Chern/Flat bands and correlation effects

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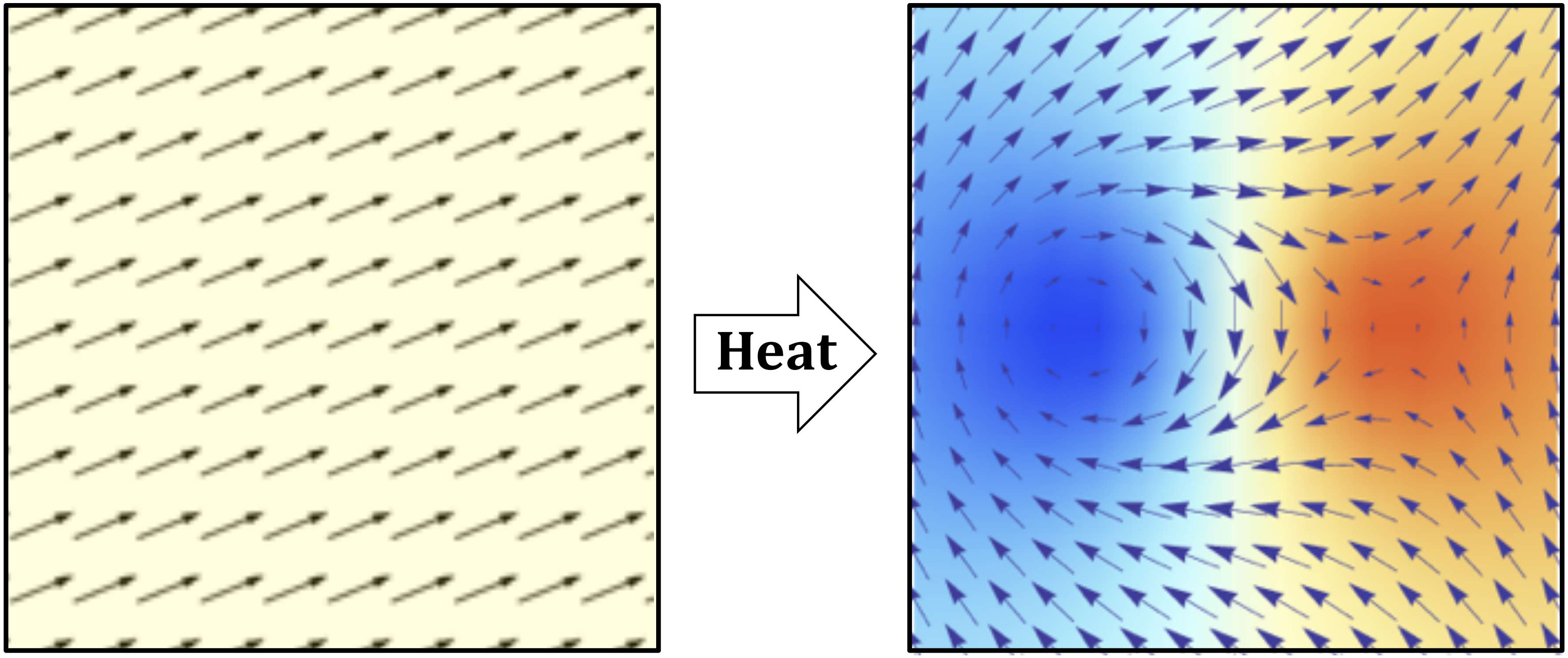
[IAS Tsinghua, 24 Apr 2018]

University of Toronto — NSERC CRSNG — CIFAR Canadian Institute for Advanced Research

Berezinskii-Kosterlitz-Thouless transition



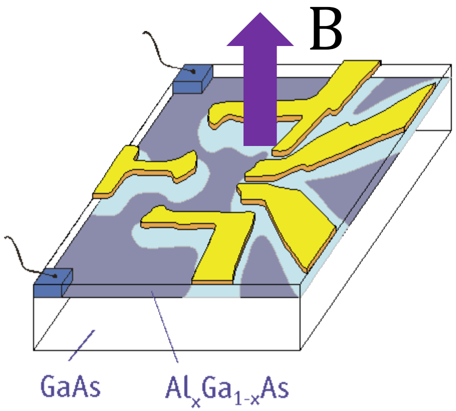
Ordered ferromagnet

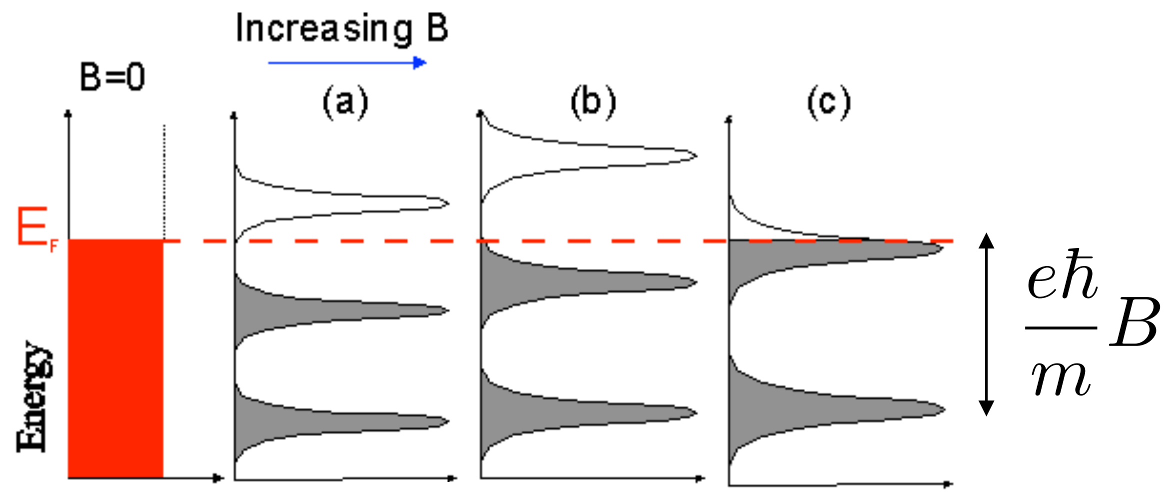
Vortex-Antivortex pair

- Proliferation of **Topological Defects: vortices, antivortices**
- Magnets, Superfluids, Superconductors, Early Universe

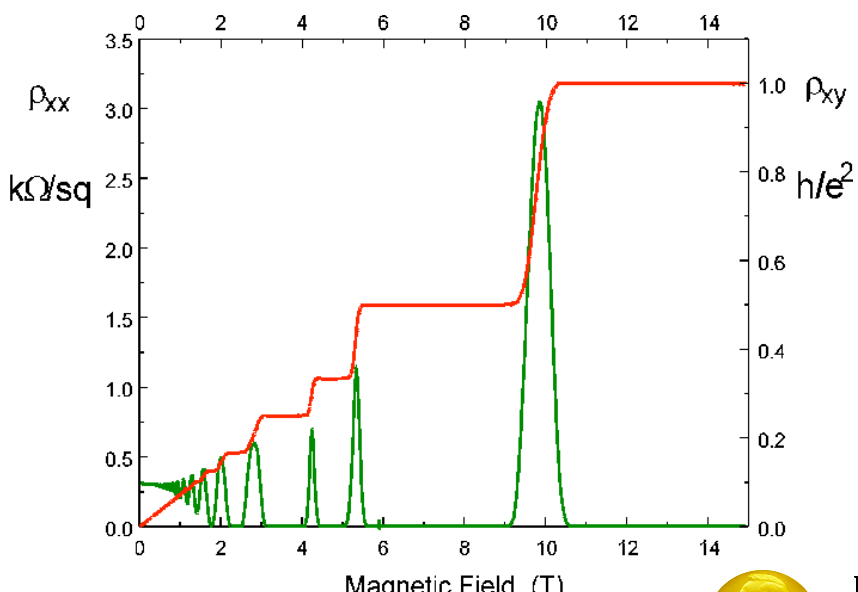
Berezinskii + Kosterlitz, Thouless
Topological ideas in condensed matter

Quantum Hall Effect





Landau levels

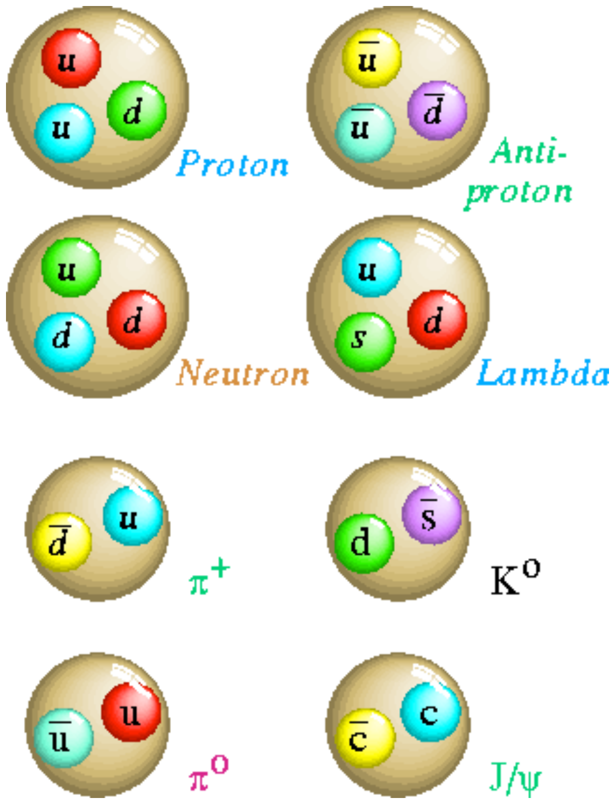


- New high mobility heterostructures
- High B field measurements

- Precise quantization in a “dirty” system!
- New standard of resistance
- **Topological ideas in quantum matter**

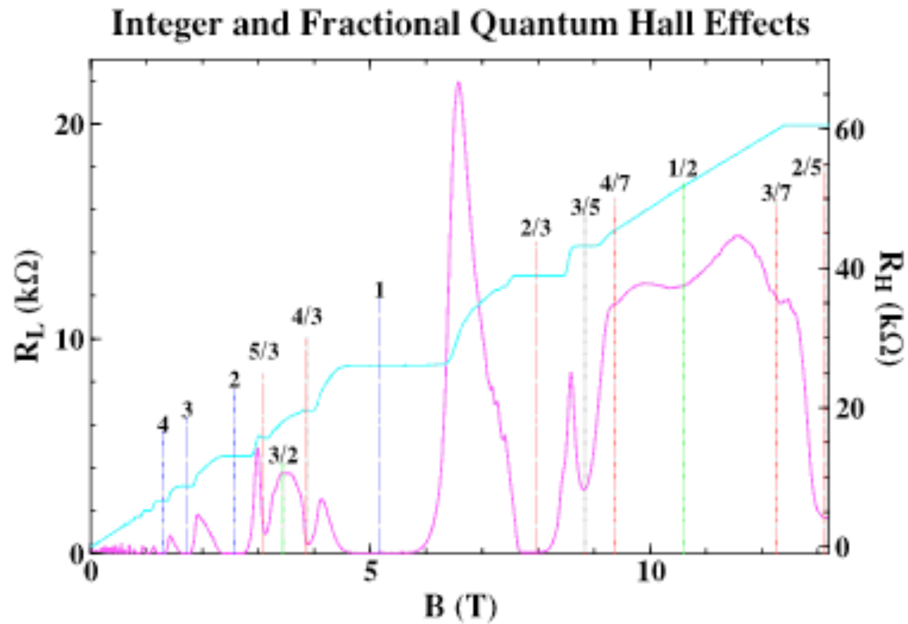
K. Von Klitzing (1985) / Tsui,Laughlin,Stormer (1988)
Thouless, Kosterlitz, Haldane (2016)

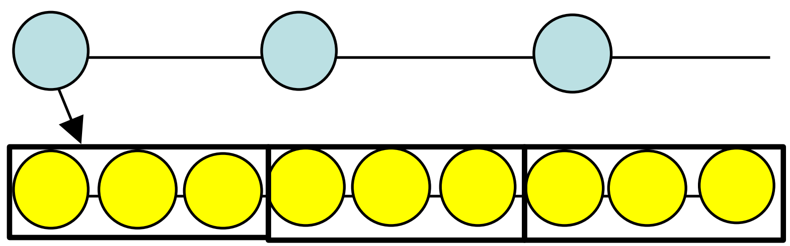
Fractionalization: Interaction driven physics in a fractionally filled Landau level



Quarks
Building blocks of hadrons, confined at low energies

(u,c,t)= +2/3
(d,s,b)= -1/3





R.B. Laughlin
J. K. Jain
X.G. Wen

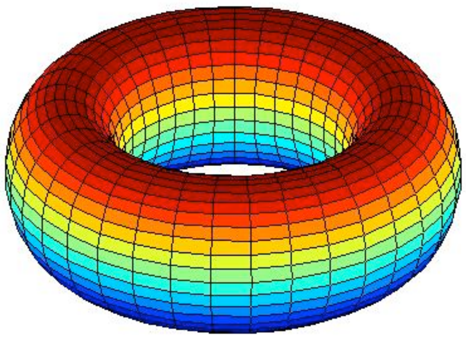
. **(1/3)-Laughlin liquid: Electron ~ 3 'partons'**
. **Electron FQHE = IQHE of fractional charges**
. **Kills enormous entropy**
. **Emergent charge-e/3 quasiparticle**

Can we realize quantum Hall effects in crystalline solids?

