Quantum transport in topological insulators:

Weak Anti-localization
&
Electron-electron interaction

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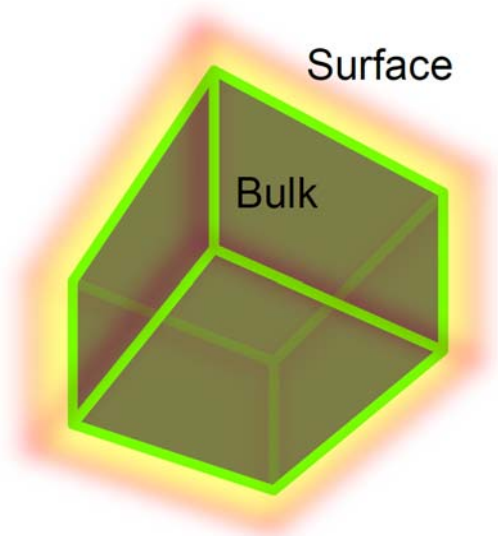
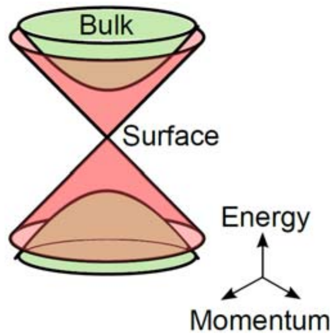
Topological insulator





always gapless

=

Super metal !

Bulk: Gapped = Insulating

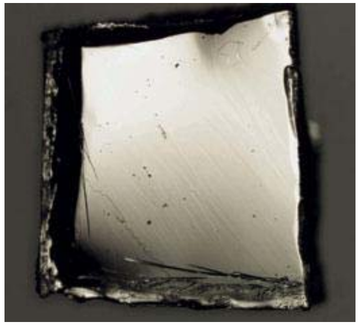
Surface: Gapless = Metallic

How about their electronic transport?

Reviews:
Hasan & Kane, RMP (2010);
Qi & Zhang, RMP (2011);
Shen, *Topological insulators* (Springer, 2012).

Bi_2Se_3 and Bi_2Te_3 in transport measurement

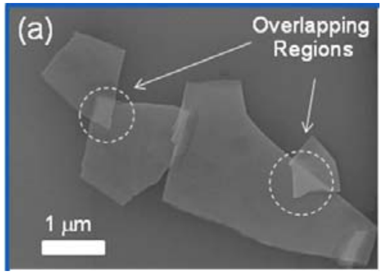
Crystal



Bi_2Se_3, Bi_2Te_3
- Thermoelectric materials
- Narrow-gap semiconductors

Moore, Nature, 2010;
Zhang et al, Nat. Phys. 5, 438 (2009);
Xia et al, Nat. Phys. 5, 398 (2009)

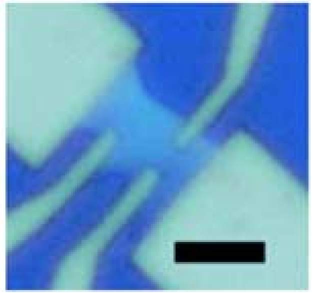
Flake



Mechanical Exfoliation (like for graphene): Teweldebrhan et al, Nano Lett.10 1209 (2010) (UC, Riverside)

Molecular Beam Epitaxy: Zhang, K.H.Wu et al, APL 95, 053114 (2009) (IOP, China); Zhang, Xue, Nature Physics 2010 (IoP&Tsinghua).

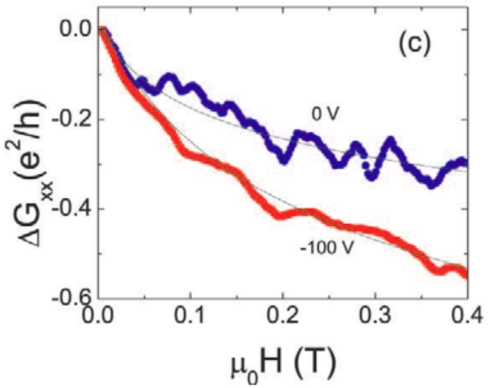
Device



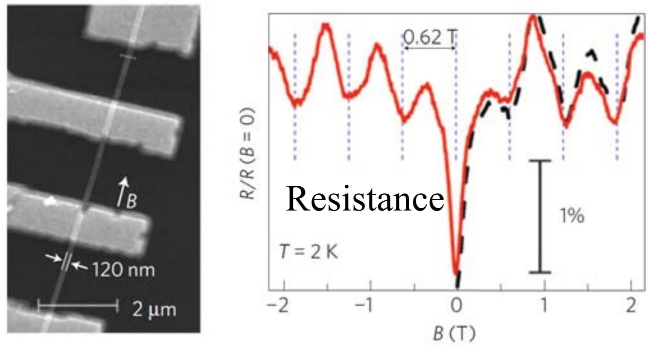
Measuring conductivity
Ong group (Princeton), PRL, 2009

What observed ?

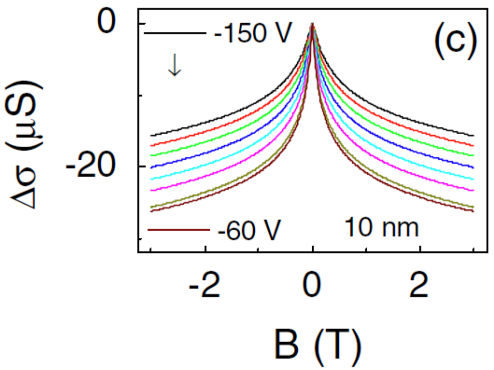
Weak antilocalization (WAL)



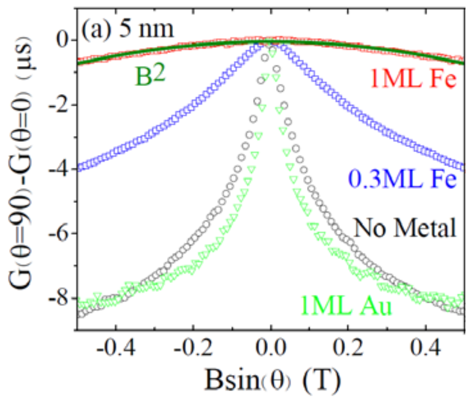
Bi_2Se_3 (Princeton)
Checkelsky, Ong, et al
PRL 103, 246601 (2009); PRL 106, 196801 (2011).



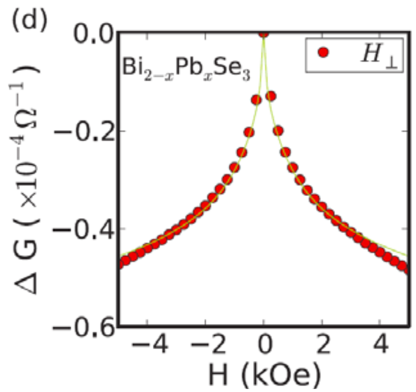
Bi_2Se_3 (Stanford)
H.L. Peng, Yi Cui et al ,
Nat. Mat. 9, 225 (2009).



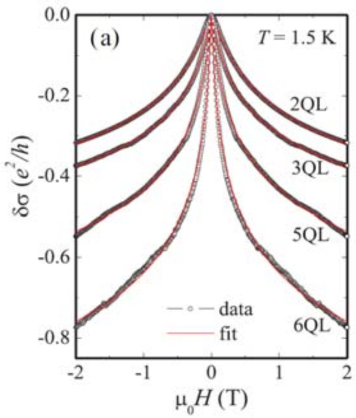
Bi_2Se_3 (IOP)
Chen, Wu, Li, Lu, et al ,
PRL 105, 176602 (2010)



Bi_2Te_3 (HKUST&HKU)
He, JN Wang, et al,
PRL 106, 166805
(2011).



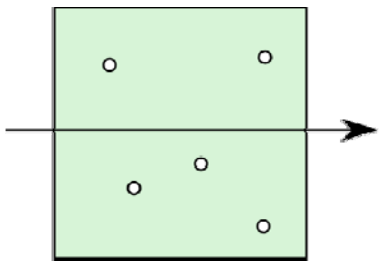
Bi_2Se_3 (PKU, Penn State)
Wang, Moses Chan, et al,
PRB 83, 245438 (2010).



Bi_2Se_3 (Tsinghua, IOP)
Liu, Yayu Wang, et al,
PRB 83, 165440
(2011); PRL 108,
036805 (2012).

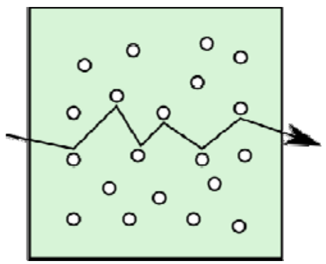
Quantum diffusion

Ballistic $\ell >> L$



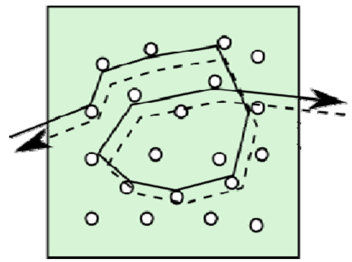
Diffusive $\ell << L$

Classical diffusive



$\ell >> \ell_\phi$

Quantum diffusive



$\ell << \ell_\phi$

Einstein's relation

$$\sigma^{sc} = \frac{ne^2\tau}{m} = e^2 N_F D = \frac{e^2}{h} k_F \ell$$

ℓ Mean free path
(due to elastic scattering by static centers)

ℓ_ϕ Phase coherence length
(due to inelastic scattering by phonons, e-e interaction,etc.)

L System size

Weak (Anti-)localization

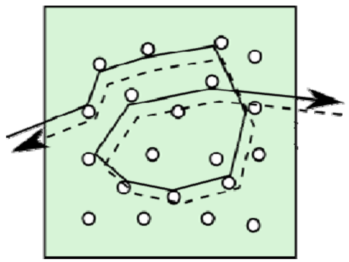
Semiclassical

Conductivity $\sigma_{xx} = \sigma^{sc}$

$+\sigma^{qi}$ $\begin{cases} \sim +\ln\dfrac{\ell_\phi}{\ell} & \text{WAL} \\ \sim -\ln\dfrac{\ell_\phi}{\ell} & \text{WL} \end{cases}$

Quantum interference

Time-reversed *scattering loops*



Quantum interference corrects conductivity

ℓ Mean free path
(due to elastic scattering, no temperature dependence)

ℓ_ϕ Phase coherence length
(due to inelastic scattering, $\ell_\phi \sim \dfrac{1}{T^{p/2}}$)

Thouless, PRL 39, 1167 (1977)

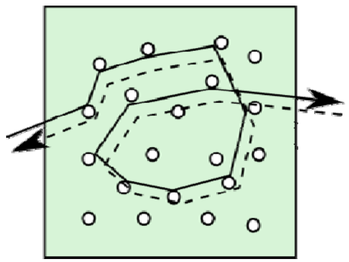
Weak (Anti-)localization

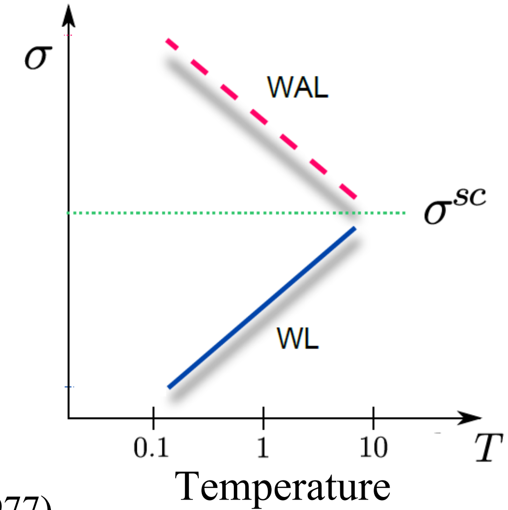
Semiclassical

Conductivity $\sigma_{xx} = \sigma^{sc} + \sigma^{qi}$ $\begin{cases} \sim +\ln \dfrac{\ell_\phi}{\ell} & \text{WAL} \\ \sim -\ln \dfrac{\ell_\phi}{\ell} & \text{WL} \end{cases}$

Quantum interference

Time-reversed scattering loops





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Weak (Anti-)localization

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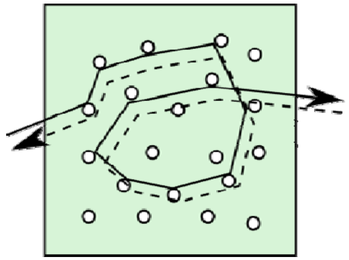
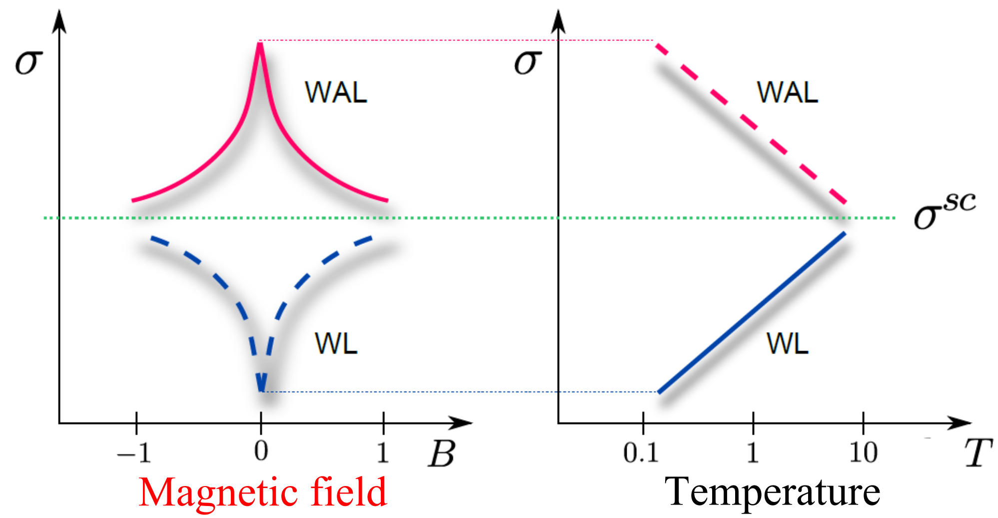
Magnetic field X

Quantum interference

Time-reversed scattering loops





Weak (Anti-)localization

Conductivity

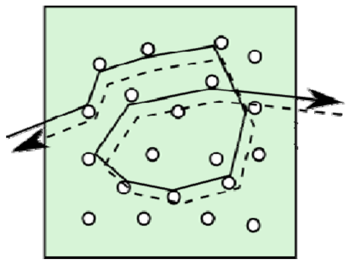
Semiclassical

$$\sigma_{xx} = \sigma^{sc} + \sigma^{qi} \begin{cases} \sim +\ln \dfrac{\ell_\phi}{\ell} & \text{WAL} \\ \sim -\ln \dfrac{\ell_\phi}{\ell} & \text{WL} \end{cases}$$

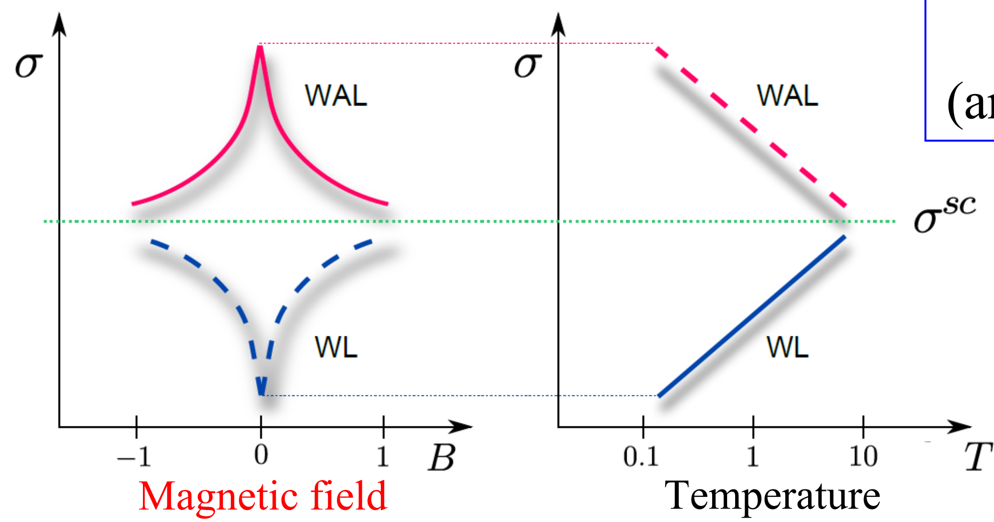
Magnetic field X

Quantum interference

Time-reversed scattering loops



Why do you call them Weak (anti-)localization ?



