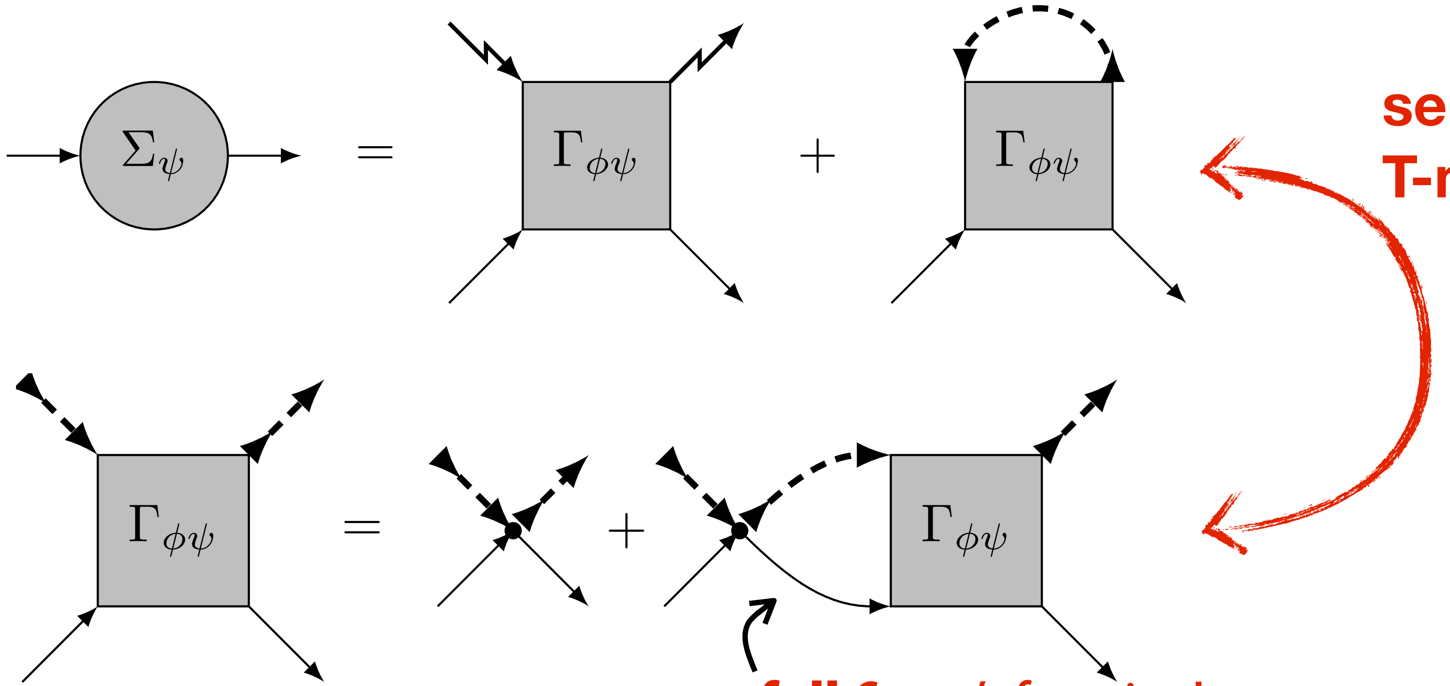
boson excited out of condensate

- captures very simple ‘entanglement’ between BEC and impurity

*recently studied in:* Li and Das Sarma, arXiv:1404.4054

# Self-consistent T-matrix

## Selfconsistent T-matrix approach



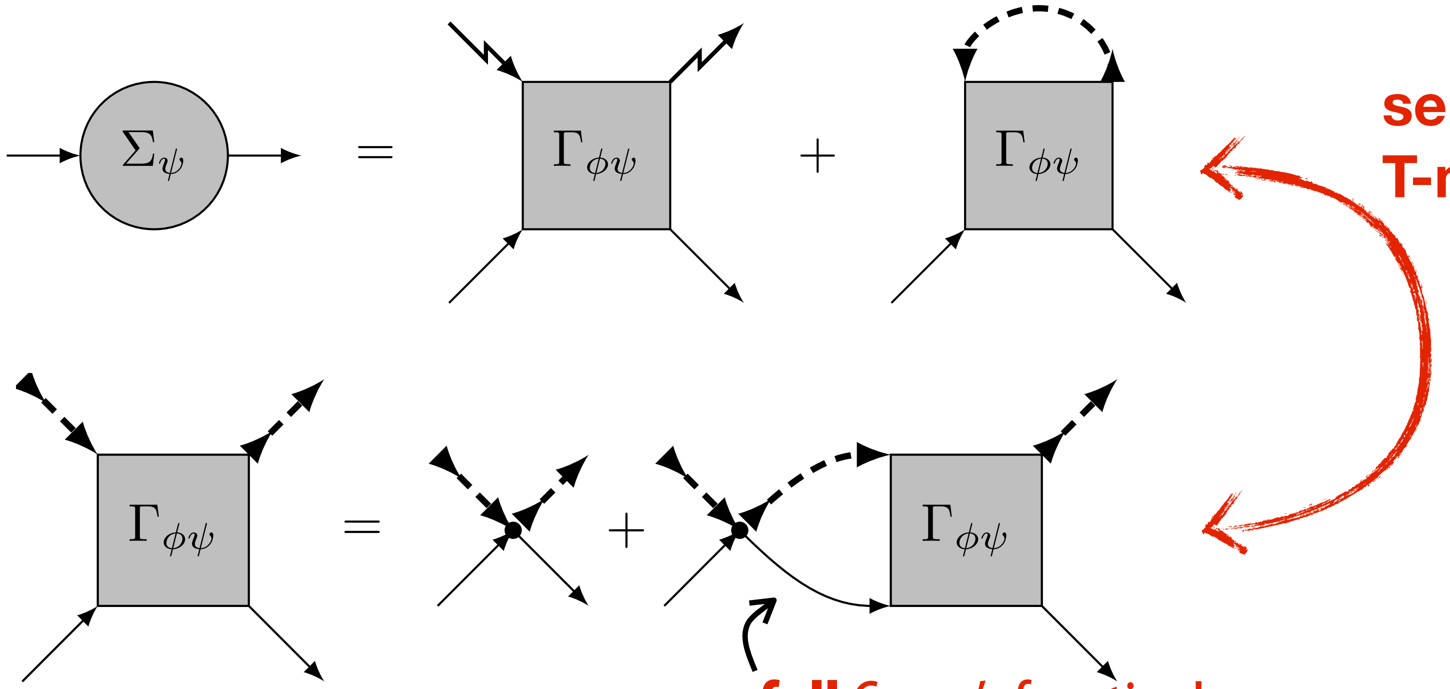
**self-consistent T-matrix** Rath, RS, PRA 88 (2013)