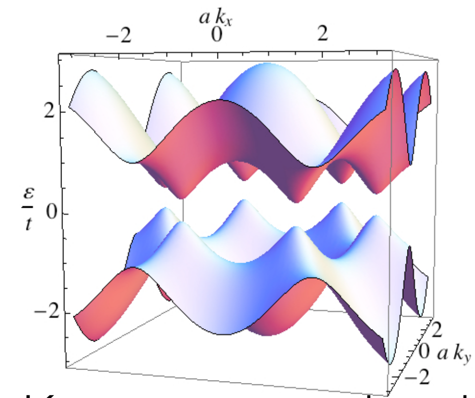
- **Without large magnetic fields**
- **Without ultralow temperatures**
- **With interaction-driven emergent phases**

Bands in Crystals – Momentum Space Topology

Crystal momentum



2D Brillouin zone: Torus



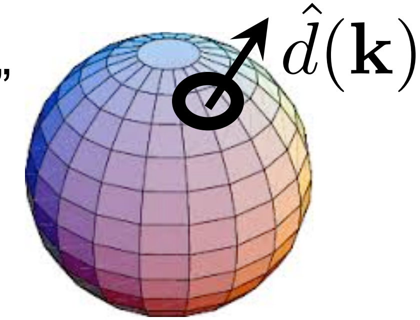
K-space energy bands
+ Bloch wavefunctions

Two-Band System

$$H(\mathbf{k}) = \vec{d}(\mathbf{k}) \cdot \vec{\sigma}$$ → pseudospin

$$E_{\pm}(\mathbf{k}) = \pm|\vec{d}(\mathbf{k})|$$

"Bloch sphere"

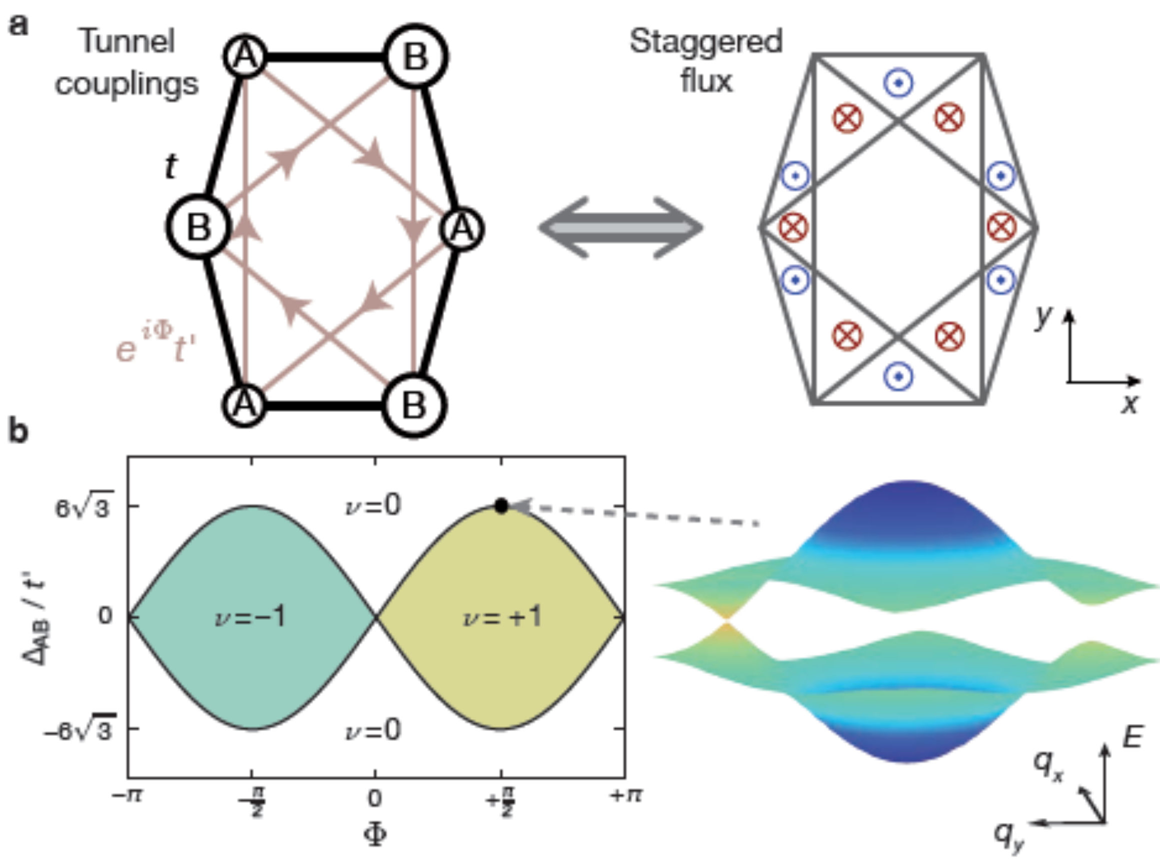


$\hat{d}(\mathbf{k})$: Information about wavefunction

$$\int \frac{dk_x dk_y}{4\pi} \hat{d}(\mathbf{k}) \cdot \partial_x \hat{d}(\mathbf{k}) \times \partial_y \hat{d}(\mathbf{k})$$

→ Topological invariant (Chern number)

Haldane model

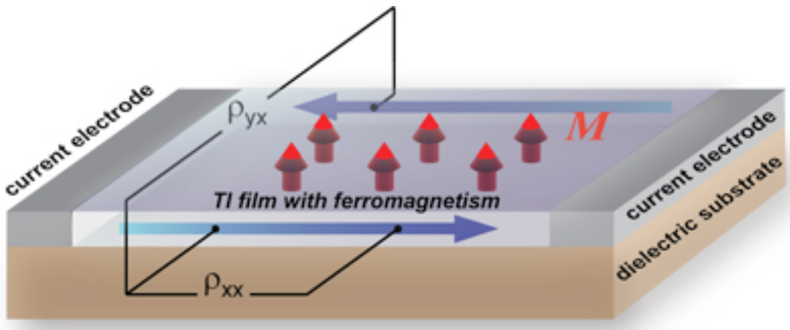


- Time-reversal broken, but "no net magnetic field"
- Bands carry nonzero Chern numbers: Quantum Hall effect
- **Chern bands ~ Landau levels**

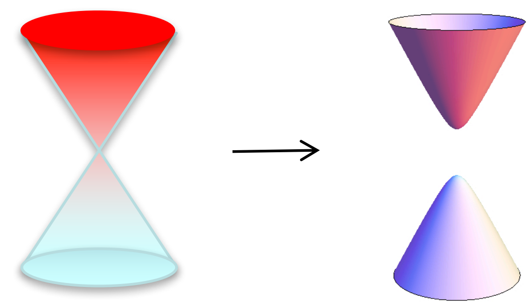
DR Hofstadter, PRB 1976; Thouless, Kohmoto, Nightingale, Nijs, PRL 1982; FDM Haldane, PRL 1988; G. Jotzu, et al (Esslinger group) Nature 2014

Quantum Anomalous Hall Effect

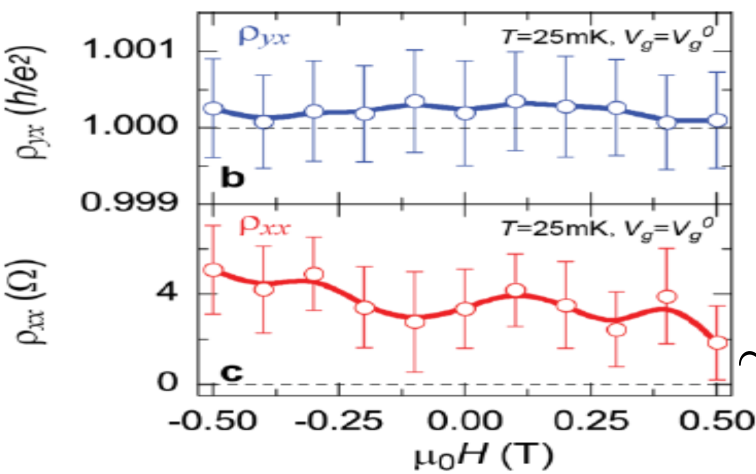
- Topological insulators: Surface Dirac Fermions
- Breaking time-reversal with dopant magnetization, no B-field!
- Spin orbit coupling is crucial: Gives a **“Lorentz force”**



R. Yu, et al (Science 2010)



Surface Dirac fermions get gapped



$$\sim 10^{-4} \frac{h}{e^2}$$

$(Bi,Sb)_2Te_3$ film doped with Cr or V

Early results

Ferromagnetic Tc ~ 10K
Hall quantization: T ~ 25 mK

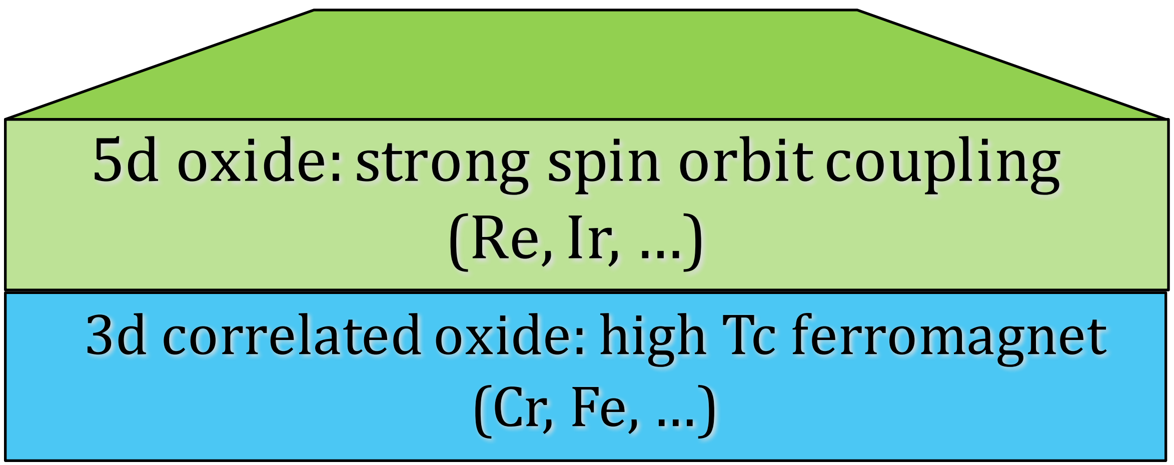
C.Z. Chang et al, Science 2013 (Xue group, Tsinghua)
C.Z. Chang et al, arXiv (M. Chan + J. Moodera groups)
A. J. Bestwick et al, arXiv (Goldhaber-Gordon group)
A. Kandala, et al, arXiv (N. Samarth group)

Can one induce nontrivial band topologies in strongly correlated materials?

