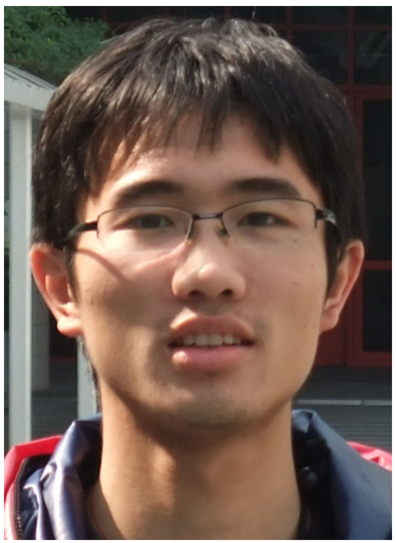
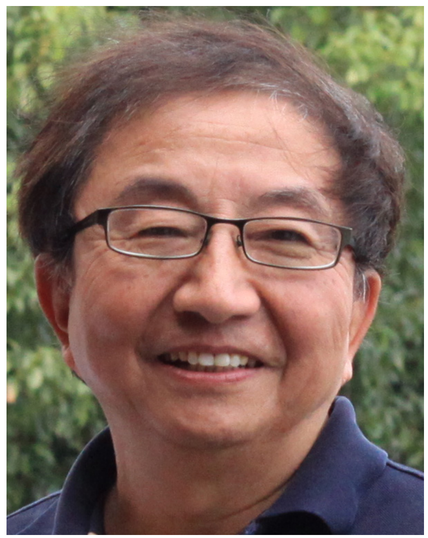
Instability of 3-band Tomonaga-Luttinger liquids: renormalization group analysis and possible application to $K_2Cr_3As_3$





Jian-Jian Miao (Student)   Fu-Chun Zhang

Yi Zhou

Zhejiang University

Reference: arXiv:1603.03651

Institute of Advanced Study, Tsinghua, Beijing, March 16, 2016

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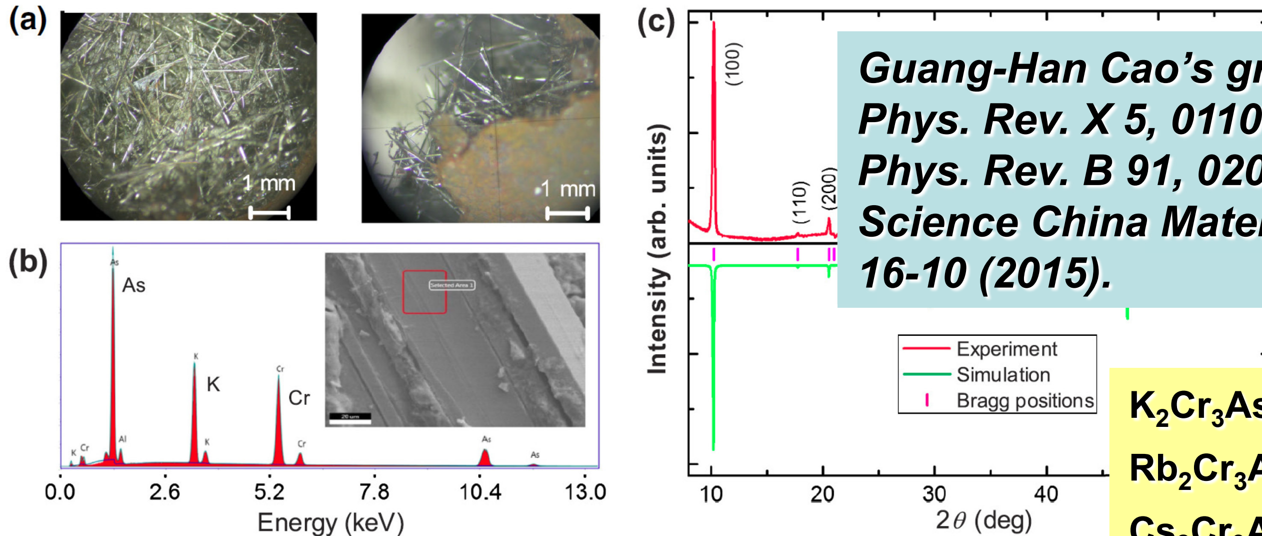
  - 973 & NSFC

Cr based superconductors under ambient pressure: $A_2Cr_3As_3$ (A=K,Rb,Cs)

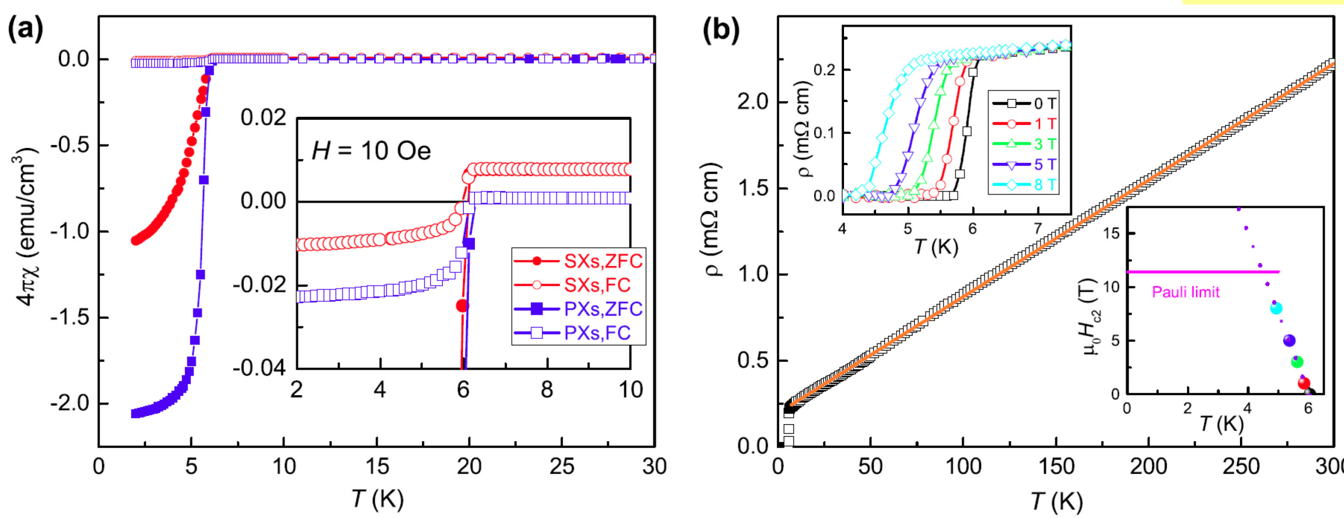


Guang-Han Cao's group in ZJU, Phys. Rev. X 5, 011013 (2015); Phys. Rev. B 91, 020506(R) (2015); Science China Materials, 58(1) 16-10 (2015).

$K_2Cr_3As_3$: T_c ~ 6.1K

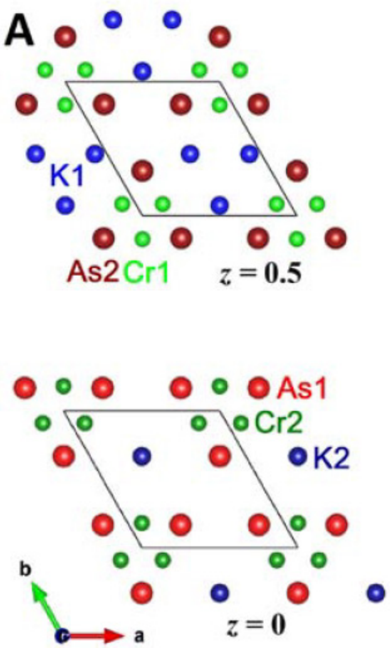
$Rb_2Cr_3As_3$: T_c ~ 4.8K

$Cs_2Cr_3As_3$: T_c ~ 2.2K

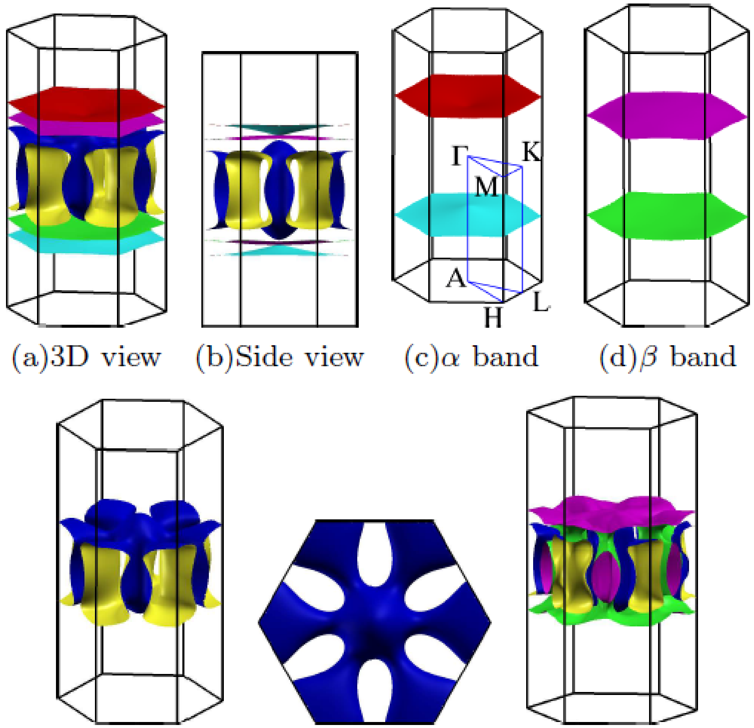


Quasi-1D crystal and electronic structure

$Cr^{+2.33}$



Guang-Han Cao's group, Phys. Rev. X 5, 011013 (2015). arXiv:1412.0067

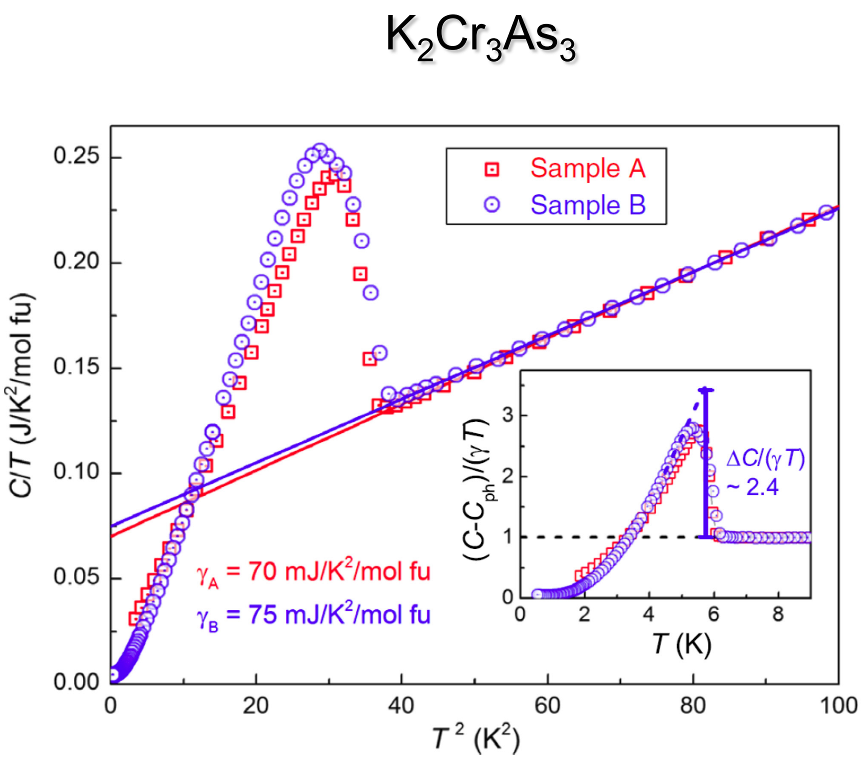


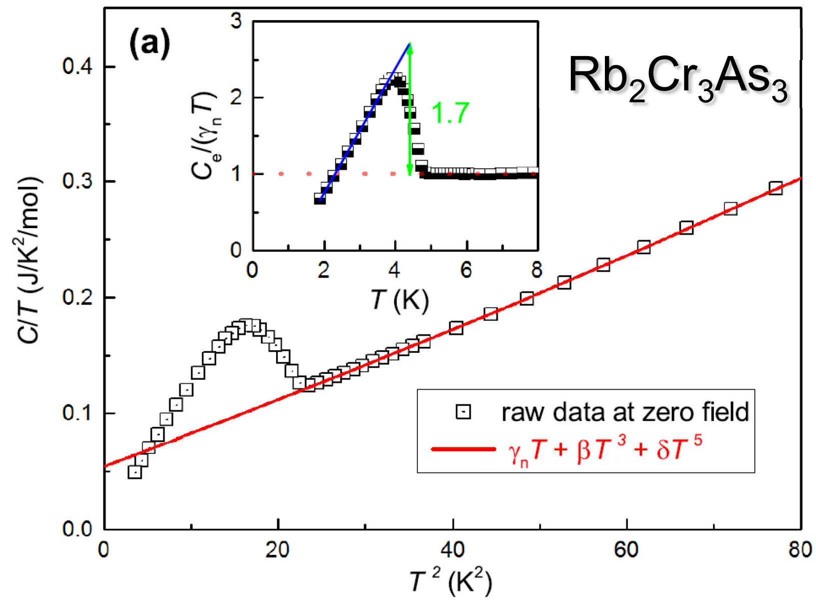
(a)3D view (b)Side view (c)α band (d)β band

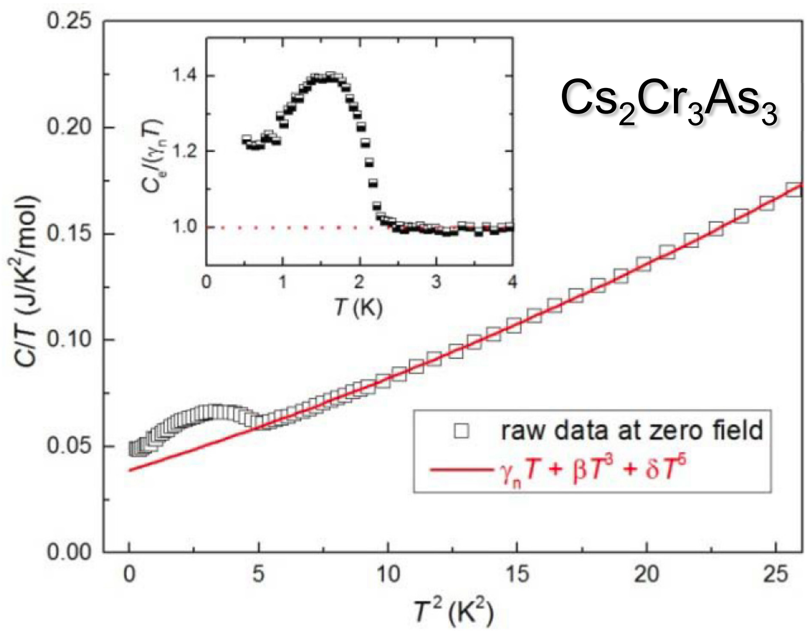
(e)γ band (f)γ band (Top view) (g)γ band, SOC