**full** Green's function!

- solved numerically using algorithm developed for *functional renormalization group approach* for RG flow of full spectral functions RS, Enss, PRA 83 (2011)

# Self-consistent T-matrix

## Selfconsistent T-matrix approach

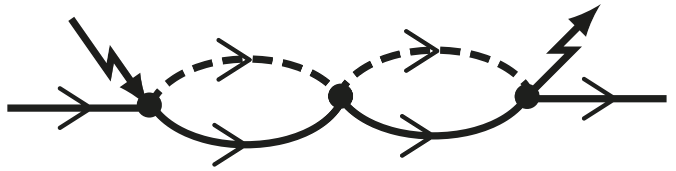


**self-consistent T-matrix** Rath, RS, PRA 88 (2013)

**full** Green's function!

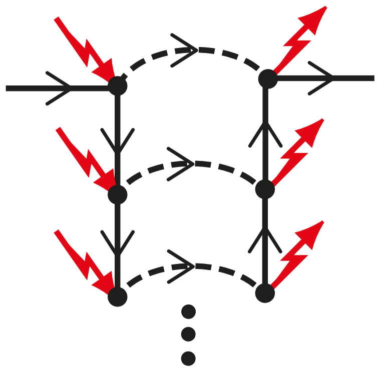
- solved numerically using algorithm developed for *functional renormalization group approach* for RG flow of full spectral functions RS, Enss, PRA 83 (2011)
- accounts for infinitely many virtual excitations of bosons out of the coherent condensate state

**Non-selfconsistent:**



- **single boson taken out of condensate**

**Selfconsistent:**

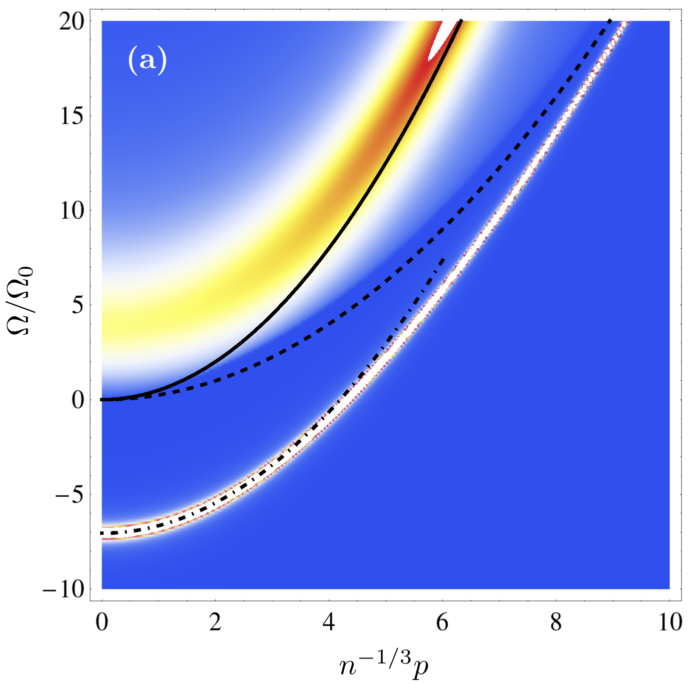


- **infinite number of bosons taken out of condensate -**
  way beyond product wave functions for BEC

