IAS, Tsinghua University, October 8th, 2014

Quantum spin liquids as soft-gap Mott insulators

Yi Zhou (Zhejiang University)

Reference: YZ and Tai-Kai Ng, Phys. Rev. B 88, 165130 (2013),
YZ, Kazushi Kanoda, and Tai-Kai Ng, submitted, invited by RMP (2014).

General motivations

- Why quantum spin liquid is interesting?
  - New states of matter in magnetic insulators
  - To understand Mott physics more

Condensed matter physics

- **The central issue in condensed matter physics is to discover and understand *new states of matters*.**

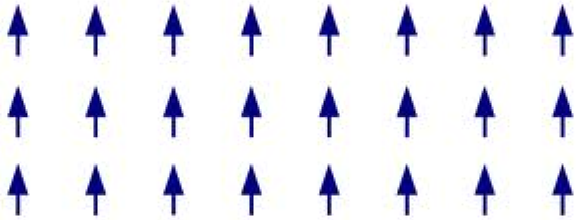
metal

insulator

superconductor

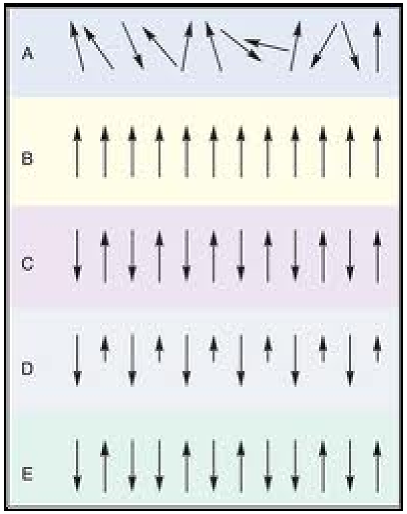
Magnetic insulators: Any new states of matter ?

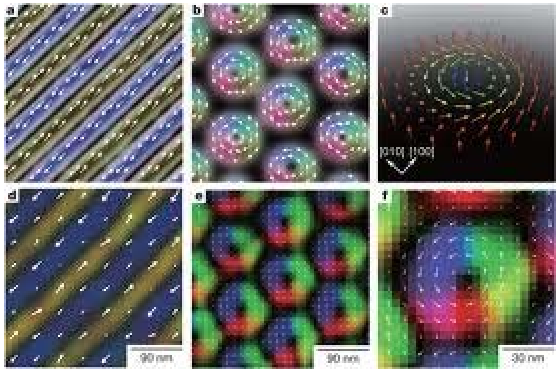
- **Magnetism is one of the oldest subjects in physics**
  - ***Historically it is associated with magnetic field generated by ordered magnetic moments.***



Magnetic insulators: Any new states of matter ?

- **Magnetic moments order differently in various materials**





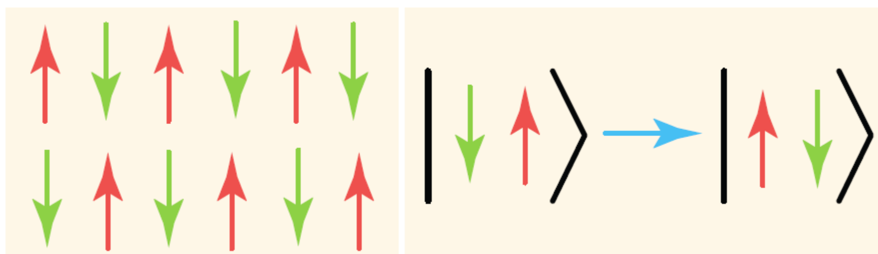
helical magnet

All these ordered states can be understood from the magnetic interaction between classical vectors (spins).

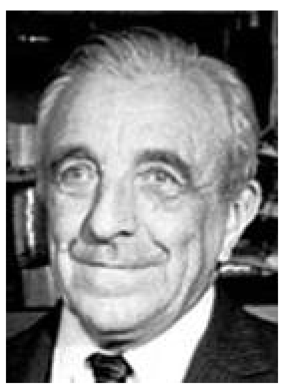
Magnetic insulators: Any new states of matter ?

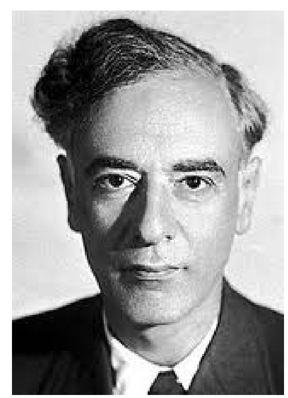
- What's the ground state for an antiferromagnet?
- The debate between Néel and Landau

AFM
ordered



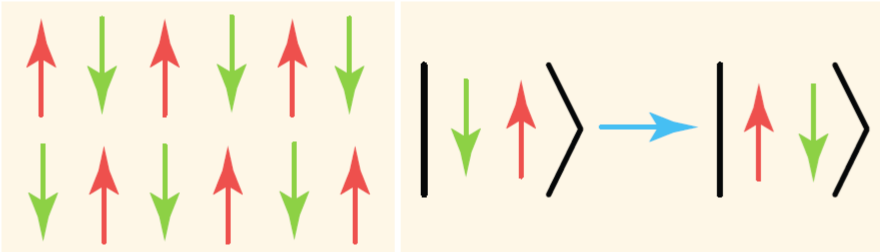
Quantum
fluctuated





Magnetic insulators: Any new states of matter ?

- **What's the ground state for an antiferromagnet?**
- **The debate between Néel and Landau**



- ***Quantum fluctuations dominate over Néel's order in one dimension → spin liquid.***
- ***Néel's order wins at 2D square lattice → AFM order.***
- ***The general situation is still unclear.***

Magnetic insulators: Any new states of matter ?

- **It was proposed that quantum spin liquid states can possess very exotic properties.**
  - **Emerged particles and fields**
- ***Emergent phenomena***
  - ***New particles and fields emerge at low-energy scales but they are totally absent in the Hamiltonian that describes the initial system.***
  - ***Different physics laws emerge at different scales.***

Definition: QSL

- ***Quantum spin liquid (QSL) is an insulator with an odd number of electrons per unit cell which does not order magnetically down to zero temperature due to quantum fluctuations.***
  - ***The quantum disorder is intrinsic, not induced by extrinsic impurities.***
  - ***Not a constructive definition.***

Features (or featureless ?)