Jiang, Cao, Cao, arXiv:1412.1309 (2014)

Specific heat: deviation from BCS





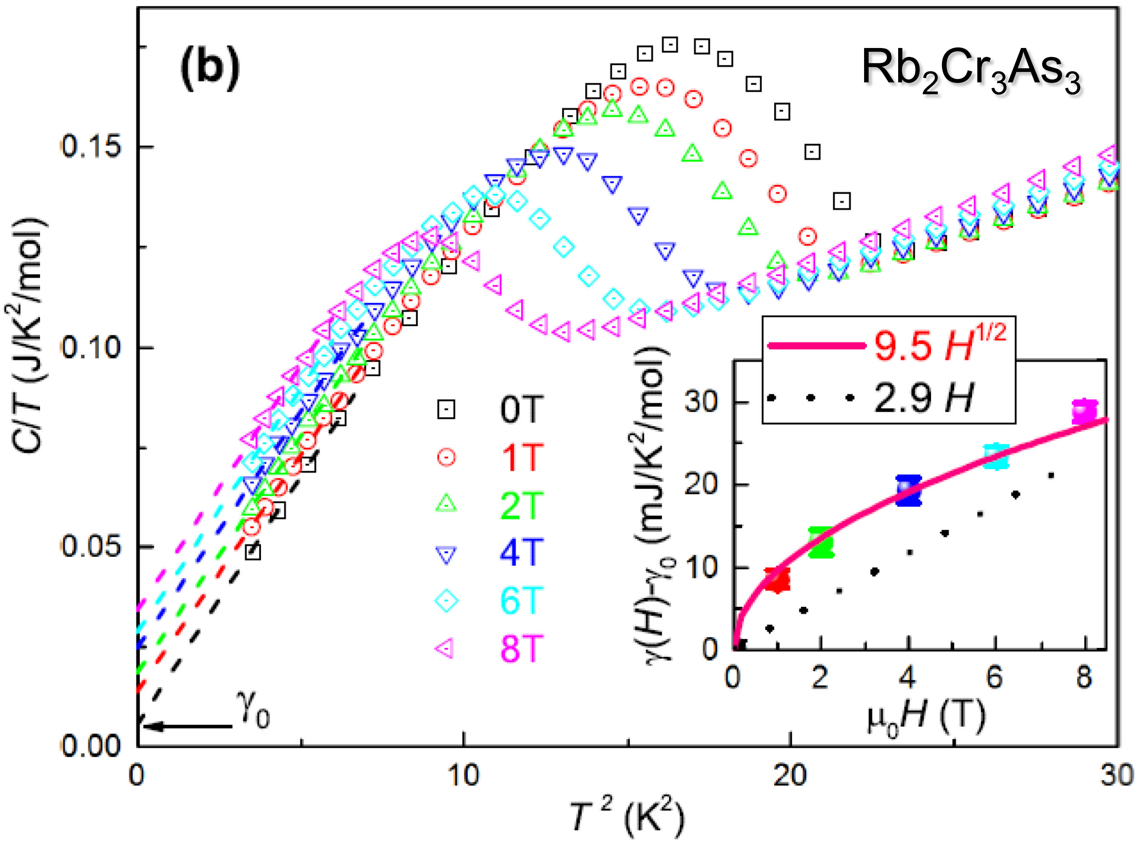


Large Sommerfeld coefficient

Guang-Han Cao's group, Phys. Rev. X 5, 011013 (2015); Phys. Rev. B 91, 020506(R) (2015); Science China Materials, 58(1) 16-10 (2015).

Magnetic field dependent γ(H)

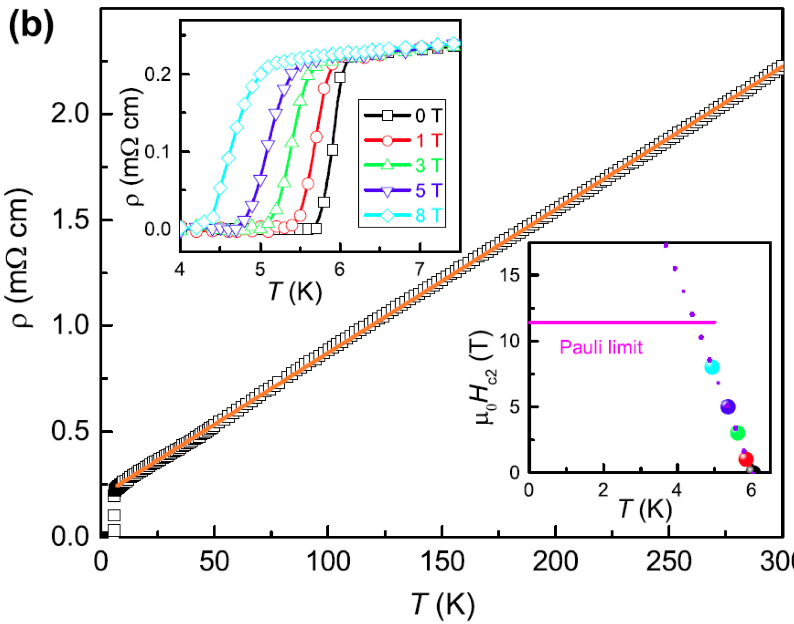
- $\gamma(H) \equiv C_v/T \propto H^{1/2}$ indicates nodes in the gap function



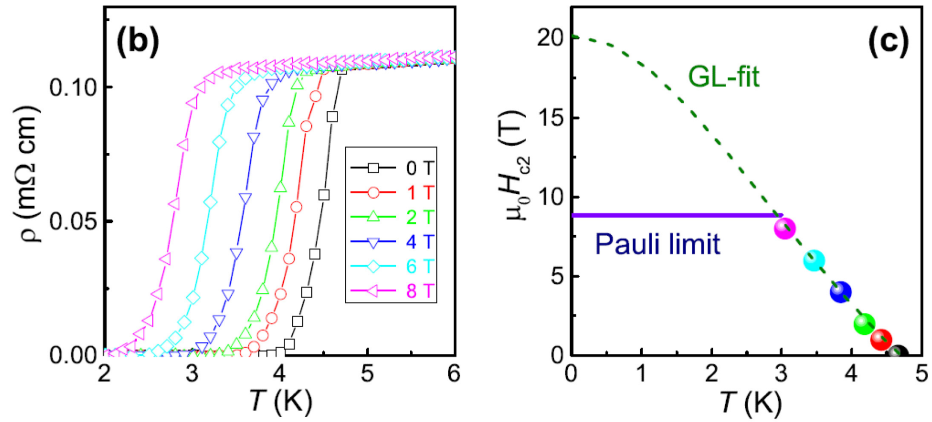
Guang-Han Cao's group, Phys. Rev. B 91, 020506(R) (2015).

Upper critical field H_{c2}

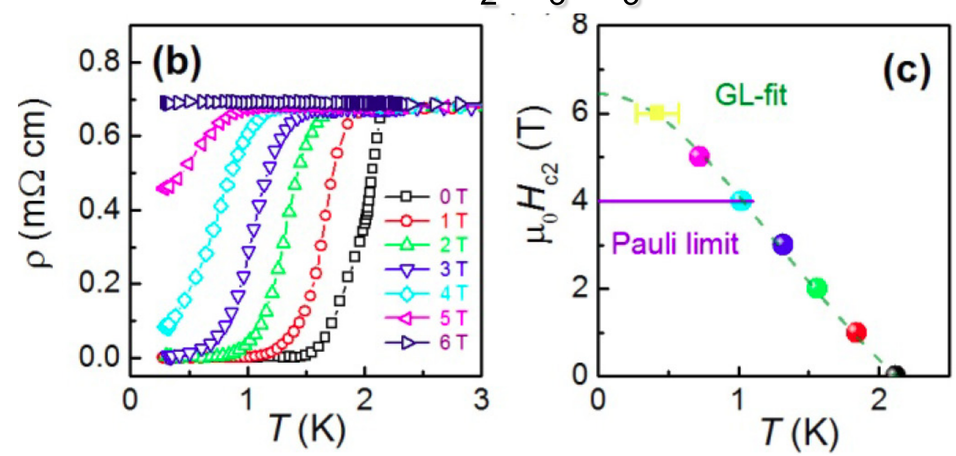
$K_2Cr_3As_3$



$Rb_2Cr_3As_3$

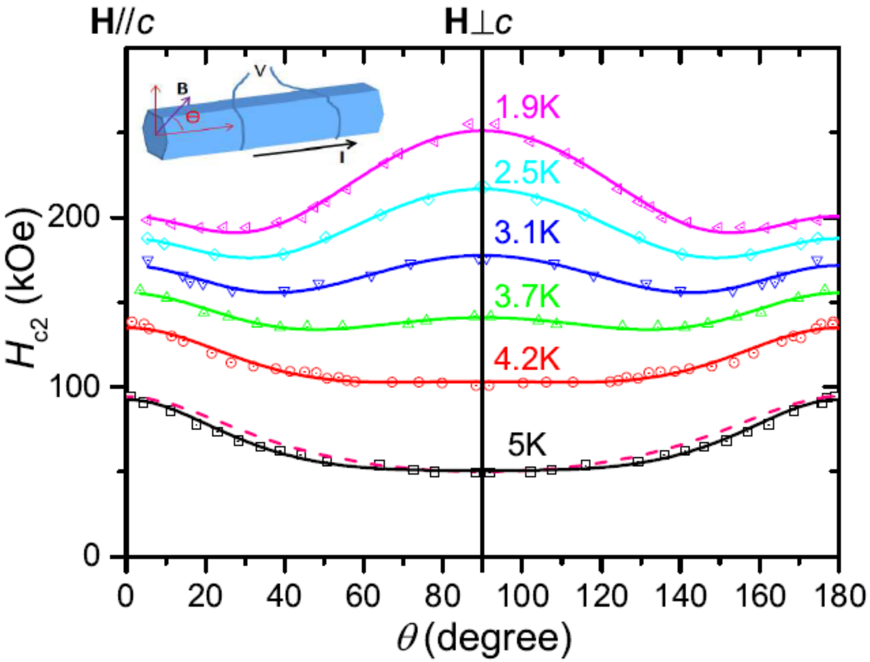


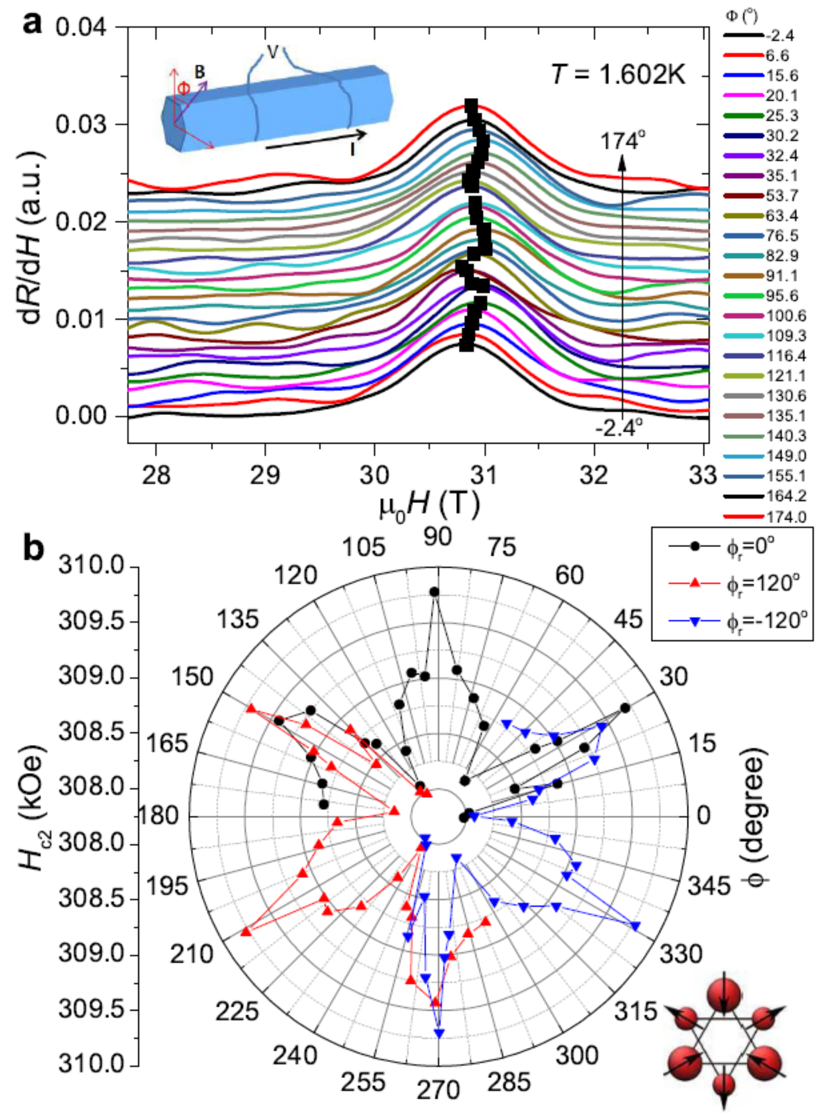
$Cs_2Cr_3As_3$



Guang-Han Cao's group, Phys. Rev. X 5, 011013 (2015); Phys. Rev. B 91, 020506(R) (2015); Science China Materials, 58(1) 16-10 (2015). Ames' group, RRB 91, 020507(R) (2015).