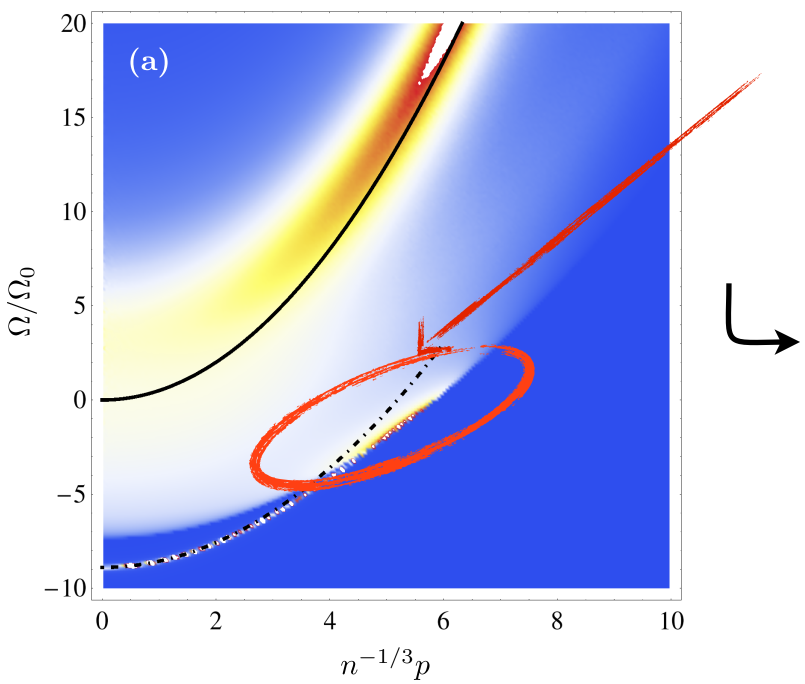
# Self-consistent T-matrix - Results

**Momentum resolved Spectral function**

## Non-selfconsistent



## Selfconsistent



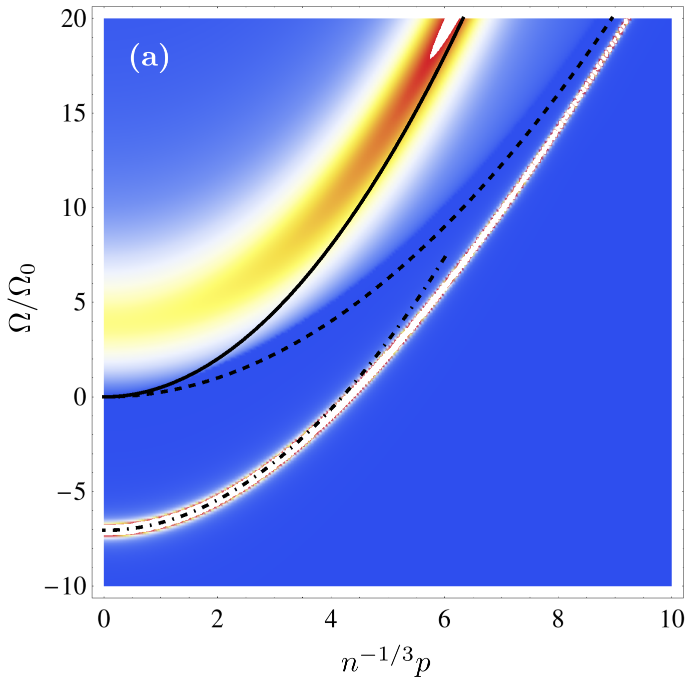
annihilation of fast attractive polaron at scattering threshold