- "Featureless" Mott insulators.
  - Lattice translational symmetry is respected.
  - The absence of long ranged magnetic order.
  - …
- Characterized by emergent phenomena and possible quantum orders

Emergent particles and fields

- **Spinon:** ***S=1/2*, charge neutral, mobile objects**
  - **The spinons *may obey Fermi or Bose statistics or even nonabelian statistics* and *there may or may not be an energy gap*;**
- **Gauge field: spin singlet fluctuations**
  - **These spinons are generally accompanied by gauge fields, *U(1)* or Z_2.**

- **How can we know which of these plausible states are “real”, especially when D>1 ?**
  - ***This is a very difficult problem.* I shall outline a few of different approaches to this problem.**

Nonlinear σ -model

- **From two-spin dynamics to a rotor model**

$$\vec{S}_i = S\vec{n}_i, \quad |\vec{n}_i| = 1 \quad \Rightarrow \quad \dot{\vec{n}}_1 = JS^2\vec{n}_2 \times \vec{n}_1, \quad \dot{\vec{n}}_2 = JS^2\vec{n}_1 \times \vec{n}_2$$

$$\boxed{\begin{aligned} & \vec{L} = \vec{n}_1 + \vec{n}_2, \quad \vec{n} = \vec{n}_1 - \vec{n}_2 \\ \Rightarrow \quad & \dot{\vec{L}} = 0, \quad \dot{\vec{n}} = -JS^2\vec{n} \times \vec{L} \end{aligned}}$$

$$\boxed{\begin{aligned} & \vec{r} = r\vec{n}, \quad \vec{L} = \vec{r} \times \vec{p} \Rightarrow \vec{r} \times \vec{L} = -r^2\vec{p} \\ & \dot{\vec{r}} = \frac{\vec{p}}{m} = -\left(\frac{1}{mr^2}\right)\vec{r} \times \vec{L}, \quad \dot{\vec{L}} = 0. \end{aligned}}$$

- **Quantum mechanics** $\vec{L}^2 = l(l+1)\hbar^2, \quad L_z = m\hbar, m = -l, \cdots, l$

Ground state: $l = 0$

Heisenberg uncertainty: $\langle \delta\Omega \rangle \langle \delta L \rangle \geq \hbar, \quad \langle \delta L \rangle \to 0 \quad \Rightarrow \quad \langle \delta\Omega \rangle \to \infty$

Nonlinear σ -model

- **Many spin problem: rotor representation**

$$H \to \frac{1}{2I}\sum_i \vec{L}_i^{\,2} + \tilde{J}\sum_{<i,j>} \vec{n}_i \cdot \vec{n}_j$$

$$\boxed{\begin{array}{l} 2I\tilde{J} >> 1 \Rightarrow \text{magnetically ordered} \\ 2I\tilde{J} << 1 \Rightarrow \text{spin liquid state} \end{array}}$$

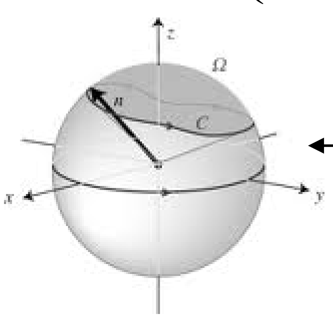
$$H \to \frac{1}{2I}\sum_i \vec{L}_i^{\,2} + \sum_{<i,j>} \tilde{J}_{ij} \vec{n}_i \cdot \vec{n}_j$$

frustration $\Rightarrow$ **quantum disordered state**

- **How does the magnitude of spin enters ?**

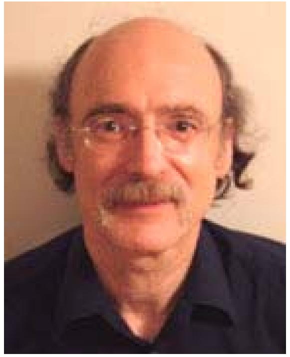
Topological term (F.D.M. Haldane)

$$S = \int dt \left(\frac{I}{2}\sum_i \dot{\vec{n}}_i^{\,2} + J\sum_{<i,j>} \vec{n}_i \cdot \vec{n}_j \right) + S_{Berry's\,Phase}$$



$$\leftarrow \quad e^{iS\Omega}$$

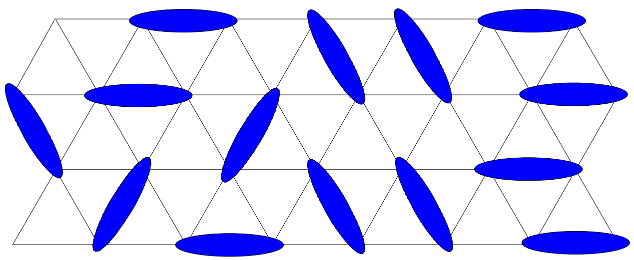
Qualitative difference between ground states of integer and half-odd-integer spin chains (Haldane conjugation)

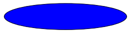


Resonating Valence Bond (RVB)

- **Issue:** ***the nonlinear σ-model approach becomes too difficult to implement in D>1.***

P.W. Anderson: Why not just "guess" the wave-function…



 $= (i, j) \equiv \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

$$|\Psi_{RVB}\rangle = \sum_{i_1 j_1 i_2 j_2 \cdots} a_{(i_1 j_1 i_2 j_2 \cdots)} |(i_1, j_1)(i_2, j_2)\cdots(i_{N/2}, j_{N/2})\rangle$$

variational parameters **superposition of spin-singlet pairs**

The term RVB was first coined by Pauling (1949) in the context of metallic materials.

RVB: Gutzwiller projection

- **Problem:** *Is there any simple way to obtain reasonable good variational parameters in RVB wave functions?*

Anderson and Zou: We may construct one from BCS wave-function…

$$\left|\Psi_{BCS}\right\rangle = \prod_k \left(u_k + v_k c^+_{k\uparrow} c^+_{-k\downarrow}\right)\left|0\right\rangle, \qquad \left|\Psi_{RVB}\right\rangle = P_G \left|\Psi_{BCS}\right\rangle$$

depend on some variational parameters, determined by Bogoliubov-de Gennes equation

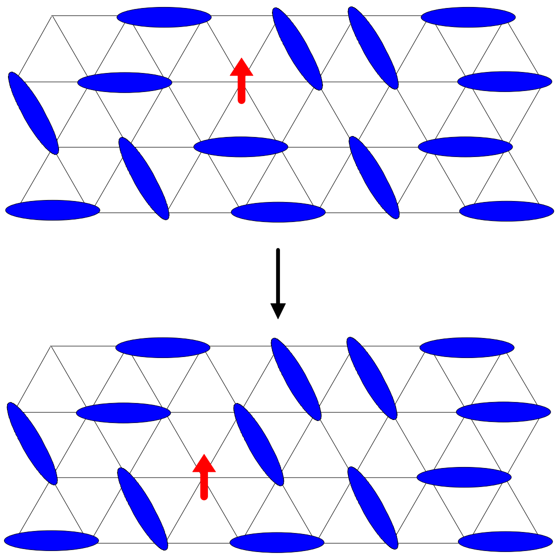
The number of electrons in a BCS wave-function is not fixed. There can be 0, 1, or 2 electrons in a lattice site. The Gutzwiller projection removes all the double occupied components. An insulator state is obtained when # of electron = # of lattice sites.