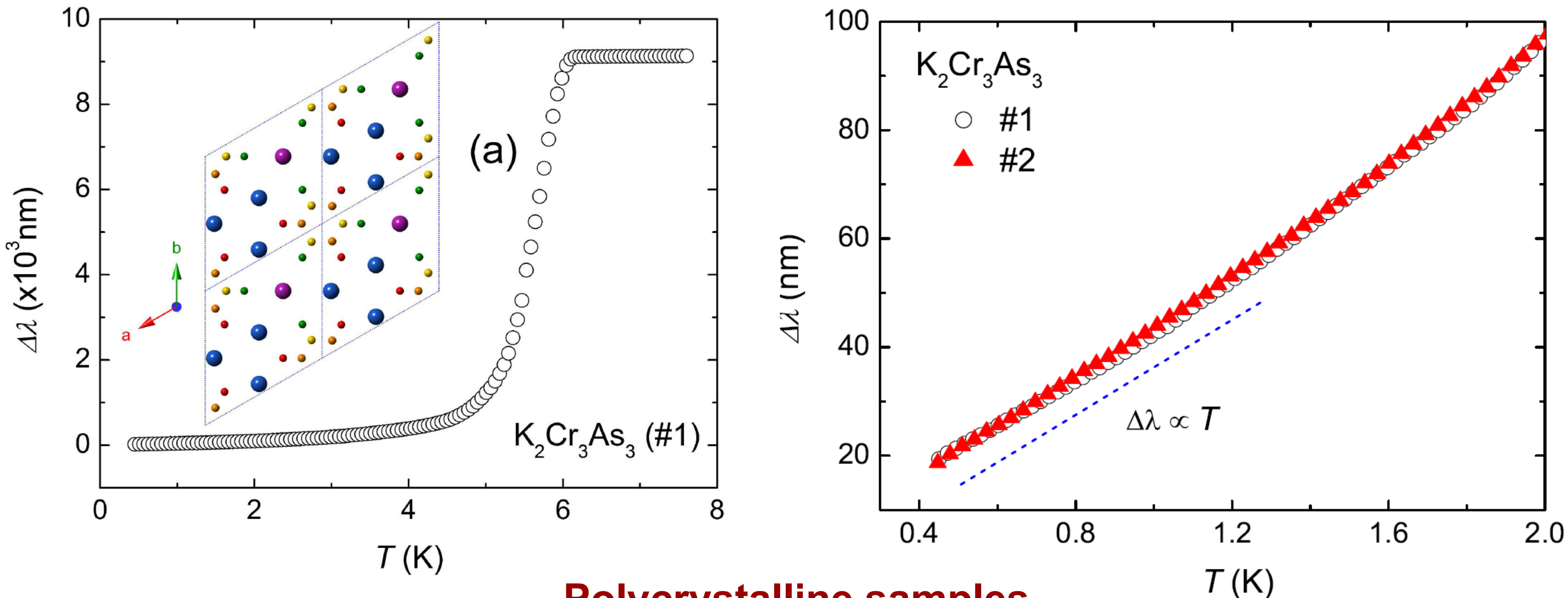
Angle resolved upper critical field





ZengWei Zhu, Guang-Han Cao et. al., arXiv:1511.06169

Penetration depth: $K_2Cr_3As_3$

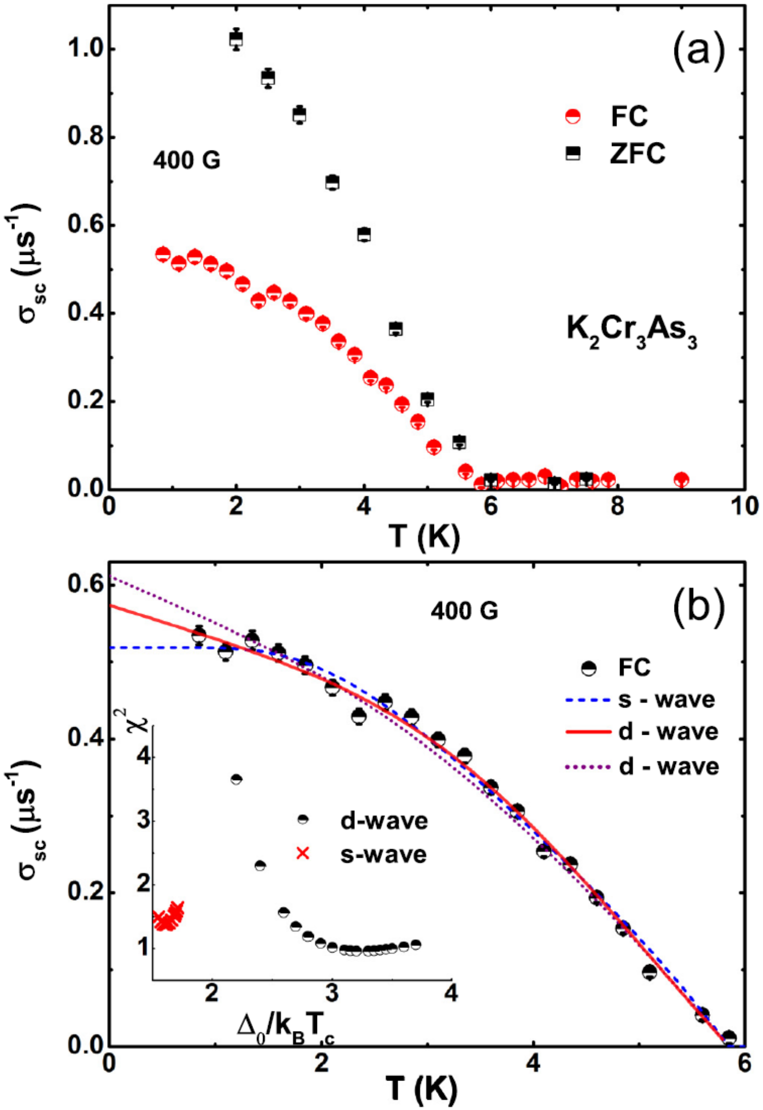


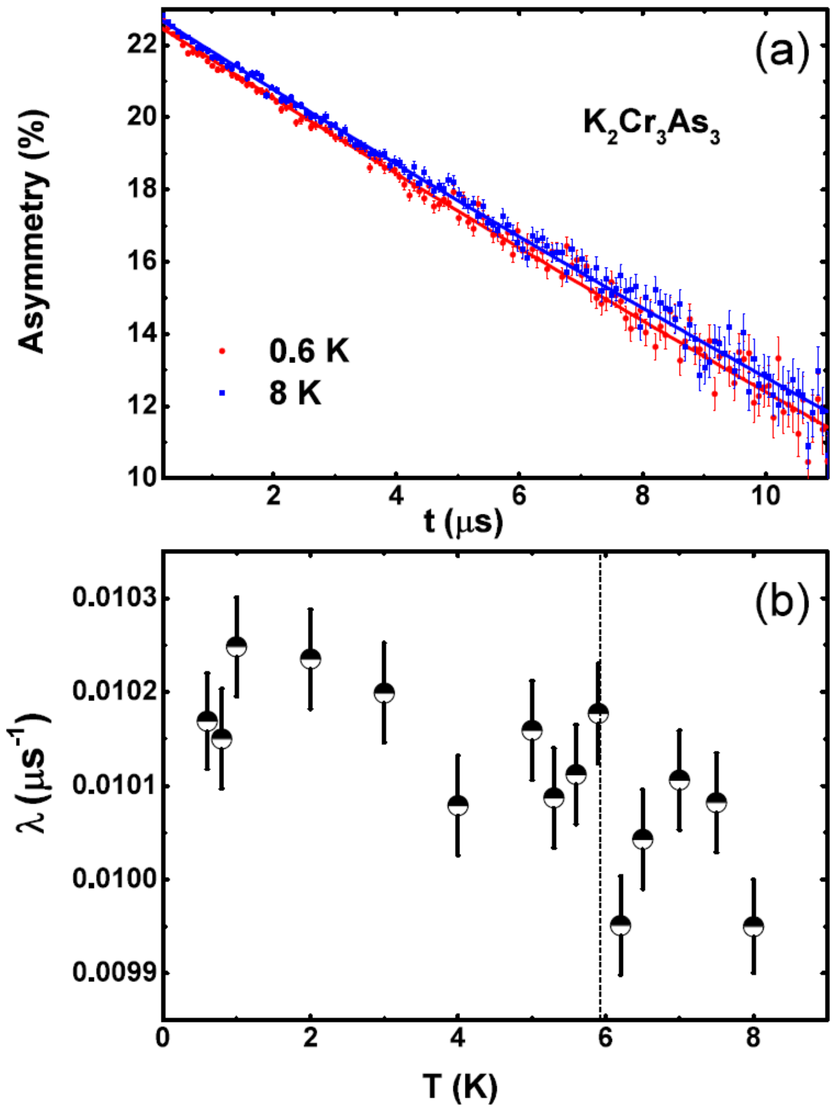
Polycrystalline samples

Linear T dependent penetration depth is an evidence for line nodal superconducting gap.

H. Q. Yuan's group in ZJU, PRB 91, 220502(R) (2015).

muSR





ISIS facilty, Phys. Rev. B 92, 134505 (2015)

Unconventional superconducting states

- **Specific heat**
  - Large Sommerfeld coefficient: band renormalization
  - Large specific-heat jump: deviation from BCS scenario

G.H. Cao et al. (2014, 2015)

- **Upper critical field**
  - Exceeding Pauli limit: $2H_P$ for $H /\!/ c$; $3.4H_P$ for $H /\!/ ab$

G.H. Cao et al. (2014, 2015), Ames' group (2015)

  - Three-fold or six-fold modulation of H_{c2}

Z.W. Zhu et al. (2015)

- **NMR/NQR**
  - Absence of Hebel-Slichter coherence peak
  - $1/T_1T \propto T^4$? or T^5 (T^5 indicates point nodal gap)

T. Imai et al. (2015); G.q. Zheng et al. (2015)

- **Penetration depth**
  - $\Delta\lambda \propto T$ (line nodal gap)

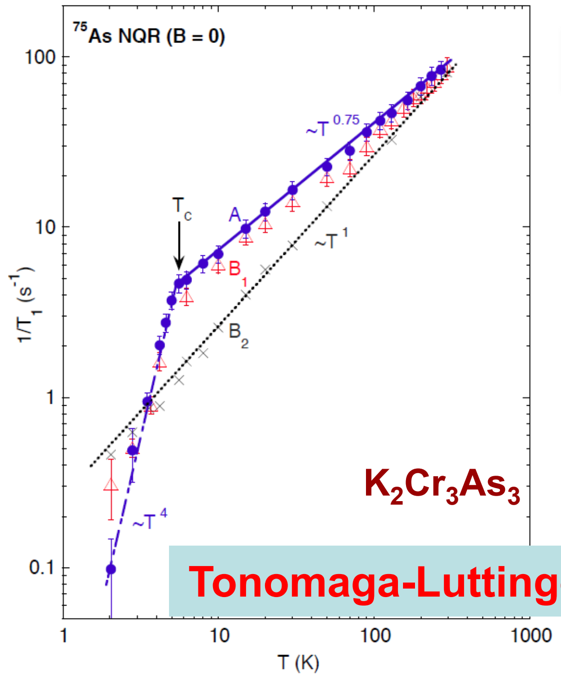
H.Q. Yuan et al. (2015)

- **muSR**
  - Weak evidence of a spontaneous internal magnetic field above T_c

ISIS facility (2015)

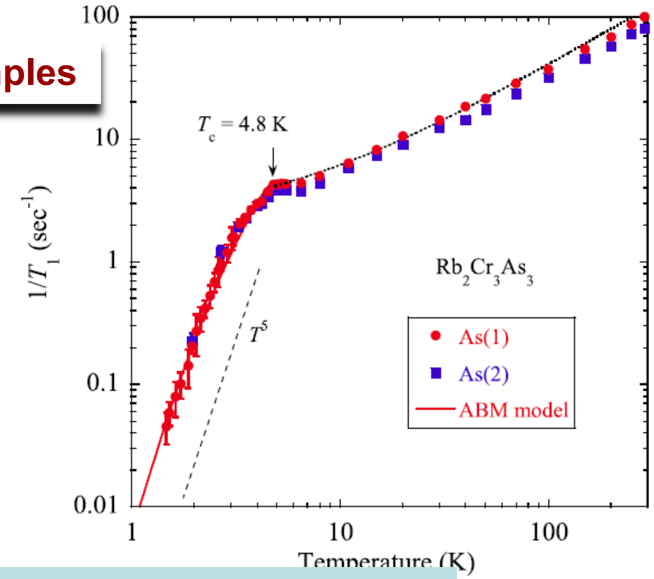
Normal state: NMR / NQR

T. Imai, et al., PRL114, 147004 (2015)

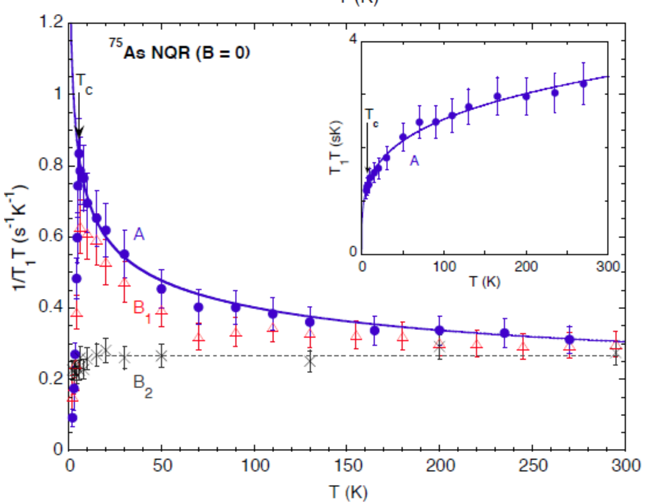


Polycrystalline samples

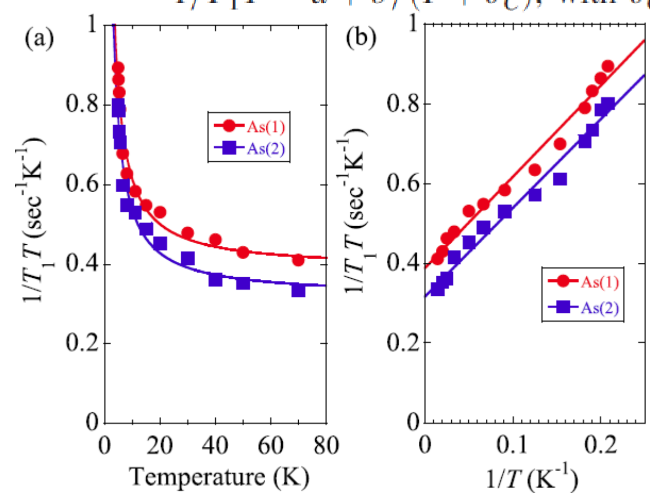
G.q. Zheng, et al., PRL115, 147002 (2015)