↳ ***momentum dependent Z-factor vanishes!***

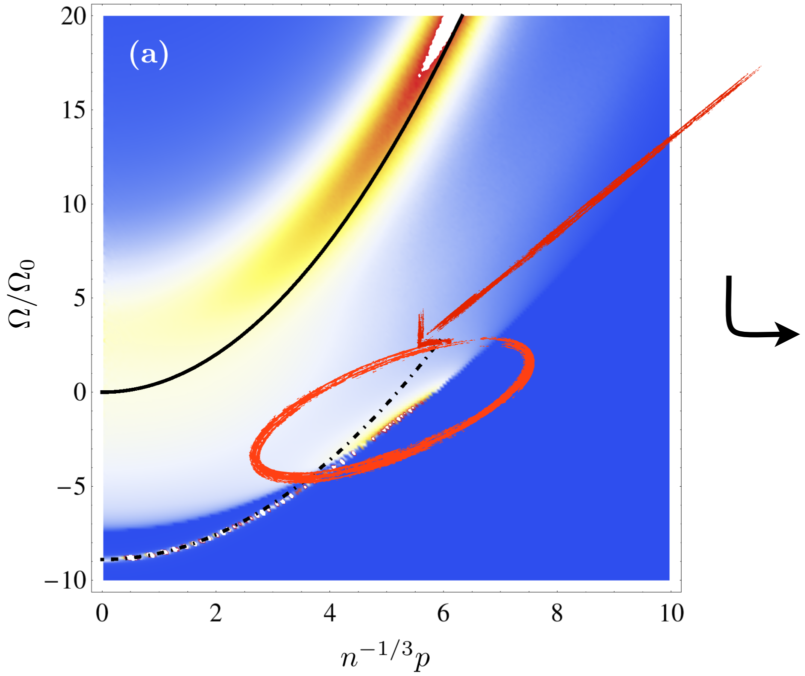
# Self-consistent T-matrix - Results

**Momentum resolved Spectral function**

**Non-selfconsistent**



**Selfconsistent**



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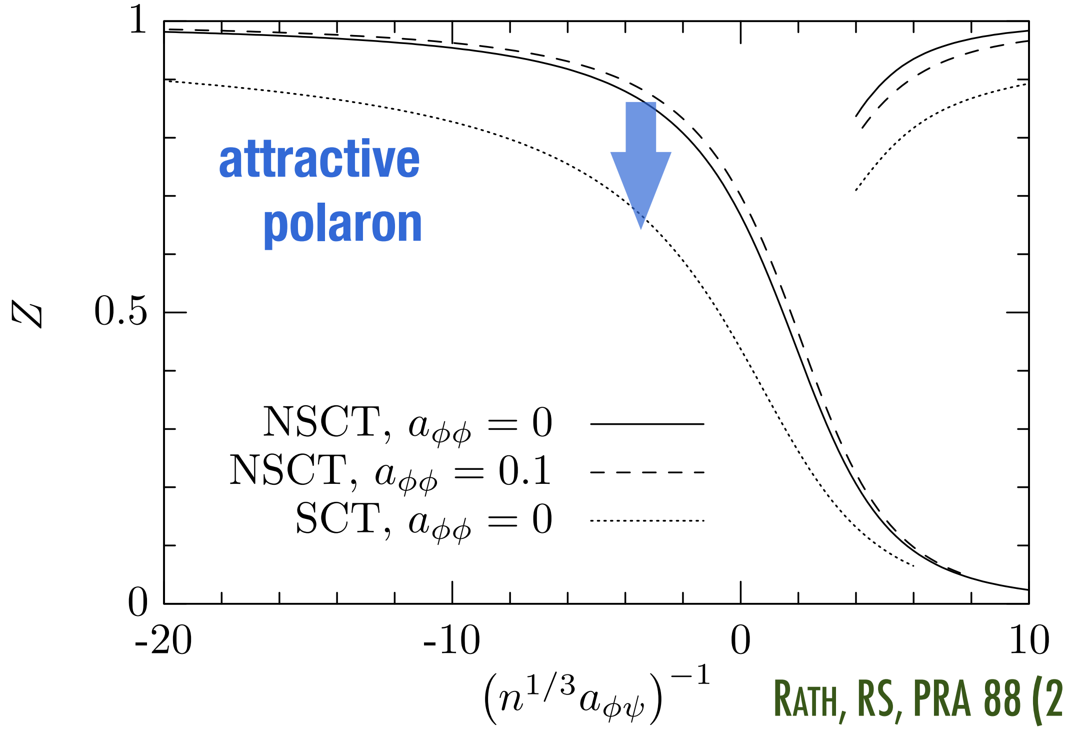
## Suppression of quasi-particle weight



**attractive polaron**

Rath, RS, PRA 88 (2013)

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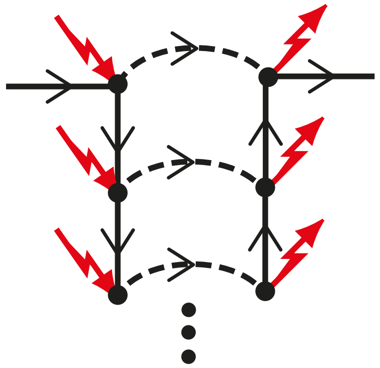


- for Fermi polaron much smaller deviation!

Prokefiev, Svistunov, PRB 77 (2008)

RS, Enss, PRA 83, 063620 (2011)

# Proposal for experimental observation

E.G. $^{40}$K/$^{41}$K MIXTURE AT B=543 G

SEE MIT GROUP: WU ET AL. PRA 84 (2012)