Weak (Anti-)localization

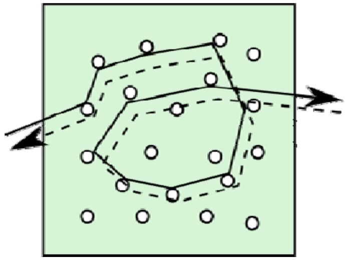
Semiclassical

Conductivity $\sigma_{xx} = \sigma^{sc} + \sigma^{qi}$

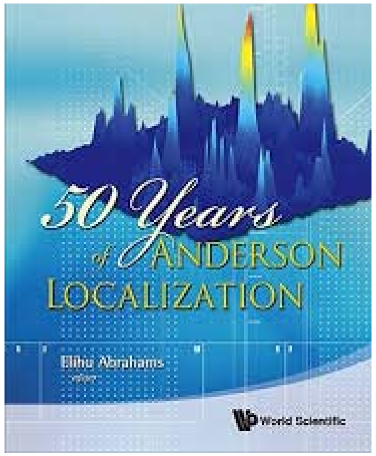
Quantum interference

$$\sigma^{qi} \begin{cases} \sim + \ln \dfrac{\ell_\phi}{\ell} & \text{WAL} \\ \sim - \ln \dfrac{\ell_\phi}{\ell} & \text{WL} \end{cases}$$

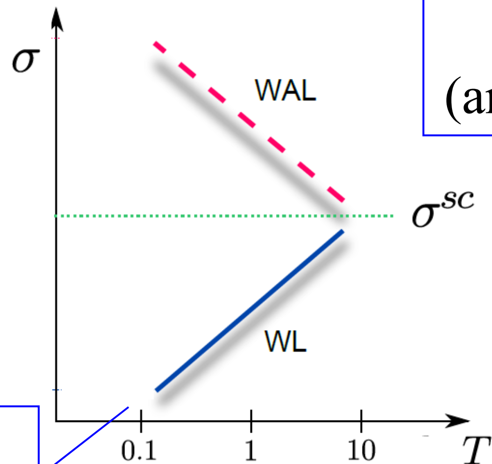
Time-reversed scattering loops



Electrons are localized



Normal metals
"Anderson localization"
No conductivity



Temperature

Why do you call them Weak (anti-)localization ?

Weak (Anti-)localization

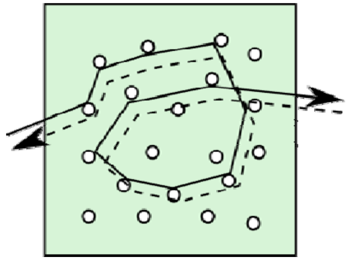
Conductivity $\sigma_{xx} = \sigma^{sc} + \sigma^{qi}$

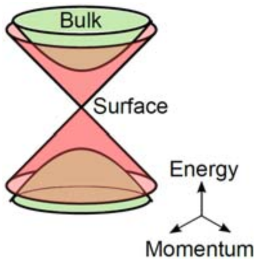
Semiclassical (σ^{sc})

Quantum interference (σ^{qi})

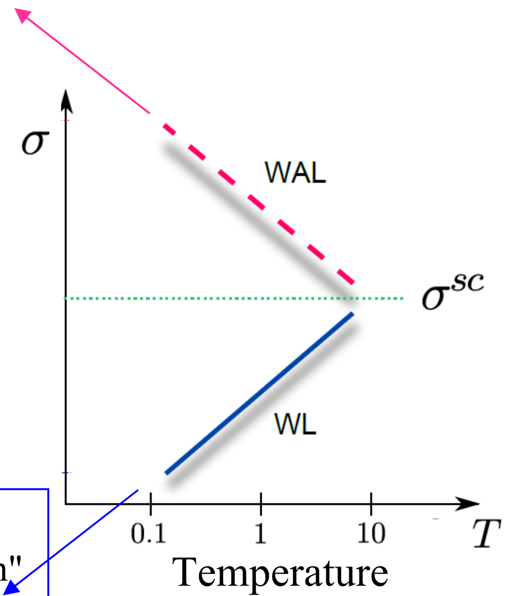
$$\sigma^{qi} \begin{cases} \sim +\ln \dfrac{\ell_\phi}{\ell} & \text{WAL} \\ \sim -\ln \dfrac{\ell_\phi}{\ell} & \text{WL} \end{cases}$$

Time-reversed scattering loops

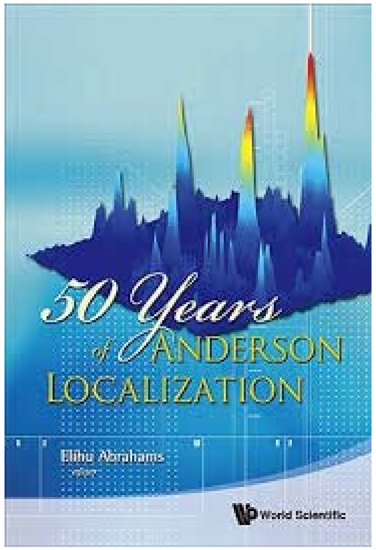




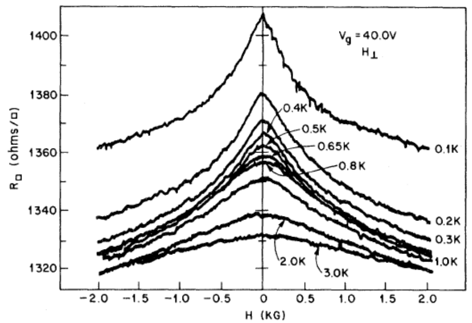
Topological insulators
(1) No gap; (2) No localization
Super metal !



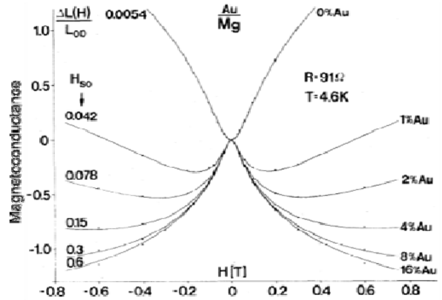
Normal metals
"Anderson localization"
No conductivity



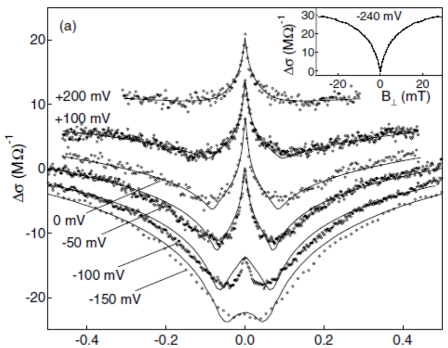
WL or WAL is everywhere



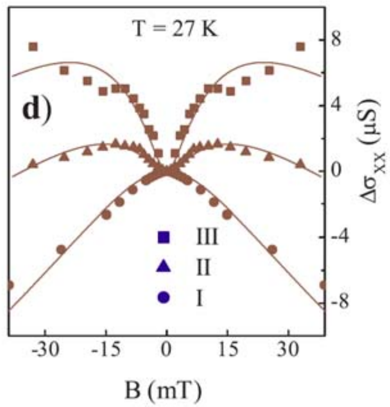
Si/SiO_2
Rosenbaum et al, PRL 47, 1758 (1981); PRL 46, 568 (1981);
Bishop, Dynes, & Tsui, PRB 26, 773 (1982).



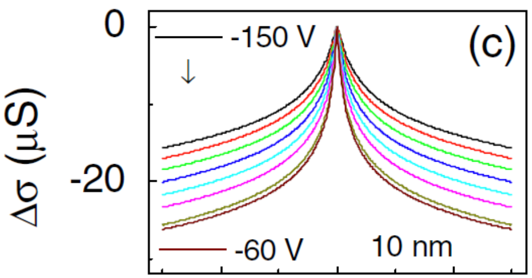
Mg
Bergmann, PRL 48, 1046 (1982).



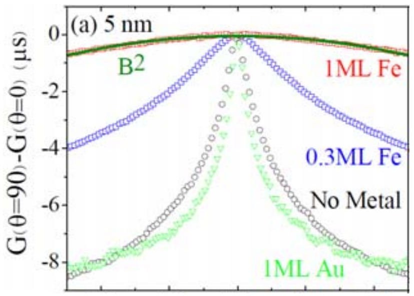
GaAs
Miller et al, PRL 90, 076807 (2003);
Neumaier et al, PRL 99, 116803 (2007).



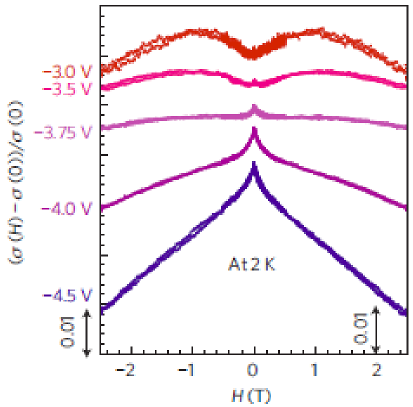
Graphene
Wu, de Heer et al, PRL 98, 136801 (2007); Tikhonenko, et al, PRL 103, 226801 (2009)



Bi_2Se_3 (IOP)
Chen, Wu, Li, Lv, et al , PRL 105, 176602 (2010)



Bi_2Te_3 HKUST & HKU
He et al, PRL 106, 166805 (2011)



MoS_2, WSe_2
HZ Lu, Di Xiao, Wang Yao & SQ Shen, PRL, 110, 016806 (2013);
Iwasa et al Science 2012; Nat. Phys. 2013 (Tokyo); Peide Ye et al ACS Nano 2013 (Purdue)

Weak localization and Weak anti-localization

Hikami, Larkin, and Nagaoka Prog. Theor. Phys. 63 7071(1980).

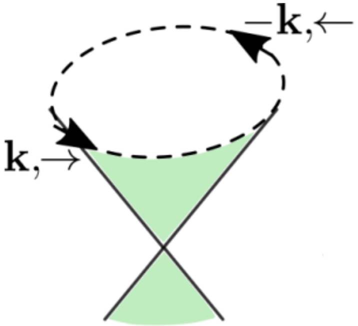
Impurity scattering	**Symmetry**	**Time-reversal**	**Spin-rotational**	**Transport**
Scalar	Orthogonal	✓	✓	WL
Spin-orbit	Symplectic	✓	✗	WAL
Magnetic	Unitary	✗		$\sigma \propto B^2$

Symmetry classes of random ensembles [F. J. Dyson, J. Math. Phys. 1962]

WAL in Topological insulators

Why WAL in TIs

(1) Surface Dirac fermions



$$H = \gamma(\sigma_x k_y - \sigma_y k_x)$$

(2) Quantum diffusion regime

$$\ell << L$$

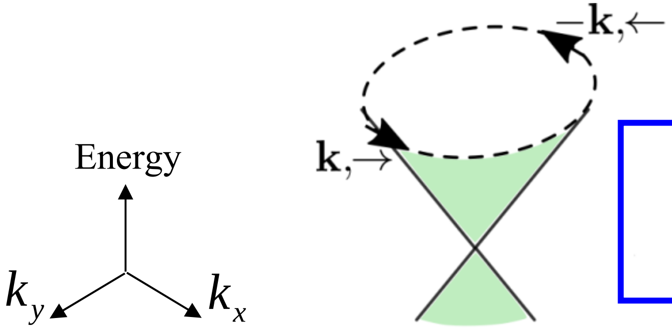
$$\ell << \ell_\phi$$

Poor mobility
Mean free path $\ell \sim 10\ \text{nm}$

Phase coherence length $\ell_\phi \sim 100 - 1000\ \text{nm}$
at ~ 1K

Berry phase of massless surface states

Surface Dirac fermions



Spin-momentum locking

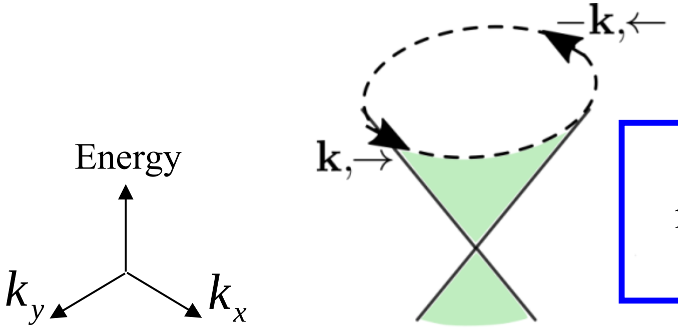
$$H = \gamma(\sigma_x k_y - \sigma_y k_x)$$

Berry phase of massless surface states

Spinor wave function $\psi_k = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -ie^{i\varphi} \end{pmatrix}$ Spin-up / Spin-down

Momentum angle $\tan\varphi \equiv \frac{k_y}{k_x}$

Surface Dirac fermions



Spin-momentum locking

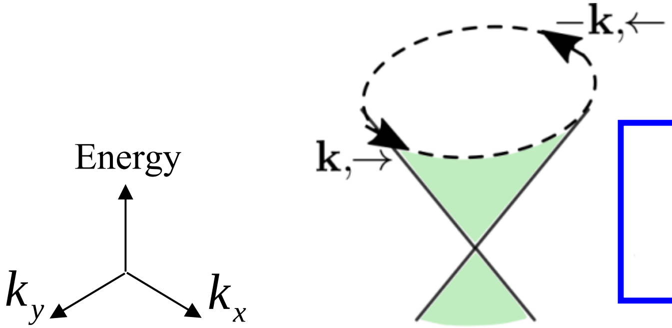
$$H = \gamma(\sigma_x k_y - \sigma_y k_x)$$

Berry phase of massless surface states

Spinor wave function
$$\psi_k = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -ie^{i\varphi} \end{pmatrix} \begin{matrix} \text{Spin-up} \\ \text{Spin-down} \end{matrix}$$

Momentum angle
$$\tan\varphi \equiv \frac{k_y}{k_x}$$

Surface Dirac fermions



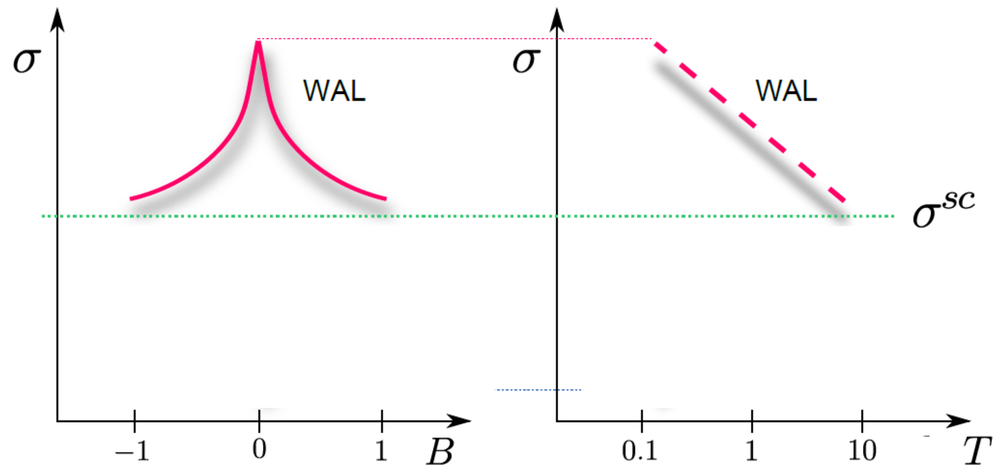
Spin-momentum locking

$$H = \gamma(\sigma_x k_y - \sigma_y k_x)$$

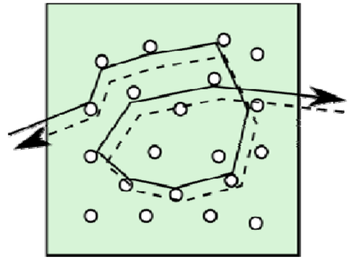
Berry phase

$$\phi \equiv -i\int_0^{2\pi} d\varphi \langle \psi_k | \frac{\partial}{\partial\varphi} | \psi_k \rangle$$
$$= -\frac{i}{2}\int_0^{2\pi} d\varphi (1, ie^{-i\varphi}) \frac{\partial}{\partial\varphi}\begin{pmatrix} 1 \\ -ie^{i\varphi} \end{pmatrix}$$
$$= -\frac{i}{2}\int_0^{2\pi} d\varphi (1, ie^{-i\varphi})\begin{pmatrix} 0 \\ e^{i\varphi} \end{pmatrix}$$
$$= \frac{1}{2}\int_0^{2\pi} d\varphi$$
$$= \pi$$