Later, physicist apply this type of wave-function to different lattice systems with different Hamiltonians and find interesting and rather good results when the $\left|\Psi_{BCS}\right\rangle$ *is chosen correctly.*

RVB: spinons and gauge fields

- **How about excited states?**

The spin excitations are electron-like (or superconductor quasiparticle-like, *S=1/2*) except that they do not carry charge, so called spinons.



$$\left|\Psi_{excited}\right\rangle = P_G \gamma^{+}_{k\sigma} \gamma^{+}_{-k\sigma'} \left|\Psi_{BCS}\right\rangle$$

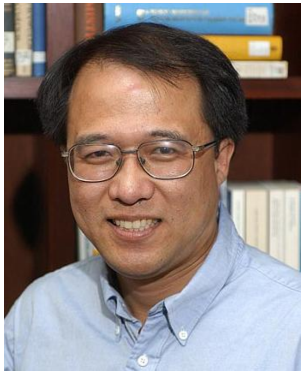
$$\gamma_{k\sigma} = u_k c_{k\sigma} + v_k c^{+}_{-k\sigma}$$

spions: S=1/2, charge neutral, mobile objects

Question: ***Can this simple picture survives Gutzwiller projection?***

RVB: spinon and gauge fields

- **Confinement of spinons**
  - *X.-G. Wen used the tool of lattice gauge theory to show that spinons may be confined with other spinons to form S=1 excitations after Gutzwiller projection.*
- **Structure of projected wave-function**
  - *X.-G. Wen also invented a new mathematical tools (Projective Symmetry Group) to classify spin-liquid states within the Gutzwiller-BCS approach.*



- **Schwinger bosons**
  - *People also use a similar approach where spins are represented by Schwinger bosons. Then spin excitations are S=1/2 bosons instead of fermions in projected BCS approach.*

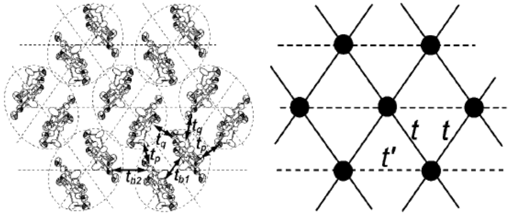
Other aspects of theory

- **Exact solvable models**
  - Kitaev honeycomb model changed our view on spin liquids singanificantly
    - A spin liquid is not necessary to be spin-rotation-invariant.
    - The statistics of spinons can be non-abelian.
    - Spin-orbital effect, etc.
  - …

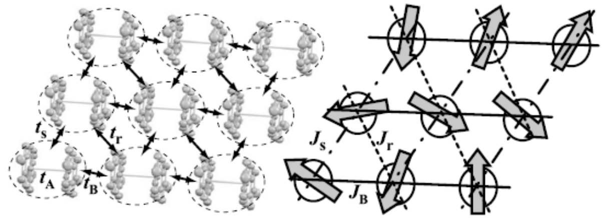
- **Other approaches**
  - Analytical methods
    - Bethe Ansatz, Bosonization, CFT, …
  - Numerical methods
    - Exact Diagonalization, DMRG, PEPS / Tensor Network, QMC, CDMFT, …
- **Spin liquids with spin S>1/2**
- **…**

for a review, see YZ, Kazushi Kanoda, Tai-Kai Ng (2014), invited by RMP

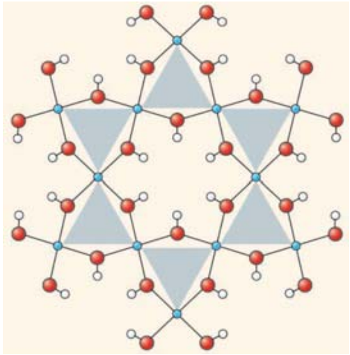
Existing *S=1/2* quantum spin liquid candidates at D>1



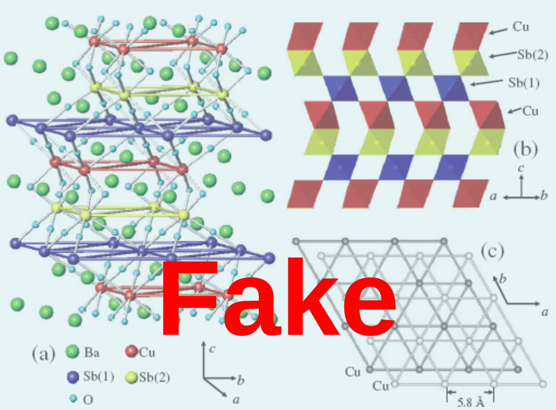
κ-(ET)$_2$Cu$_2$(CN)$_3$ (2003)



Pd-(dmit)$_2$(EtMe$_3$Sb) (2008)

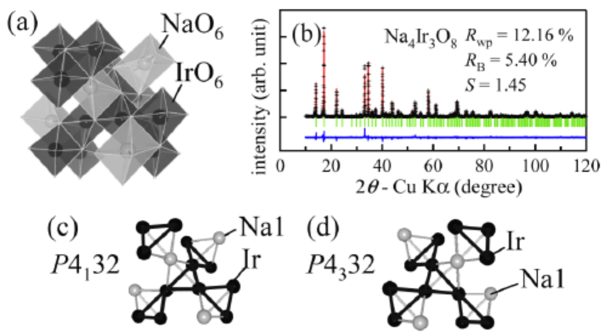


ZnCu$_3$(OH)$_6$Cl$_2$ (2007)

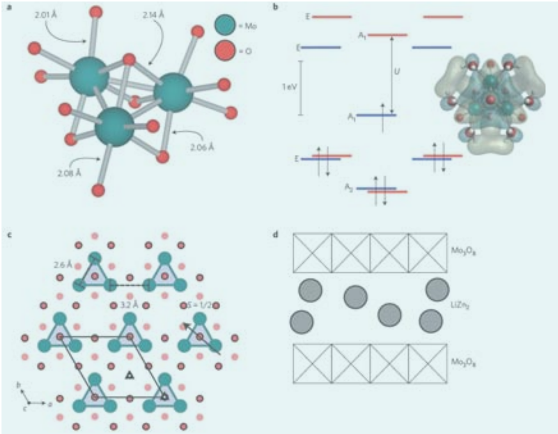


Fake

Ba$_3$CuSb$_2$O$_9$ (2011)
quasi 1D?



Na$_4$Ir$_3$O$_8$ (2007)



LiZn$_2$Mo$_3$O$_8$?(2012)

TABLE III Spin liquid materials summary

YZ, Kazushi Kanoda, Tai-Kai Ng (2014), invited by RMP

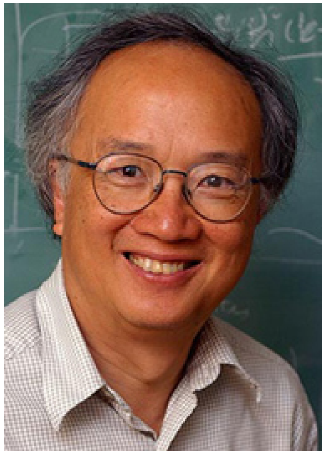
Material	Triangular, κ-$(\mathbf{ET})_2\mathbf{Cu}_2(\mathbf{CN})_3$	Triangular $\mathbf{M[Pd(dmit)_2]_2}$	Kagome $\mathbf{ZnCu_3(OH)_6Cl_2}$	Hyper-Kagome, $\mathbf{Na_4Ir_3O_8}$
Susceptibility	A broad peak at 60 K, Finite at 2 K, J =250 K (*1)	A broad peak at 50 K, Finite at 2 K, $J = 220 \sim 280$ K (*7)	Curie-Weiss + upturn, Θ_W = -300 K, J =230 K, (*11, *12)	Curie-Weiss Θ_W = -650 K (*19, *20)
Specific heat	Gapless, γ =15 mJ/K^2 mol, Field-independent (*2)	Gapless, γ=20 mJ/K^{2}mol, Field-independent (*8)	Gapless, $C \sim T^\alpha$ at low-T, $\alpha \lesssim 1$ (*13), $\alpha = 1.3$ (*14), Field-dependent broad peak (*11, *13)	Gapless, $C \sim T^2$ (*19), $C \sim \gamma T + \beta T^{2.4}$, γ = 2 mJ/K^{2}mol (*20), Field-independent (*19, *20)
Thermal conductivity	Gapped; $\Delta = 0.46$ K (*3)	Gapless; finite κ/T (*9)		
NMR shift	Not precisely resolved (*4)	Not precisely resolved (*10)	A broad peak at 50 K for ^{17}O (*14), at 25-50 K for ^{35}Cl (*15), Finite at low-T (*14)	
NMR $1/T_1$	Inhomogeneous $1/T_1$, Power law, ^{1}H $1/T_1; \sim T$ / $\sim T^2$ at $T <$ 0.3K (two components) (*1), ^{13}C $1/T_1; \sim 1/T^{1.5}$ at $T < 0.2$ K (stretched exponential) (*4)	Inhomogeneous $1/T_1$, Power law, ^{13}C $1/T^2$ at $<$ 0.5 K (stretched exponential) (*10)	Power law, $1/T_1 \sim T^\alpha$ at low-T $\alpha \sim 0.73$ for ^{17}O (*14), $\alpha \sim$0.5 for ^{63}Cu (*15), Field-induced spin freezing (*16)	
μSR	No internal field at 0 T (*5,*6)		No internal field at 0 T (*17)	
Neutron			Inelastic scattering $\sim$ no excitation gap (*11), Fractionalized excitations with a continuum (*18)	