- **Transition metal oxide layers:** Quantum anomalous Hall effects

(Di Xiao, Zhu, Okamoto, Y Ran, Nagaosa, Okamoto, Nat Comm 2011; Fiete, PRB 2011, PRL 2013)

Multicomponent (111) bilayer thin films
Predict high Tc quantum anomalous Hall effect



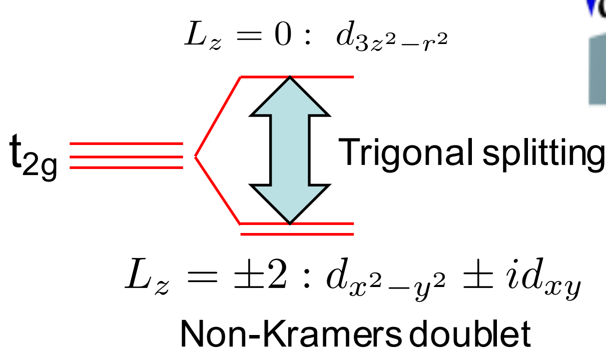
DFT+Monte Carlo

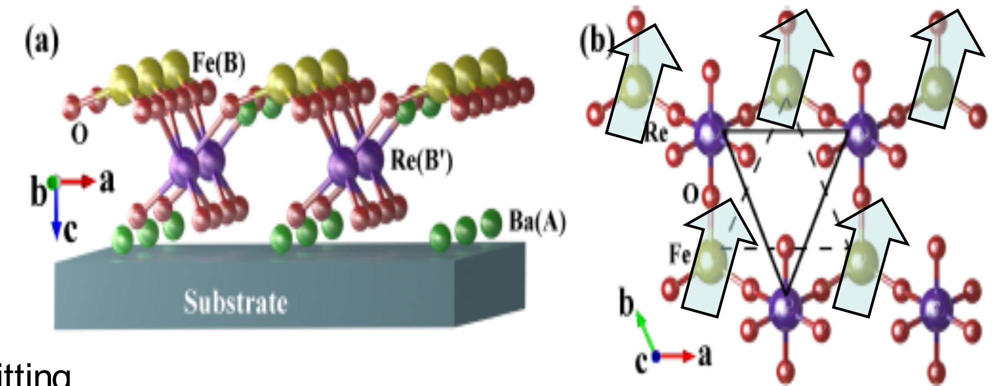
QAH gap ~ 100 meV
Ferromagnetic Tc ~ 300 K

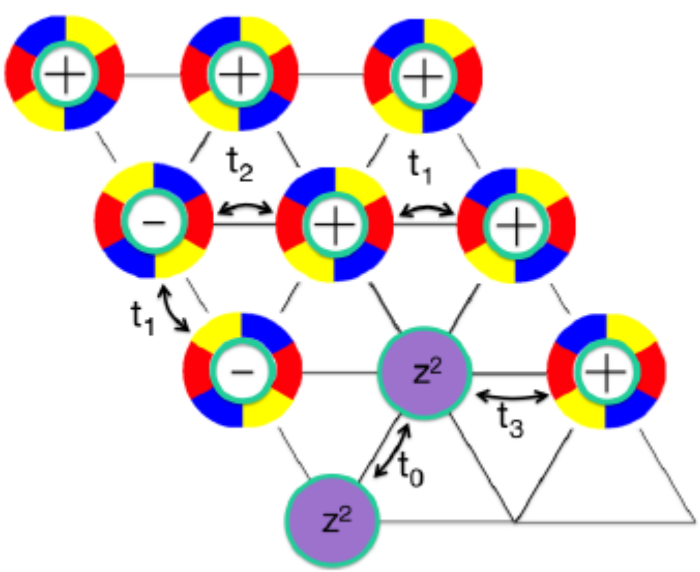
A. M. Cook, **AP** (***PRL*** 2014); S. Baidya, U.Waghmare, **AP**,T.Saha-Dasgupta (PRB 2015+16)

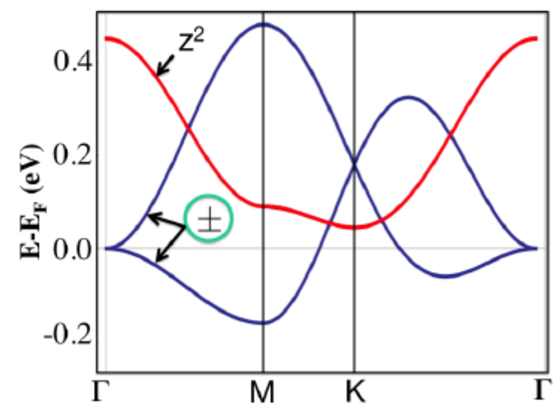
Half-metallic double perovskite (111) bilayer

- Triangular half-metal







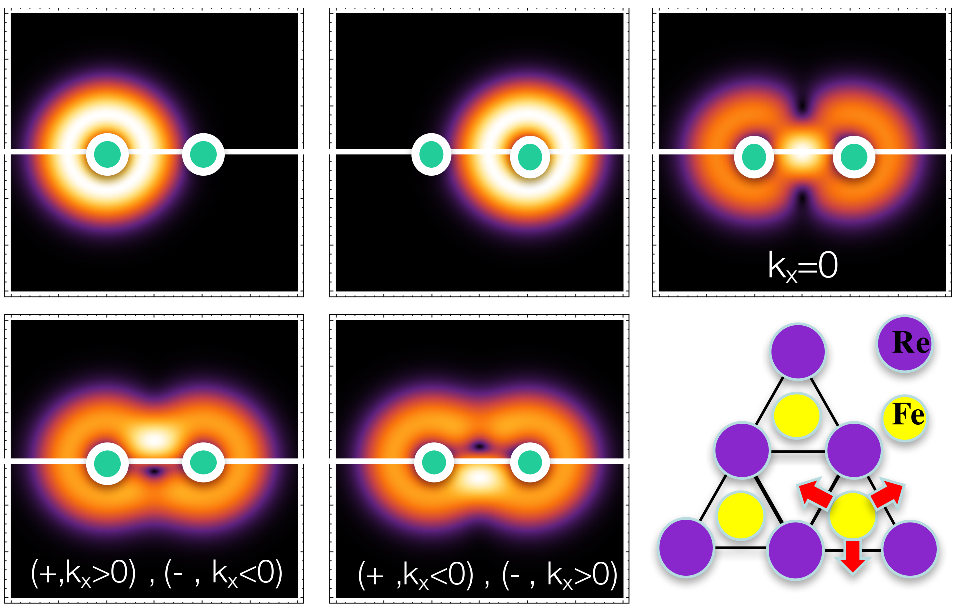


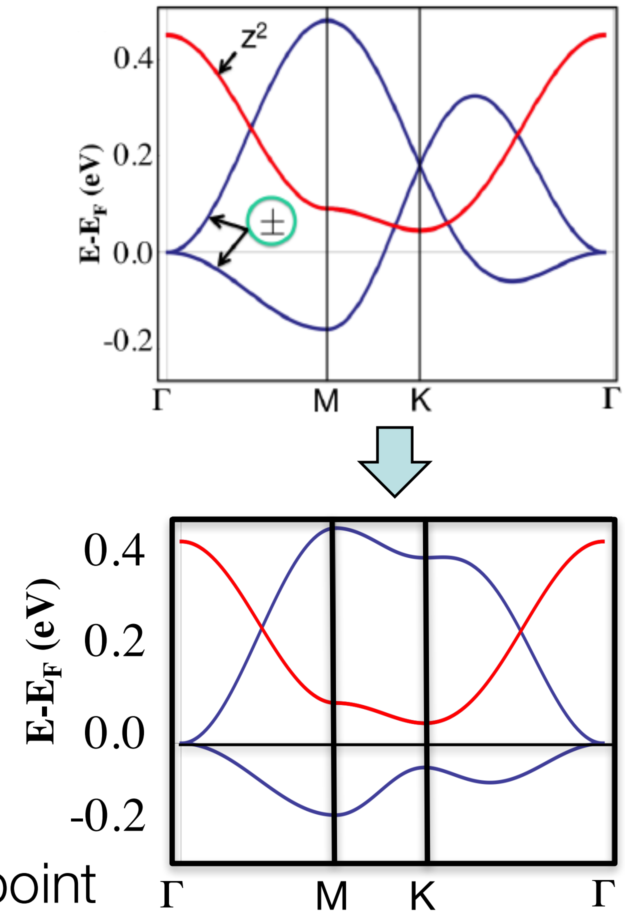
Dirac points at K: Inversion + “Time-reversal”
Quadratic band touching at Γ: C3 + “Time reversal”

S. Baidya, U. Waghmare, AP, T. Saha-Dasgupta (PRB 2016)

“Orbital Dipoles” and the “Orbital Rashba effect”

[“Orbital Rashba”, Jung-Hoon Han group, PRL 2011, PRB 2012]





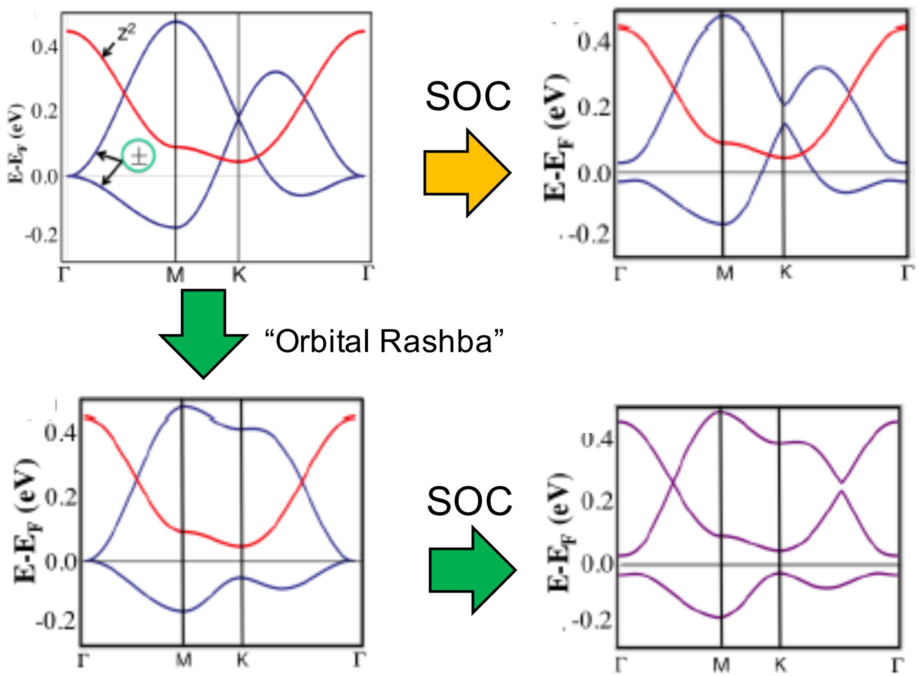
Inversion-symmetry breaking

- “Orbital Rashba” effect gaps out K Dirac point
- Half-semimetal

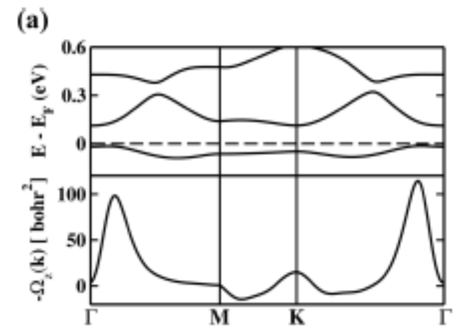
S. Baidya, U. Waghmare, AP, T. Saha-Dasgupta (PRB 2016)