WAL of massless surface states

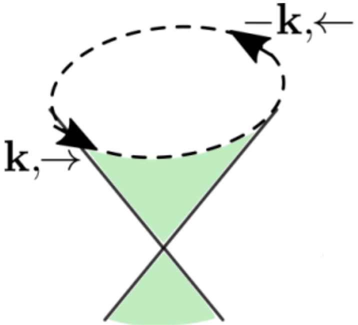


$$\sigma^{qi} \sim +\ln\frac{\ell_\phi}{\ell}$$



Destructive Quantum Interference

Suppress Back Scattering



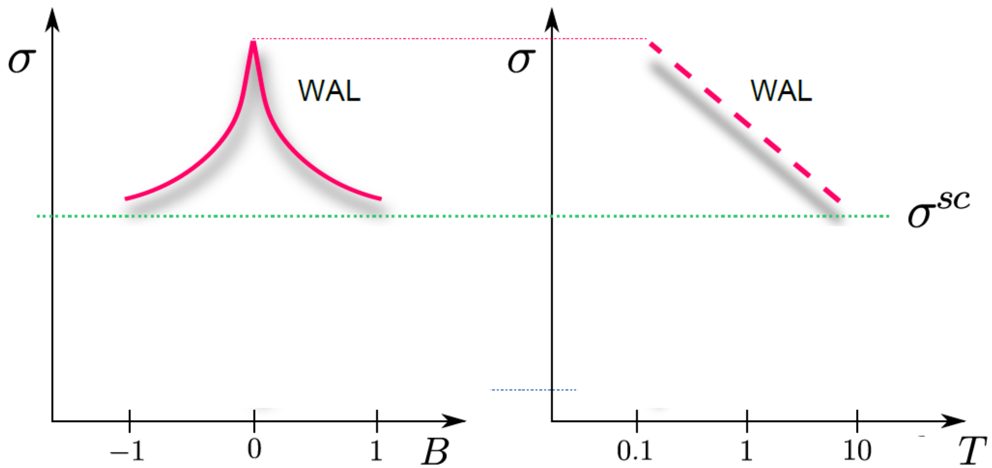
$$H=\gamma(\sigma_x k_y - \sigma_y k_x)$$

Berry phase $\phi = -i\int_0^{2\pi} d\varphi \langle \psi_k | \frac{\partial}{\partial \varphi} | \psi_k \rangle = \pi$

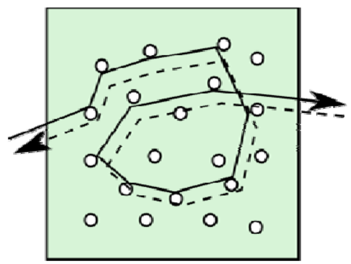
Suzuura & Ando, PRL 89, 266603 (2002)

Berry phase is the reason

WAL of massless surface states



$$\sigma^{qi} \sim +\ln\frac{\ell_\phi}{\ell}$$



Destructive Quantum Interference

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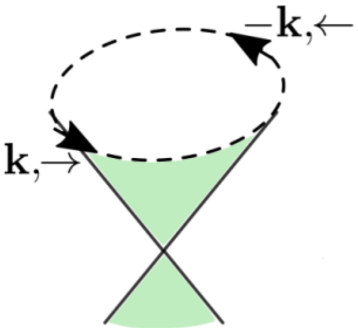


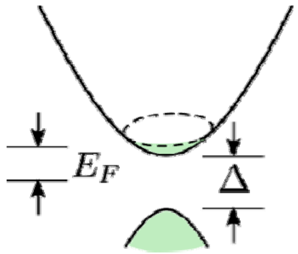
$$H = \gamma(\sigma_x k_y - \sigma_y k_x)$$

Berry phase φ … $_k\rangle = \pi$

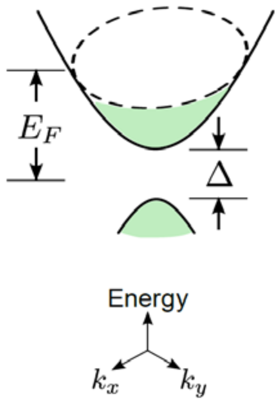
Suzuu… (2002)

Can we change Berry phase?

Berry phase

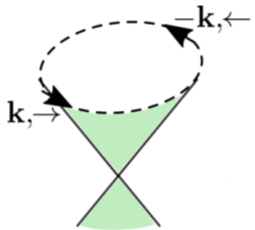


Large-gap



Berry phase

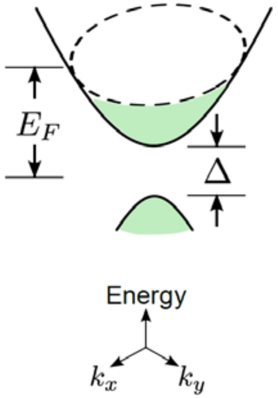
$$\phi = \pi\left(1 \pm \frac{\Delta}{2E_F}\right) = \begin{cases} 2\pi, & \dfrac{\Delta}{2E_F} = 1 \quad \text{WL} \\ \pi, & \dfrac{\Delta}{2E_F} = 0 \quad \text{WAL} \end{cases}$$



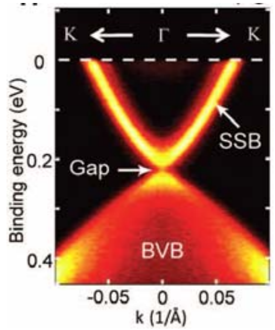
gapless

Based on the Berry phase argument, we predicted a crossover from WAL to WL

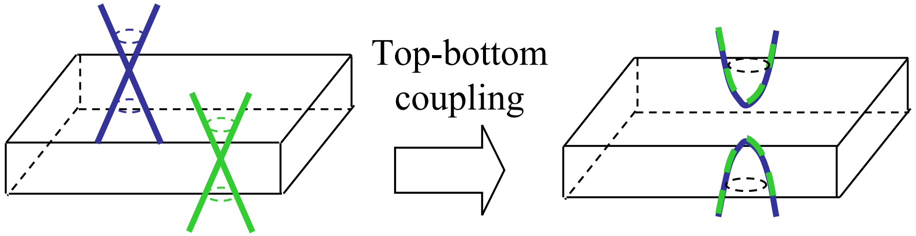
Massive Dirac fermions



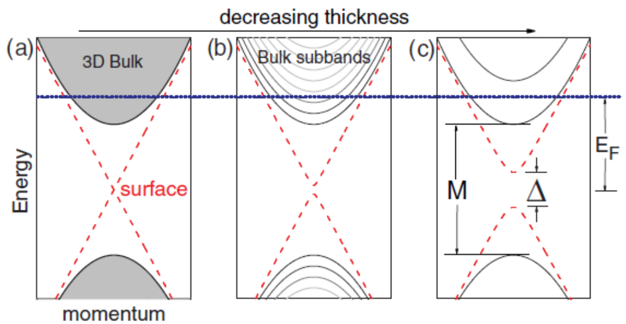
(1) Magnetically doped surface states
[Chen, Shen et al, Science 2010;
Wray, Hasan et al, Nat. Phys. 2011]



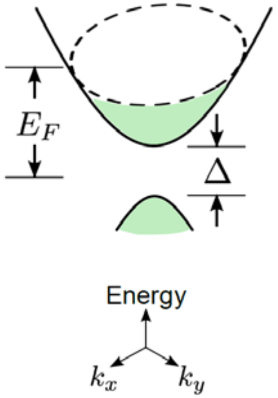
(2) Thin film finite size effect
[Lu, Shan, Yao, Niu & Shen PRB 81, 115407 (2010);
Liu, Zhang et al, PRB(R) 2010; Linder et al, PRB 2009.]



(3) 2D Bulk subband
[Lu & Shen, PRB 84, 125138 (2011)]



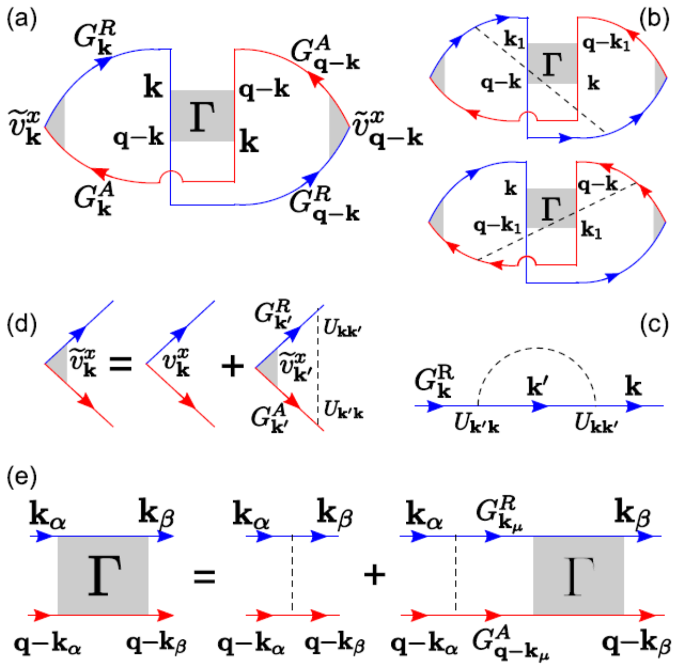
Model



Dirac model $$H = \gamma(\sigma_x k_y - \sigma_y k_x) + \frac{\Delta}{2}\sigma_z$$

Disorder potential $$U(r) = \sum_i u_i \delta(r - R_i)$$

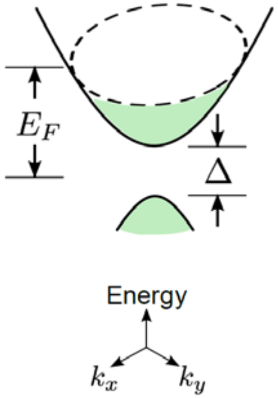
Lu, Shi & Shen,
PRL, 107 076801 (2011)



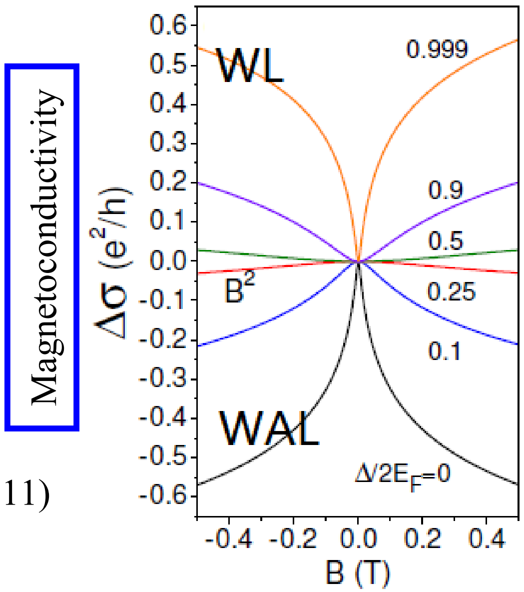
We calculate magnetoconductivity using Feynman diagrams

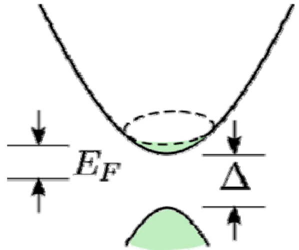
Maximally crossed diagram:
Langer and Neal, PRL 16, 984 (1966)
Ladder diagram correction to velocity:
Shon and Ando, JPSJ 67, 2421 (1998)
The dressed Hikami boxes:
McCann, Kechedzhi, Fal'ko, Suzuura, Ando, and Al'tshuler, PRL 97, 146805 (2006)
Reviews for non-Dirac Fermions
Bergmann Phys. Rep. 1984
Lee and Ramakrishnan, RMP 1985

WAL-WL crossover

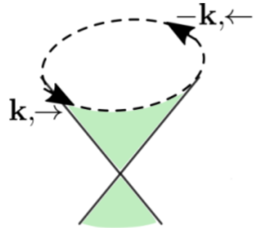


Lu, Shi & Shen,
PRL, 107 076801 (2011)

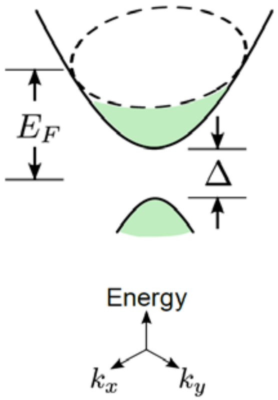




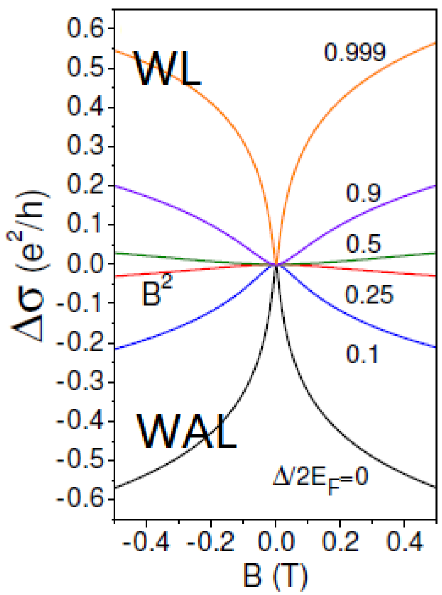
$$\phi = \pi\left(1 \pm \frac{\Delta}{2E_F}\right) = \begin{cases} 2\pi, & \dfrac{\Delta}{2E_F} = 1 \\ \pi, & \dfrac{\Delta}{2E_F} = 0 \end{cases}$$

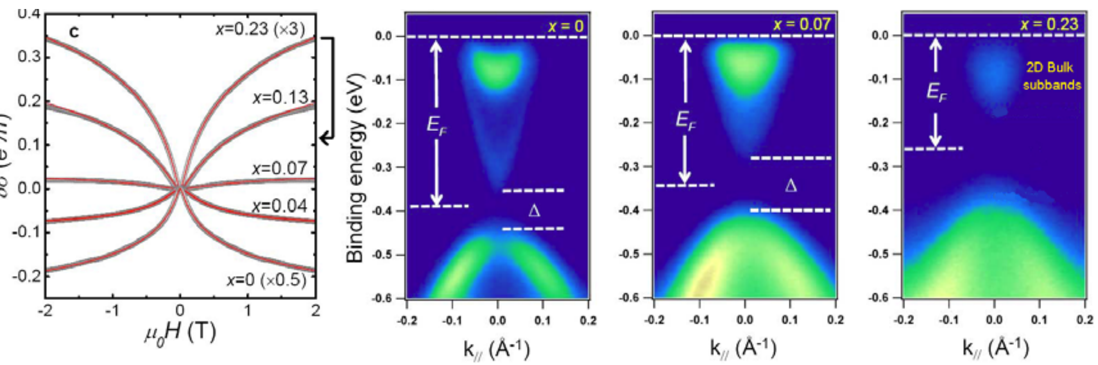


WAL-WL crossover



Lu, Shi & Shen,
PRL, 107 076801 (2011)



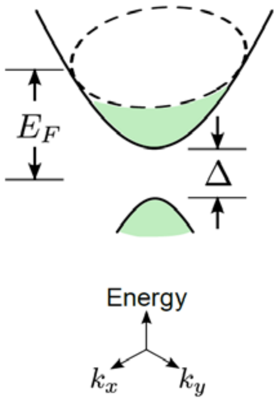


Liu, Yayu Wang et al,
PRL 108, 036805 (2012)
(Tsinghua & IOP, China)
3QL $Bi_{2-x}Cr_xSe_3$

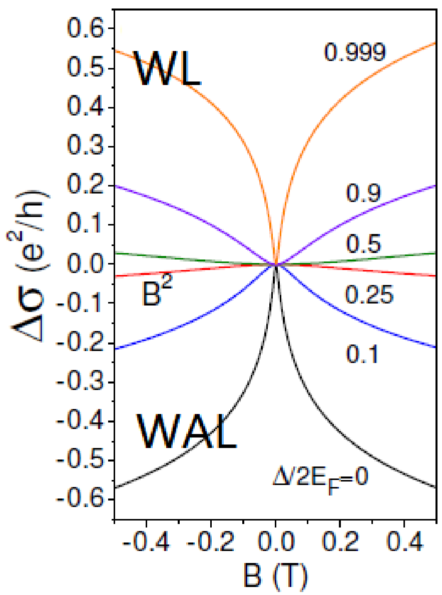
Finite size effect: Lu, Shan, Yao, Niu & Shen PRB 81, 115407 (2010); Liu, Zhang et al, PRB(R) 2010; Linder et al, PRB 2009.