All existing QSL candidates are fermion-like gapless systems.

Experimental detection of spin liquid states?

- **How to identify and characterize spin liquid states in established materials?**

Patrick A. Lee: All these materials may be described by some kind of projected BCS (or Fermi liquid) state at low temperature.



P.A. Lee is perhaps the strongest believer of spin liquid states. He works most closely with experimentalist to show that spin liquids exist in nature.

He noticed a common feature of most of the spin liquid candidates discovered so far: they are all closed to the metal to insulator transition.

$\Rightarrow$ Question: **What is the physical significance of this observation?**

Further experimental proposals

- **Optical conductivity: gapless spinons, power law behavior**

  Tai-Kai Ng and P. A. Lee, PRL 99, 156402 (2007).

  Expts: *1) κ-ET organic salt, S. Elsässer, et. al. (U. Stutggart group), PRB 86, 155150 (2012).*

  2) Herbertsmithite, D. V. Pilon, et. al. (MIT group), Phys. Rev. Lett. 111, 127401 (2013).

- **GMR-like setup: oscillatory coupling between two FMs via a QSL spacer**

  M. R. Norman and T. Micklitz, PRL 102, 067204 (2009).

- **Thermal Hall effect: different responses between magnons and spinons**

  H. Katsura, N. Nagaosa, and P. A. Lee, Phys. RRL 104, 066403 (2010). in contradiction to expts.

- **Sound attenuation: spinon-phonon interaction, spinon lifetime, gauge fields**

  YZ and Patrick A. Lee, PRL. 106, 056402 (2011).

- **ARPES: electron spectral function for a QSL with spinon FS or Dirac cone**

  E. Tang, M.P.A. Fisher and Patrick A. Lee, PRB, 045119 (2013).

- **Neutron scattering: spin chirality, DM interaction, Kagome lattice**

  N. Nagaosa and Patrick A. Lee, PRB 87, 064423 (2013).

- **Spinon transport: measure spin current flow through M-QSL-M junction**

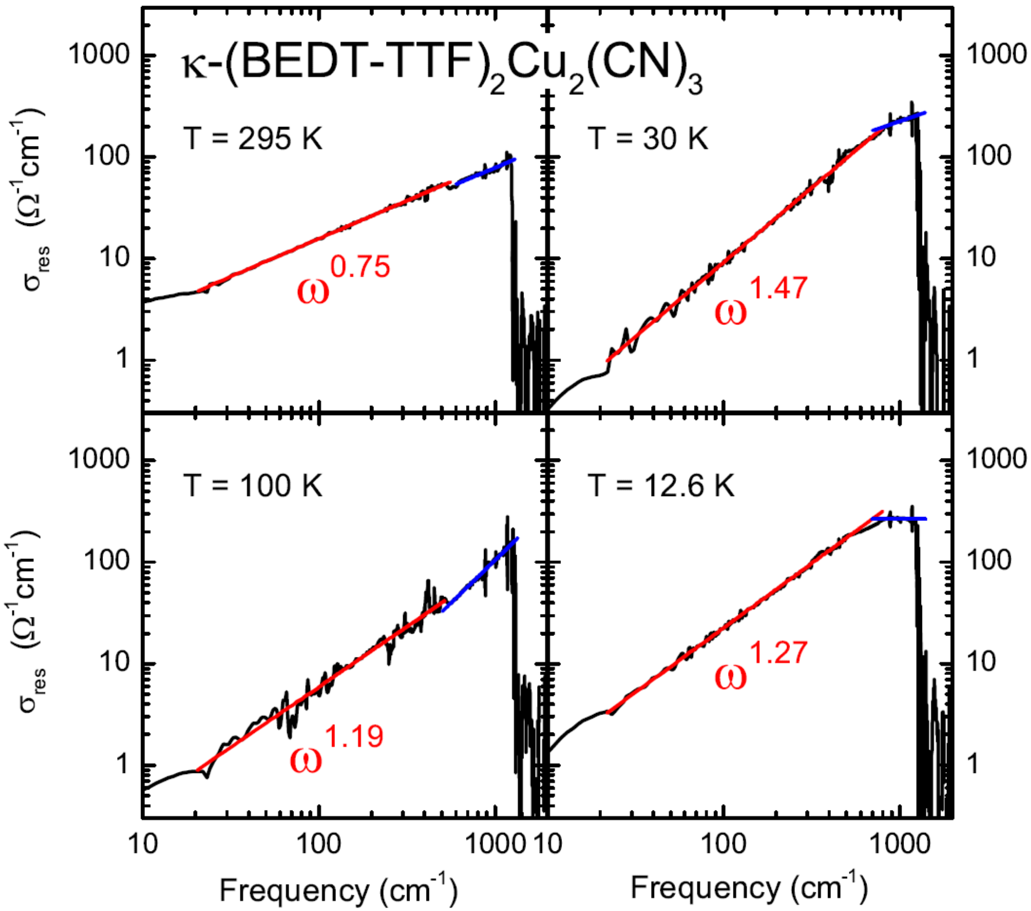
  C.Z. Chen, Q.F.Sun, Fa Wang and X.C. Xie, PRBB 88, 041405(R) (2013).

Motivation 1: A generic framework to compare theoretical predictions of QSLs to experimental data is still missing at the phenomenological level.