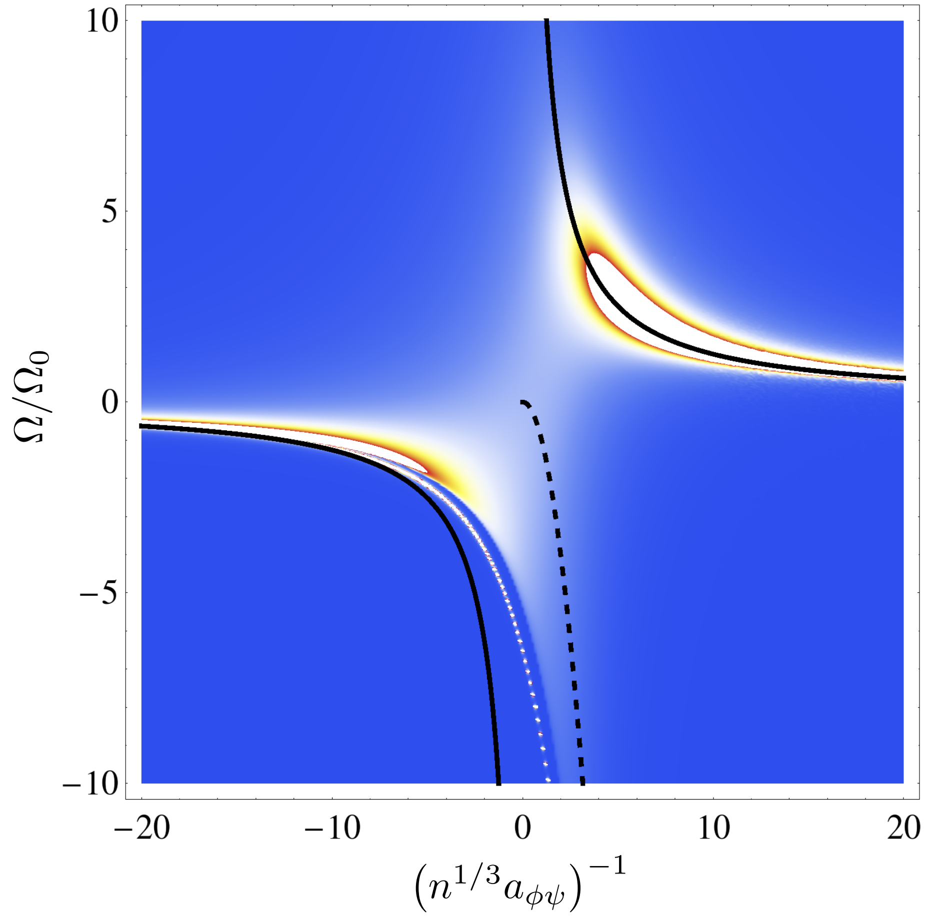
## Challenge

- Efimov effect + statistics: Bose-Fermi mixtures unstable due to enhanced three-body recombination

SEE E.G. RS, RATH, ZWERGER, EJB 85 (2012)

EFIMOV IN SPIN-ORBIT BOSONS: SHI, CUI, ZHAI PRL 112 (2014)

- possible BEC deformation due to large interactions



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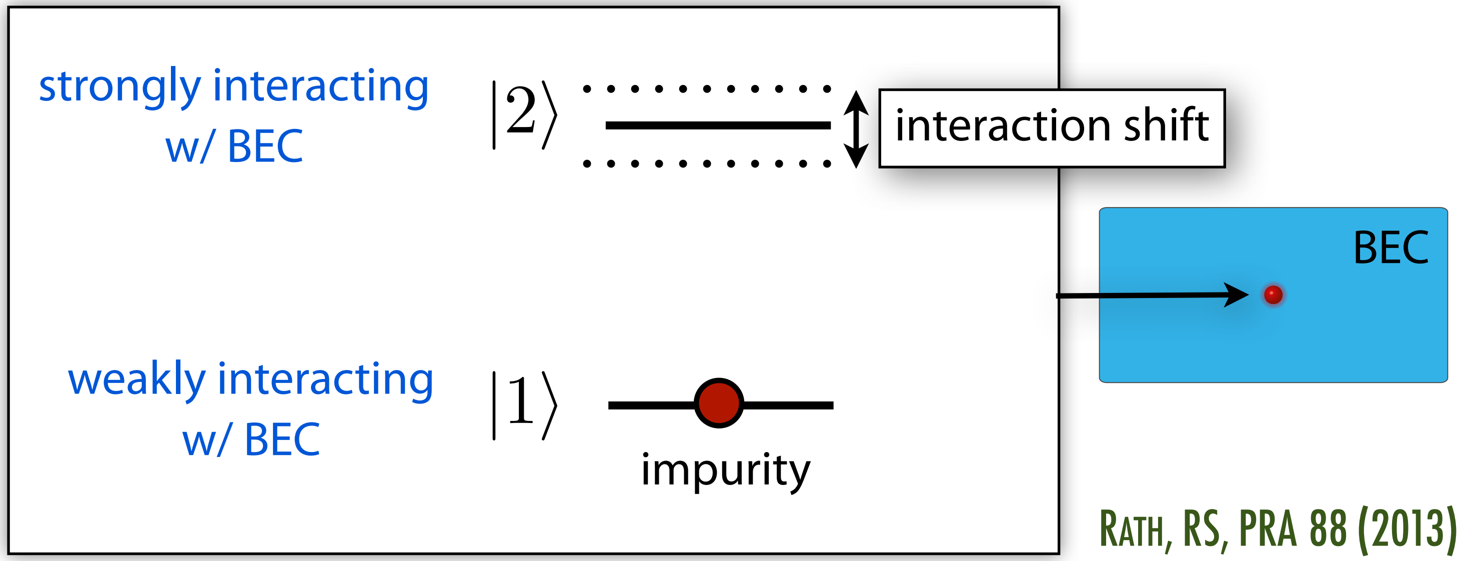
## Resolution: Inverse RF spectroscopy