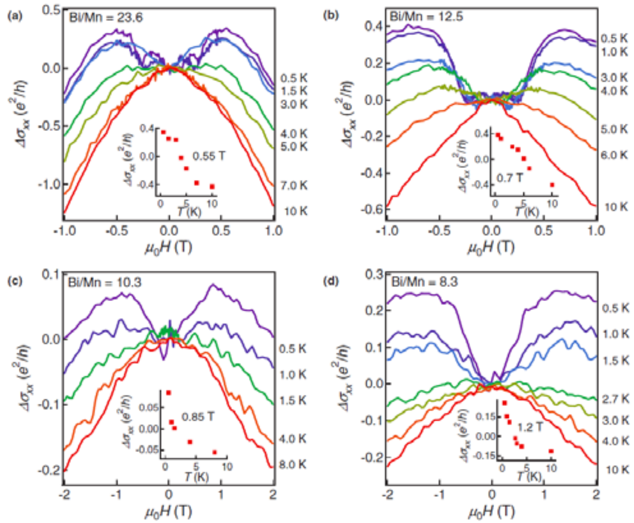
Magnetic doping: Chen et al, Science 2010; Wray et al Nat. Phys. 2011.

WAL-WL crossover



Lu, Shi & Shen,
PRL, 107 076801 (2011)





Zhang, Samarth et al, (Penn State)
Mn-Bi2Se3
WAL-WL crossover as a signature of ferromagnetic phase transition with Tc~5K

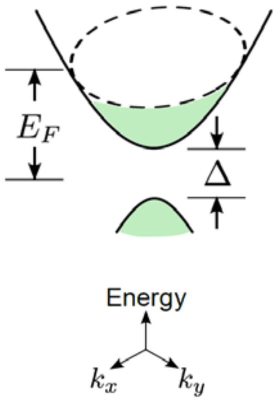
PHYSICAL REVIEW B **86**, 205127 (2012)

Interplay between ferromagnetism, surface states, and quantum corrections in a magnetically doped topological insulator

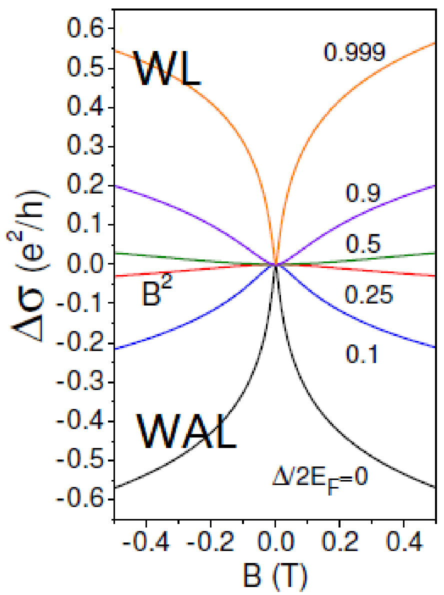
Duming Zhang,[1] Anthony Richardella,[1] David W. Rench,[1] Su-Yang Xu,[2] Abhinav Kandala,[1] Thomas C. Flanagan,[1] Haim Beidenkopf,[2] Andrew L. Yeats,[3] Bob B. Buckley,[3] Paul V. Klimov,[3] David D. Awschalom,[3] Ali Yazdani,[2] Peter Schiffer,[1] M. Zahid Hasan,[2] and Nitin Samarth[1,*]

partment of Physics and Materials Research Institute, The Pennsylvania State University, University Park, Pennsylvania 16802-6300,
[2]*Department of Physics, Princeton University, Princeton, New Jersey 08544, USA*
[3]*Center for Spintronics and Quantum Computation, University of California, Santa Barbara, California 93106, USA*

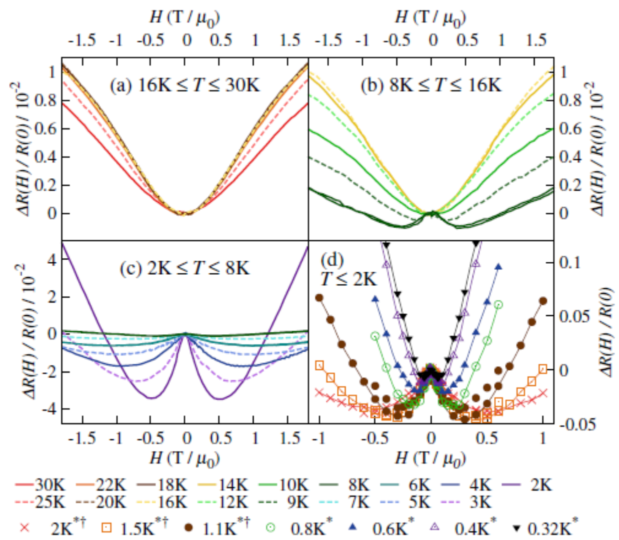
WAL-WL crossover



Lu, Shi & Shen,
PRL, 107 076801 (2011)

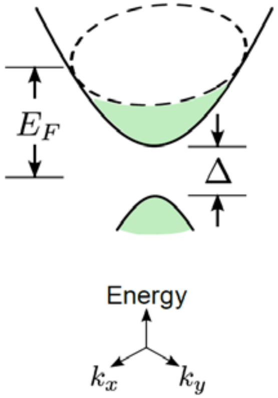


Yang, Kapitulnik (Stanford)
PRB 88, 081407(R) (2013)
Editors' suggestion
EuS/Bi2Se3
Europium Sulfide

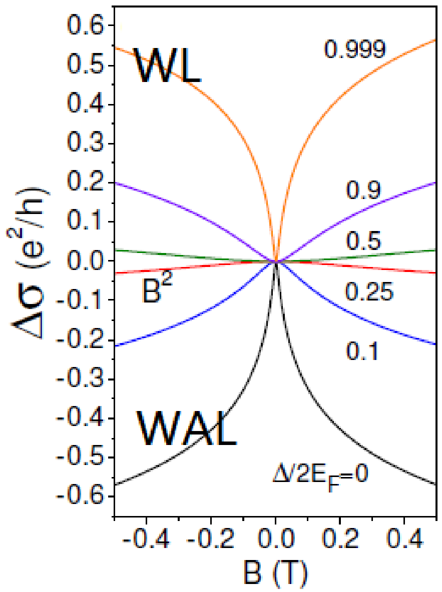


Also:
EuS/Bi2Se3
Wei, Moodera, et al
(MIT),
APS March
meeting 2013

WAL-WL crossover



Lu, Shi & Shen,
PRL, 107 076801 (2011)

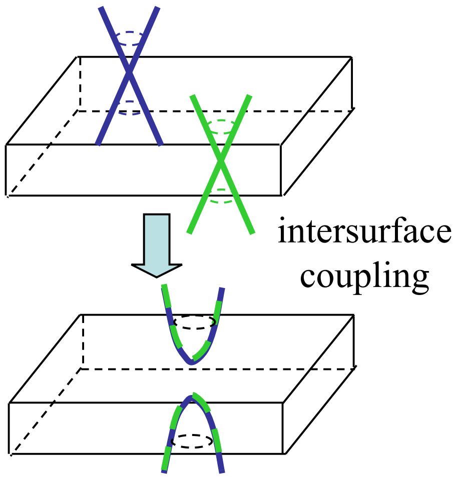


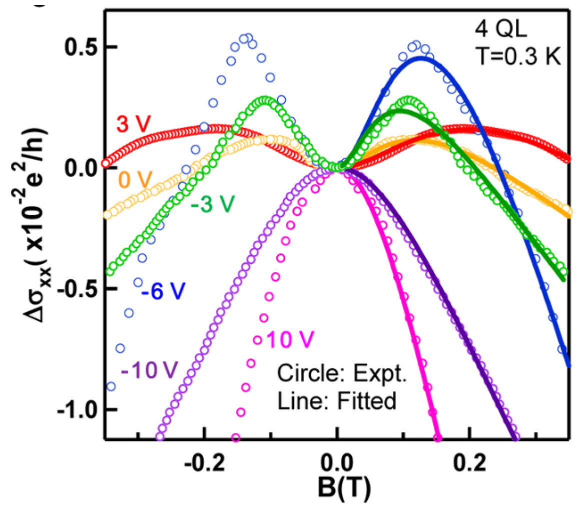
contributions as a function of gate voltage by fitting the measured MC curves to the two-component HLN theory:[42]

$$\Delta\sigma(B) = \sum_{i=0,1} \frac{\alpha_i e^2}{\pi h}\left[\psi\left(\frac{l_B^2}{l_{\phi i}^2} + \frac{1}{2}\right) - \ln\left(\frac{l_B^2}{l_{\phi i}^2}\right)\right] \quad (2)$$

(42) Lu, H.-Z.; Shen, S.-Q. *Phys. Rev. B* **2011**, *84* (12), 125138.

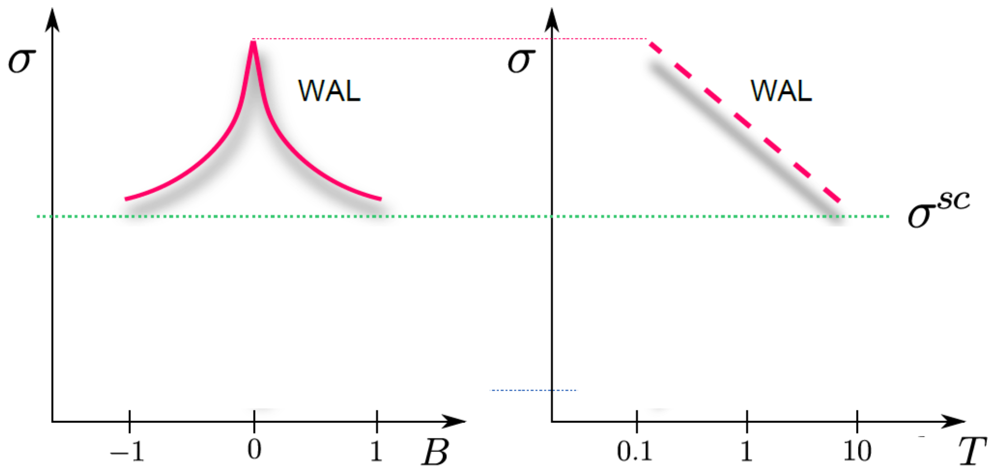
Finite size effect:
Lu, Shan, Yao, Niu & Shen PRB 81, 115407 (2010); Liu, Zhang et al, PRB(R) 2010; Linder et al, PRB 2009.

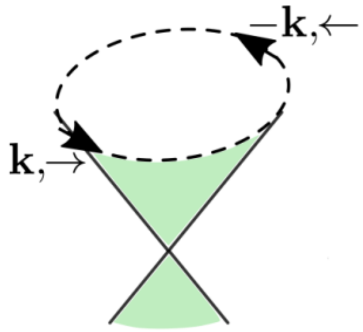




Lang, K. L. Wang (UCLA)
Nano Lett. (2013)
4 QL $Bi_{1.14}Sb_{0.86}Te_3$

WAL

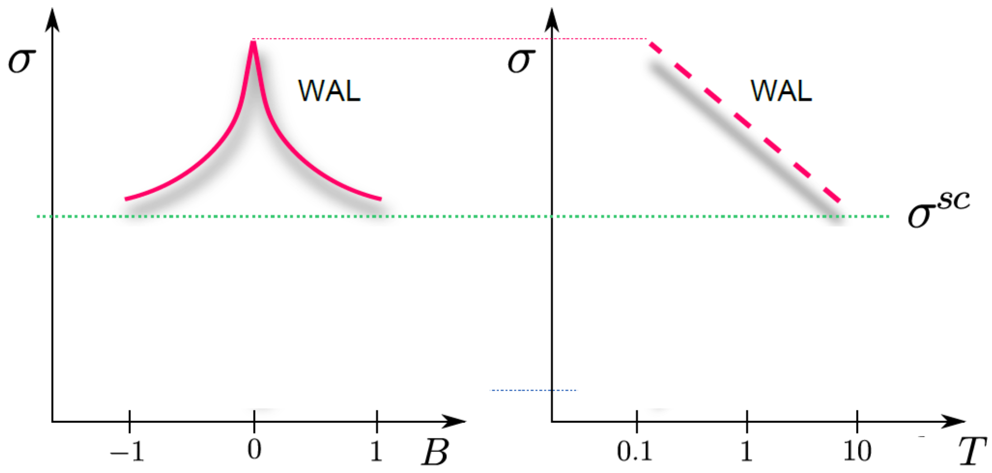




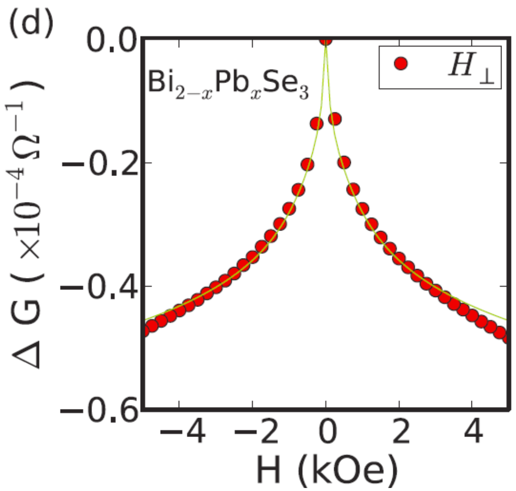
Mechanism of WAL:

Berry phase
of
Surface Dirac fermions

WAL

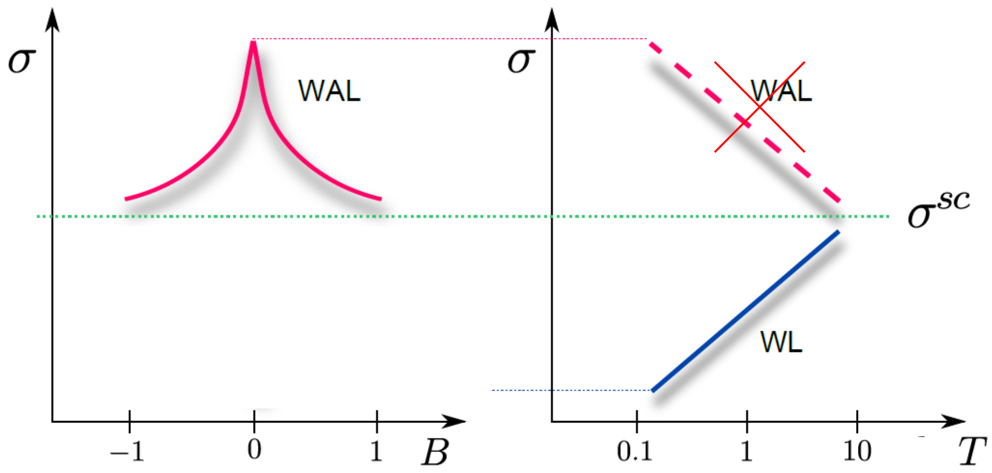


Wang, Moses Chan (Penn state), et al, PRB 83, 245438 (2010). Bi2Se3 and Pb-Bi2Se3 (Tsinghua, IOP)

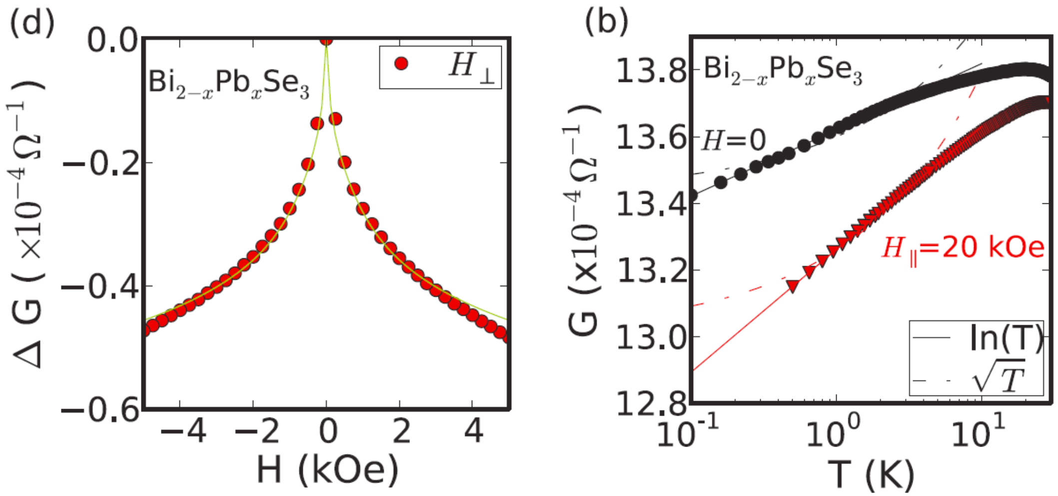


How about temperature dependence in experiments?