Large gap quantum anomalous Hall insulator

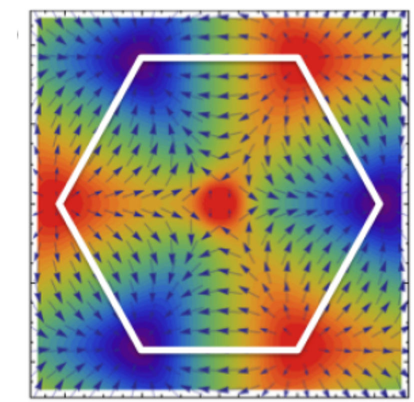
SOC: $-\lambda \langle S_z \rangle L_z$



QAH gap ~ 100 meV
Ferromagnetic Tc ~ 250 K



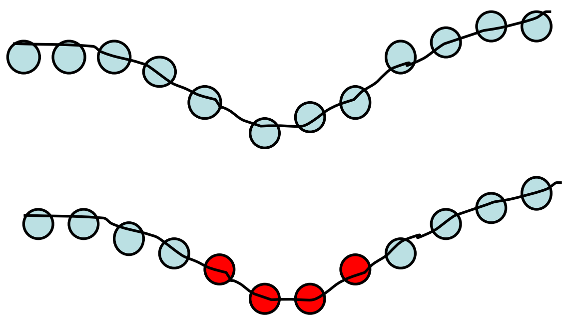
ab initio: GGA+SOC



k-space skyrmion

S. Baidya, U. Waghmare, AP, T. Saha-Dasgupta (PRB 2016)

Fractionalization in Chern bands



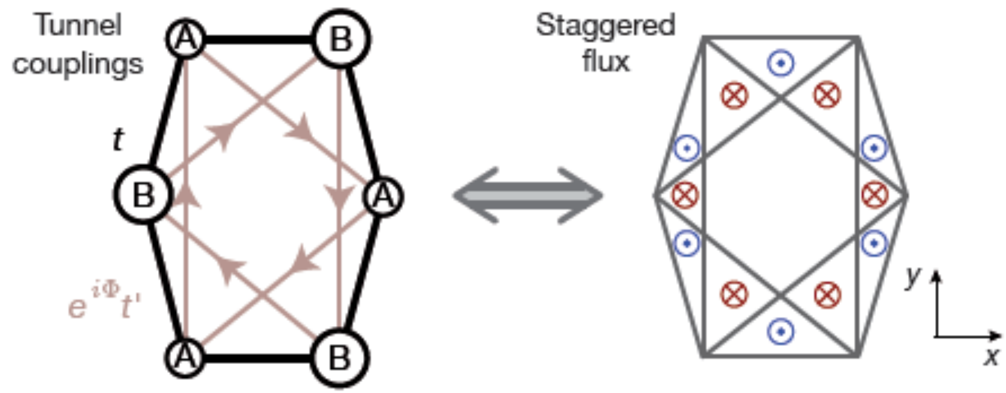
- C=1 Chern band ~ Landau level
- Can 1/3-filled Chern band show FQHE?
- Clear alternative: Zero entropy metal

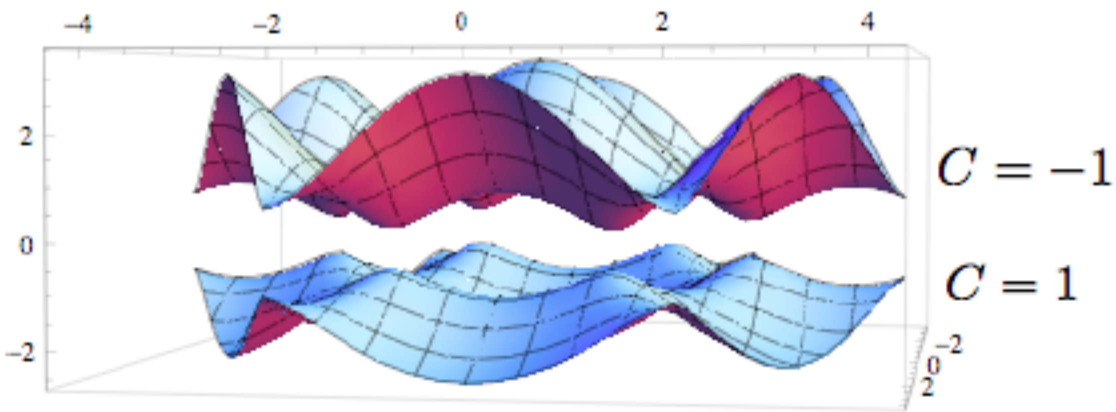
- **Band flattening** ~ more analogous to Landau levels
- Interactions drive fractional quantum Hall effect
- **Regime**: Dispersion << Interactions << Band gap

- ν=1/2 : Lattice version of bosonic Laughlin liquid
- ν=1/3 : Lattice version of fermionic Laughlin state

Sorensen, Demler, Lukin, PRL 2005; K. Sun, Z.-C. Gu, H. Katsura, S. Das Sarma, PRL 2011; Neupert, Santos, Chamon, Mudry, PRL 2011; E. Tang, J.-W. Mei, X.-G. Wen, PRL 2011; D. Sheng, et al, Nat Comm 2011

Half-filled Haldane Chern insulator with strong correlation





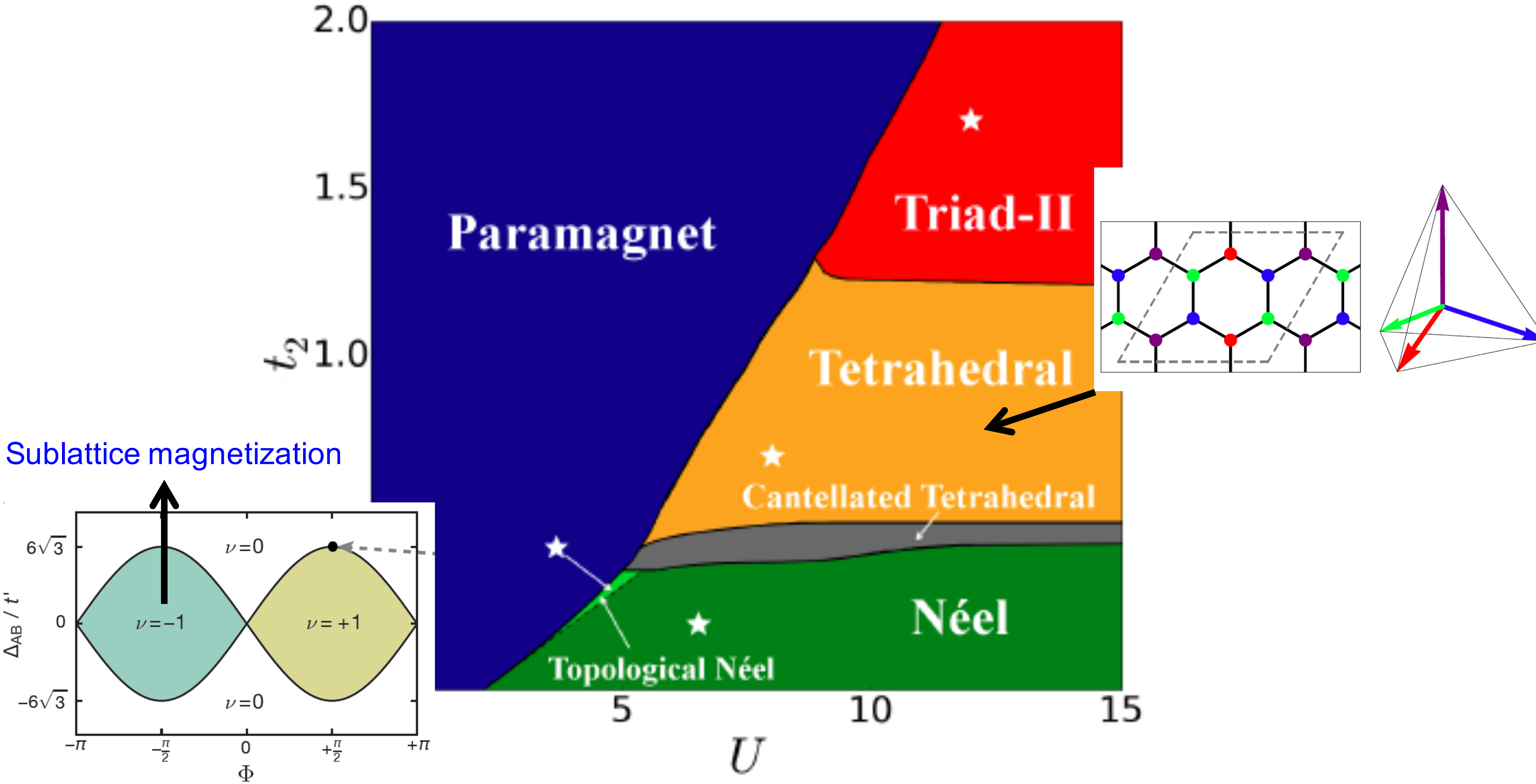
- Bulk gap
- Nontrivial Chern numbers

- Consider 2 copies: $\uparrow, \downarrow$
- Fill up lower Chern band: $\sigma_{xy} = 2e^2/h$ ("2" due to spin)
- Crank up interactions between $\uparrow$ and $\downarrow$: "Haldane-Hubbard" model

Theoretical proposals -

J. He, et al, PRB 2011, 2012 - Slave rotor mean field theory: Chiral spin liquid (ν=1/2 bosonic Laughlin state)
J. Maciejko, A. Ruegg, PRB 2013 - Exotic Z_2 CI* phase at intermediate U
D. Prychynenko, S. Huber arXiv:1410.2001 – Topological SDW/CDW order (with sublattice imbalance)

Mean Field Phase diagram: Novel broken symmetries



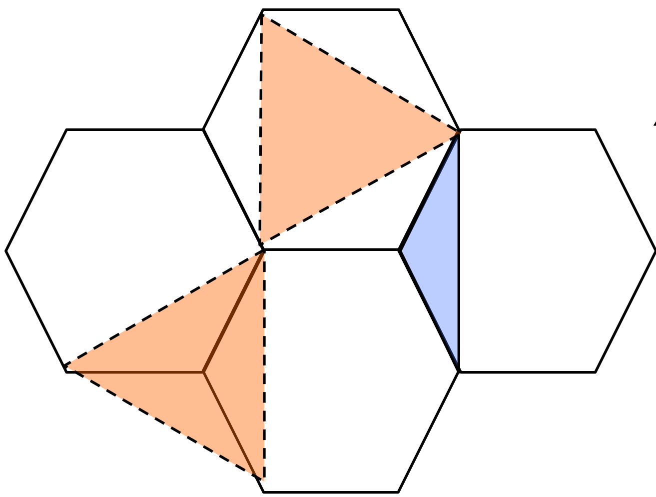
First careful mean field study - W. Zheng, H. Shen, Zhong Wang, Hui Zhai (PRB 2015)

* Predicted mean field topological Neel state (see: K. Jiang, S. Zhou, Xi Dai, Z. Wang, PRL 2018)
* Predicted fluctuation-induced 1st order – confirmed: M. Troyer group, (PRB 2016)

Mean field extended to larger t_2: V. S. Arun, R. Sohal, C. Hickey, AP (PRB 2016)

Strong coupling limit: Chiral spin interactions

$$H_{\rm spin} = \frac{4t_1^2}{U} \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + \frac{4t_2^2}{U} \sum_{\langle\langle ij \rangle\rangle} \mathbf{S}_i \cdot \mathbf{S}_j - \frac{24 t_1^2 t_2}{U^2} \sum_{\rm small-\triangle} \hat{\chi}_\triangle \sin \Phi_\triangle - \frac{24 t_2^3}{U^2} \sum_{\rm big-\triangle} \hat{\chi}_\triangle \sin \Phi_\triangle$$