strongly interacting w/ BEC

$|2\rangle$

interaction shift

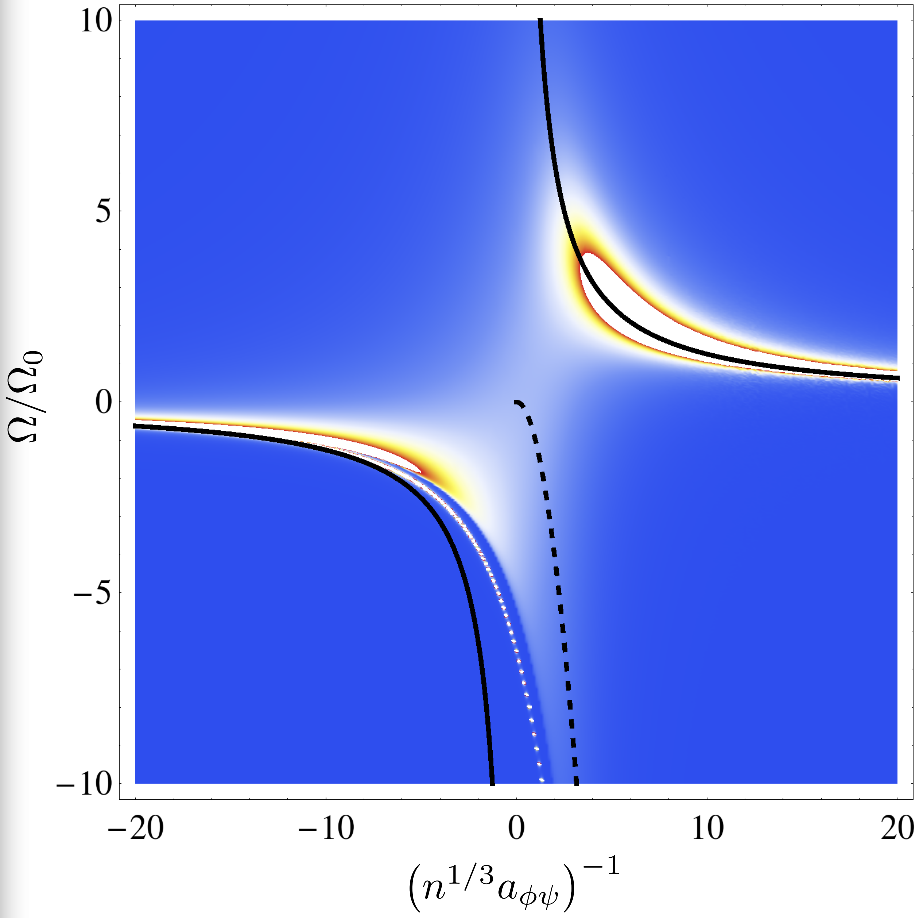
BEC

weakly interacting w/ BEC

$|1\rangle$

impurity

RATH, RS, PRA 88 (2013)



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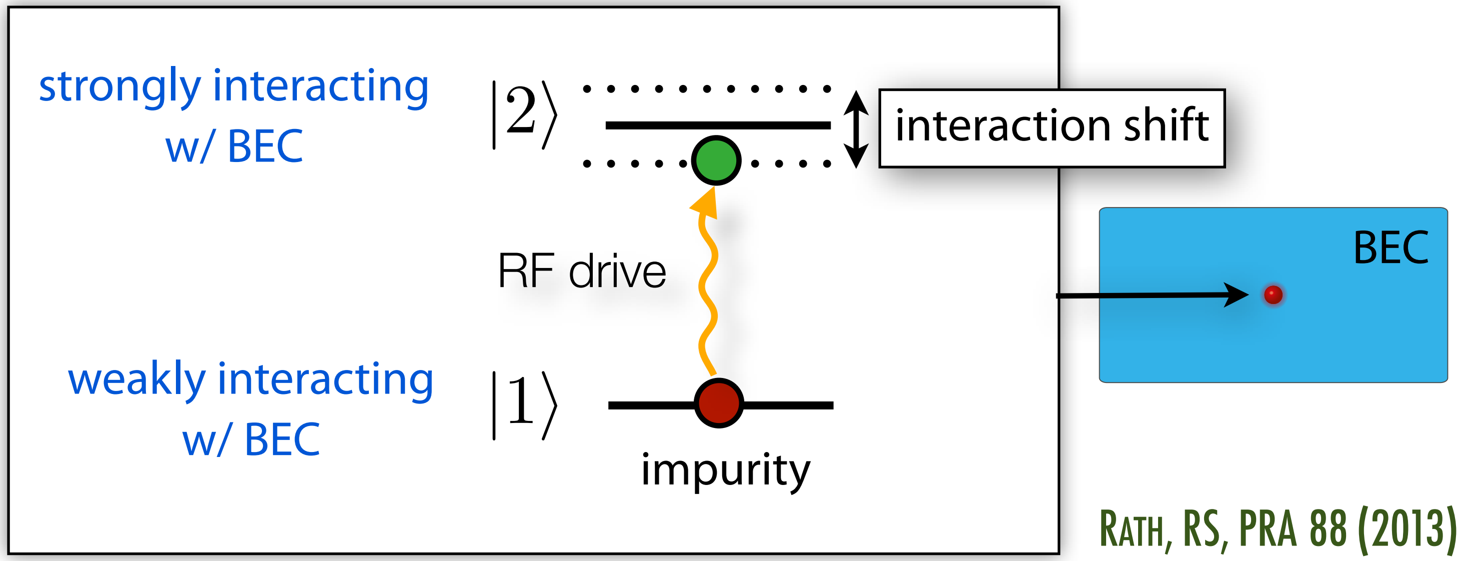
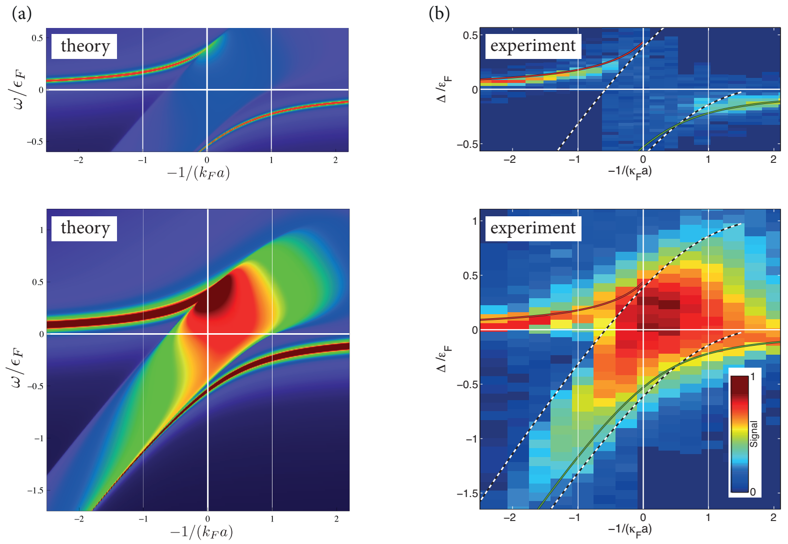