Dilemma in topological insulators

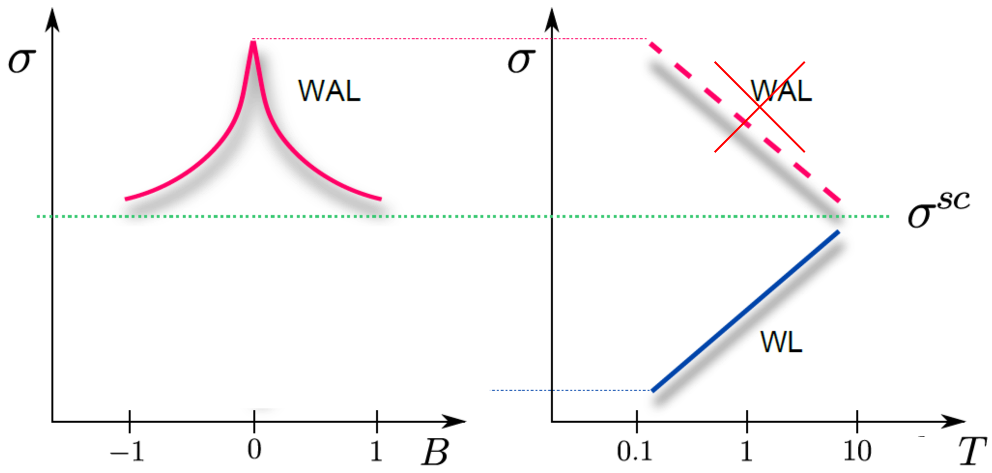


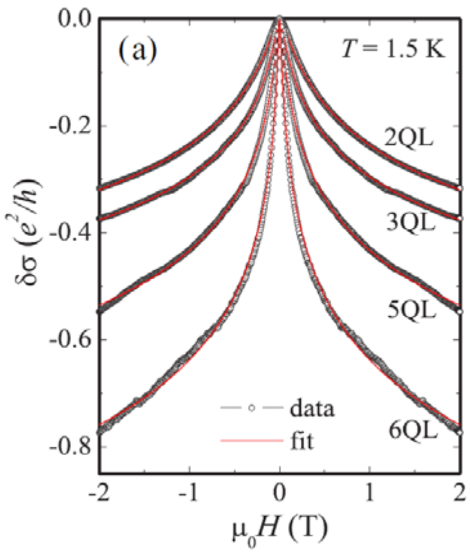
Wang, Moses Chan (Penn state), et al, PRB 83, 245438 (2010). Bi2Se3 and Pb-Bi2Se3 (Tsinghua, IOP)

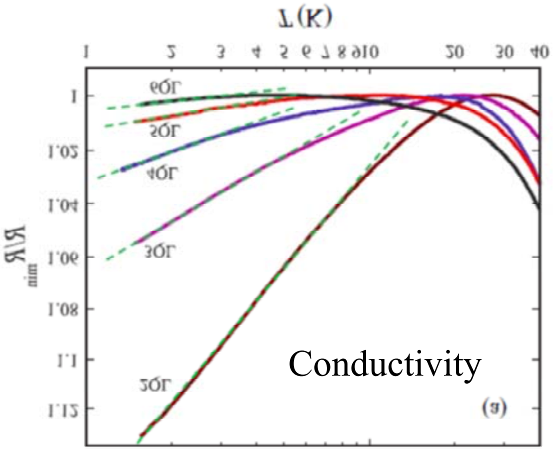


A sad truth

Dilemma in topological insulators

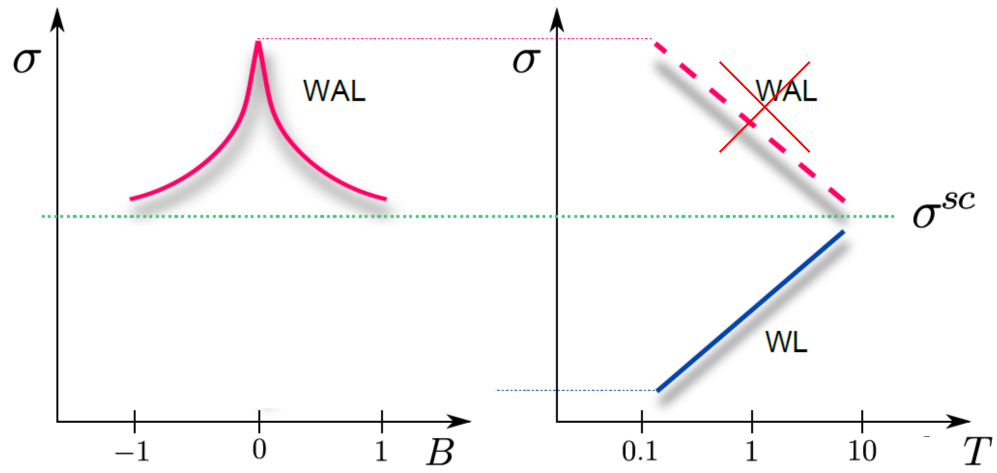


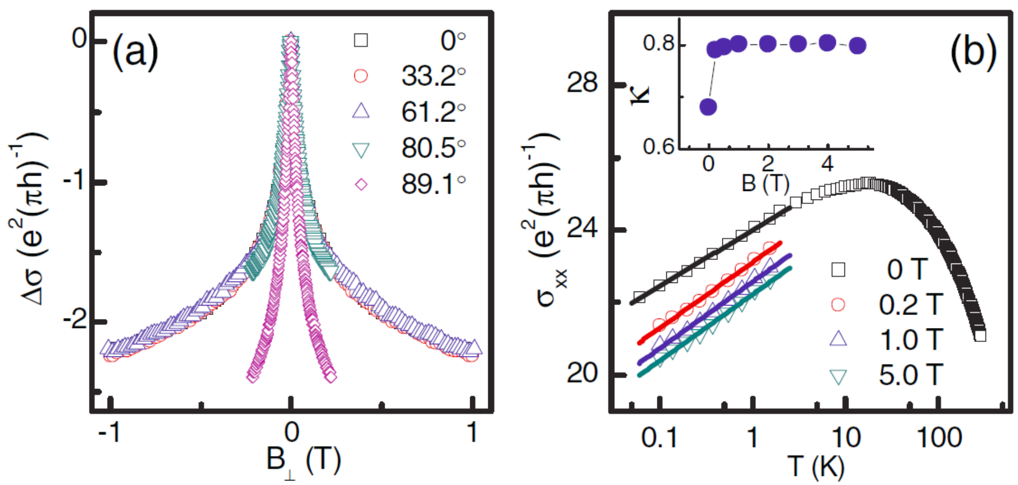




Liu, Yayu Wang (Tsinghua), et al, PRB 83, 165440 (2011)
Bi2Se3 ultrathin films

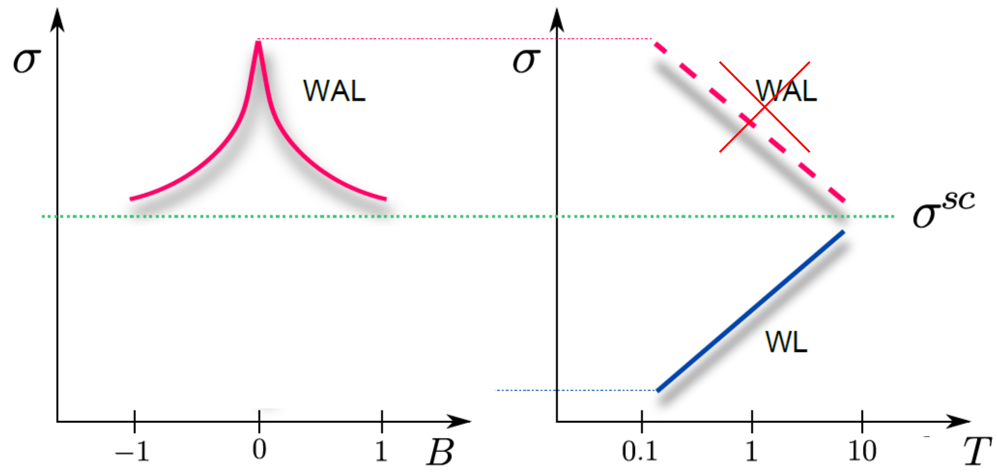
Dilemma in topological insulators

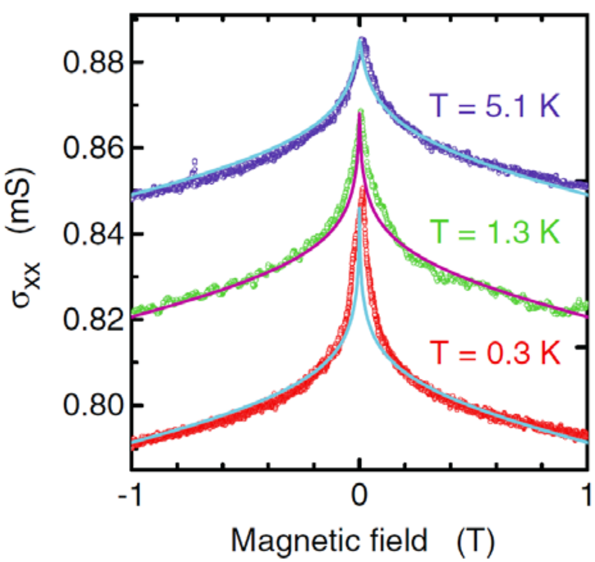


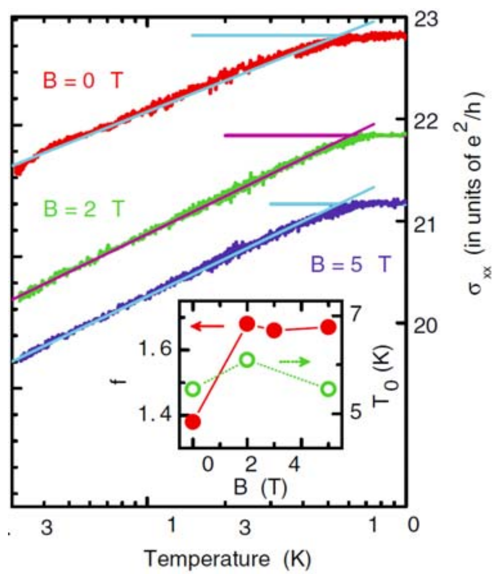


Chen, Y.Q. Li, K. H. Wu, L. Lu, et al (IOP), PRB, 83, 241304 (R) (2011) Bi2Se3

Dilemma in topological insulators

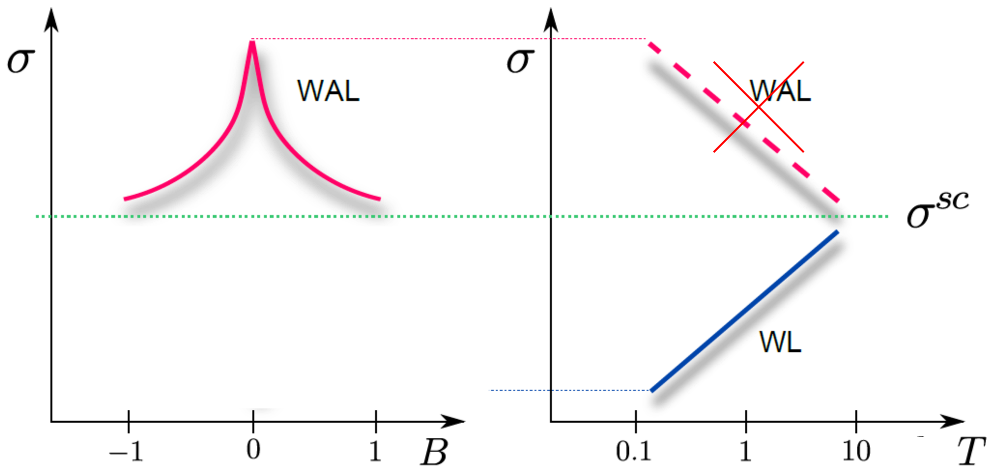


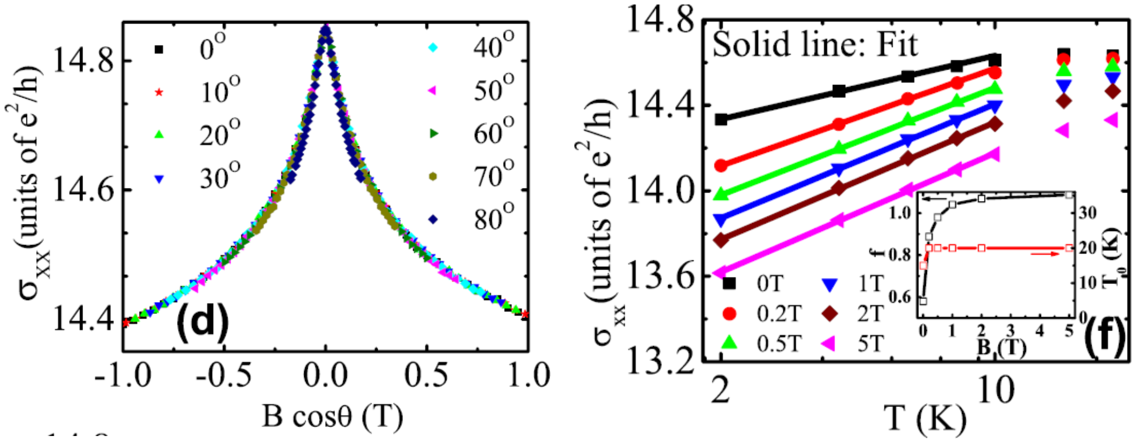




Takagaki et al (Berlin), PRB 85, 115314 (2012) Bi2Se3

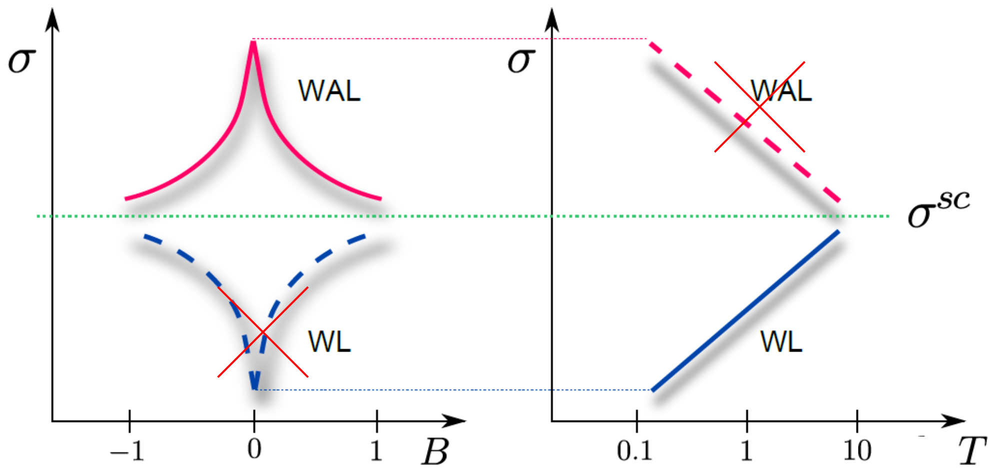
Dilemma in topological insulators



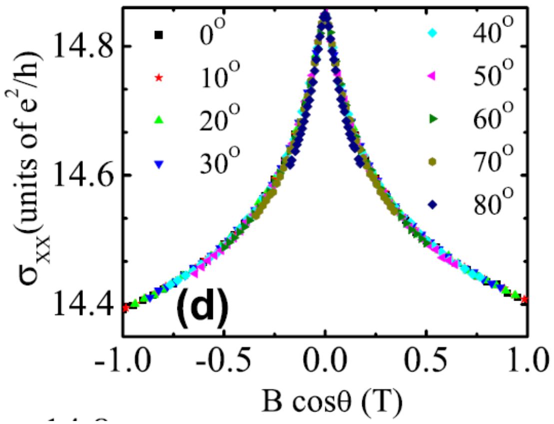


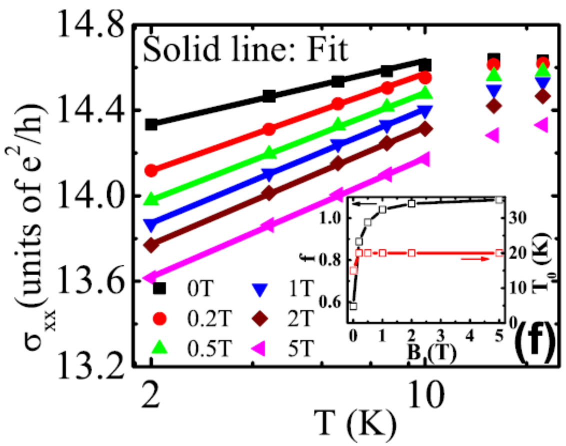
Roy et al (Austin), APL 102, 163118 (2013);
Bi2Te3 ultrathin film