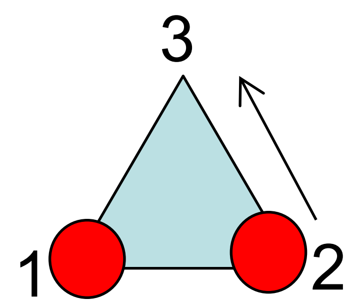


$$\chi_\triangle = \vec{S}_1 \cdot \vec{S}_2 \times \vec{S}_3$$

Scalar spin chirality

Boson language: Correlated hopping

$$i n_1 (b_2^\dagger b_3 - b_3^\dagger b_2)$$



Strong coupling limit: Chiral spin interactions

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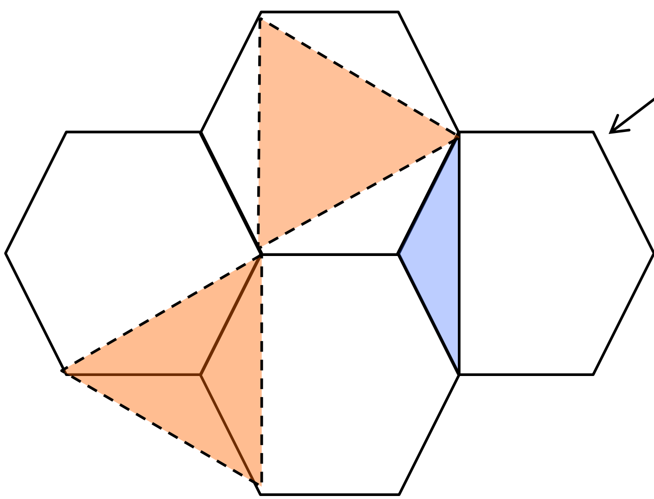
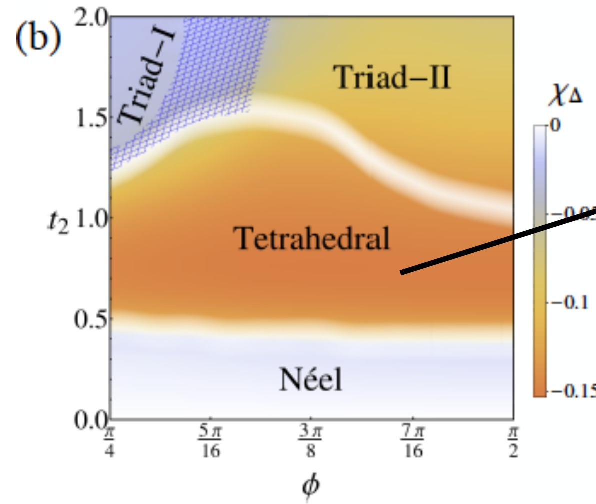
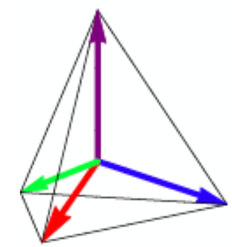
$$\chi_\triangle = \vec{S}_1\cdot\vec{S}_2\times\vec{S}_3$$

Scalar spin chirality



U/t = 10





Exact Diagonalization with N=18,24,32

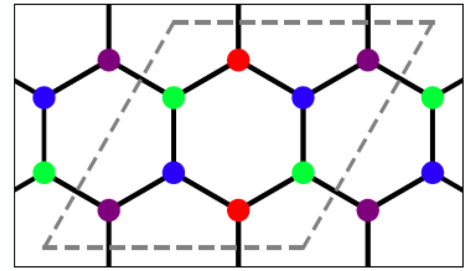
- Classical ground states
- Exact diagonalization
- Ground state: magnetically ordered!

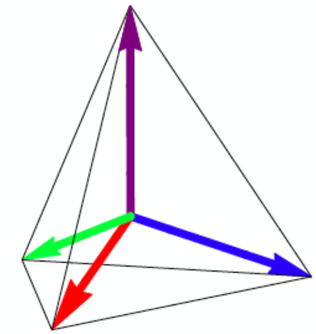
C. Hickey, L. Cincio, Z. Papic, AP (PRL 2016)

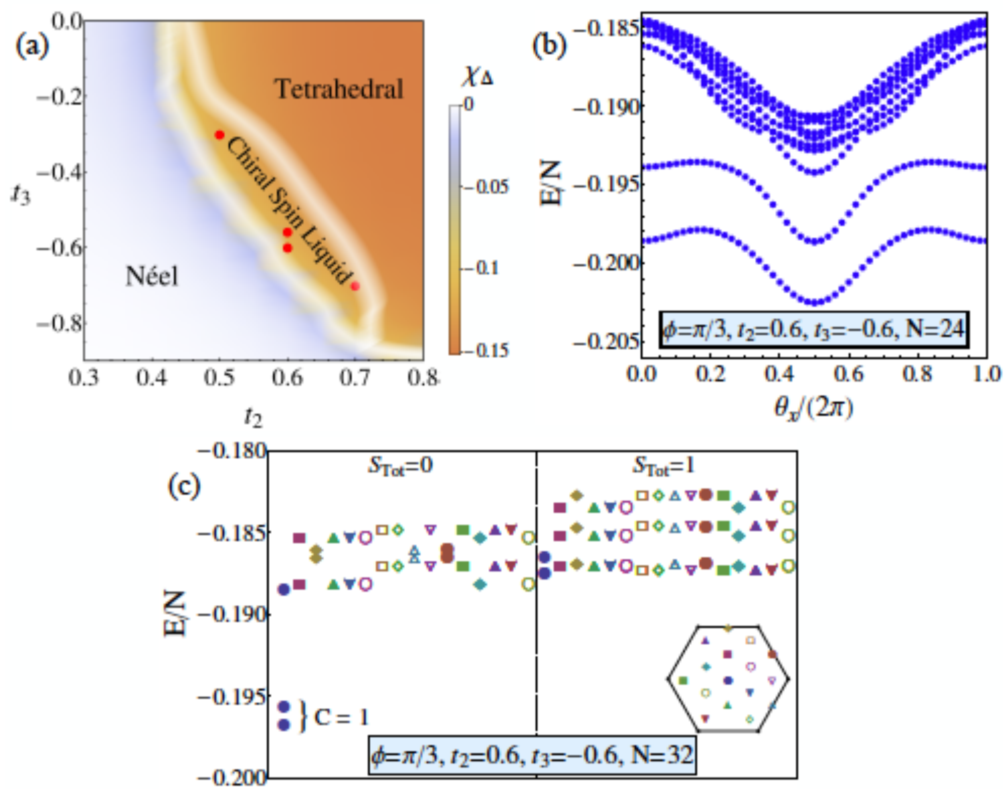
Cf: Nielsen, Sierra, Cirac, Nat. Comm. 2013 – Square lattice Chern insulator yields a CSL Mott insulator
Cf: B. Bauer, et al, Nature Communications 5, 5137 (2014) – Kagome chiral terms give CSL Mott insulator

Melting the Tetrahedral Order

Add 3rd neighbor AFM interaction
Frustrates Tetrahedral order
Creates a CSL retaining large chirality!





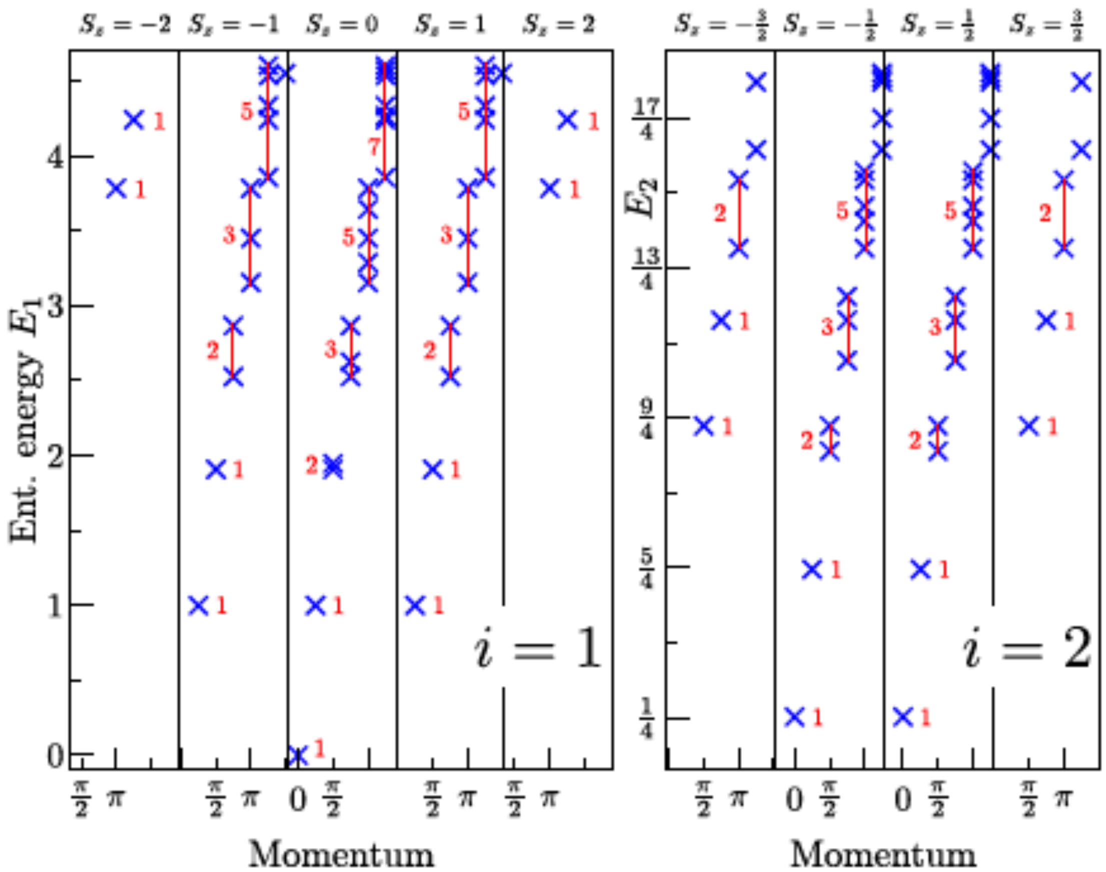


Exact diagonalization results: N=18,24,32

C. Hickey, L. Cincio, Z. Papic, AP (PRL 2016)

Infinite cylinder DMRG results

Y. Zhang, et al PRB 2013
L. Cincio and G. Vidal, PRL 2013



Agrees with chiral semion

C. Hickey, L. Cincio, Z. Papic, AP (PRL 2016)

Spin crystallization transition out of the CSL

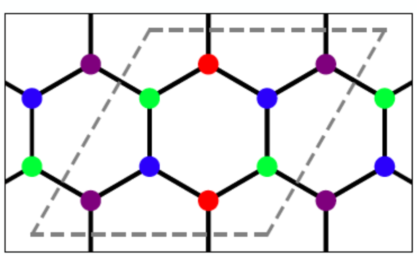
- **Low energy theory of CSL: Pure topological Chern Simons theory**

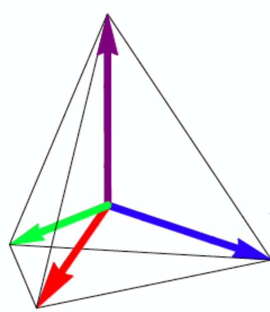
$$\mathcal{L}_{\rm CS} = \frac{1}{2\pi}\epsilon^{\alpha\beta\lambda} a_\alpha \partial_\beta a_\lambda \qquad \text{U(1) level k=2}$$

- Need gapped excitations in CSL: Semions
- Like to examine continuous transition to a magnetically ordered state

Minimally couple bosonic spinons to CS gauge field

- In the CSL: Bosonic spinons bind π-flux to give semions
- Out of CSL: Condensing spinons can yield magnetic order

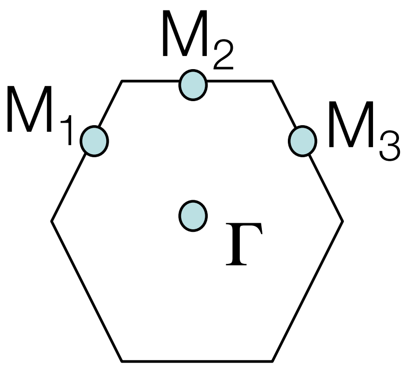




Adiabatic spinon transport: Berry Fluxes

Hofstadter Problem {
- Triangular loops: π/2
- Hexagonal loops: π

Spin crystallization transition out of the CSL



$$\mathcal{L}_{\mathrm{CS},\phi} = \frac{1}{2\pi}\epsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda + |(\partial_\mu - i a_\mu)\phi_{i\alpha}|^2 + r|\phi_{i\alpha}|^2$$

$$\rho_i = \phi^*_{i\alpha}\phi_{i\alpha}$$
$$\vec{\mathcal{S}}_i = \phi^*_{i\alpha}\vec{\sigma}_{\alpha\beta}\phi_{i\beta}$$

Interactions:

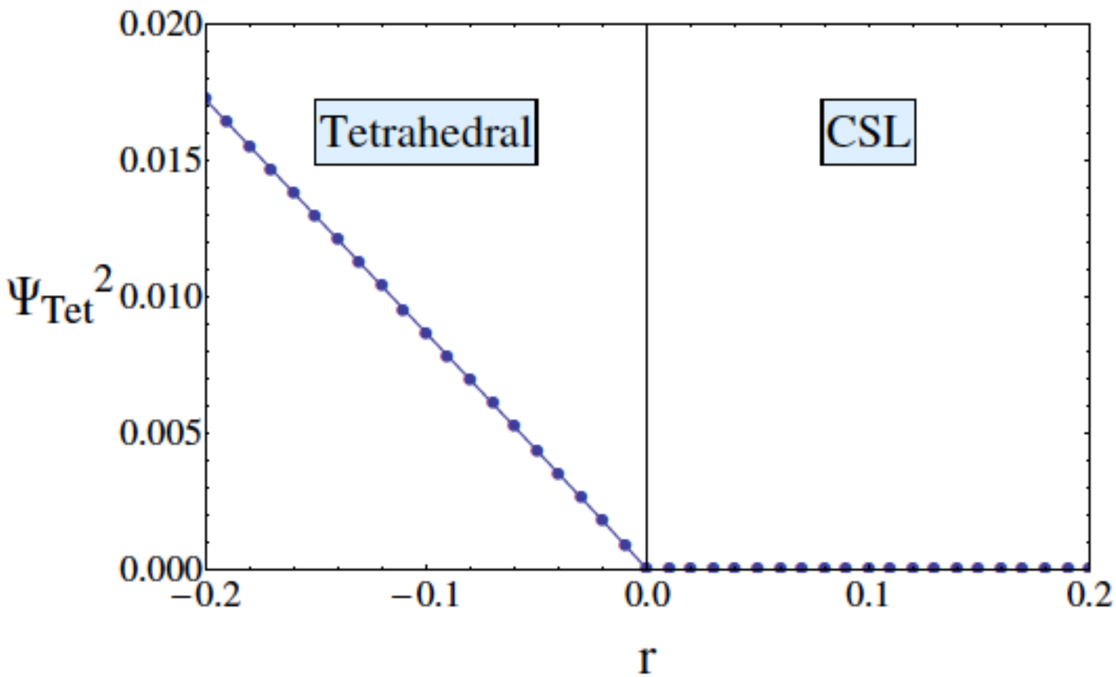
u_2<0 favors multimode

$$\mathcal{L}^{(1)}_{\mathrm{int}} = u_1\Big(\sum_i \rho_i\Big)^2 + u_2 \sum_{i\neq j} \rho_i\rho_j + u_3 \sum_{i\neq j} \vec{\mathcal{S}}_i \cdot \vec{\mathcal{S}}_j$$
$$+ u_4 \sum_{[ijk\ell]} \phi^*_{i\alpha}\phi^*_{j\beta}\phi_{k\alpha}\phi_{\ell\beta} + u_5 \sum_{i\neq j} \phi^*_{i\alpha}\phi^*_{i\beta}\phi_{j\alpha}\phi_{j\beta}$$

Spin crystallization transition out of the CSL

Chern-Simons-Higgs theory

Mean field Landau theory

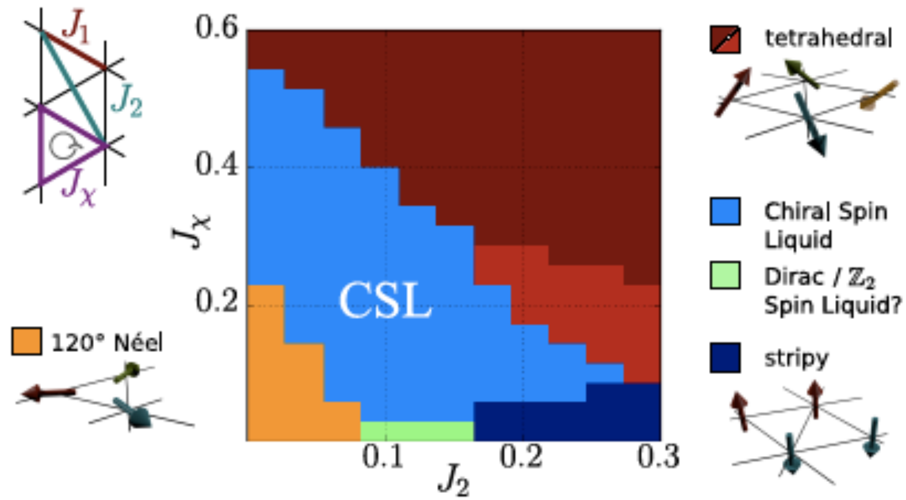


- CSL arises from frustration induced melting of tetrahedral order
- How generic is this idea?

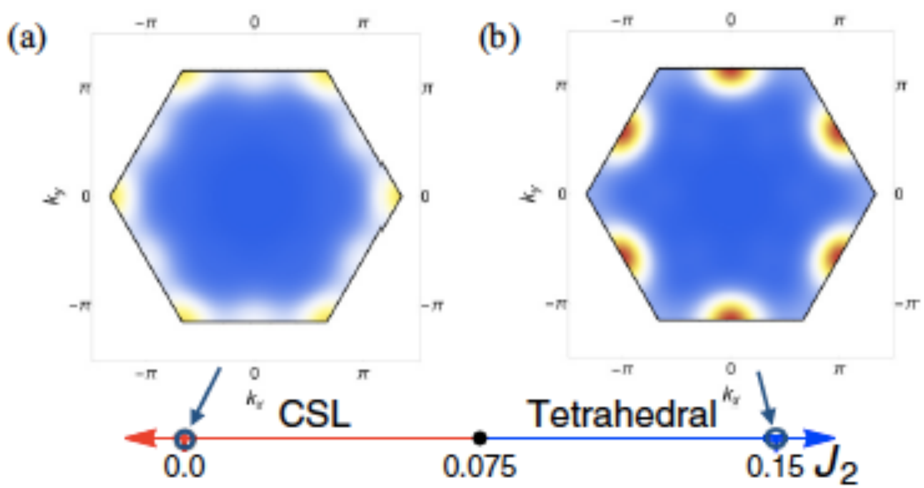
Generalization to other lattices

Triangular lattice

$$\mathcal{H} = J_1 \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \vec{S}_i \cdot \vec{S}_j + J_\chi \sum_{i,j,k \in \triangle} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$

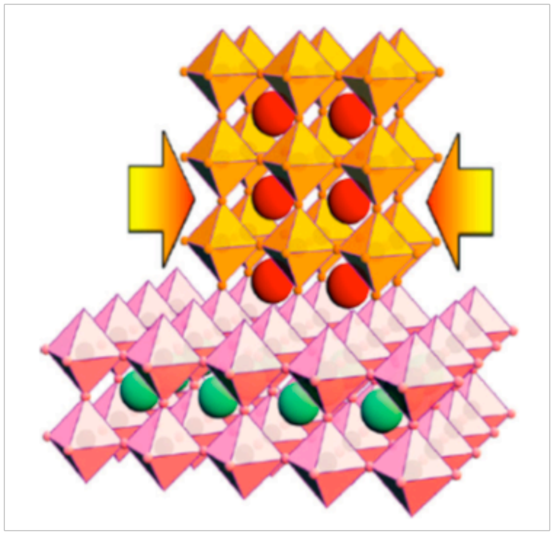


VMC/ED: Wietek, A. Laeuchli (PRB 2017)
DMRG: Saadatmand, McCulloch (PRB 2017)
DMRG: S.S. Gong, Zhu,Zhu, D.N. Sheng. K. Yang (PRB 2017)

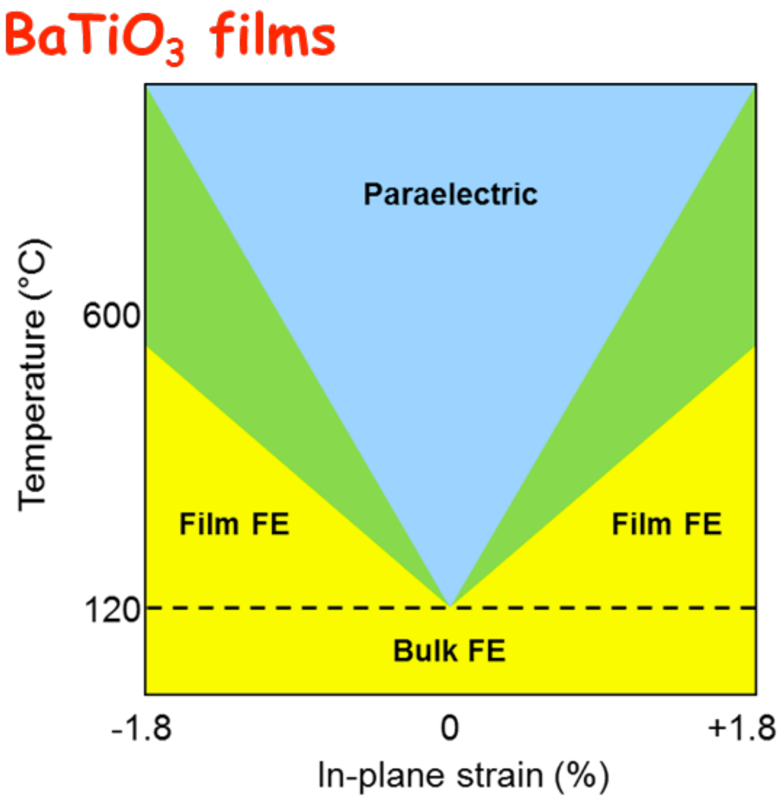


ED/DMRG:
C. Hickey, L. Cincio, Z. Papic, AP (PRB 2017)

Strain in correlated materials

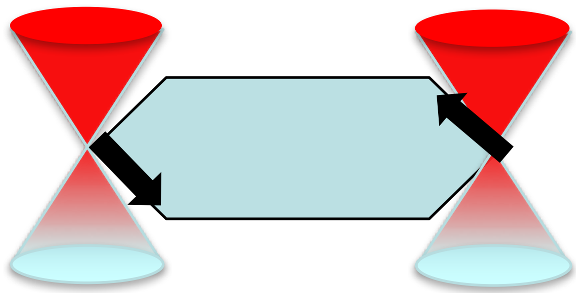


D.G. Schlom *Rev. Mater. Res.* 37 589 (2007)



- **Strain impacts electronic bands**
- **How do interactions modify the resulting phases?**

Strain induced "Landau levels" in graphene

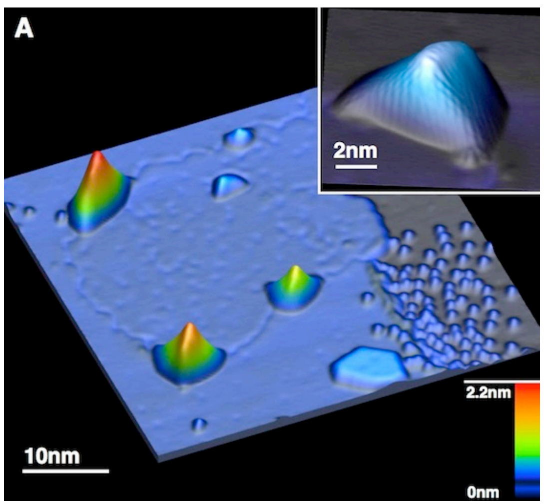


Uniform strain: Moves Dirac points

$$\vec{p} \rightarrow \vec{p} - \text{“}e\vec{A}\text{”}$$

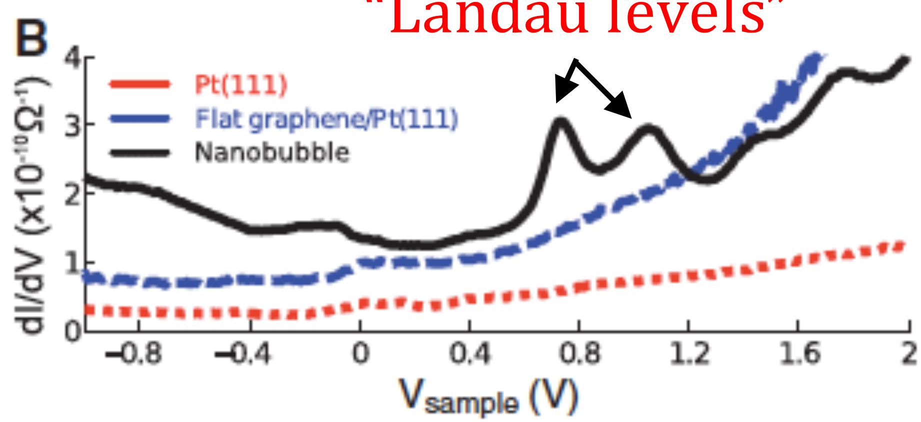
Non-uniform strain: $\vec{A}(\mathbf{r})$

Graphene



- Time-reversal invariant
- Pseudomagnetic field: 300 Tesla!