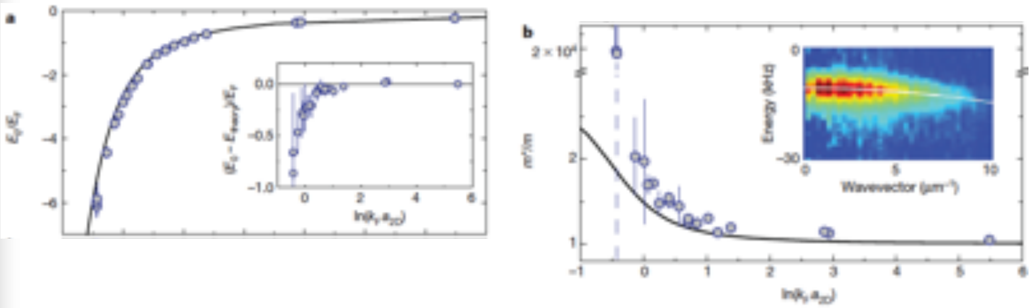
RATH, RS, PRA 88 (2013)

- maps out impurity spectral function
  - similar procedure proposed and used for fermions

THEORY: RS, ENSS, PRA 83, 063620 (2011)
MASSIGNAN, BRUUN, EPJD 65 (2011)
RS, ENSS, PIETILA, DEMLER, PRA 85, 021602 (2012)

EXPERIMENTS: KOHSTALL ET AL., NATURE 485, 615 (2012)
KOSCHORREK ET AL., NATURE 485, 619 (2012)





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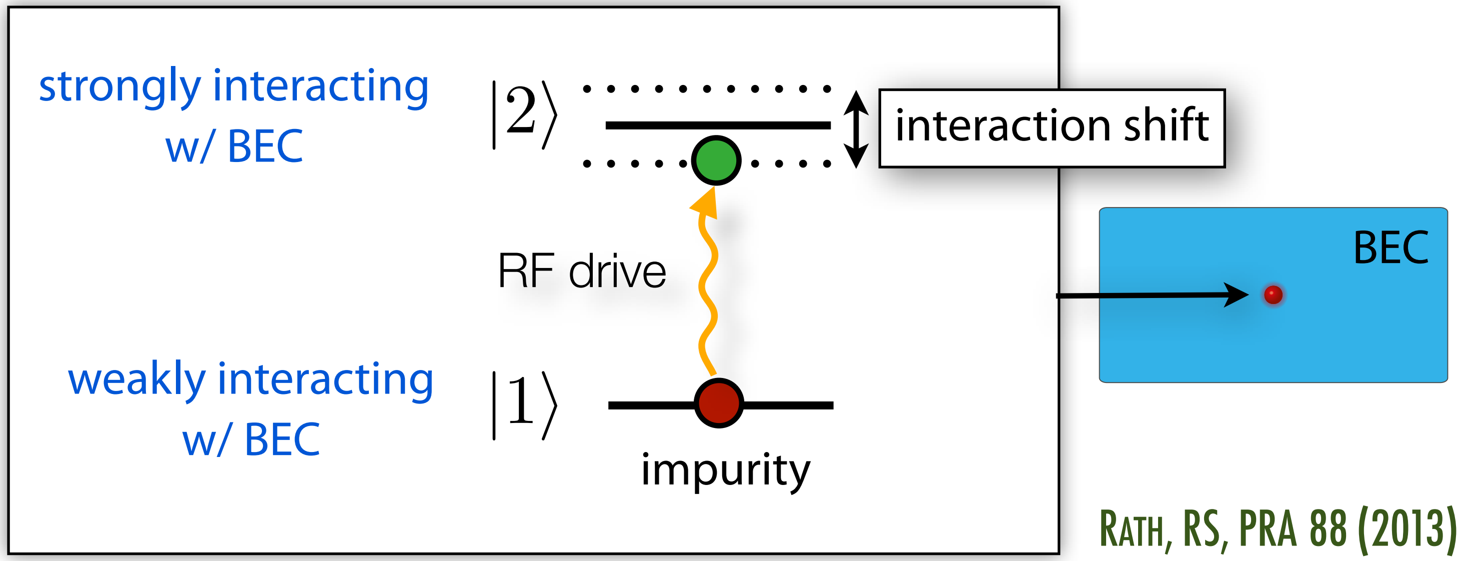