PHYSICAL REVIEW B **86**, 155150 (2012)

Power-law dependence of the optical conductivity observed in the quantum spin-liquid compound κ-(BEDT-TTF)$_2$Cu$_2$(CN)$_3$

Sebastian Elsässer, Dan Wu, and Martin Dressel*
Physikalisches Institut, Universität Stuttgart, Pfaffenwaldring 57, D-70550 Stuttgart, Germany
John A. Schlueter
Material Science Division, Argonne National Laboratory, Argonne, Illinois 60439-4831, USA



$$\sigma(\omega) \sim \omega^{\eta}$$

$$\eta = \begin{cases} 3.33, & \omega > \dfrac{k_B T}{\hbar}, \dfrac{1}{\tau_{el}} \\ 2, & \omega < \dfrac{k_B T}{\hbar}, \dfrac{1}{\tau_{el}} \end{cases}$$

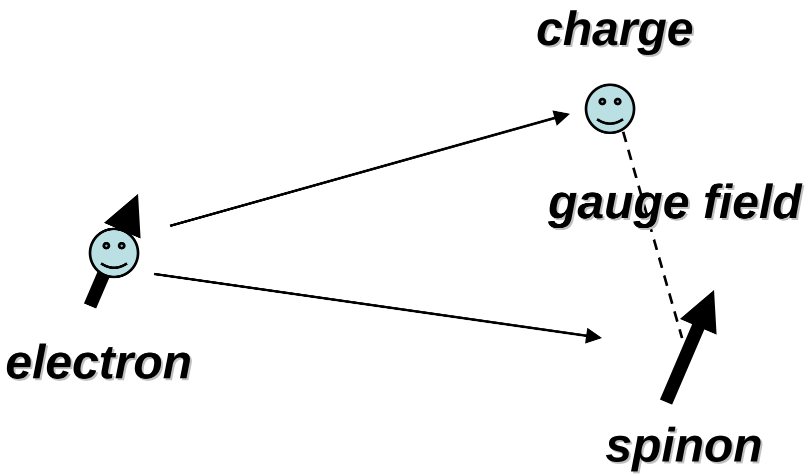
T.K.Ng and P.A.Lee (2007)

$1\,\mathrm{cm}^{-1} \sim 3\times10^{10}\,\mathrm{Hz}$

$\hbar \sim 6.6\times10^{-16}\,\mathrm{eV\,s}$

$1\,\mathrm{cm}^{-1} \sim 2\times10^{-5}\,\mathrm{eV} \sim 0.2\,\mathrm{K}$

Difficulties in QSL theory



$$\hat{c}^{+} = \hat{f}^{+}\hat{h},$$
$$\hat{f} \rightarrow e^{i\theta}\hat{f},$$
$$\hat{h} \rightarrow e^{i\theta}\hat{h}.$$

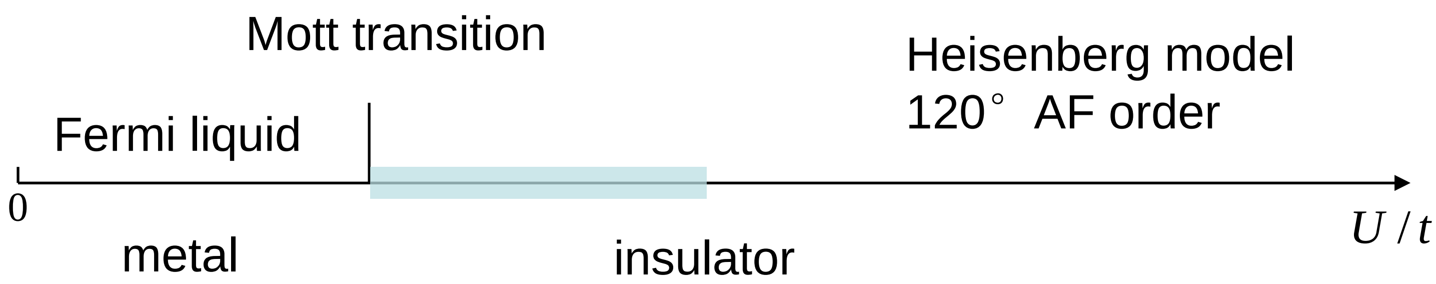
U(1) gauge structure

- What's the low energy effective theory for QSLs?
  - Confinement vs. deconfinement
  - Spinons are *not well defined* quasiparticles even though the *U(1)* gauge field is deconfined. $\Sigma'' \propto \omega^{2/3}$
- Lack of Renormalization-group scheme to illustrate possible effective theories as in Fermi liquid theory.

Motivation 2: Could we construct an effective theory with "electrons" or "dressed electrons" (quasiparticles) directly?

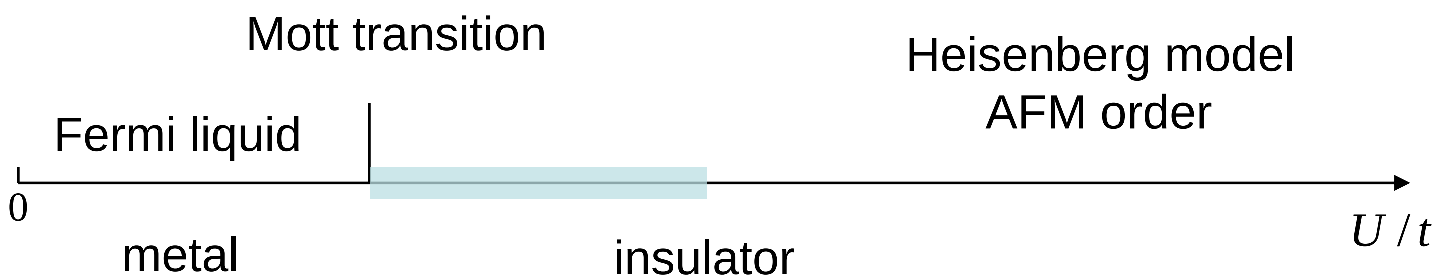
Spin liquids in the vicinity of metal-insulator transition

- **Pressure effect**
  - ***κ*-(ET)$_2$Cu$_2$(CN)$_3$** : Z_2 QSL → superconductor
  - ***Pd-(dmit)$_2$(EtMe$_3$Sb)*** : *U(1)* QSL → metal
  - ***Na$_4$Ir$_3$O$_8$*** : QSL → metal
- **Importance of charge fluctuations**

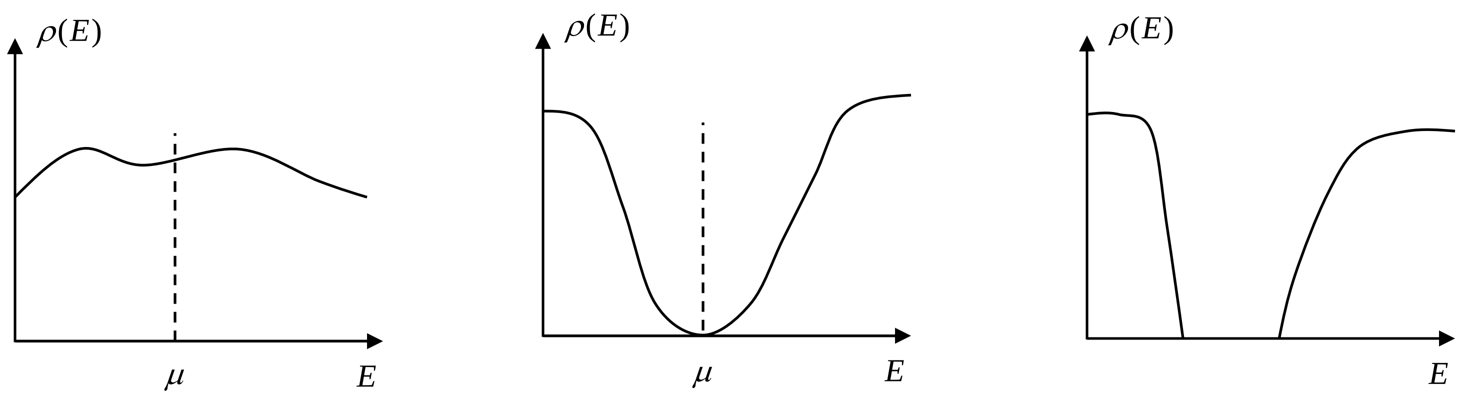


Schematic phase diagram on triangular lattice

QSL as a soft-gap Mott insulator



Schematic DOS (from optical conductivity and other expts.)



$$\rho(E) \propto |E-\mu|^{\eta_\pm}$$

Questions

- What kind of charge fluctuations should be the key to QSLs?
- Can we formulate a phenomenological theory for QSLs in the vicinity of Mott transition starting from the metallic side?