Dilemma in topological insulators



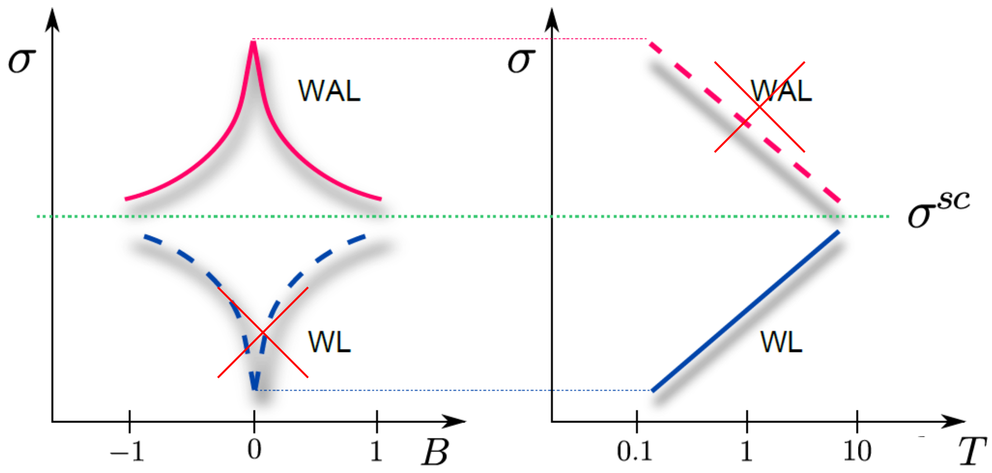
WL?





Roy et al (Austin), APL 102, 163118 (2013);
Bi2Te3 ultrathin film

Dilemma in topological insulators



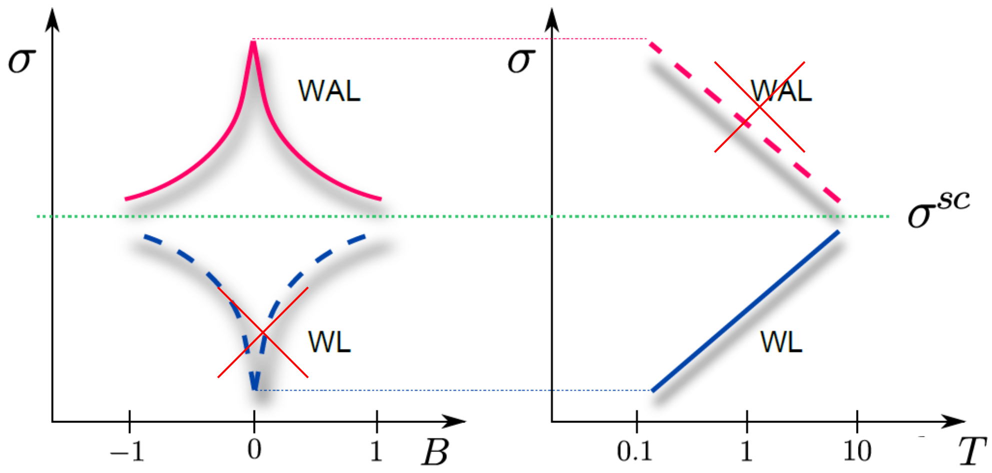
An embarrasing situation

states but, crucially, the topological properties of the bulk insulator do not allow the metallic surface state to vanish — it cannot become localized or gapped. These two theoretical predictions, about the electronic

This protection of the surface metal from Anderson localization (that is, formation of an insulating state as a result of strong disorder[20]) is one of the key differences between the surface of the topological insulator and the 'accidental' surface states present in other materials, such as the

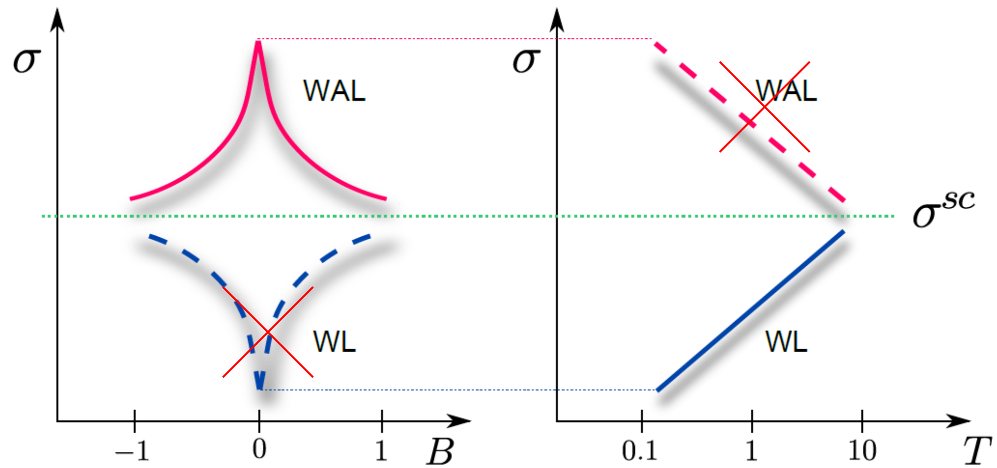
J. E. Moore, "The birth of topological insulators", Nature 464, 194 (2010)

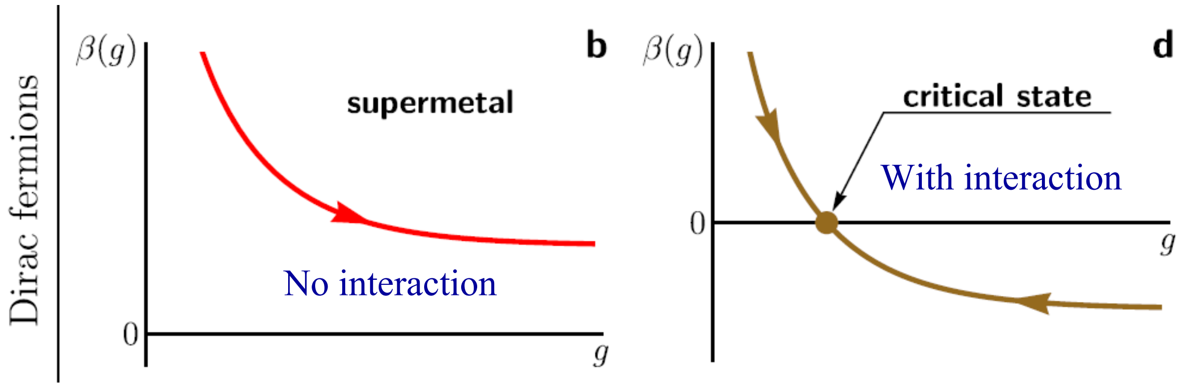
Dilemma in topological insulators



What missed?

Dilemma in topological insulators



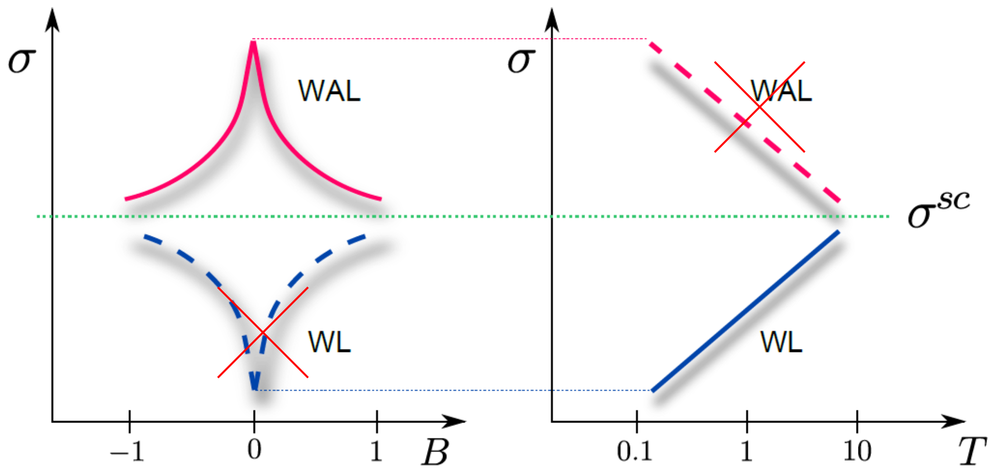


Based on one-loop RG argument: Interaction ‘‘kills’’ the supermetal.

$$\beta(g) = \frac{dg}{d\ln L} \begin{cases} > 0, & metal \\ < 0, & localized \end{cases}$$

Ostrovsky, Gornyi, & Mirlin, PRL 105, 036803 (2010).

Dilemma in topological insulators



Electron-electron interaction ?

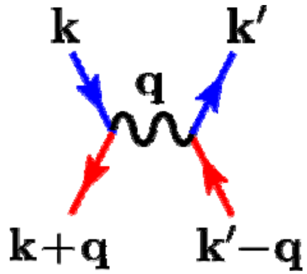
Altshuler-Aronov effect ?

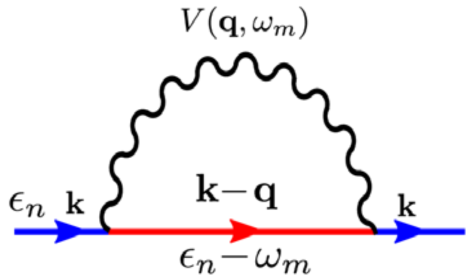
Solid State Communications **30**, 115 (1979).

Wang, Moses Chan (Penn state), et al, PRB 83, 245438 (2010).
Liu, Yayu Wang (Tsinghua), et al, PRB 83, 165440 (2011).

Altshuler-Aronov effect

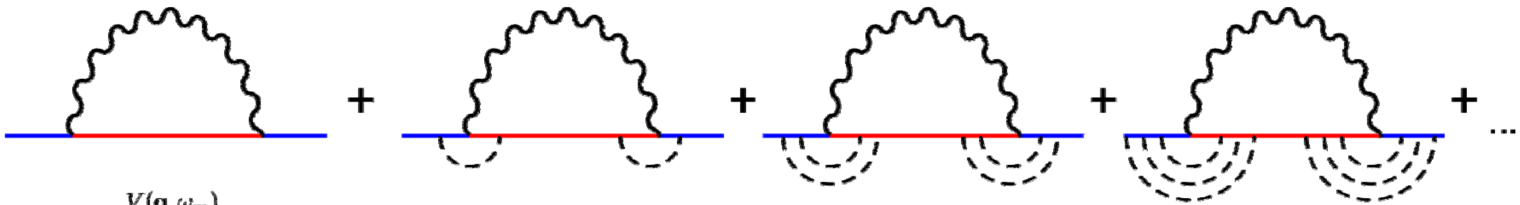
Electron-electron interaction



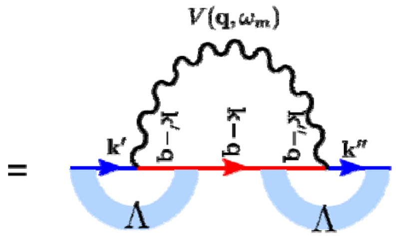


Fock self-energy (exchange interaction)

Altshuler-Aronov effect



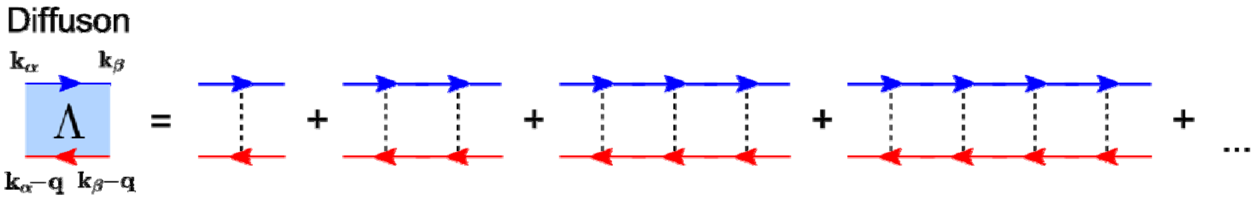
Disorder scattering

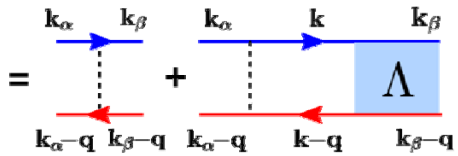


Weak e-e interaction

+

Strong disorder scattering



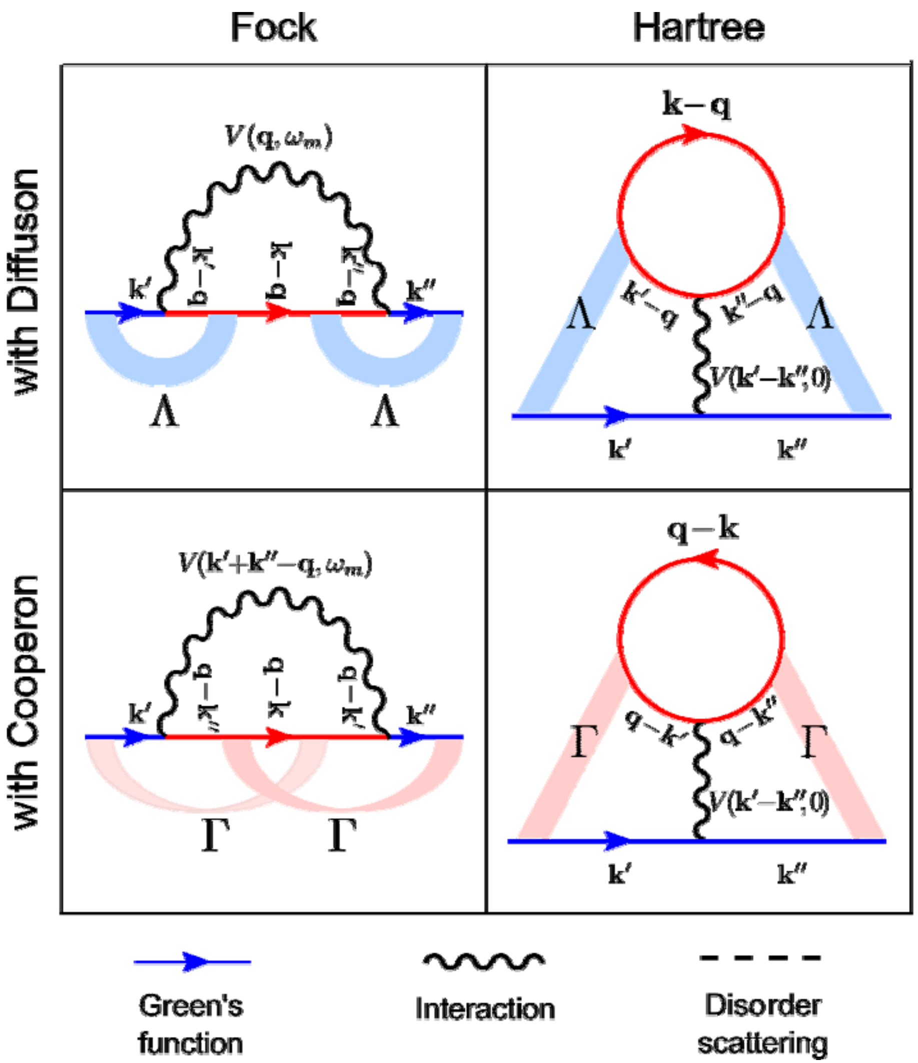


$$\propto \int_{1/\ell_T}^{1/\ell} \frac{d^2\vec{q}}{|\omega_m|+Dq^2} \propto \begin{cases} \sqrt{T}, & 3\mathrm{D} \\ \ln T, & 2\mathrm{D} \\ 1/\sqrt{T}, & 1\mathrm{D} \end{cases}$$

$$1/\ell_T \propto \sqrt{T}$$

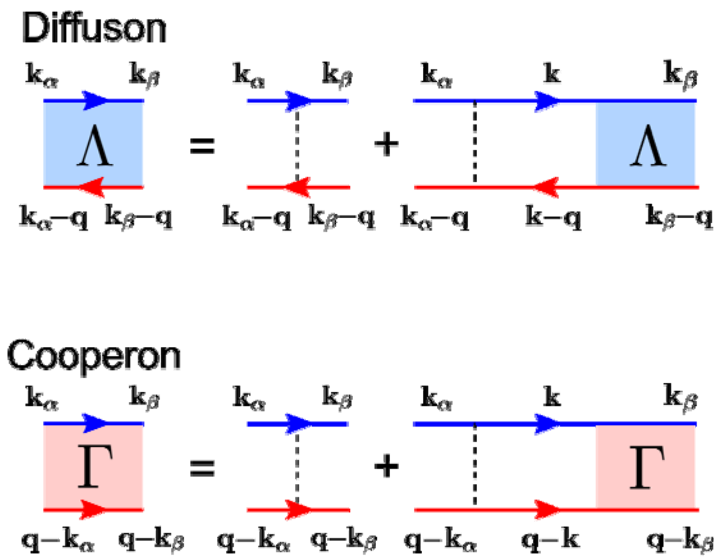
Thermal diffusion length

Altshuler-Aronov effect



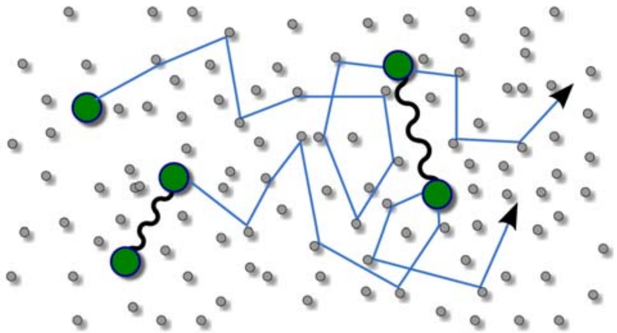
First-order diagrams:

Altshuler, Aronov, and Lee, Phys. Rev. Lett. 44, 1288 (1980).
H. Fukuyama,
J. Phys. Soc. Jpn. 48, 2169 (1980).

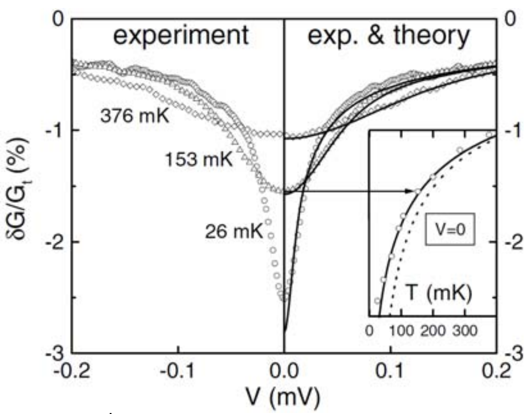


Altshuler-Aronov effect

Conventional electrons $\frac{p^2}{2m}$



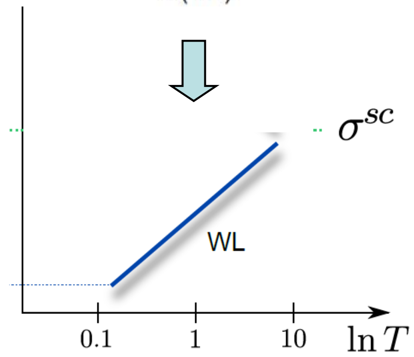
Electron-electron interaction
+
Disorder scattering



Suppress the density of states at the Fermi energy

Pierre et al,
PRL 86, 1590 (2001).

Conductivity correction from e-e interaction

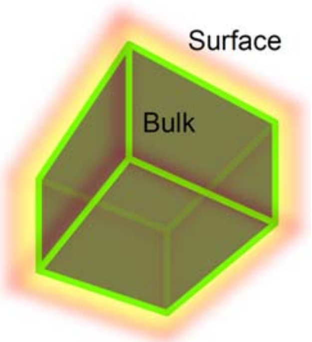


WL-like temperature dependence
(Thermal diffusion, so conductivity decreases with decreasing T)

In topological insulator ?

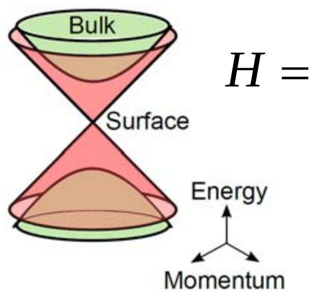
Altshuler-Aronov theory was established for **conventional electrons**. $\dfrac{p^2}{2m}$

However



Surface states:
massless Dirac fermions

Bulk states:
massive Dirac fermions



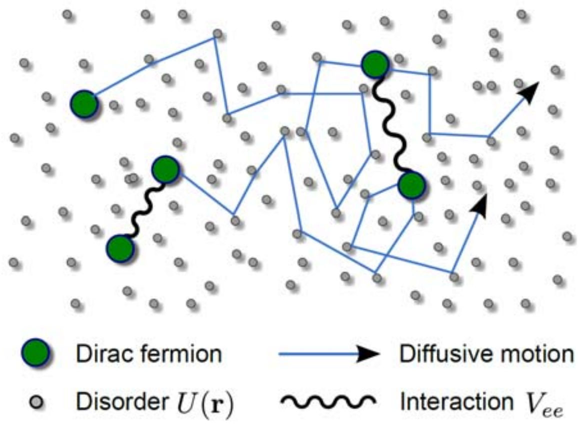
$$H = \gamma(\sigma_x k_y - \sigma_y k_x)$$

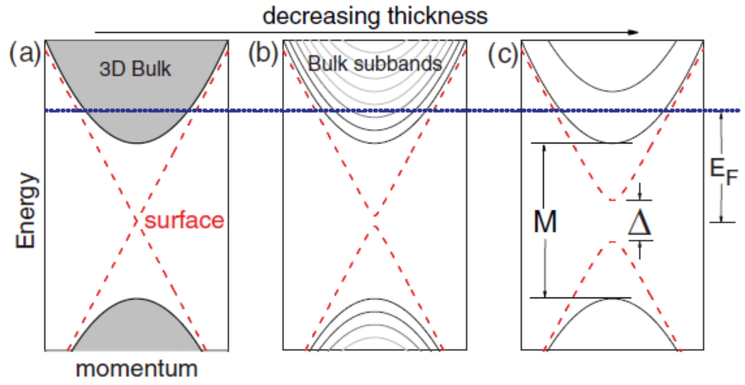
Questions:

AA effect works for Dirac fermions?

A material-dependent analysis?

Model





Dirac model $$H=\gamma(\sigma_x k_y-\sigma_y k_x)+\frac{\Delta}{2}\sigma_z$$

Disorder potential $$U(r)=\sum_i u_i\delta(r-R_i)$$

Now we need to include interaction

Electron-electron interaction $$V_{ee}=\frac{1}{2}\sum_{k,k',q}v(q)(\phi_k^+\cdot\phi_{k+q})(\phi_{k'}^+\cdot\phi_{k'-q})c_{k'}^+c_k^+c_{k+q}c_{k'-q}$$

$$v(q)=\frac{e^2}{2\varepsilon_0\varepsilon_r q},\quad \phi_k=\begin{pmatrix}\cos\frac{\theta}{2}\\ -i\sin\frac{\theta}{2}e^{i\varphi_k}\end{pmatrix}$$

Lu & Shen, PRL 112, 146601 (2014)

Feynman diagrams

Semiclassical (Drude)

σ^{sc}

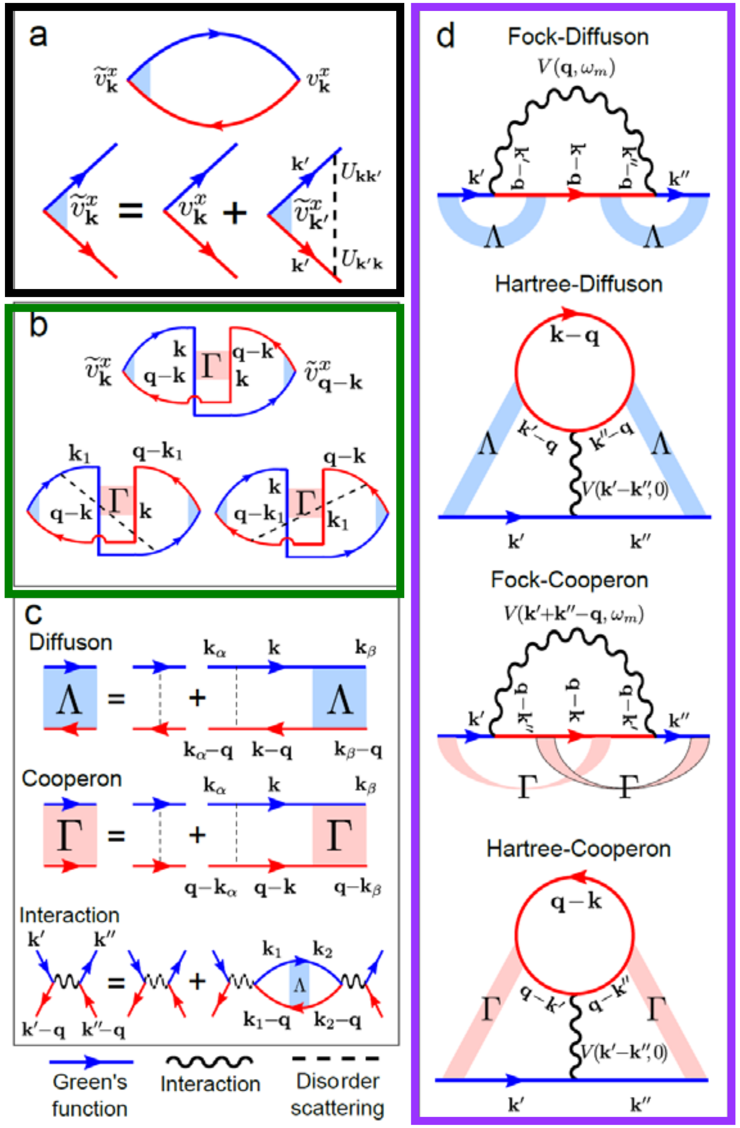
Quantum interference (WL / WAL)

σ^{qi}

Maximally crossed diagrams

Bergmann Phys. Rep. 1984

McCann, Kechedzhi, Fal'ko, Suzuura, Ando, and Al'tshuler, PRL 97, 146805 (2006)



Altshuler, Aronov, and Lee, Phys. Rev. Lett. 44, 1288 (1980).

H. Fukuyama, J. Phys. Soc. Jpn. 48, 2169 (1980).

Electron-electron interaction (AA effect)

σ^{ee}

Now we need to include interaction

Lu & Shen, PRL 112, 146601 (2014)

Temperature and magnetic field dependence

Quantum interference

$$\sigma^{qi} = \frac{e^2}{\pi h}\sum_{i=0,1}\alpha_i[\psi(\frac{1}{2}+\frac{\ell_B^2}{\ell_{\phi i}^2}) - \ln\frac{\ell_B^2}{\ell^2}]$$

Electron-electron interaction

$$\sigma^{ee} = \frac{e^2}{\pi h}(1-\eta_{\Lambda\Gamma}F)\ln\frac{2\ell^2}{\ell_T^2} - \frac{e^2}{\pi h}\eta_\Gamma F\psi(\frac{1}{2}+\frac{\ell_T^2}{\ell_{B\phi}^2})$$

Variables:

(1) Perpendicular magnetic field B

$$\ell_B = \sqrt{\frac{\hbar}{4eB}}$$ Magnetic length

(2) Temperature T

$$\ell_T = \sqrt{\frac{D\hbar}{2\pi k_B T}}$$ Thermal diffusion length

$$\ell_\phi \sim \frac{1}{T^{p/2}}$$ Phase coherence length

Dirac model parameters:

(1) Mass $\frac{\Delta}{2E_F}$

(2) Velocity $\gamma = v\hbar$

Sample parameters:

(1) Mean free path ℓ

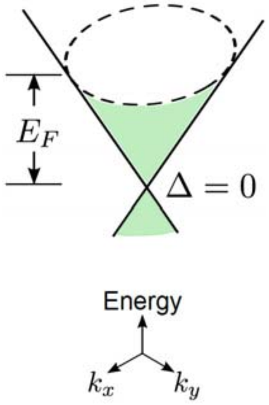
(2) Phase coherence length ℓ_ϕ

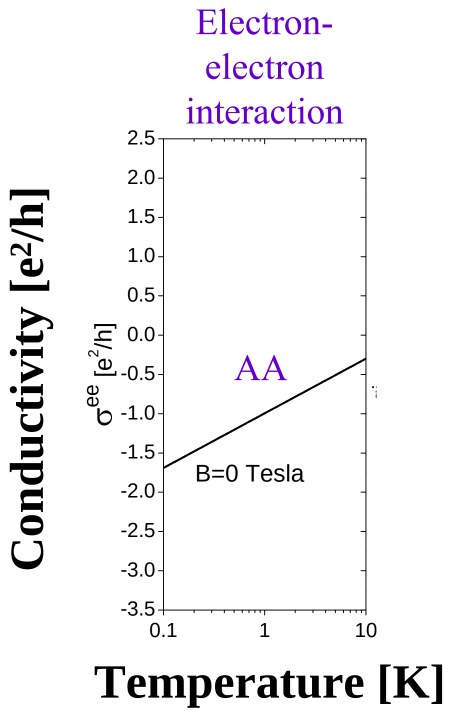
(3) Screening factor $F(\varepsilon_r, \gamma)$

↑ Relative dielectric constant

Lu & Shen, PRL 112, 146601 (2014)

Conductivity vs. Temperature

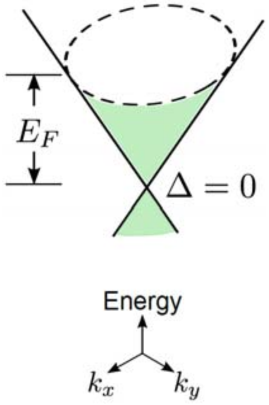


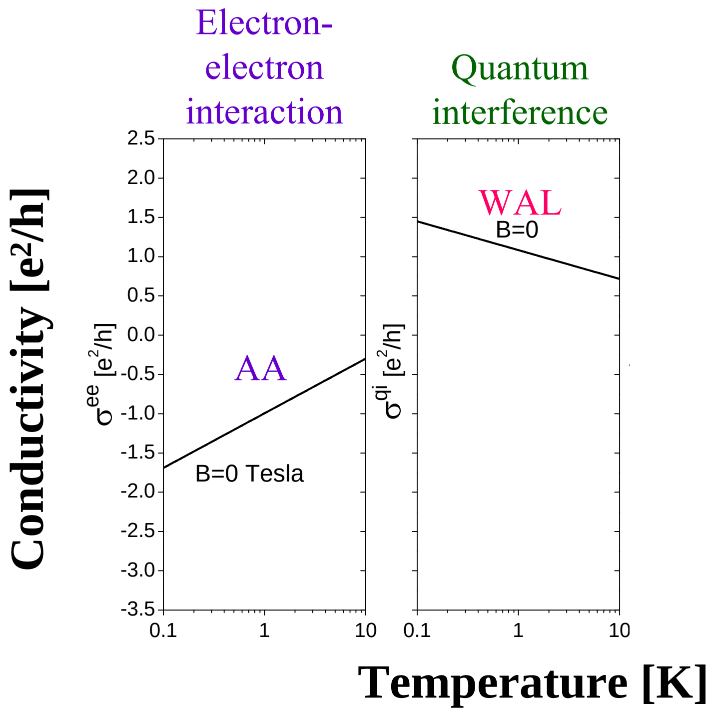


We do have the Altshuler-Aronov effect

Lu & Shen, PRL 112, 146601 (2014)