strongly interacting w/ BEC

$|2\rangle$

interaction shift

RF drive

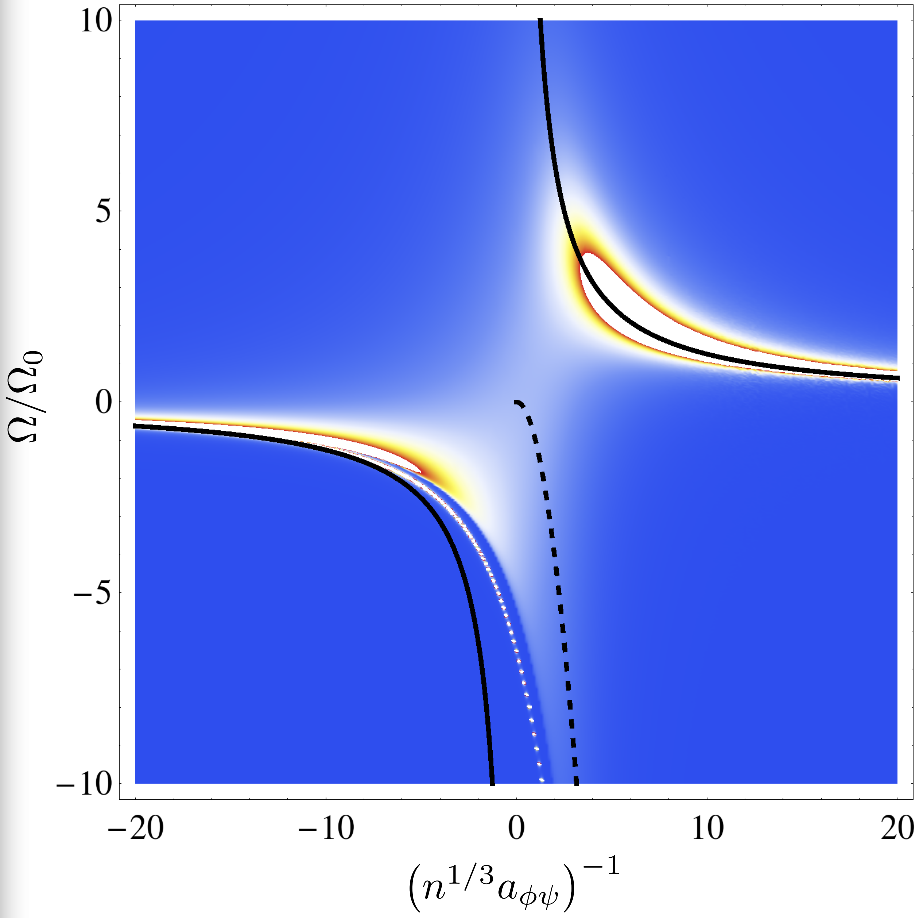
weakly interacting w/ BEC

$|1\rangle$

impurity

BEC

RATH, RS, PRA 88 (2013)



- maps out impurity spectral function
  - similar procedure proposed and used for fermions
- Efimov states off-resonant
- BEC deformation irrelevant as

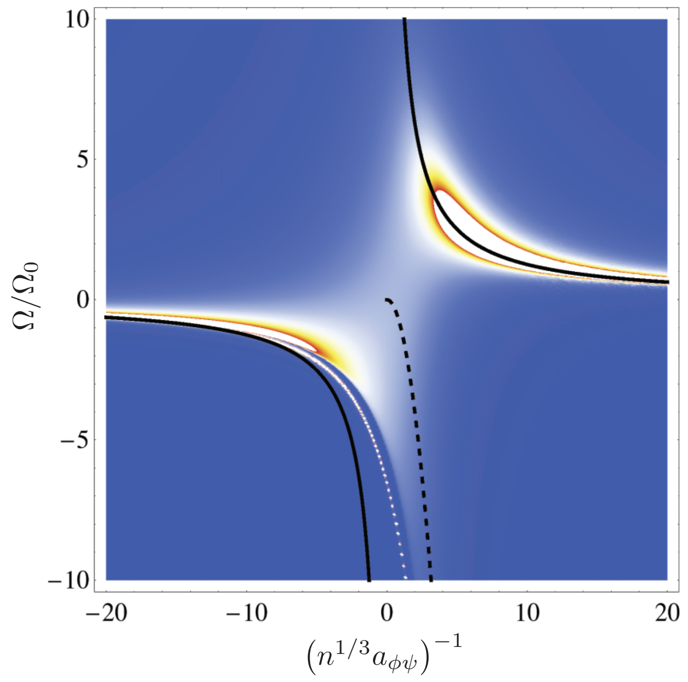
$$\tau_{BEC} = \frac{\hbar}{\mu_B} = \frac{\hbar}{g_B n_B} \gg \tau_{\text{Rabi}}$$

# Summary

- Using a field theoretical approach we studied impurity in ultracold BEC close to Feshbach resonance

- Impurity is dressed by bosonic excitations ⇨ Bose polaron

- Spectrum exhibits two distinct quasi-particle branches
  - ➡ attractive polaron
  - ➡ repulsive (Fröhlich) polaron



- We predict finite lifetime of repulsive polaron:
  Quantum simulation of Fröhlich model challenging with ultracold atoms

- We propose experimental procedure to measure polaron properties via radio-frequency spectroscopy

# Outlook

## ① non-equilibrium physics

- Our theory describes the polaron right after the drive to the final state
  Here the repulsive polaron is in a highly excited, non-equilibrium state
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**Thank you!**