Fermi Liquid theory

Quasiparticles

- When electron-electron interactions are adiabatically turned on, the *low energy excited states* of interacting *N*-electron systems evolve in a continuous way, and therefore remain one-to-one correspondence with the states of noninteracting *N*-electron systems.
- **Assumption**: The same labeling scheme through fermion occupation number can be applied to fermionic QSLs.

Interaction between quasiparticles

$$\delta E = \sum_{\mathbf{p}\sigma}\left(\frac{p^2}{2m^*} - \mu\right)\delta n_{\mathbf{p}\sigma} + \frac{1}{2}\sum_{\mathbf{pp}'\sigma\sigma'} f_{\mathbf{pp}'}^{\sigma\sigma'}\delta n_{\mathbf{p}\sigma}\delta n_{\mathbf{p}'\sigma'} + O\left(\delta n^3\right)$$

$$\delta E = E - F_0, \quad \delta n_{\mathbf{p}\sigma} = n_{\mathbf{p}\sigma} - n_{\mathbf{p}\sigma}^0$$

Quasiparticle energy

$$\tilde{\varepsilon}_{\mathbf{p}\sigma} = \frac{p^2}{2m^*} + \sum_{\mathbf{p}'\sigma'} f_{\mathbf{pp}'}^{\sigma\sigma'}\delta n_{\mathbf{p}'\sigma'}$$

Landau parameters

Spin symmetric and antisymmetric decomposition

$$f_{\mathbf{pp'}}^{\sigma\sigma'} = f_{\mathbf{pp'}}^{s}\delta_{\sigma\sigma'} + f_{\mathbf{pp'}}^{a}\sigma\sigma'$$

Isotropic systems

3D:

$$f_{\mathbf{pp'}}^{s(a)} = \sum_{l=0}^{\infty} f_l^{s(a)} P_l(\cos\theta)$$

2D:

$$f_{\mathbf{pp'}}^{s(a)} = \sum_{l=0}^{\infty} f_l^{s(a)} \cos(l\theta)$$

Dimensionless Landau parameters

$$\boxed{F_l^{s(a)} = N(0) f_l^{s(a)}}$$

Ideas

- **Effective theory with *chargeful* quasiparticles**
  - **"Building blocks" are chargeful quasiparticles instead of spinons.**
  - **Both Fermi liquids and quantum spin liquids can be described within the same framework.**
- **Physical quantities will be renormalized**
  - **By the interaction between quasiparticles.**
- **Electrically insulating but thermally conducting state**
  - **Can be achieved in the framework of Landau's Fermi liquid theory with properly chosen Landau parameters.**

Quasi-particle transport (I)

Particle (charge or mass) current carried by quasi-particles

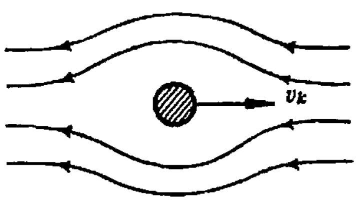
$$\mathbf{J} = \sum_{\mathbf{p}} \delta\tilde{n}_{\mathbf{p}} \mathbf{v}_{\mathbf{p}} = \sum_{\mathbf{p}} \delta n_{\mathbf{p}} \mathbf{j}_{\mathbf{p}}, \quad \mathbf{j}_{\mathbf{p}} = \mathbf{v}_{\mathbf{p}} - \sum_{\mathbf{p}'} f^{s}_{\mathbf{pp}'} \frac{\partial n^0}{\partial \varepsilon_{\mathbf{p}'}} \mathbf{v}_{\mathbf{p}'}$$

$$\delta\tilde{n}_{\mathbf{p}} = n_{\mathbf{p}} - \tilde{n}^0_{\mathbf{p}}$$

$\tilde{n}^0_{\mathbf{p}} = n_F(\tilde{\varepsilon}_{\mathbf{p}} - \mu)$ is local equilibrium occupation number

$$\Rightarrow \quad \boxed{\mathbf{J} = \frac{m}{m^*}\left(1 + \frac{F_1^s}{d}\right)\mathbf{J}^0}$$

dimension



where $\mathbf{J}^0$ is the current carried by corresponding non-interacting quasiparticles.

Quasi-particle transport (I)

$$\boxed{\mathbf{J} = \frac{m}{m^*}\left(1+\frac{F_1^s}{d}\right)\mathbf{J}^0}$$

***Galilean invariance* is broken by the periodic crystal potential**

$$\Rightarrow \quad \frac{m^*}{m} \neq 1+\frac{F_1^s}{d}$$

charge current is always renormalized by the interaction.

The electron system will be electrically insulating when

$$1+F_1^s/d \to 0 \quad \text{or} \quad m/m^* \to 0$$

Quasi-particle transport (II)

Thermal current carried by quasi-particles

$$\mathbf{J}_Q = \sum_{\mathbf{p}} \delta\tilde{n}_{\mathbf{p}}(\varepsilon_{\mathbf{p}} - \mu)\mathbf{v}_{\mathbf{p}} = \sum_{\mathbf{p}} (\varepsilon_{\mathbf{p}} - \mu)\mathbf{v}_{\mathbf{p}} \left(\delta n_{\mathbf{p}} - \sum_{\mathbf{p}'} \frac{\partial n^0}{\partial \varepsilon_{\mathbf{p}}} f^s_{\mathbf{pp}'} \delta n_{\mathbf{p}'} \right)$$

$$\Rightarrow \quad \boxed{\mathbf{J}_Q = \frac{m}{m^*} \mathbf{J}^0_Q}$$

Thermal current is renormalized by the factor m/m^*

A quantum spin liquid phase will be achieved through

$$\boxed{1 + \frac{F^s_1}{d} \to 0, \quad \frac{m}{m^*} \neq 0}$$

strong charge (current) fluctuations in the $l = 1$ ***channel***

Framework of effective theory

Effective theory = Landau Fermi-liquid-type theory with chargeful quasiparticles + singular Landau parameters

Elementary excitations (particle-hole excitations) described by Landau transport equation is chargeless at $q=0$ and $\omega=0$, but charges are recovered at finite q and ω.