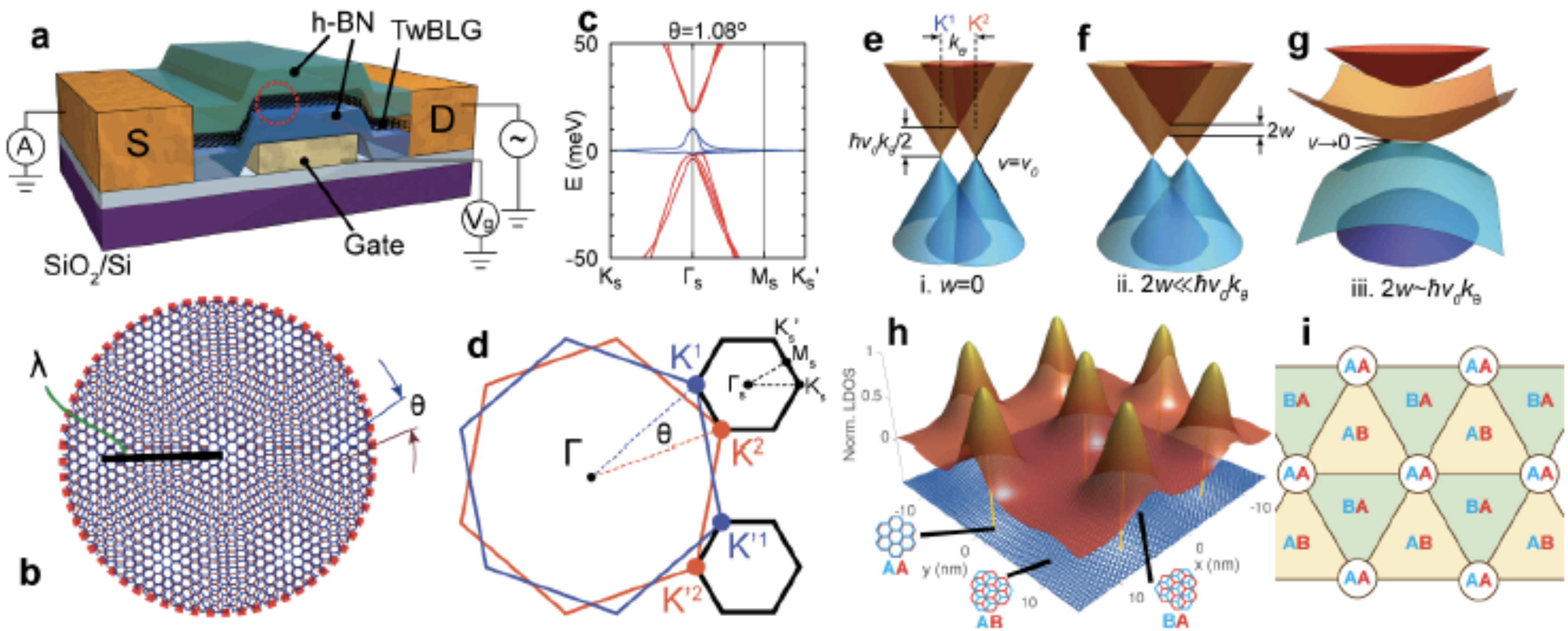
"Landau levels"



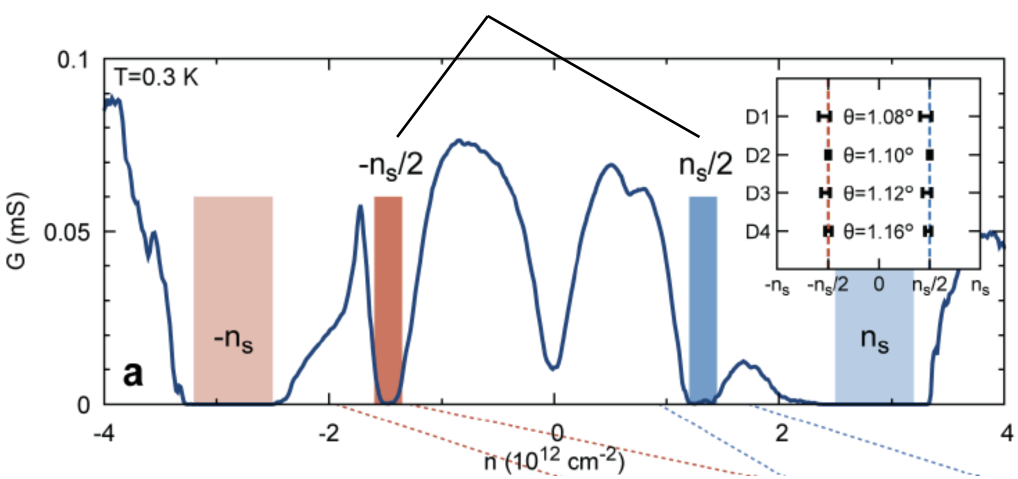
F. Guinea, Katsnelson, Geim (Nat Phys 2010)
N. Levy, et al (Science 2010)

Interactions can induce fractionalized phases: Ghaemi, Cayssol, Sheng, Vishwanath (PRL 2012)

Flat bands in twisted bilayer graphene



Mott insulators?



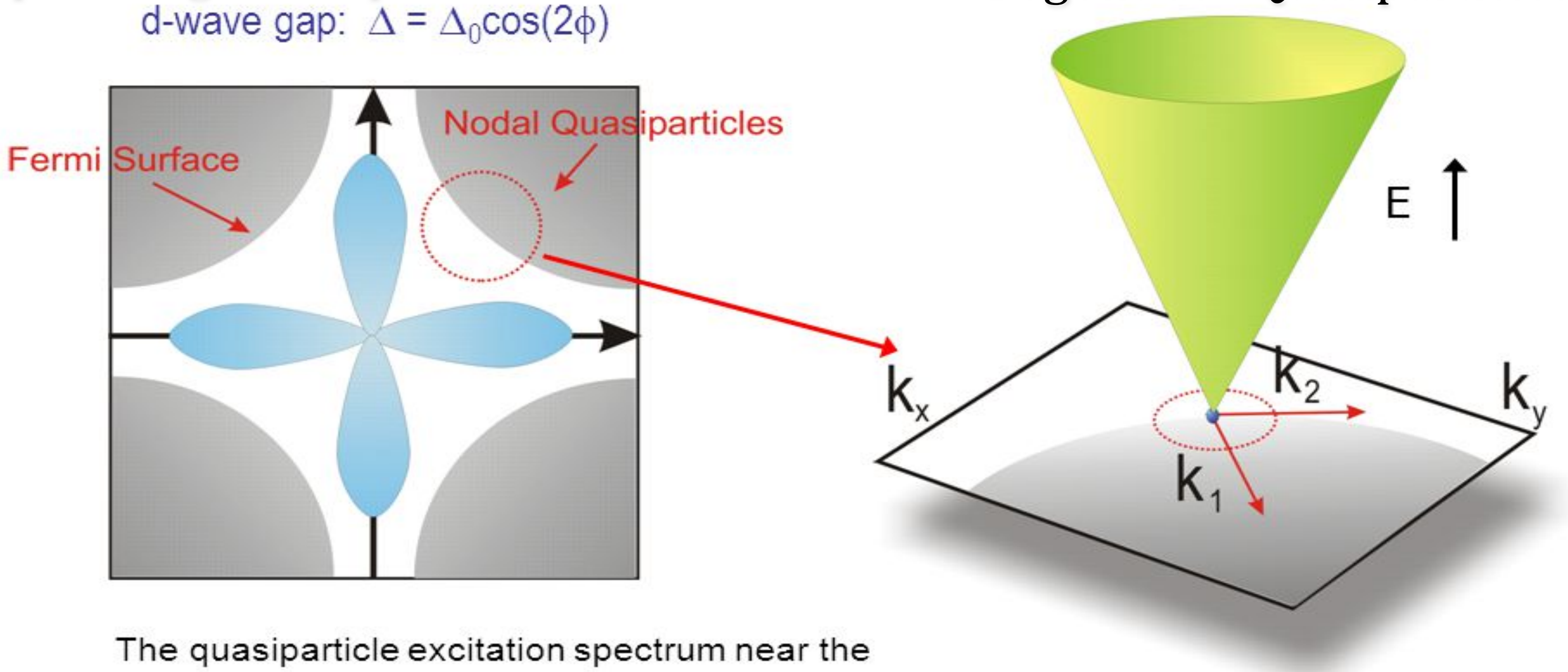
Superconductivity at Tc ~ 1K upon doping

P. Jarillo-Herrero (Nature 2018)
P. Jarillo-Herrero (Nature 2018)

Strain-induced Landau levels in Correlated Matter

Cuprate high Tc superconductors

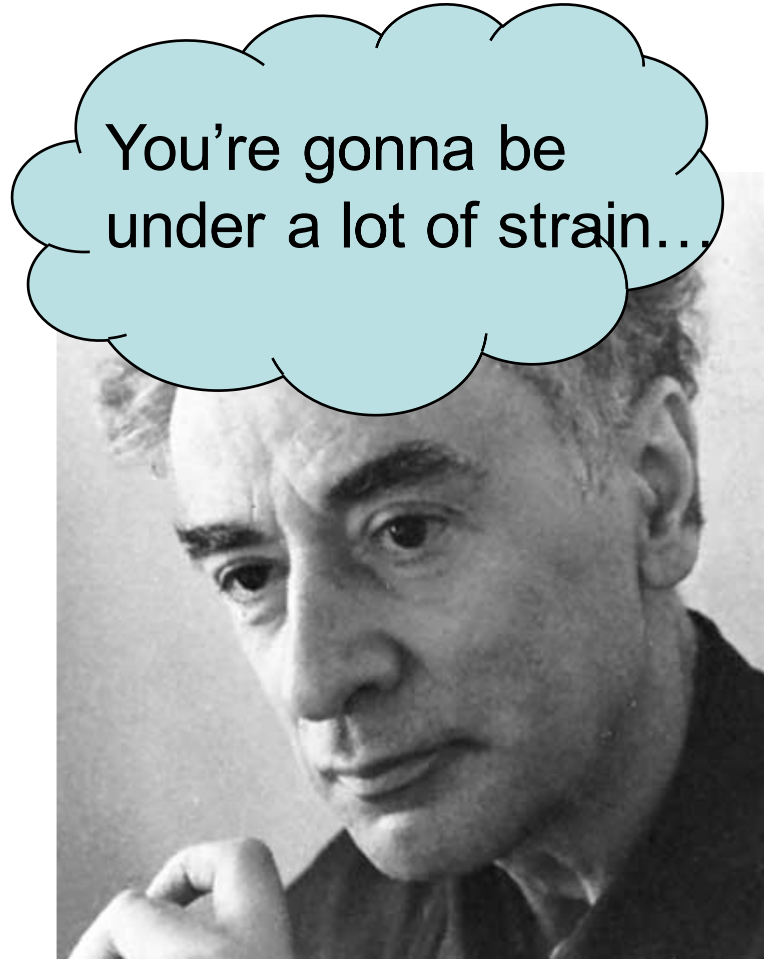
Bogoliubov Quasiparticles



The quasiparticle excitation spectrum near the nodes takes the form of a 'Dirac cone' :

$$E_{\vec{k}} = \sqrt{v_F^2 k_1^2 + v_\Delta^2 k_2^2}$$

Bogoliubov quasiparticles in d-wave SCs



Bogoliubov quasiparticles in strained d-wave SCs

Linearized BdG Hamiltonian

$$H_0 = \int d^2\mathbf{r} \left[-i\sigma^z \vec{v}_f \cdot \vec{\nabla} - i\sigma^x \vec{v}_\Delta \cdot \vec{\nabla} \right]$$