Tonomaga-Luttinger Liquid vs. Critical spin fluctuations

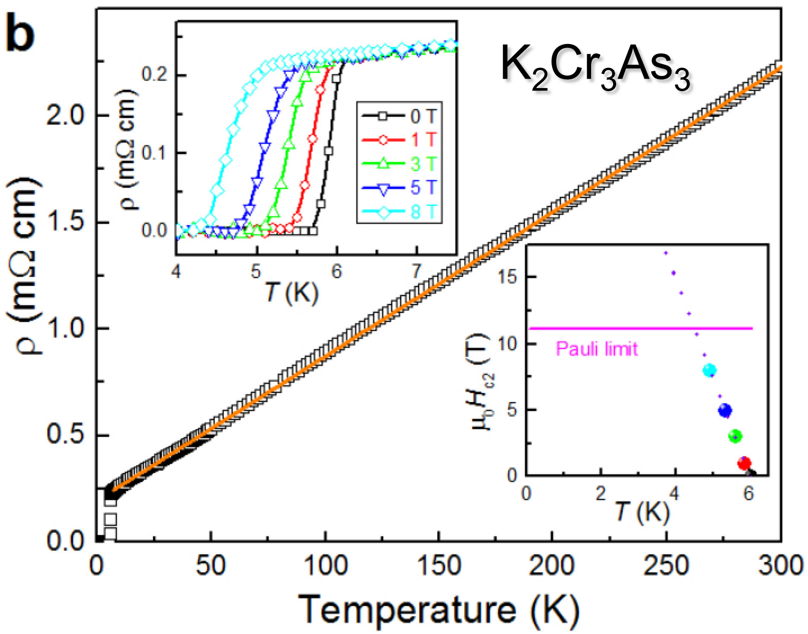


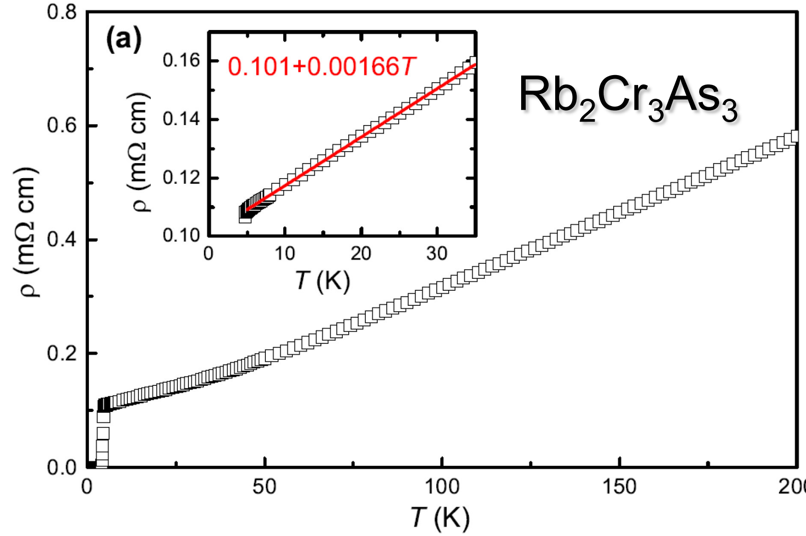
$1/T_1T = a + b/(T + \theta_C)$, with $\theta_C \sim 0$ K

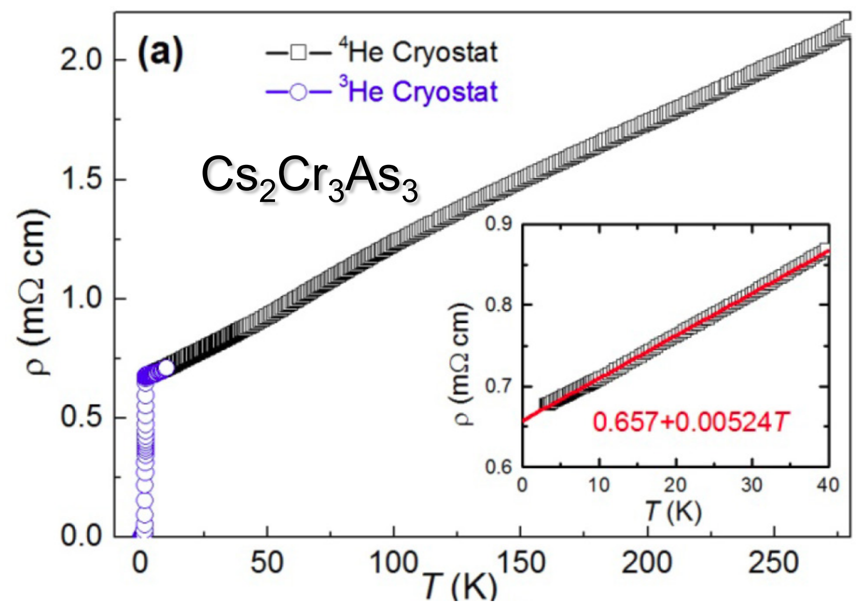


Normal state: transport, polycrystal

Fermi liquid: $\rho_0 + AT^2$





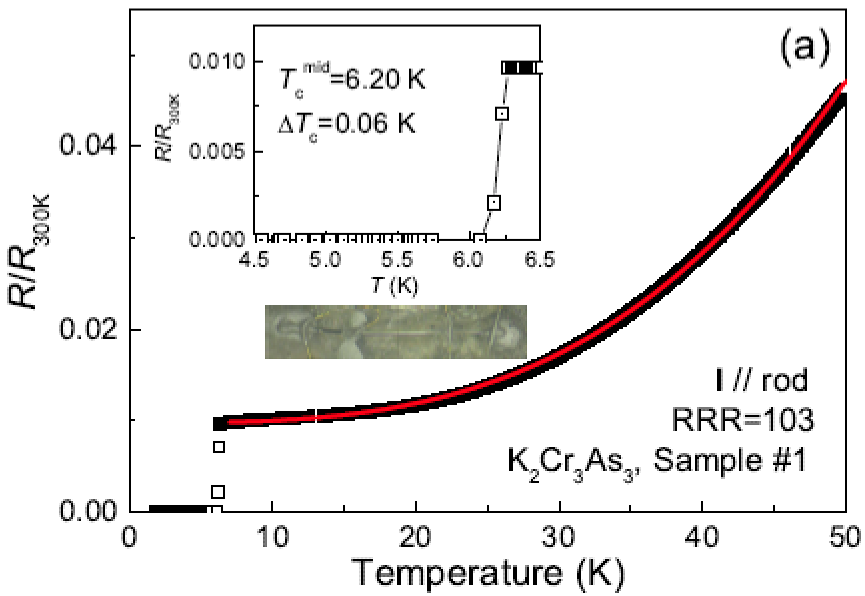


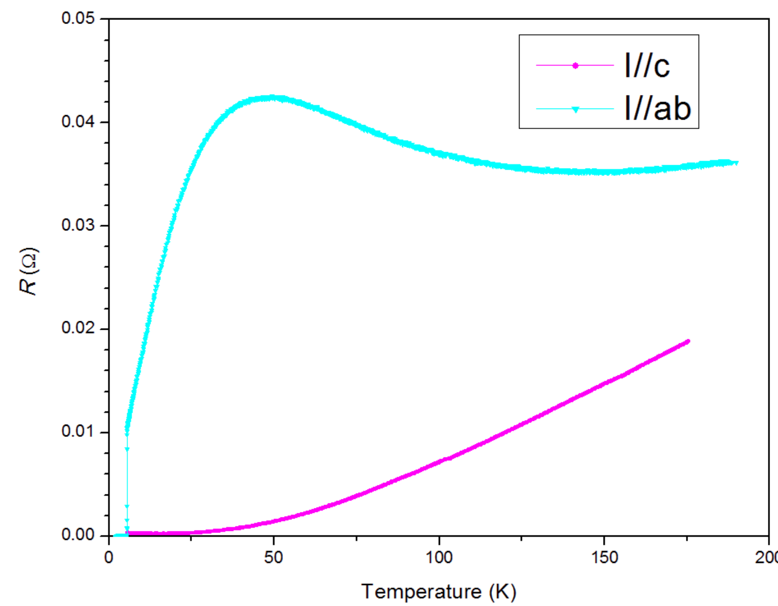
Polycrystalline samples

Guang-Han Cao's group, Phys. Rev. X 5, 011013 (2015); Phys. Rev. B 91, 020506(R) (2015); Science China Materials, 58(1) 16-10 (2015).

Normal state: smectic metal

Single crystal





$$\rho_c = \rho_0 + AT^{\alpha}, \alpha \approx 3$$

Guang-Han Cao's group (unpublished)

Previous theoretical studies

Yi Zhou, Chao Cao, and Fu-Chun Zhang, arXiv:1502.03928

- Effective Hamiltonian in 3D
- Superconducting instability by RPA

Other groups

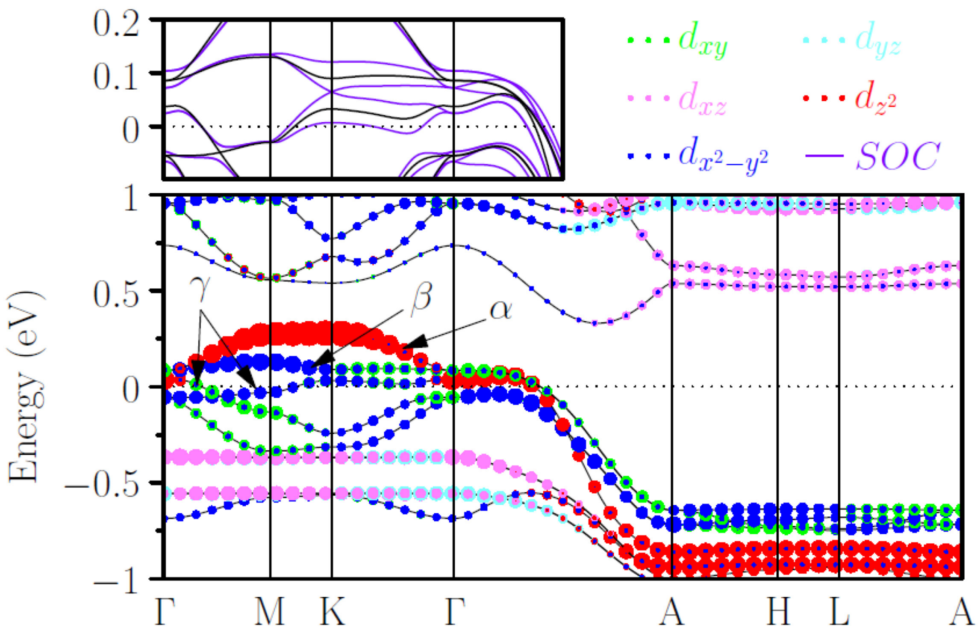
- Strong coupling approach

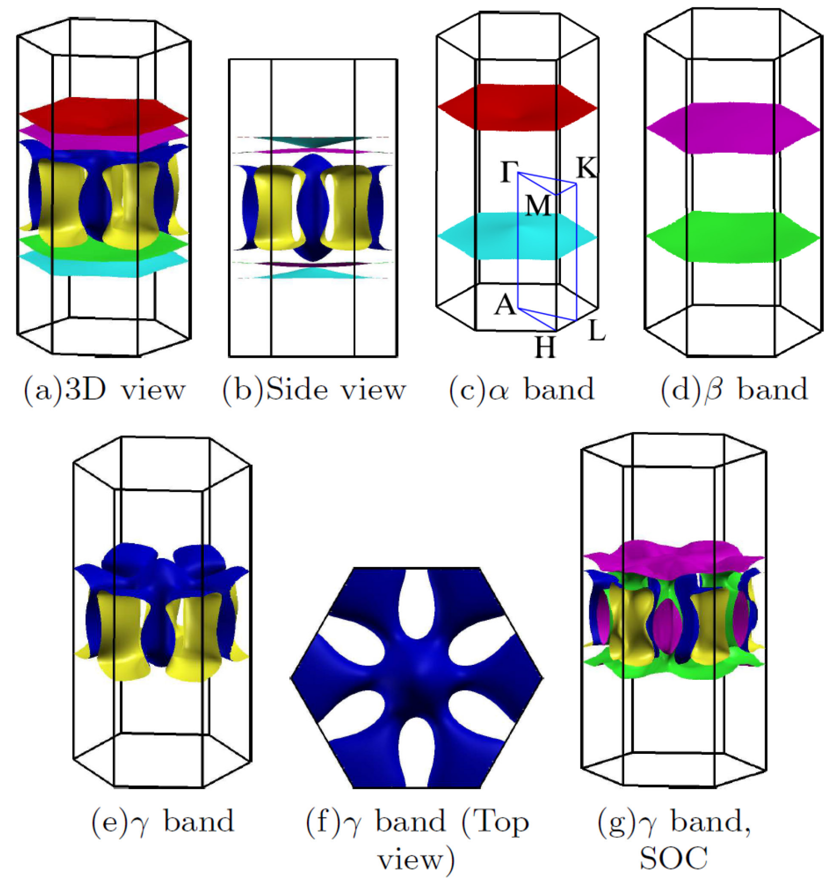
  Xianxin Wu, Fan Yang, Congcong Le, Heng Fan, Jiangping Hu, arXiv:1503.06707

- Effective Hamiltonian for a single-tube Cr2As3

  Hanting Zhong, Xiao-Yong Feng, Hua Chen, Jianhui Dai, arXiv:1503.08965

Extract tight-binding model from DFT





- ➢ **At least three orbitals per unit cell are required to catch the low energy electronic features.**
- ➢ **A three-orbital model is sufficient to describe both 1D and 3D features for superconductivity.**

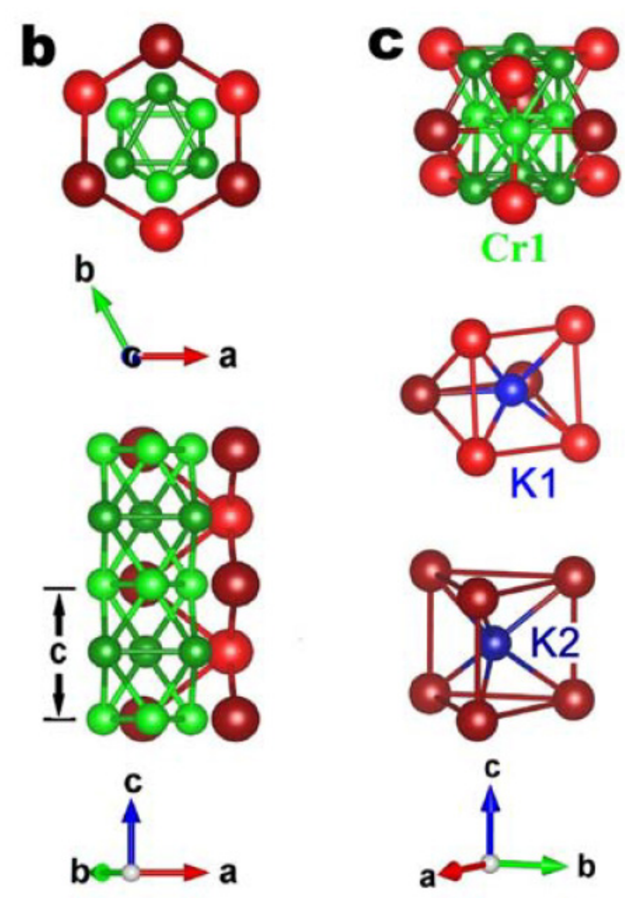
Jiang, Cao, Cao, arXiv:1412.1309 (2014)

Three-orbital model

TABLE I: Character table for D_{3h} group.