Thermodynamic quantities

Specific heat ratio

$$\gamma = \frac{C_V}{T} = \frac{m^*}{m}\gamma^{(0)}$$

Pauli susceptibility

$$\chi_P = \frac{m^*}{m}\frac{1}{1+F_0^a}\chi_P^{(0)}$$

Wilson ratio

$$R_W = \frac{4\pi^2 k_B^2 \chi_P}{3(g\mu_B)^2\gamma} = \frac{1}{1+F_0^a}$$

Electromagnetic responses

Charge response function

$$\chi_d(\mathbf{q},\omega) = \frac{\chi_{0d}(\mathbf{q},\omega)}{1-\left(F_0^s + \dfrac{F_1^s(\mathbf{q},\omega)}{F_1^s(\mathbf{q},\omega)+d}\dfrac{\omega^2}{v_F^2 q^2}\right)\dfrac{\chi_{0d}(\mathbf{q},\omega)}{N(0)}}$$

Transverse current response function

$$\chi_t(\mathbf{q},\omega) = \frac{\chi_{0t}(\mathbf{q},\omega)}{1-\dfrac{F_1^s(\mathbf{q},\omega)}{F_1^s(\mathbf{q},\omega)+d}\dfrac{\chi_{0t}(\mathbf{q},\omega)}{N(0)}}$$

χ_{0d}, χ_{0t} : **for a Fermi liquid with effective mass *m** in the absence of Landau interactions**

A. J. Leggett, Phys. Rev. 140, A1869 (1965); Phys. Rev. 147, 119 (1966).

Longitudinal current response function

$$\chi_l(\mathbf{q},\omega) = \frac{\omega^2}{q^2}\chi_d(\mathbf{q},\omega)$$

AC conductivity

$$\sigma_{l(t)}(\mathbf{q},\omega) = \frac{e^2}{i\omega}\chi_{l(t)}(\mathbf{q},\omega)$$

In the limit $\quad 1 + F_1^s / d \to 0$

Expand it as $\quad \dfrac{1 + F_1^s(\mathbf{q},\omega)/d}{N(0)} \sim \alpha + \beta\omega^2 + \gamma_t q_t^2 + \gamma_l q_l^2$

$$U > U_c \Rightarrow \alpha = 0 \;\&\; \gamma_l = 0 \qquad \longleftarrow \quad \textbf{incompressibility}$$

The other possibility $F_0^s \to \infty$ would result in complete vanishing of charge response.

Dielectric function

$$\boxed{\varepsilon(\mathbf{q},\omega)=1-\frac{4\pi e^2}{q^2}\chi_d(\mathbf{q},\omega)\sim 1-4\pi\beta e^2 v_F^2 N(0)^2+O(q^2)}$$

Insulator: no screening effect even though we start with the metallic side.

AC conductivity

For q=0 and small ω,

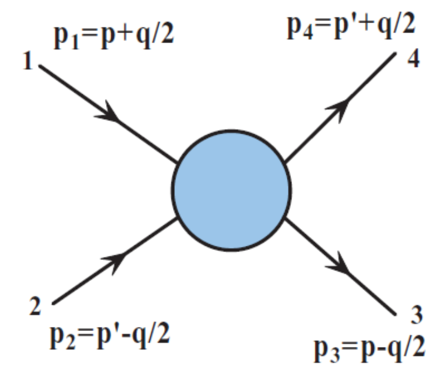
$$\boxed{\sigma(\omega)=\frac{\omega\sigma_0(\omega)}{\omega+[i/\beta e^2 N(0)^2]\sigma_0(\omega)}}$$

$$\Rightarrow \quad \boxed{\mathrm{Re}\left[\sigma(\omega)\right]\propto\omega^2\,\mathrm{Re}\left[\sigma_0(\omega)\right]}$$

Power law AC conductivity inside the Mott gap.

Quasipatricle scattering amplitude

$$A_{\mathbf{pp'}}(\mathbf{q},\omega) - \sum_{\mathbf{p''}} f_{\mathbf{pp''}} \chi_{0\mathbf{p''}}(\mathbf{q},\omega) A_{\mathbf{p''p'}}(\mathbf{q},\omega) = f_{\mathbf{pp'}}$$



where $$\chi_{0\mathbf{p''}}(\mathbf{q},\omega) = \frac{n^0_{\mathbf{p}-\mathbf{q}/2} - n^0_{\mathbf{p}+\mathbf{q}/2}}{\omega + \xi_{\mathbf{p}-\mathbf{q}/2} - \xi_{\mathbf{p}+\mathbf{q}/2}} \approx \frac{\mathbf{q}\cdot\mathbf{v}_{\mathbf{p}}}{\mathbf{q}\cdot\mathbf{v}_{\mathbf{p}} - \omega} \frac{\partial n^0_{\mathbf{p}}}{\partial \xi_{\mathbf{p}}}$$

Assuming that the scattering is dominating in the $l=1$ channel,

$$f_{\mathbf{pp'}} \sim \frac{\mathbf{p}\cdot\mathbf{p'}}{p_F^2} f_1^s$$

Using $\frac{1+F_1^s(\mathbf{q},\omega)/d}{N(0)} \sim \beta\omega^2 + \gamma_t q_t^2$, after some algebra, we obtain

$$\boxed{A_{\mathbf{pp'}}(\mathbf{q},\omega) \approx \frac{d}{N(0)} \frac{\mathbf{p}\cdot\mathbf{p'}}{p_F^2} \frac{1}{-ig\dfrac{\omega}{v_F q} + \gamma_t q^2}, \quad g = \begin{cases} 1, & d=2 \\ \dfrac{\pi}{2}, & d=3 \end{cases}}$$

Thermal conductivity

Thermal resistivity for a Fermi liquid

$$\frac{1}{\kappa}=\frac{1}{4}\sum_{1,2,3,4}W(1,2;3,4)n_1^0 n_2^0(1-n_3^0)(1-n_4^0)(\phi_1+\phi_2-\phi_3-\phi_4)^2\left(\sum_1 \phi_1\xi_1\mathbf{v}_1\cdot\mathbf{u}\frac{\partial n_1^0}{\partial\varepsilon_1}\right)$$
$$\times\delta(\varepsilon_1+\varepsilon_2-\varepsilon_3-\varepsilon_4)\delta_{\sigma_1+\sigma_2,\sigma_3+\sigma_4}\delta_{\mathbf{p}_1+\mathbf{p}_2,\mathbf{p}_3+\mathbf{p}_4}$$

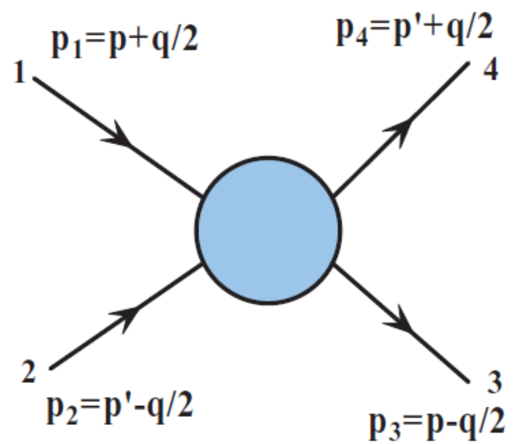
C.J. Pethick, Phys. Rev. 177, 393 (1969)

$$n_i=n_0-\phi_i\frac{\partial n_i^0}{\partial\varepsilon_i},\quad \mathbf{v}_i \text{ is quasiparticle velocity}, \mathbf{u} \text{ is an arbitrary unit vector along } \nabla T$$

Trial functions: $\phi_i=\xi_i\mathbf{v}_i\cdot\mathbf{u},\quad \xi_i=\varepsilon_i-\mu$

Scattering probability:

$$W(1,2;3,4)=2\pi\,|A_{\mathbf{pp}'}(\mathbf{q},\omega)|^2$$
$$\omega=\varepsilon_{\mathbf{p}+\mathbf{q}/2}-\varepsilon_{\mathbf{p}-\mathbf{q}/2}=\varepsilon_{\mathbf{p}'+\mathbf{q}/2}-\varepsilon_{\mathbf{p}'-\mathbf{q}/2}$$



Thermal resistivity due to inelastic scattering between quasiparticles

$$\frac{1}{\kappa_{in}} \propto \left(\frac{k_B T}{\varepsilon_F}\right)^{(d-1)/3}$$

Thermal resistivity due to elastic impurity scattering

At low temperature, the inelastic scattering is cut off by elastic impurity scattering rate $\frac{1}{\tau_0}$

$$\frac{\kappa_{el}}{T} = \frac{1}{d}\gamma^* v_F^2 \tau_0$$

$$\Rightarrow \quad \boxed{\frac{\kappa}{T} \propto \max\left[\frac{\hbar}{k_B^3}\left(\frac{k_B T}{\varepsilon_F}\right)^{(4-d)/3}, \frac{d}{\gamma^* v_F^2}\frac{1}{\tau_0}\right]^{-1}}$$

Consistent with U(1) gauge theory,

Cody P. Nave and Patrick A. Lee, Phys. Rev. B 76 235124 (2007).