Strain: Time-reversal invariant perturbation

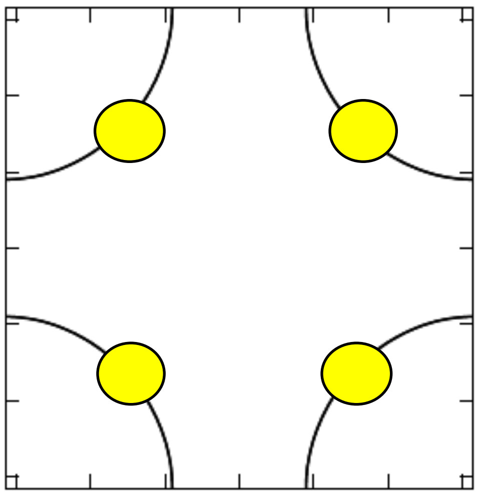
Hopping

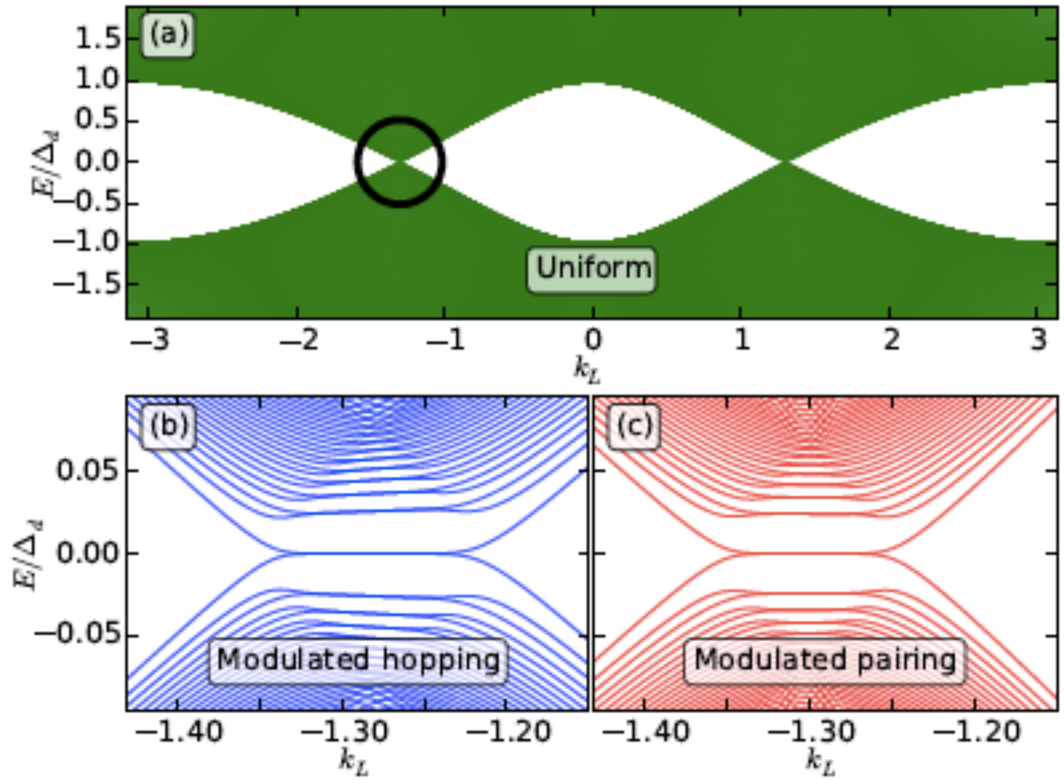
$$\delta H_1 = -\frac{1}{2} \sum_{\mathbf{R},\eta,\alpha} \delta t_\eta(\mathbf{R}) (c^\dagger_{\mathbf{R},\alpha} c_{\mathbf{R}+\eta,\alpha} + \text{h.c.})$$

Pairing

$$\delta H_2 = \frac{1}{8} \sum_{\mathbf{R},\eta} \delta\Delta_\eta(\mathbf{R}) (c^\dagger_{\mathbf{R}\uparrow} c^\dagger_{\mathbf{R}+\eta,\downarrow} - c^\dagger_{\mathbf{R}\downarrow} c^\dagger_{\mathbf{R}+\eta,\uparrow} + \text{h.c.})$$

Bogoliubov Landau levels on a (10) oriented d-wave SC strip

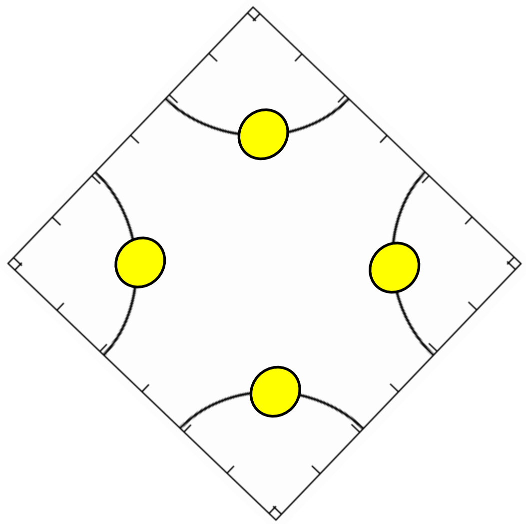


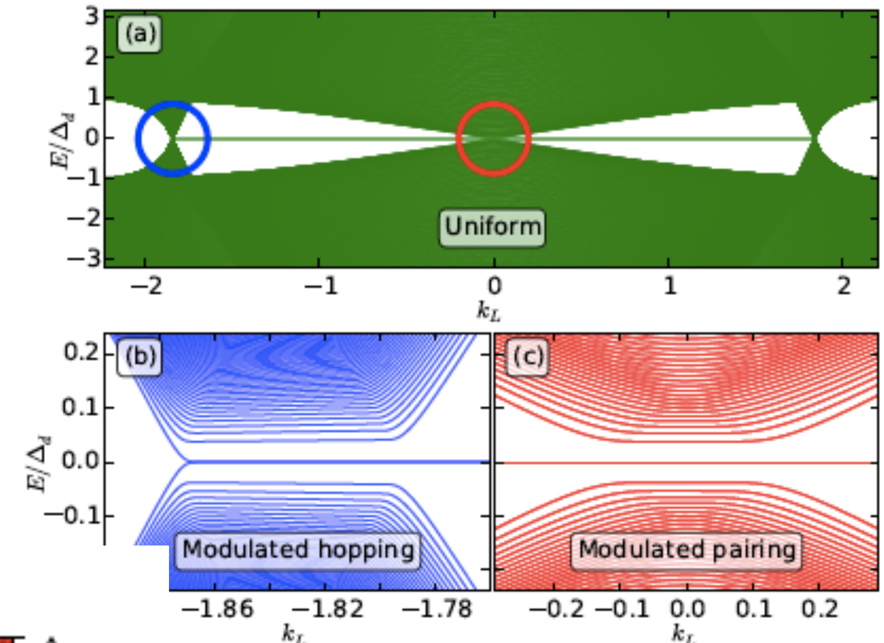


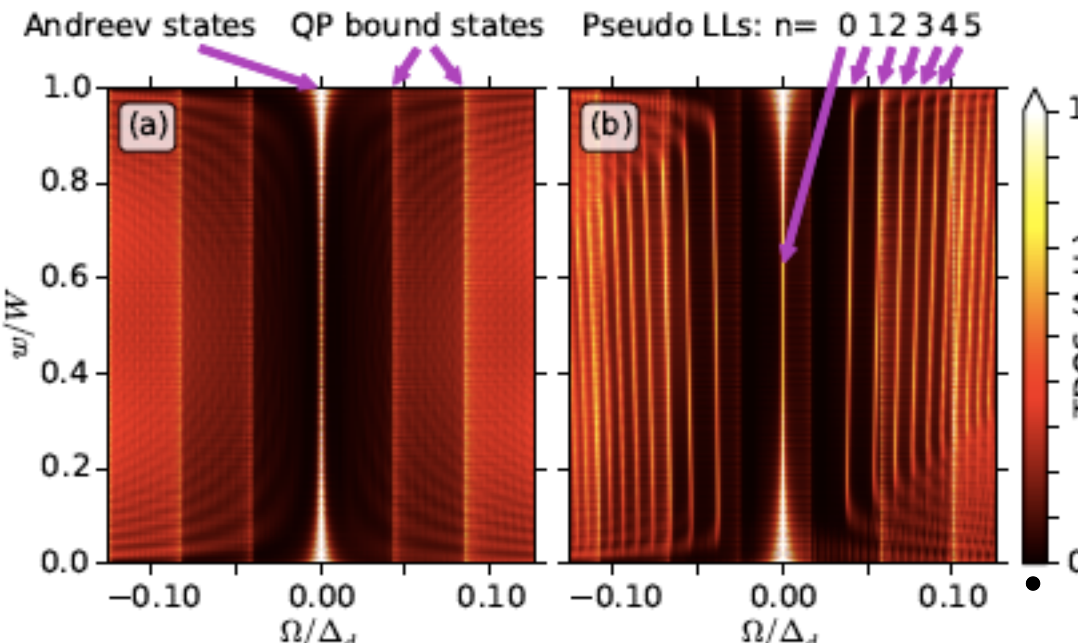
- Strain induced Landau levels
- Impact of residual interactions?

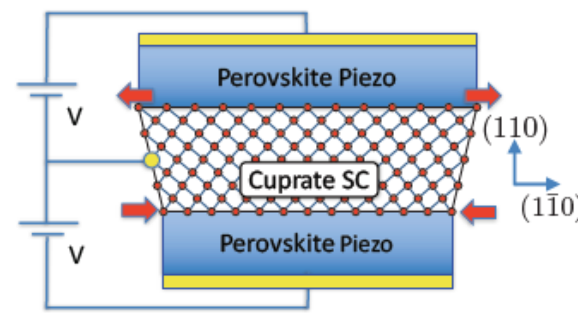
(G. Massarelli, G. Wachtel, J. Wei, AP, PRB 2017)

Bogoliubov Landau levels on a (11) oriented d-wave SC strip









- Strain induced Landau levels
- Impact of residual interactions?

(G. Massarelli, G. Wachtel, J. Wei, AP, PRB 2017)

Collaborators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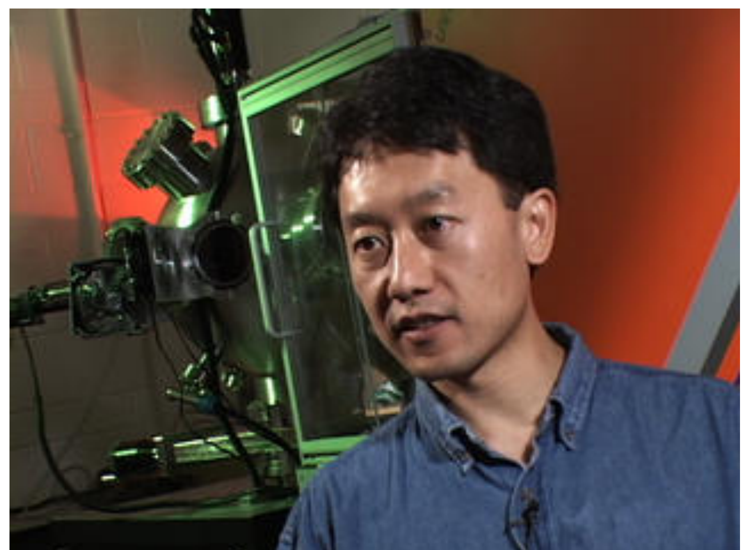
John Wei

Summary

- Correlated materials are potential candidates for high Tc QAH
- Correlations in topological bands can induce FQH states
- Frustration of non-coplanar magnets: Chiral spin liquids
- Strain: Tunable knob to control superconductor band topology