$D_{3h} = D_3 \times \sigma_h (\bar{6}m2)$			E	$2C_3$	$3C_2$	σ_h	$2S_3$	$3\sigma_v$
x^2+y^2, z^2	$x(x^2-3y^2)$	A_1'	1	1	1	1	1	1
	$y(3x^2-y^2)$	A_2'	1	1	-1	1	1	-1
(x^2-y^2, xy)	$(x,y), (xz^2, yz^2)$	E'	2	-1	0	2	-1	0
$yz(3x^2-y^2)$		A_1''	1	1	1	-1	-1	-1
$xz(x^2-3y^2)$	z, z^3	A_2''	1	1	-1	-1	-1	1
(xz, yz)	$(z(x^2-y^2), zxy)$	E''	2	-1	0	-2	1	0

Crystal: D_{3h} point group



- ➢ Selected by the principle of symmetry.
- ➢ Too many atomic orbitals, more than 10 atomic orbitals per unit cell.
- ➢ Three relevant molecular (per Cr_6As_6 cluster) orbitals: A'_1 and E' states.
- ➢ Neglect spin-orbit coupling at first for simplicity.

Yi Zhou, Chao Cao, and Fu-Chun Zhang, arXiv:1502.03928

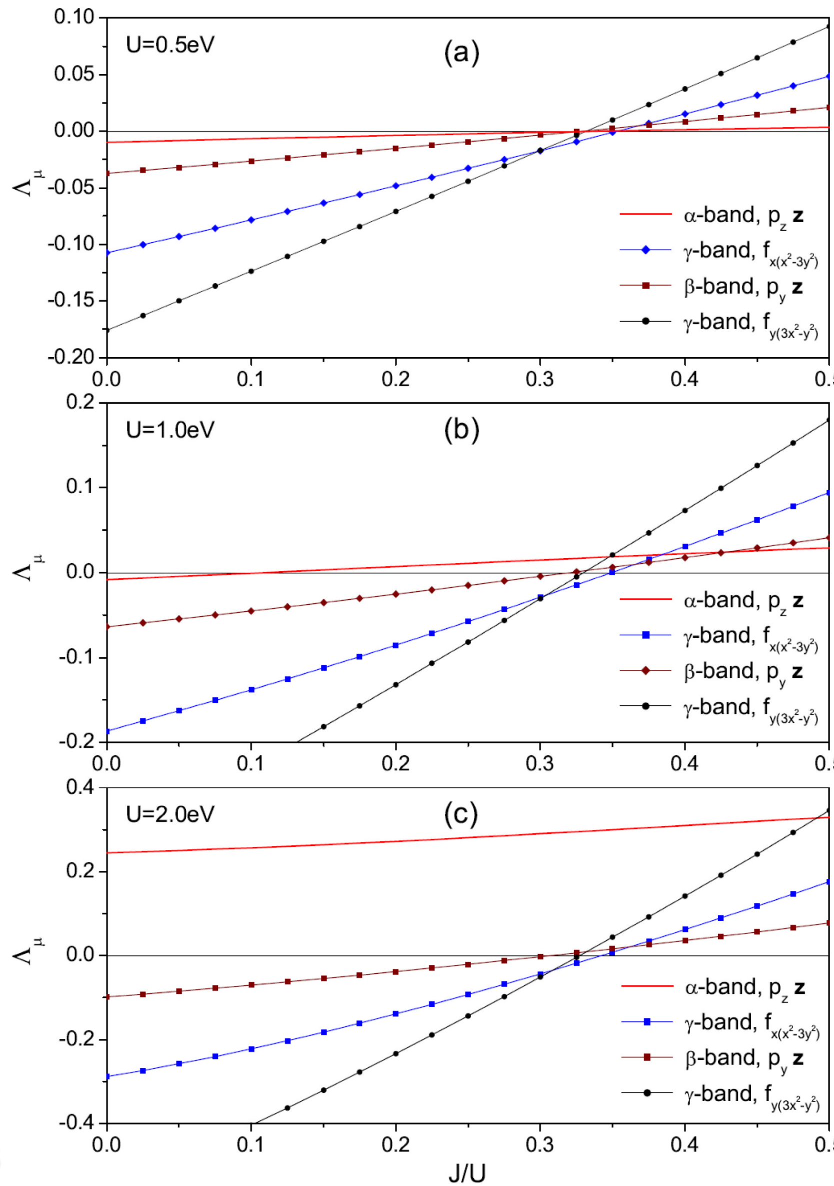
Superconducting pairing instability

Parameters:

$$U_1 = U_1' = U,\ U_2' = U_2 \text{ and } J' = J$$

The results are similar when $0.5 < J'/J < 2.$

- All the dominant states are spin-triplet states.
- For small *U*, the pairing arises from 3D γ-band, and has spatial *f*-wave symmetry when *J/U>1/3*.
  - Driving force: Hund's coupling
  - Line nodes in the gap function
  - DOS at Fermi level at γ-band is the largest
- For large *U*, a fully gapped *p*-wave state dominates at the quasi-1D α-band.



Yi Zhou, Chao Cao, and Fu-Chun Zhang, arXiv:1502.03928

Issues

- How the superconducting state is related to the **smectic metal** normal state?
- Superconducting instability in a single-tube $K_2Cr_3As_3$?
  - Spin-orbit coupling will mix spin-singlet and spin-triplet SC pairing through broken inversion symmetry.
  - Which component (singlet or triplet) will be more important?
- Why Rb compound exhibit different temperature dependence in 1/T1 from K compound in the normal state?

1D three-band Hubbard model

Hamiltonian

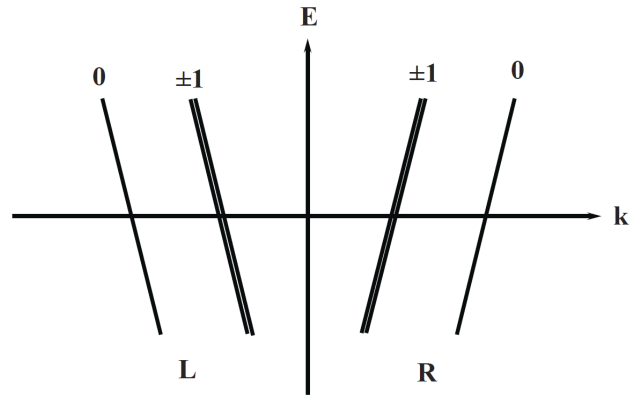
$$H^F = H_0^F + H_{int}^F$$

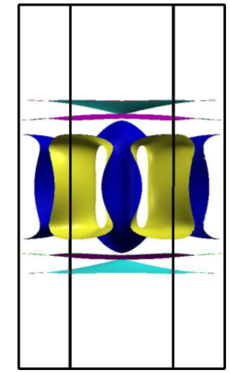
Non-interacting part

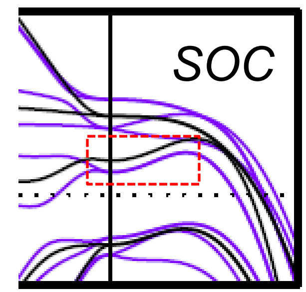
$$H_0^F = \sum_{km\sigma} \xi_{km} c_{km\sigma}^{\dagger} c_{km\sigma}$$

$m = 0, \quad A_1'$ orbital

$m = \pm 1, \quad$ degenerate E' orbitals







On-site repulsion

$$H_{int}^F = \frac{1}{2}\sum_{im}\sum_{\sigma\neq\sigma'} U n_{im\sigma} n_{im\sigma'} + \frac{1}{2}\sum_{i\sigma\sigma'}\sum_{m\neq m'} U' n_{im\sigma} n_{im'\sigma'} - \sum_{i}\sum_{m\neq m'} J\left(\vec{S}_{im}\cdot\vec{S}_{im'} + \frac{1}{4} n_{im} n_{im'}\right) + \frac{1}{2}\sum_{i\sigma}\sum_{m\neq m'} J' c_{im\sigma}^{\dagger} c_{im\bar{\sigma}}^{\dagger} c_{im'\bar{\sigma}} c_{im'\sigma}$$

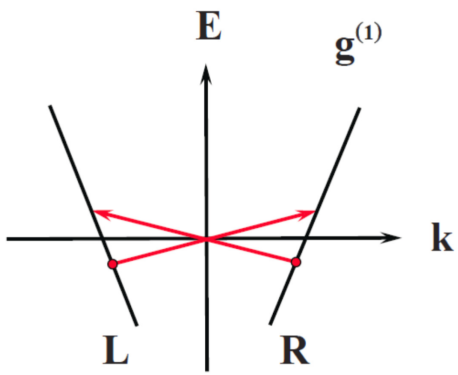
Irreducible representations for D_{3h} group $\Rightarrow \quad U = U' + 2J$

We also set $\quad J = J'$

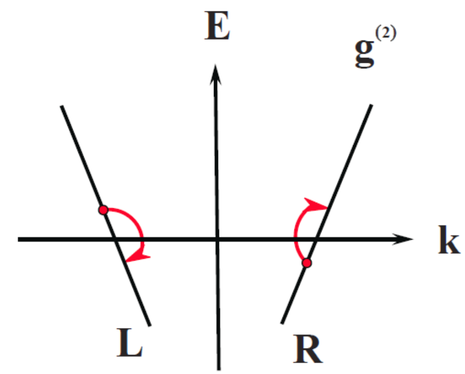
Scattering processes and "g-ology"

Single-band scattering processes

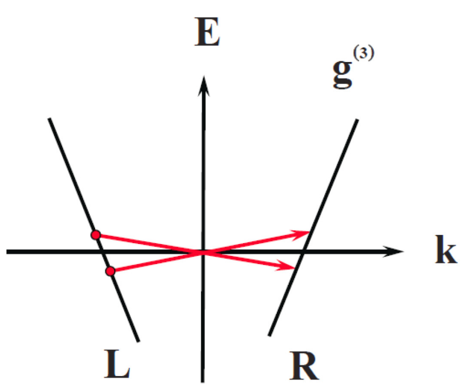
back scattering



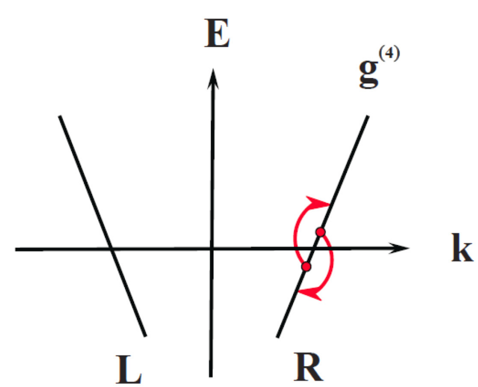
forward scattering (intra-chirality)



Umklapp scattering



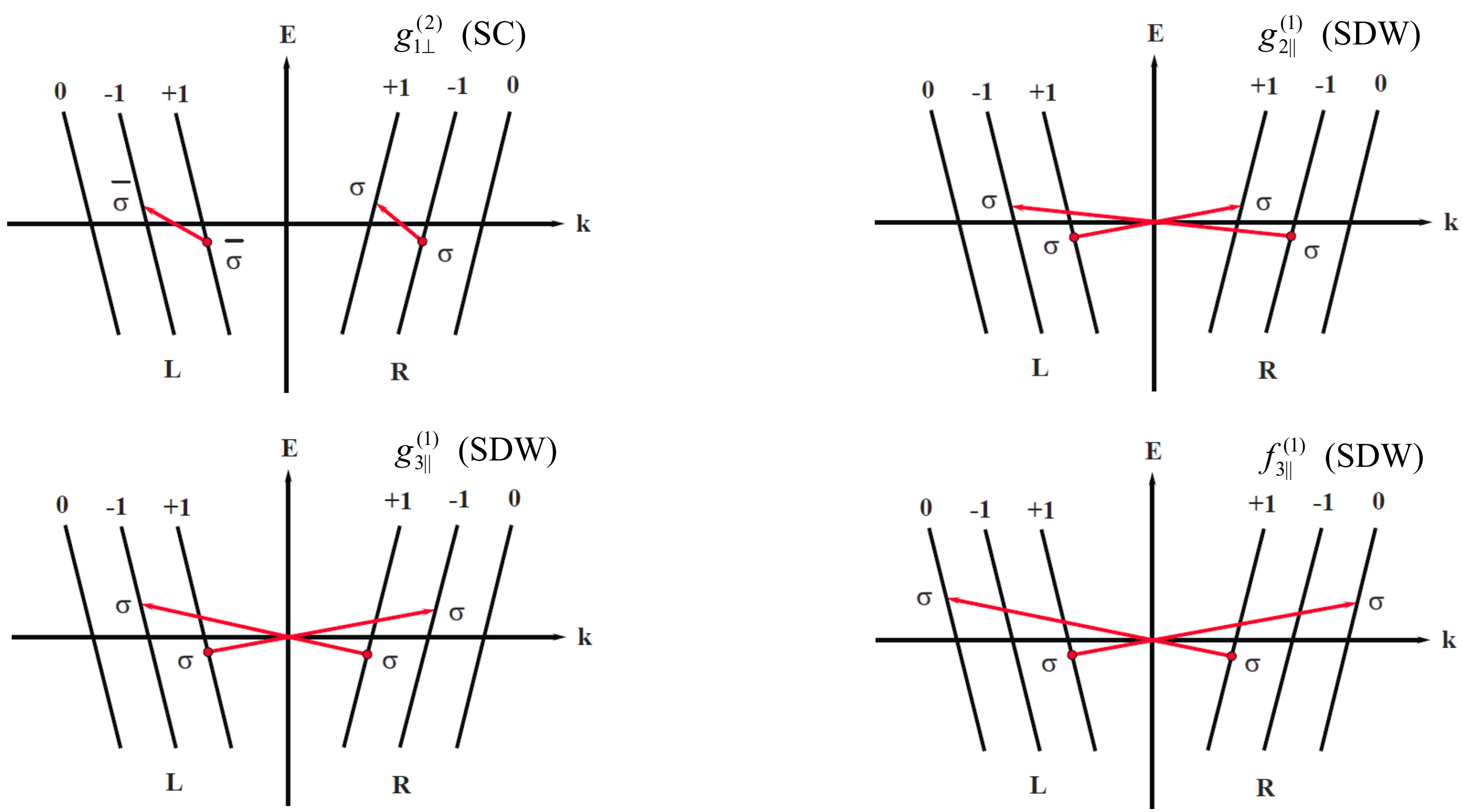
forward scattering (inter-chirality)