Collective modes

Charge sector: density fluctuations

$$\delta n_{\rm p} = -\frac{\partial n^0_{\rm p}}{\partial \varepsilon_{\rm p}} \nu_{\rm p}$$

energy shift in the direction $\hat{p}$

spherical symmetry $\Longrightarrow$

$$\nu_{\rm p} = \sum_l \sum_{m=-l}^{l} Y_l^m(\theta_{\rm p}, \phi_{\rm p}) \nu_l^m$$

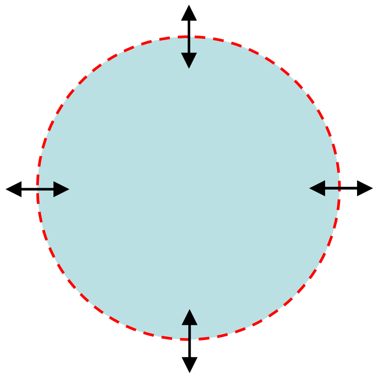
Equation of motion: linearized Landau transport equation

$$\frac{\partial \delta n_{\rm p}}{\partial t} + \vec{v}_{\rm p} \cdot \nabla_{\rm r} \left(\delta n_{\rm p} - \frac{\partial n^0_{\rm p}}{\partial \varepsilon_{\rm p}} \delta \varepsilon_{\rm p} \right) = I[n_{\rm p'}]$$

collision integral

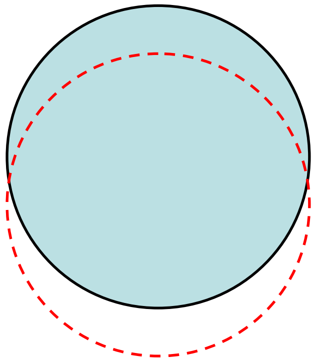
Density fluctuation modes in a system with spherical symmetry

$m = 0$



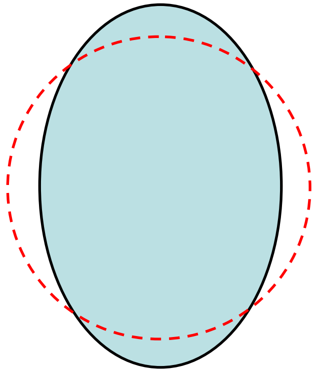
longitudinal mode

$m = 1$



transverse mode

$m = 2$



quadrupolar mode

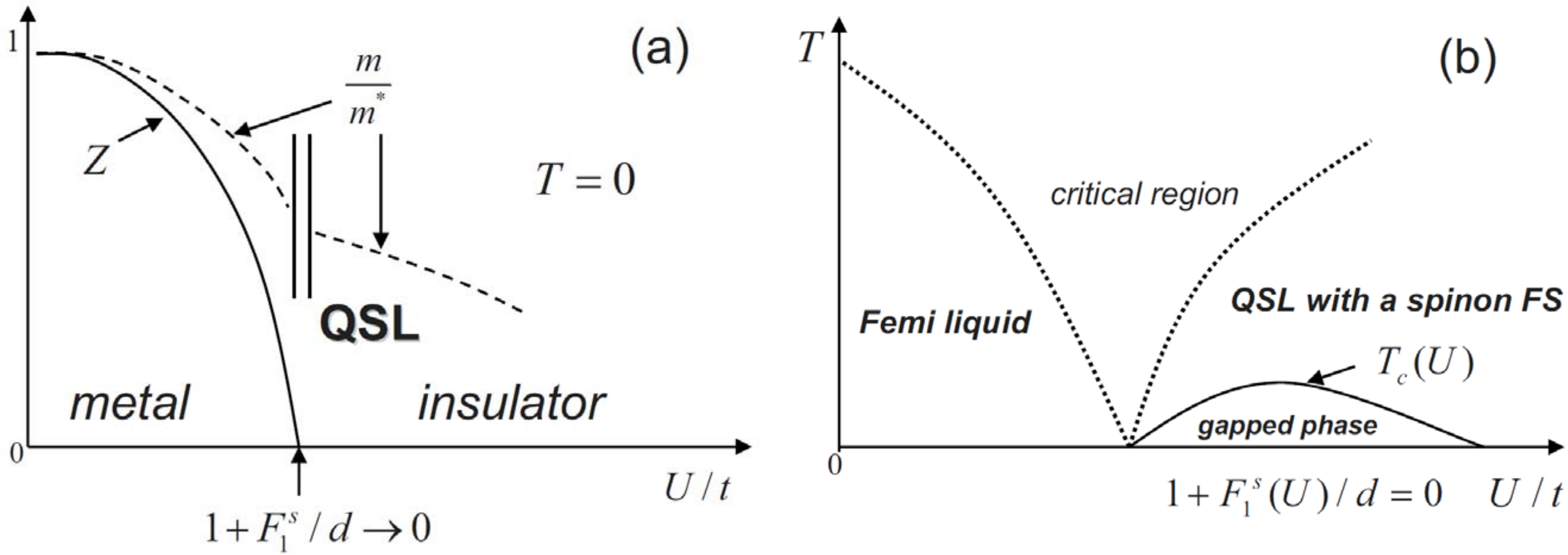
Zero sound modes in the spin liquid phase: $1+\frac{F_1^s}{3}=0$

Three channel model with only F_0^s, F_1^s and F_2^s : a weakly damping zero sound mode exists when $F_2^s > 10/3$.

$$s \equiv \frac{\omega}{q v_F}$$

$$s = \begin{cases} \sqrt{\frac{175}{F_2^s}}, F_2^s >> \frac{10}{3}, \\ 1 + 2e^{\frac{20-11F_2^s}{3F_2^s-10}}, 0 < F_2^s - \frac{10}{3} << 1 \end{cases}$$

Schematic phase diagram



Different from Brinkman-Rice picture

Summary

- Mott transition driven by current fluctuations
  - An alternative picture of metal-insulator transitions to Brinkman-Rice
  - QSL as a soft gap Mott insulator
- Phenomenological theory for both Fermi liquids and quantum spin liquids in the vicinity of Mott transition.
  - FL: both electrically and thermally conducting
  - QSL: electrically insulating but well thermally conducting
- Mott physics : characterized by many intrinsic in-gap excitations in such quantum spin liquids.
  - Wilson ratio ~ 1, ac conductivity, dielectric function, thermal conductivity
  - There exist collective modes as well as “quasiparticles”

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Thank you for attention !