Three-band scattering processes

chirality		band				spin	
$g^{(1)}$	$\psi_p^\dagger\psi_{\bar p}^\dagger\psi_p\psi_{\bar p}$	g_1	$\psi_m^\dagger\psi_{\bar m}^\dagger\psi_m\psi_{\bar m}$	f_1	$\psi_m^\dagger\psi_0^\dagger\psi_m\psi_0 + h.c.$	$g_\parallel$	$\psi_\sigma^\dagger\psi_\sigma^\dagger\psi_\sigma\psi_\sigma$
$g^{(2)}$	$\psi_p^\dagger\psi_{\bar p}^\dagger\psi_{\bar p}\psi_p$	g_2	$\psi_m^\dagger\psi_{\bar m}^\dagger\psi_{\bar m}\psi_m$	f_2	$\psi_m^\dagger\psi_0^\dagger\psi_0\psi_m + h.c.$	$g_\perp$	$\psi_\sigma^\dagger\psi_{\bar\sigma}^\dagger\psi_{\bar\sigma}\psi_\sigma$
$g^{(3)}$	$\psi_p^\dagger\psi_p^\dagger\psi_{\bar p}\psi_{\bar p}$	g_3	$\psi_m^\dagger\psi_m^\dagger\psi_{\bar m}\psi_{\bar m}$	f_3	$\psi_m^\dagger\psi_m^\dagger\psi_0\psi_0 + h.c.$		
$g^{(4)}$	$\psi_p^\dagger\psi_p^\dagger\psi_p\psi_p$	g_4	$\psi_m^\dagger\psi_m^\dagger\psi_m\psi_m$	g	$\psi_0^\dagger\psi_0^\dagger\psi_0\psi_0$		

Four dominant scattering processes at incommensurate filling



Bosonization for 1D systems

Abelian bosonization

$$\psi_{pm\sigma} = \frac{\eta_{m\sigma}}{\sqrt{2\pi a}} e^{ipk_{Fm}x} e^{-ip\varphi_{pm\sigma}}$$

$$\{\eta_{m\sigma}, \eta_{m'\sigma'}\} = 2\delta_{mm'}\delta_{\sigma\sigma'}$$

Chiral and non-chiral fields

$$\varphi_{pm\sigma} = \phi_{m\sigma} - p\theta_{m\sigma}$$

$$\nabla\phi_{m\sigma} \propto n_{m\sigma} = \psi^{\dagger}_{Rm\sigma}\psi_{Rm\sigma} + \psi^{\dagger}_{Lm\sigma}\psi_{Lm\sigma}$$

$$\nabla\theta_{m\sigma} \propto j_{m\sigma} = \psi^{\dagger}_{Rm\sigma}\psi_{Rm\sigma} - \psi^{\dagger}_{Lm\sigma}\psi_{Lm\sigma}$$

Charge and spin degrees of freedom

$$\phi_{m\sigma} = \frac{1}{\sqrt{2}}\left(\phi_{cm} + \sigma\phi_{sm}\right)$$

$$\theta_{m\sigma} = \frac{1}{\sqrt{2}}\left(\theta_{cm} + \sigma\theta_{sm}\right)$$

Gauge choice for Klein factors

$$\eta_{m\sigma}\eta_{\bar{m}\sigma} = im\sigma$$

$$\eta_{m\sigma}\eta_{m\bar{\sigma}} = i\sigma$$

$$\eta_{m\sigma}\eta_{\bar{m}\bar{\sigma}} = im$$

$$\eta_{0\sigma}\eta_{m\sigma} = im\sigma$$

$$\eta_{0\sigma}\eta_{0\bar{\sigma}} = i\sigma$$

$$\eta_{0\sigma}\eta_{m\bar{\sigma}} = im$$

Non-interacting bosonic Hamiltonian

$$H_0^B = \frac{1}{2\pi}\int dx \sum_{\substack{\mu = c, s \\ \nu = 0, \pm 1}} v_{\mu\nu}\left[K_{\mu\nu}\left(\nabla\theta_{\mu\nu}\right)^2 + \frac{1}{K_{\mu\nu}}\left(\nabla\phi_{\mu\nu}\right)^2\right]$$

Luttinger liquid fixed point: gives rise to smectically metallic behaviors in normal state.

Renormalized Fermi velocities and Luttinger parameters

$$v_{c(s)\pm 1} = v_F\left\{1 - \frac{\left[\left(+(-)\,g_{4\perp}^{(2)}\right) - \left(g_{2\parallel}^{(2)} + (-)\,g_{2\perp}^{(2)}\right)\right]^2}{\left(2\pi v_F\right)^2}\right\}^{1/2}$$

$$v_{c(s)0} = v_F\left\{1 - \frac{\left[\left(+(-)\,g_{4\perp}^{(2)}\right) + 2\left(g_{2\parallel}^{(2)} + (-)\,g_{2\perp}^{(2)}\right)\right]^2}{\left(2\pi v_F\right)^2}\right\}^{1/2}$$

$$K_{c(s)\pm 1} = \left\{\frac{1 - \frac{1}{2\pi v_F}\left[\left(+(-)\,g_{4\perp}^{(2)}\right) - \left(g_{2\parallel}^{(2)} + (-)\,g_{2\perp}^{(2)}\right)\right]}{1 + \frac{1}{2\pi v_F}\left[\left(+(-)\,g_{4\perp}^{(2)}\right) - \left(g_{2\parallel}^{(2)} + (-)\,g_{2\perp}^{(2)}\right)\right]}\right\}^{1/2}$$

$$K_{c(s)0} = \left\{\frac{1 - \frac{1}{2\pi v_F}\left[\left(+(-)\,g_{4\perp}^{(2)}\right) + 2\left(g_{2\parallel}^{(2)} + (-)\,g_{2\perp}^{(2)}\right)\right]}{1 + \frac{1}{2\pi v_F}\left[\left(+(-)\,g_{4\perp}^{(2)}\right) + 2\left(g_{2\parallel}^{(2)} + (-)\,g_{2\perp}^{(2)}\right)\right]}\right\}^{1/2}$$

Interacting bosonic Hamiltonian

with all possible 13 running coupling constants

$$
\begin{aligned}
H^B_{int} = &- g^{(1)}_{1\perp}\frac{4}{(2\pi a)^2}\int dx\cos\left(\frac{2}{\sqrt{3}}\tilde{\phi}_{s-1}+\frac{4}{\sqrt{6}}\tilde{\phi}_{s0}\right)\cos\left(2\tilde{\theta}_{s+1}\right)\\
&+ g^{(1)}_{2\parallel}\frac{4}{(2\pi a)^2}\int dx\cos\left(2\tilde{\phi}_{c+1}\right)\cos\left(2\tilde{\phi}_{s+1}\right)\\
&+ g^{(1)}_{2\perp}\frac{4}{(2\pi a)^2}\int dx\cos\left(2\tilde{\phi}_{c+1}\right)\cos\left(\frac{2}{\sqrt{3}}\tilde{\phi}_{s-1}+\frac{4}{\sqrt{6}}\tilde{\phi}_{s0}\right)\\
&+ g^{(1)}_{3\parallel}\frac{4}{(2\pi a)^2}\int dx\cos\left(2\tilde{\theta}_{c+1}\right)\cos\left(2\tilde{\theta}_{s+1}\right)\\
&+ g^{(1)}_{3\perp}\frac{4}{(2\pi a)^2}\int dx\cos\left(2\tilde{\theta}_{c+1}\right)\cos\left(\frac{2}{\sqrt{3}}\tilde{\phi}_{s-1}+\frac{4}{\sqrt{6}}\tilde{\phi}_{s0}\right)\\
&+ g^{(1)}_{4\perp}\frac{4}{(2\pi a)^2}\int dx\cos\left(2\tilde{\phi}_{s+1}\right)\cos\left(\frac{2}{\sqrt{3}}\tilde{\phi}_{s-1}+\frac{4}{\sqrt{6}}\tilde{\phi}_{s0}\right)\\
&- g^{(2)}_{1\perp}\frac{4}{(2\pi a)^2}\int dx\cos\left(2\tilde{\phi}_{c+1}\right)\cos\left(2\tilde{\theta}_{s+1}\right)\\
&+ g^{(2)}_{3\perp}\frac{4}{(2\pi a)^2}\int dx\cos\left(2\tilde{\theta}_{c+1}\right)\cos\left(2\tilde{\phi}_{s+1}\right)\\
&- f^{(1)}_{1\perp}\frac{8}{(2\pi a)^2}\int dx\left[\cos\tilde{\phi}_{s+1}\cos\left(-\frac{1}{\sqrt{3}}\tilde{\phi}_{s-1}+\frac{4}{\sqrt{6}}\tilde{\phi}_{s0}\right)\cos\tilde{\theta}_{s+1}\cos\sqrt{3}\tilde{\theta}_{s-1}+(\cos\to\sin)\right]\\
&+ f^{(1)}_{3\parallel}\frac{8}{(2\pi a)^2}\int dx\left[\cos\tilde{\theta}_{c+1}\cos\sqrt{3}\tilde{\theta}_{c-1}\cos\tilde{\theta}_{s+1}\cos\sqrt{3}\tilde{\theta}_{s-1}+(\cos\to\sin)\right]\\
&+ f^{(1)}_{3\perp}\frac{8}{(2\pi a)^2}\int dx\left[\cos\tilde{\theta}_{c+1}\cos\sqrt{3}\tilde{\theta}_{c-1}\cos\tilde{\phi}_{s+1}\cos\left(-\frac{1}{\sqrt{3}}\tilde{\phi}_{s-1}+\frac{4}{\sqrt{6}}\tilde{\phi}_{s0}\right)+(\cos\to\sin)\right]\\
&+ f^{(2)}_{3\perp}\frac{8}{(2\pi a)^2}\int dx\left[\cos\tilde{\theta}_{c+1}\cos\sqrt{3}\tilde{\theta}_{c-1}\cos\tilde{\phi}_{s+1}\cos\sqrt{3}\tilde{\phi}_{s-1}+(\cos\to\sin)\right]\\
&+ g^{(1)}_{\perp}\frac{2}{(2\pi a)^2}\int dx\cos\left(-\frac{4}{\sqrt{3}}\tilde{\phi}_{s-1}+\frac{4}{\sqrt{6}}\tilde{\phi}_{s0}\right)
\end{aligned}
$$

Order parameters

Definition

$$O_{ph}^{ij} = \sum_{mm'\sigma\sigma'} \lambda^i_{mm'} \sigma^j_{\sigma\sigma'} \psi^\dagger_{Rm\sigma} \psi_{Lm'\sigma'}$$

$$O_{pp}^{ij} = \sum_{mm'\sigma\sigma'} \lambda^i_{mm'} \sigma \sigma^j_{\sigma\sigma'} \psi^\dagger_{Rm\sigma} \psi^\dagger_{Lm'\bar{\sigma}'}$$

λ^i : Gell - Mann matrices

σ^j : Pauli matrices

Examples for bosonization

SDW

$$O_{ph}^{13} \propto e^{-i2k_F x + i\left(\frac{1}{\sqrt{3}}\tilde{\phi}_{c-1} + \frac{2}{\sqrt{6}}\tilde{\phi}_{c0}\right)} \left[\cos\left(\frac{1}{\sqrt{3}}\tilde{\phi}_{s-1} + \frac{2}{\sqrt{6}}\tilde{\phi}_{s0}\right) \sin\tilde{\theta}_{c+1} \cos\tilde{\theta}_{s+1} + i\left(\cos \leftrightarrow \sin\right)\right]$$

Spin-triplet superconducting state

$$O_{pp}^{23} \propto e^{i\left(\frac{1}{\sqrt{3}}\tilde{\theta}_{c-1} + \frac{2}{\sqrt{6}}\tilde{\theta}_{c0}\right)} \left[\cos\left(\frac{1}{\sqrt{3}}\tilde{\phi}_{s-1} + \frac{2}{\sqrt{6}}\tilde{\phi}_{s0}\right) \cos\tilde{\phi}_{c+1} \cos\tilde{\theta}_{s+1} - i\left(\cos \leftrightarrow \sin\right)\right]$$

Renormalization group

Operator product expansion (OPE)

$$\frac{dg_k}{dl} = (d - \Delta_k)\, g_k - \sum_{ij} C_{ij}^k g_i g_j$$

tree level one-loop

Renormalized coupling constants

$$y_i = \frac{g_i}{\pi v_F}$$
$$x_i = \frac{f_i}{\pi v_F}$$

Tree-level RG equations

$$\frac{dg_{1\perp}^{(1)}}{dl} = \left[2 - \left(\frac{1}{3}K_{s-1} + \frac{2}{3}K_{s0} + K_{s+1}^{-1}\right)\right] g_{1\perp}^{(1)}$$

$$\frac{dg_{2\parallel}^{(1)}}{dl} = \left[2 - \left(K_{c+1} + K_{s+1}\right)\right] g_{2\parallel}^{(1)}$$

$$\frac{dg_{2\perp}^{(1)}}{dl} = \left[2 - \left(K_{c+1} + \frac{1}{3}K_{s-1} + \frac{2}{3}K_{s0}\right)\right] g_{2\perp}^{(1)}$$

$$\frac{dg_{3\parallel}^{(1)}}{dl} = \left[2 - \left(K_{c+1}^{-1} + K_{s+1}^{-1}\right)\right] g_{3\parallel}^{(1)}$$

$$\frac{dg_{3\perp}^{(1)}}{dl} = \left[2 - \left(K_{c+1}^{-1} + \frac{1}{3}K_{s-1} + \frac{2}{3}K_{s0}\right)\right] g_{3\perp}^{(1)}$$

$$\frac{dg_{4\perp}^{(1)}}{dl} = \left[2 - \left(K_{c+1} + \frac{1}{3}K_{s-1} + \frac{2}{3}K_{s0}\right)\right] g_{4\perp}^{(1)}$$

$$\frac{dg_{1\perp}^{(2)}}{dl} = \left[2 - \left(K_{c+1} + K_{s+1}^{-1}\right)\right] g_{1\perp}^{(2)}$$

$$\frac{dg_{3\perp}^{(2)}}{dl} = \left[2 - \left(K_{c+1}^{-1} + K_{s+1}\right)\right] g_{3\perp}^{(2)}$$

$$\frac{df_{1\perp}^{(1)}}{dl} = \left[2 - \left(\frac{1}{4}K_{s+1} + \frac{1}{12}K_{s-1} + \frac{2}{3}K_{s0} + \frac{1}{4}K_{s+1}^{-1} + \frac{3}{4}K_{s-1}^{-1}\right)\right] f_{1\perp}^{(1)}$$

$$\frac{df_{3\parallel}^{(1)}}{dl} = \left[2 - \left(\frac{1}{4}K_{c+1}^{-1} + \frac{3}{4}K_{c-1}^{-1} + \frac{1}{4}K_{s+1}^{-1} + \frac{3}{4}K_{s-1}^{-1}\right)\right] f_{3\parallel}^{(1)}$$

$$\frac{df_{3\perp}^{(1)}}{dl} = \left[2 - \left(\frac{1}{4}K_{c+1}^{-1} + \frac{3}{4}K_{c-1}^{1} + \frac{1}{4}K_{s+1} + \frac{1}{12}K_{s-1} + \frac{2}{3}K_{s0}\right)\right] f_{3\perp}^{(1)}$$

$$\frac{df_{3\perp}^{(2)}}{dl} = \left[2 - \left(\frac{1}{4}K_{c+1}^{-1} + \frac{3}{4}K_{c-1}^{-1} + \frac{1}{4}K_{s+1} + \frac{3}{4}K_{s-1}\right)\right] f_{3\perp}^{(2)}$$

$$\frac{dg_{\perp}^{(1)}}{dl} = \left[2 - \left(\frac{4}{3}K_{s-1} + \frac{2}{3}K_{s0}\right)\right] g_{\perp}^{(1)}$$

Renormalize coupling constants

$$y_i = \frac{g_i}{\pi v_F}$$

$$x_i = \frac{f_i}{\pi v_F}$$

Expanding Luttinger paramters

$$K_{\mu\nu} = 1 - y_{\mu\nu}$$

with

$$y_{c\pm 1} = \frac{1}{2}\left[\left(y_{4\perp}^{(2)}\right) - \left(y_{2\parallel}^{(2)} + y_{2\perp}^{(2)}\right)\right]$$

$$y_{c0} = \frac{1}{2}\left[\left(y_{4\perp}^{(2)}\right) + 2\left(y_{2\parallel}^{(2)} + y_{2\perp}^{(2)}\right)\right]$$

$$y_{s\pm 1} = \frac{1}{2}\left[\left(-y_{4\perp}^{(2)}\right) - \left(y_{2\parallel}^{(2)} - y_{2\perp}^{(2)}\right)\right]$$

$$y_{s0} = \frac{1}{2}\left[\left(-y_{4\perp}^{(2)}\right) + 2\left(y_{2\parallel}^{(2)} - y_{2\perp}^{(2)}\right)\right]$$

Tree-level RG equations

$$\frac{dy_{1\perp}^{(1)}}{dl} = \left(y_{2\|}^{(2)} - y_{2\perp}^{(2)}\right) y_{1\perp}^{(1)},$$

$$\frac{dy_{2\|}^{(1)}}{dl} = -y_{2\|}^{(2)} y_{2\|}^{(1)},$$

$$\frac{dy_{2\perp}^{(1)}}{dl} = -y_{2\perp}^{(2)} y_{2\perp}^{(1)},$$

$$\frac{dy_{3\|}^{(1)}}{dl} = y_{2\|}^{(2)} y_{3\|}^{(1)},$$

$$\frac{dy_{3\perp}^{(1)}}{dl} = \left(-y_{4\perp}^{(2)} + y_{2\|}^{(2)}\right) y_{3\perp}^{(1)},$$

$$\frac{dy_{4\perp}^{(1)}}{dl} = -y_{2\perp}^{(2)} y_{4\perp}^{(1)},$$

$$\frac{dy_{1\perp}^{(2)}}{dl} = \left(y_{4\perp}^{(2)} - y_{2\perp}^{(2)}\right) y_{1\perp}^{(2)},$$

$$\frac{dy_{3\perp}^{(2)}}{dl} = \left(-y_{4\perp}^{(2)} + y_{2\perp}^{(2)}\right) y_{3\perp}^{(2)},$$

$$\frac{dx_{1\perp}^{(1)}}{dl} = \left(y_{2\|}^{(2)} - y_{2\perp}^{(2)}\right) x_{1\perp}^{(1)},$$

$$\frac{dx_{3\|}^{(1)}}{dl} = y_{2\|}^{(2)} x_{3\|}^{(1)},$$

$$\frac{dx_{3\perp}^{(1)}}{dl} = \left(-y_{4\perp}^{(2)} + y_{2\|}^{(2)}\right) x_{3\perp}^{(1)},$$

$$\frac{dx_{3\perp}^{(2)}}{dl} = \left(-y_{4\perp}^{(2)} + y_{2\perp}^{(2)}\right) x_{3\perp}^{(2)},$$

$$\frac{dy_{\perp}^{(1)}}{dl} = -y_{4\perp}^{(2)} y_{\perp}^{(1)}.$$

Relevant coupling constants

$J < U/3$ $\quad x_{3\|}^{(1)},\ y_{3\|}^{(1)}$ and $y_{1\perp}^{(2)}$

$J > U/3$ $\quad y_{2\|}^{(1)}$ and $y_{1\perp}^{(2)}$

Fixed points (hypersurface)

$J < U/3$

$$y_{3\|}^{(1)} = y_{3\|}^{(1)*}$$
$$y_{1\perp}^{(2)} = y_{1\perp}^{(2)*}$$
$$x_{3\|}^{(1)} = x_{3\|}^{(1)*}$$

$J > U/3$

$$y_{2\|}^{(1)} = y_{2\|}^{(1)*}$$
$$y_{1\perp}^{(2)} = y_{1\perp}^{(2)*}$$

- Insufficient to determine the ground state.
- One-loop correction will change these results.

One-loop RG equations (spin rotational symmetry has been applied for simplicity)

$$\frac{dy_{1\perp}^{(1)}}{dl} = -\left(y_{1\perp}^{(1)}\right)^2 - y_{2\perp}^{(1)} y_{1\perp}^{(2)} + y_{3\parallel}^{(1)} y_{3\perp}^{(1)},$$

$$\frac{dy_{2\parallel}^{(1)}}{dl} = \frac{1}{2} y_{1\perp}^{(1)} y_{2\parallel}^{(1)} - y_{2\perp}^{(1)} y_{4\perp}^{(1)},$$

$$\frac{dy_{2\perp}^{(1)}}{dl} = -\frac{1}{2} y_{1\perp}^{(1)} y_{2\perp}^{(1)} - y_{1\perp}^{(1)} y_{1\perp}^{(2)} - y_{2\parallel}^{(1)} y_{4\perp}^{(1)},$$

$$\frac{dy_{3\parallel}^{(1)}}{dl} = -\frac{1}{2} y_{1\perp}^{(1)} y_{3\parallel}^{(1)} + y_{1\perp}^{(1)} y_{3\perp}^{(1)},$$

$$\frac{dy_{3\perp}^{(1)}}{dl} = -\left(y_{4\perp}^{(1)} + \frac{1}{2} y_{1\perp}^{(1)}\right) y_{3\perp}^{(1)} + y_{1\perp}^{(1)} y_{3\parallel}^{(1)} - y_{4\perp}^{(1)} y_{3\perp}^{(2)},$$

$$\frac{dy_{4\perp}^{(1)}}{dl} = \frac{1}{2} y_{1\perp}^{(1)} y_{4\perp}^{(1)} - y_{2\parallel}^{(1)} y_{2\perp}^{(1)} - y_{3\perp}^{(1)} y_{3\perp}^{(2)},$$

$$\frac{dy_{1\perp}^{(2)}}{dl} = \left(y_{4\perp}^{(1)} - \frac{1}{2} y_{1\perp}^{(1)}\right) y_{1\perp}^{(2)} - y_{1\perp}^{(1)} y_{2\perp}^{(1)},$$

$$\frac{dy_{3\perp}^{(2)}}{dl} = \left(-y_{4\perp}^{(1)} + \frac{1}{2} y_{1\perp}^{(1)}\right) y_{3\perp}^{(2)} - y_{3\perp}^{(1)} y_{4\perp}^{(1)},$$

$$\frac{dx_{1\perp}^{(1)}}{dl} = -\left(x_{1\perp}^{(1)}\right)^2 + x_{3\parallel}^{(1)} x_{3\perp}^{(1)},$$

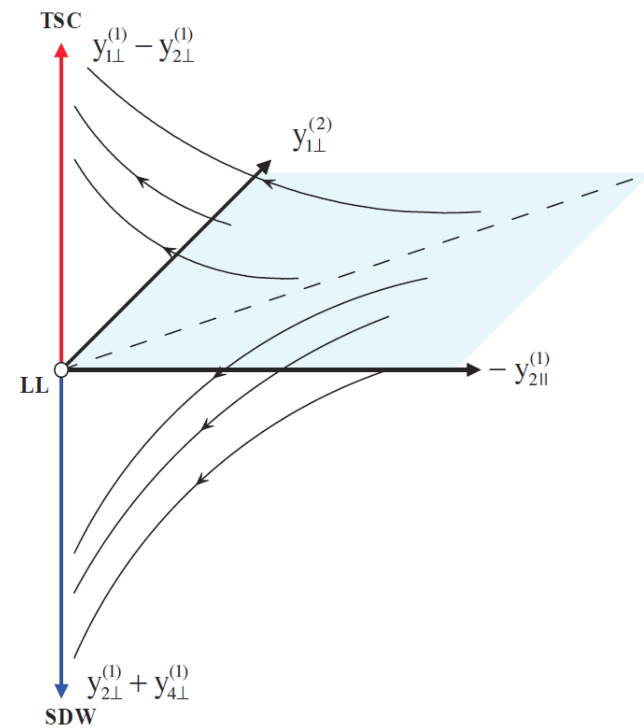
$$\frac{dx_{3\parallel}^{(1)}}{dl} = -\frac{1}{2} x_{1\perp}^{(1)} x_{3\parallel}^{(1)} + x_{1\perp}^{(1)} x_{3\perp}^{(1)},$$

$$\frac{dx_{3\perp}^{(1)}}{dl} = -\left(y_{\perp}^{(1)} + \frac{1}{2} x_{1\perp}^{(1)}\right) x_{3\perp}^{(1)} + x_{1\perp}^{(1)} x_{3\parallel}^{(1)},$$

$$\frac{dx_{3\perp}^{(2)}}{dl} = \left(-y_{\perp}^{(1)} + \frac{1}{2} x_{1\perp}^{(1)}\right) x_{3\perp}^{(2)},$$

$$\frac{dy_{\perp}^{(1)}}{dl} = -\left(y_{\perp}^{(1)}\right)^2.$$

RG flow



Sketched RG flow for $J > U/3$

Relevant scaling field and corresponding ground states

$J < U/3$

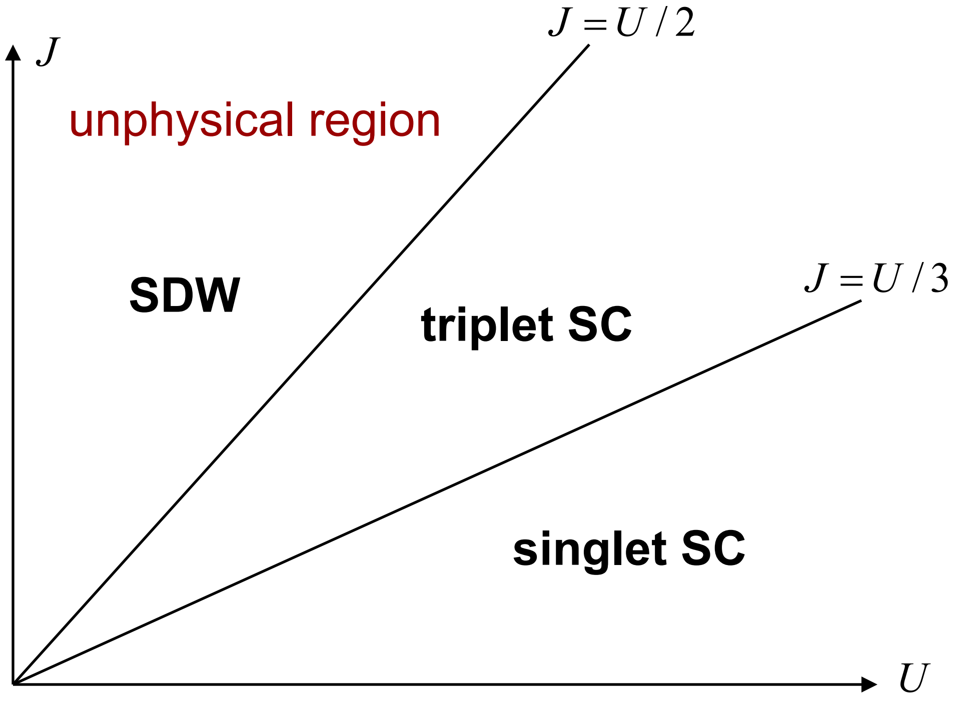
Scaling field	$x_{1\perp}^{(1)} + x_{3\perp}^{(1)}$	$y_{1\perp}^{(1)} + y_{3\perp}^{(1)}$	$y_{1\perp}^{(1)} - y_{2\perp}^{(1)}$
Instability	SDW	SDW	SSC
Order parameter	O_{ph}^{43}, O_{ph}^{63}	O_{ph}^{13}	O_{pp}^{20}
Saddle point	$\frac{1}{2}\tilde{\phi}_{s+1} - \frac{1}{\sqrt{12}}\tilde{\phi}_{s-1} + \frac{2}{\sqrt{6}}\tilde{\phi}_{s0} = 0\left(\frac{\pi}{2}\right)$ $\frac{1}{2}\tilde{\theta}_{c+1} + \frac{3}{\sqrt{12}}\tilde{\theta}_{c-1} = \frac{\pi}{2}\,(0)$ $\frac{1}{2}\tilde{\theta}_{s+1} + \frac{3}{\sqrt{12}}\tilde{\theta}_{s-1} = 0\left(\frac{\pi}{2}\right)$	$\frac{1}{\sqrt{3}}\tilde{\phi}_{s-1} + \frac{2}{\sqrt{6}}\tilde{\phi}_{s0} = 0\left(\frac{\pi}{2}\right)$ $\tilde{\theta}_{c+1} = \frac{\pi}{2}\,(0)$ $\tilde{\theta}_{s+1} = 0\left(\frac{\pi}{2}\right)$	$\frac{1}{\sqrt{3}}\tilde{\phi}_{s-1} + \frac{2}{\sqrt{6}}\tilde{\phi}_{s0} = 0\left(\frac{\pi}{2}\right)$ $\tilde{\phi}_{c+1} = \frac{\pi}{2}\,(0)$ $\tilde{\theta}_{s+1} = \frac{\pi}{2}\,(0)$

$J > U/3$

Scaling field	$y_{2\perp}^{(1)} + y_{4\perp}^{(1)}$	$y_{1\perp}^{(1)} - y_{2\perp}^{(1)}$
Instability	SDW	TSC
Order parameter	$O_{ph}^{03} + \frac{\sqrt{3}}{2}O_{ph}^{83}$	O_{pp}^{23}
Saddle point	$\frac{1}{\sqrt{3}}\tilde{\phi}_{s-1} + \frac{2}{\sqrt{6}}\tilde{\phi}_{s0} = 0\left(\frac{\pi}{2}\right)$ $\tilde{\phi}_{c+1} = \frac{\pi}{2}\,(0)$ $\tilde{\phi}_{s+1} = \frac{\pi}{2}\,(0)$	$\frac{1}{\sqrt{3}}\tilde{\phi}_{s-1} + \frac{2}{\sqrt{6}}\tilde{\phi}_{s0} = 0\left(\frac{\pi}{2}\right)$ $\tilde{\phi}_{c+1} = 0\left(\frac{\pi}{2}\right)$ $\tilde{\theta}_{s+1} = 0\left(\frac{\pi}{2}\right)$

Phase diagram

- **Determined by initial coupling constants, say, the microscopic model.**



$$U = U' + 2J$$

Normal state: TLL fixed point

NMR: Spin-lattice relaxation rate

$$\frac{1}{T_1} = A_f^2 T \sum_q \frac{Im\chi\left(q,\omega\right)}{\omega}$$

Three-band model

$$\begin{aligned}\frac{1}{T_1} \propto\ & A_1 T \\ & + A_2 T^{\frac{1}{2}\left[\left(K_{c+1}+\frac{1}{3}K_{c-1}+\frac{2}{3}K_{c0}\right)+\left(K_{s+1}+\frac{1}{3}K_{s-1}+\frac{2}{3}K_{s0}\right)\right]-1} \\ & + A_3 T^{\frac{1}{2}\left[\left(\frac{4}{3}K_{c-1}+\frac{2}{3}K_{c0}\right)+\left(\frac{4}{3}K_{s-1}+\frac{2}{3}K_{s0}\right)\right]-1}. \qquad \text{(C3)}\end{aligned}$$

Spin-rotational symmetric system

$$\frac{1}{T_1} \propto A\ T + B\ T^{1-\frac{U}{2\pi v_F}}$$

U>0 and U<0 **will result in different low temperature behaviors !**

Possible superconducting ground states

0<J<U/3

$y_{1\perp}^{(1)} - y_{2\perp}^{(1)}$
SSC
O_{pp}^{20}
$\begin{aligned} \frac{1}{\sqrt{3}}\tilde{\phi}_{s-1} + \frac{2}{\sqrt{6}}\tilde{\phi}_{s0} &= 0\left(\frac{\pi}{2}\right) \\ \tilde{\phi}_{c+1} &= \frac{\pi}{2}\,(0) \\ \tilde{\theta}_{s+1} &= \frac{\pi}{2}\,(0) \end{aligned}$

spin-singlet, odd parity, orbital antisymmetric

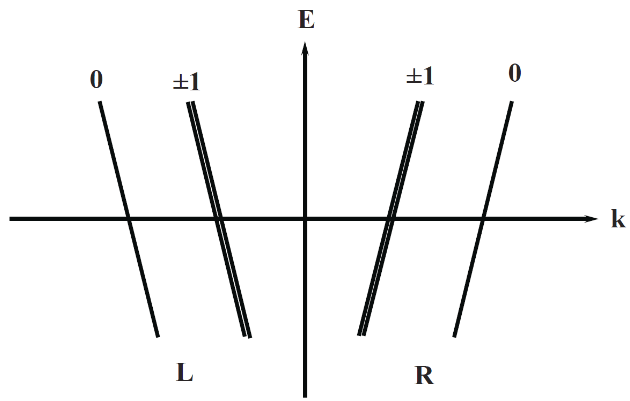
U/3<J<U/2

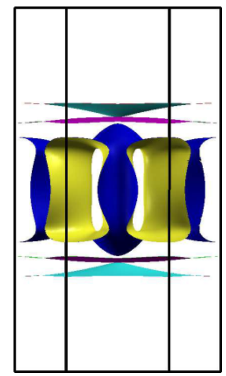
$y_{1\perp}^{(1)} - y_{2\perp}^{(1)}$
TSC
O_{pp}^{23}
$\begin{aligned} \frac{1}{\sqrt{3}}\tilde{\phi}_{s-1} + \frac{2}{\sqrt{6}}\tilde{\phi}_{s0} &= 0\left(\frac{\pi}{2}\right) \\ \tilde{\phi}_{c+1} &= 0\left(\frac{\pi}{2}\right) \\ \tilde{\theta}_{s+1} &= 0\left(\frac{\pi}{2}\right) \end{aligned}$

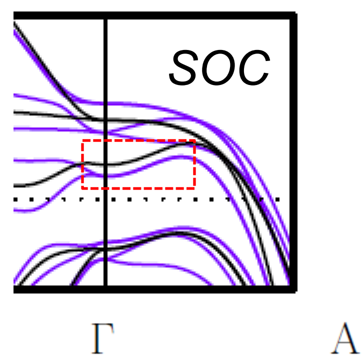
spin-triplet, even parity, orbital antisymmetric

Discussion: Lifted degeneracy

- The two-fold degenerate of *E'* bands will be lifted by *inter-chain coupling*.







Interacting bosonic Hamiltonian when $k_{F-1} \neq k_{F1}$

$$
\begin{aligned}
H^B_{int} = &- g^{(1)}_{1\perp}\frac{4}{(2\pi a)^2}\int dx \cos\left(\frac{2}{\sqrt{3}}\tilde{\phi}_{s-1}+\frac{4}{\sqrt{6}}\tilde{\phi}_{s0}\right)\cos\left(2\tilde{\theta}_{s+1}\right) \\
&+ g^{(1)}_{2\parallel}\frac{4}{(2\pi a)^2}\int dx \cos\left(2\Delta k_F x+2\tilde{\phi}_{c+1}\right)\cos\left(2\tilde{\phi}_{s+1}\right) \\
&+ g^{(1)}_{2\perp}\frac{4}{(2\pi a)^2}\int dx \cos\left(2\Delta k_F x+2\tilde{\phi}_{c+1}\right)\cos\left(\frac{2}{\sqrt{3}}\tilde{\phi}_{s-1}+\frac{4}{\sqrt{6}}\tilde{\phi}_{s0}\right) \\
&+ g^{(1)}_{3\parallel}\frac{4}{(2\pi a)^2}\int dx \cos\left(2\tilde{\theta}_{c+1}\right)\cos\left(2\tilde{\theta}_{s+1}\right) \\
&+ g^{(1)}_{3\perp}\frac{4}{(2\pi a)^2}\int dx \cos\left(2\tilde{\theta}_{c+1}\right)\cos\left(\frac{2}{\sqrt{3}}\tilde{\phi}_{s-1}+\frac{4}{\sqrt{6}}\tilde{\phi}_{s0}\right) \\
&+ g^{(1)}_{4\perp}\frac{4}{(2\pi a)^2}\int dx \cos\left(2\tilde{\phi}_{s+1}\right)\cos\left(\frac{2}{\sqrt{3}}\tilde{\phi}_{s-1}+\frac{4}{\sqrt{6}}\tilde{\phi}_{s0}\right) \\
&- g^{(2)}_{1\perp}\frac{4}{(2\pi a)^2}\int dx \cos\left(2\Delta k_F x+2\tilde{\phi}_{c+1}\right)\cos\left(2\tilde{\theta}_{s+1}\right) \\
&+ g^{(2)}_{3\perp}\frac{4}{(2\pi a)^2}\int dx \cos\left(2\tilde{\theta}_{c+1}\right)\cos\left(2\tilde{\phi}_{s+1}\right) \\
&- f^{(1)}_{1\perp}\frac{8}{(2\pi a)^2}\int dx \left[\cos\tilde{\phi}_{s+1}\cos\left(-\frac{1}{\sqrt{3}}\tilde{\phi}_{s-1}+\frac{4}{\sqrt{6}}\tilde{\phi}_{s0}\right)\cos\tilde{\theta}_{s+1}\cos\sqrt{3}\tilde{\theta}_{s-1}+(\cos\to\sin)\right] \\
&+ f^{(1)}_{3\parallel}\frac{8}{(2\pi a)^2}\int dx \left[\cos\tilde{\theta}_{c+1}\cos\sqrt{3}\tilde{\theta}_{c-1}\cos\tilde{\theta}_{s+1}\cos\sqrt{3}\tilde{\theta}_{s-1}+(\cos\to\sin)\right] \\
&+ f^{(1)}_{3\perp}\frac{8}{(2\pi a)^2}\int dx \left[\cos\tilde{\theta}_{c+1}\cos\sqrt{3}\tilde{\theta}_{c-1}\cos\tilde{\phi}_{s+1}\cos\left(-\frac{1}{\sqrt{3}}\tilde{\phi}_{s-1}+\frac{4}{\sqrt{6}}\tilde{\phi}_{s0}\right)+(\cos\to\sin)\right] \\
&+ f^{(2)}_{3\perp}\frac{8}{(2\pi a)^2}\int dx \left[\cos\tilde{\theta}_{c+1}\cos\sqrt{3}\tilde{\theta}_{c-1}\cos\tilde{\phi}_{s+1}\cos\sqrt{3}\tilde{\phi}_{s-1}+(\cos\to\sin)\right] \\
&+ g^{(1)}_{\perp}\frac{2}{(2\pi a)^2}\int dx \cos\left(-\frac{4}{\sqrt{3}}\tilde{\phi}_{s-1}+\frac{4}{\sqrt{6}}\tilde{\phi}_{s0}\right),
\end{aligned}
$$

Fixed points (hypersurface)

$J < U/3$

- **SDW** $y^{(1)}_{3\parallel} = y^{(1)*}_{3\parallel}$
- **SSC** $y^{(2)}_{1\perp} = y^{(2)*}_{1\perp}$
- **SDW** $x^{(1)}_{3\parallel} = x^{(1)*}_{3\parallel}$

$J > U/3$

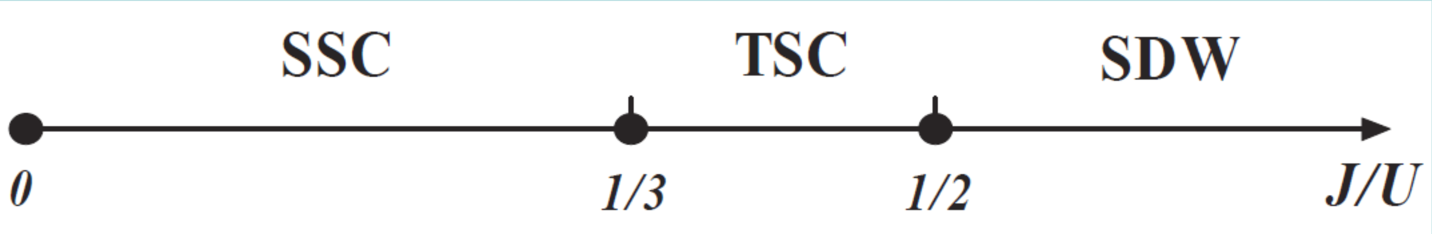
- **SDW** $y^{(1)}_{2\parallel} = y^{(1)*}_{2\parallel}$
- **TSC** $y^{(2)}_{1\perp} = y^{(2)*}_{1\perp}$

Consequences of lifted degeneracy

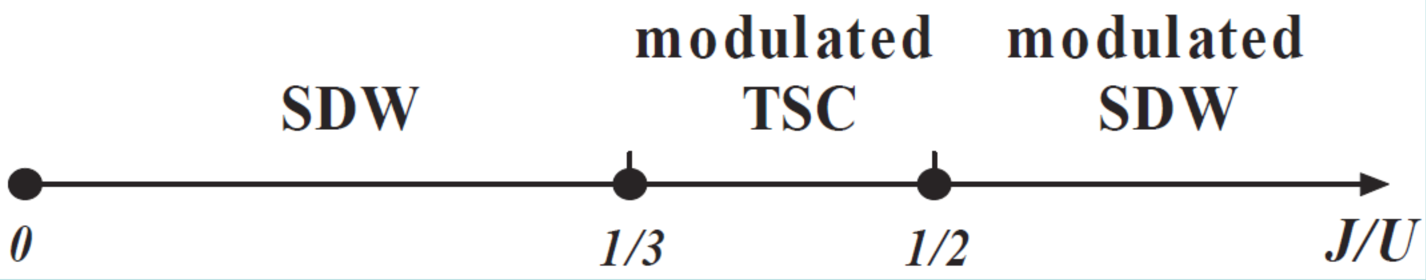
- SSC be suppressed, SDW have chance to dominate for $J<U/3$
- Interband pairing will be modulated by a phase factor
  - Possible FFLO state?
  - Unstable when the degeneracy lift becomes significant.

Phase diagrams

Degenerate bands $k_{F-1} = k_{F1}$



Nearly-degenerate bands $k_{F-1} \neq k_{F1}$



Take home message

- One-loop RG analysis to 1D three-band Hubbard model
  - Assumption 1: Two of the three bands are (nearly) degenerate
  - Assumption 2: Incommensurate electron filling
- Results:
  - *0<J<U/3*, spin-singlet SC (degenerate bands) or spin density wave (non-degenerate bands)
  - *U/3<J<U/2*, spin-triplet SC (nearly degenerate bands)
  - *J>U/2* (unphysical region), SDW
- Possible application to superconductor $K_2Cr_3As_3$
  - Take Luttinger liquid normal state as the starting point
  - Inter-chain coupling will lift band degeneracy and SDW will dominate over SSC when *J/U* <1/3; while TSC will always dominate over SDW when *J/U* >1/3.
  - Inter-chain coupling will determine the spatially pairing symmetry

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Thank you！