Conductivity vs. Temperature

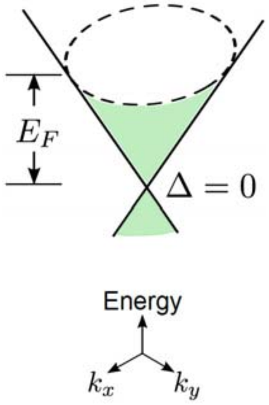


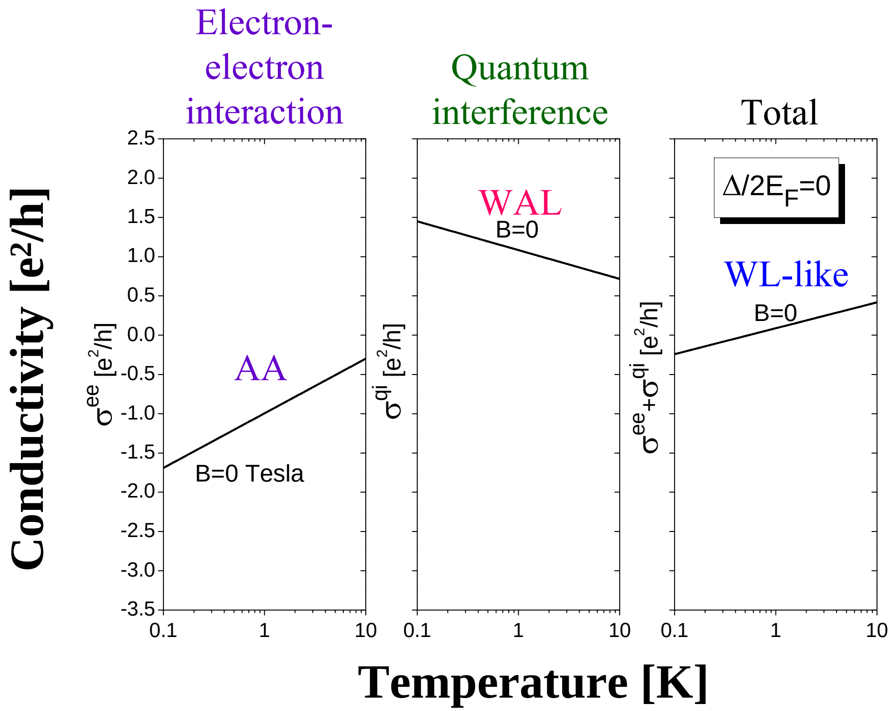


... and Weak Anti-Localization

Lu & Shen, PRL 112, 146601 (2014)

Conductivity vs. Temperature

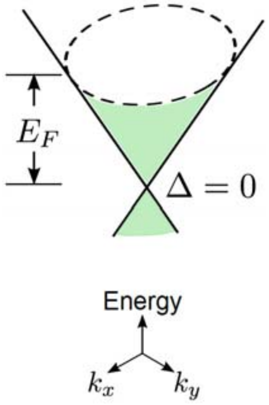


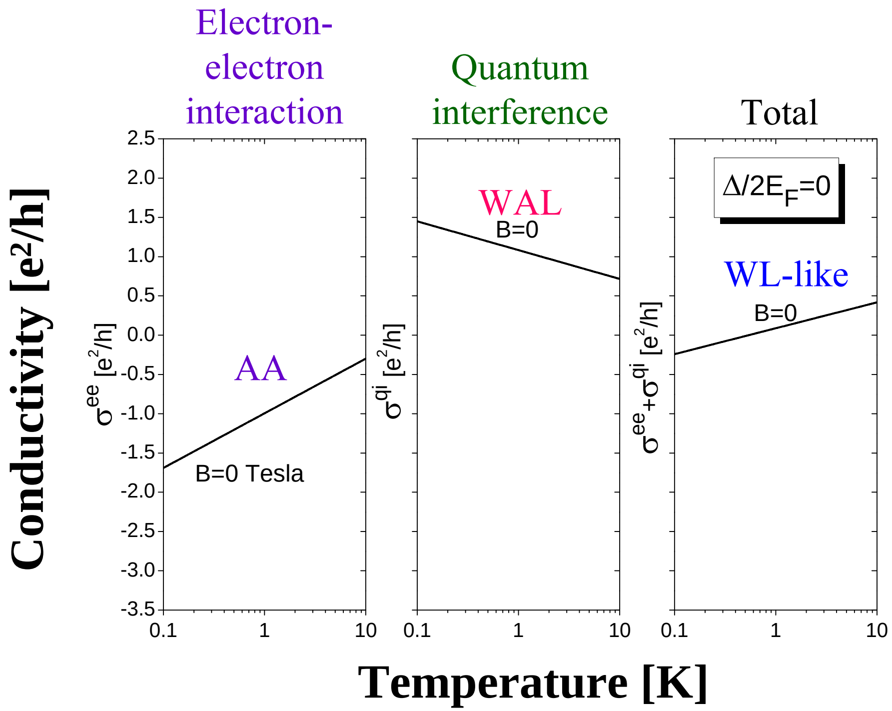


Together, a Weak Localization-like temperature dependence

Lu & Shen, PRL 112, 146601 (2014)

Conductivity vs. Temperature





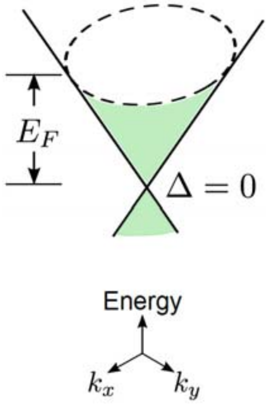
Why interaction dominates the temperature dependence ?

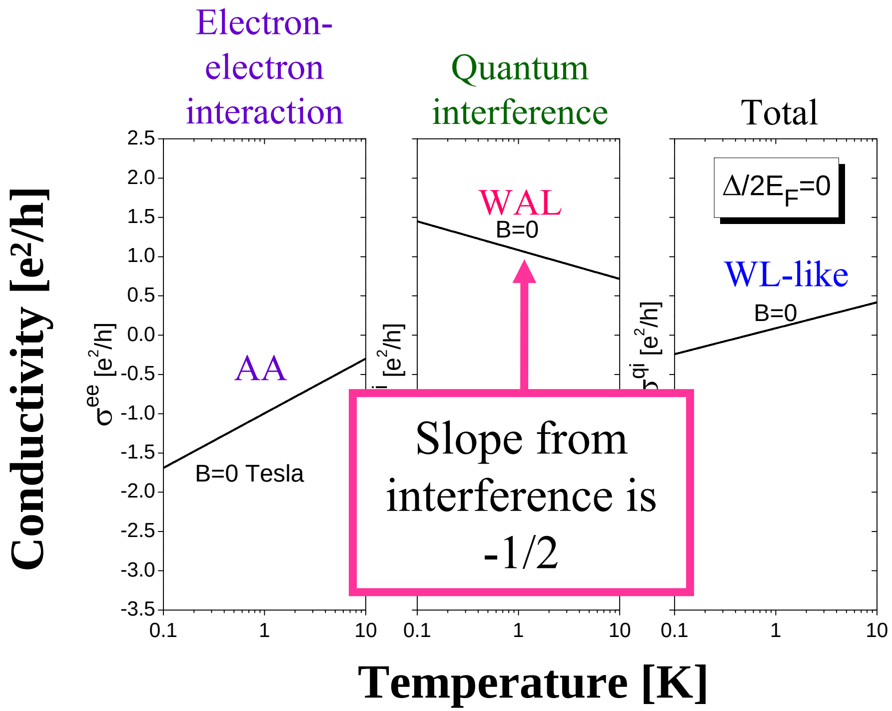
Check the slope

$$\kappa \equiv \frac{\pi h}{e^2} \frac{\partial \sigma}{\partial \ln T}$$

Lu & Shen, PRL 112, 146601 (2014)

Conductivity vs. Temperature





$$\kappa^{qi} = \alpha p = -\frac{1}{2} \times 1$$

A universal value
McCann et al, PRL 97, 146805 (2006); Lu, Shi & Shen, PRL, 107 076801 (2011).

p=1

$\ell_\phi \sim T^{-p/2}$

Peng, Cui et al (Stanford), Nat. Mater. 9, 225 (2010); Checkelsky, Ong et al (Princeton), PRL 106, 196801 (2011).

Lu & Shen, PRL 112, 146601 (2014)

Phase coherence length

$$\ell_\phi \sim \frac{1}{T^{p/2}}$$

p depends on dimensionality and decoherence mechanisms

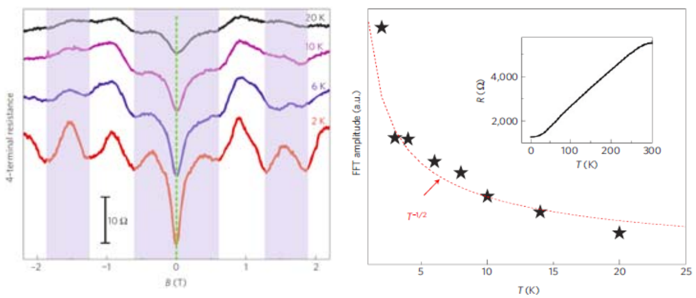
p in disordered metals

	EEI or EM	E-ph
1D	2/3	
2D	1	3
3D	3/2	

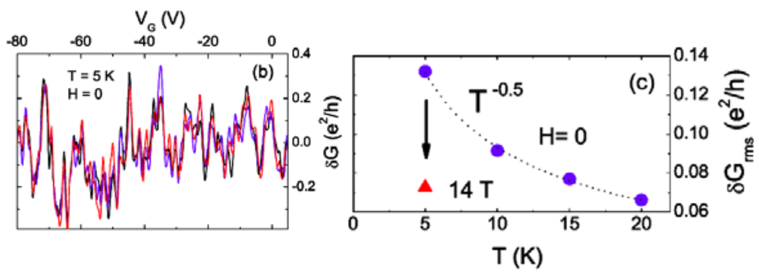
Altshuler et al, J. Phys. C 15, 7367 (1982); Fukuyama, JPSJ, 53, 3299, (1984); Lee & Ramakrishnan, RMP 57, 287 (1985).

In experiments, p is relaxed to be a fitting parameter

$p \sim 1$ in topological insulators

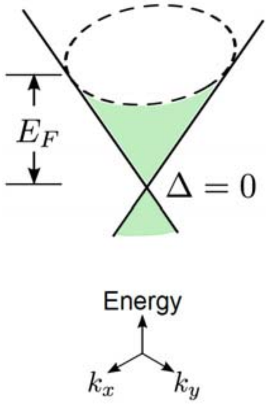


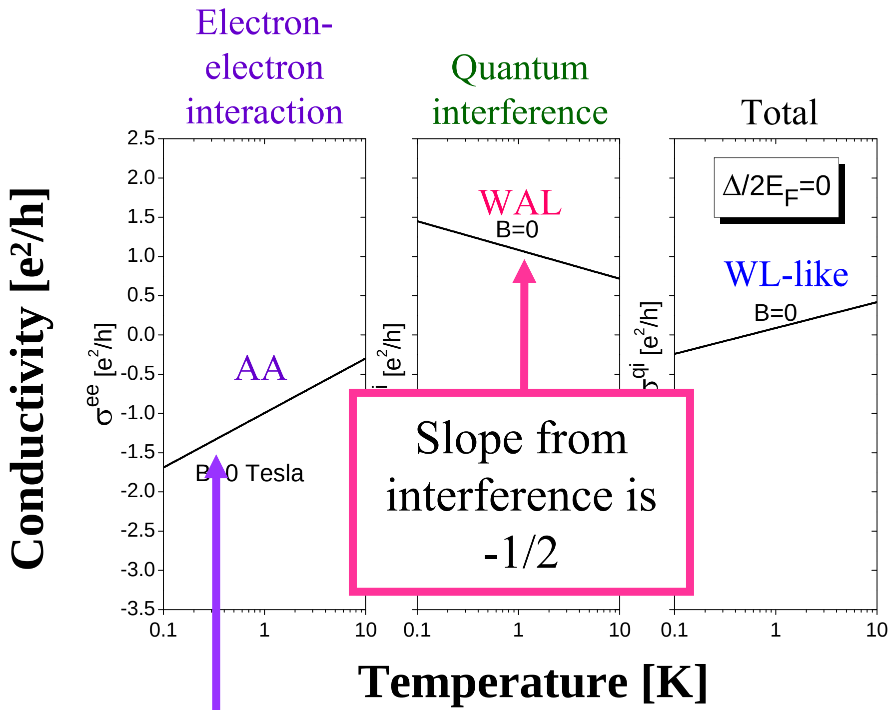
AB oscillation
Peng, Yi Cui et al (Stanford), Nat. Mater. 9, 225 (2010).



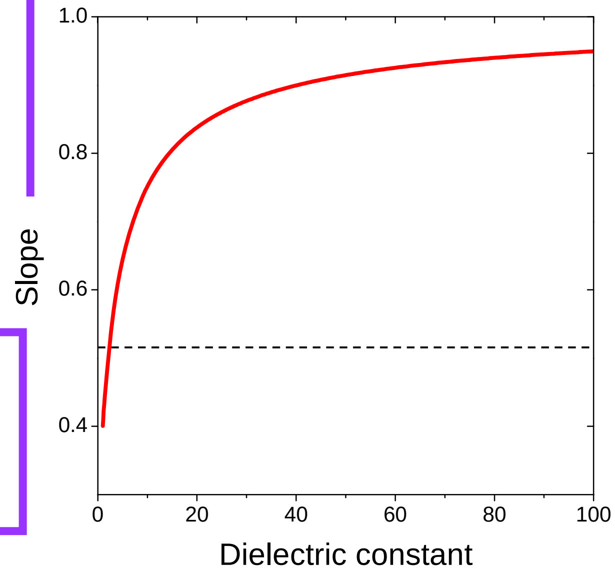
Universal conductance fluctuation
Checkelsky, Ong et al (Princeton), PRL 106, 196801 (2011)

Conductivity vs. Temperature





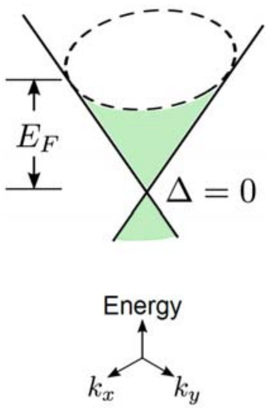
$$\kappa^{ee} \equiv \frac{\pi h}{e^2}\frac{\partial \sigma^{ee}}{\partial \ln T}$$

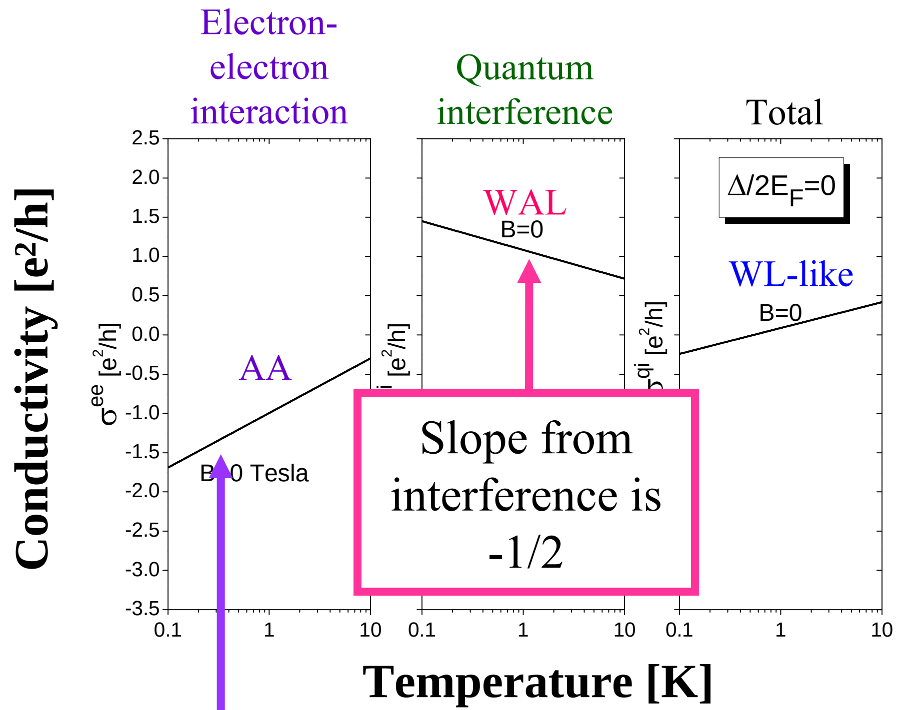


Slope from interaction depends on Dielectric constant

Lu & Shen, PRL 112, 146601 (2014)

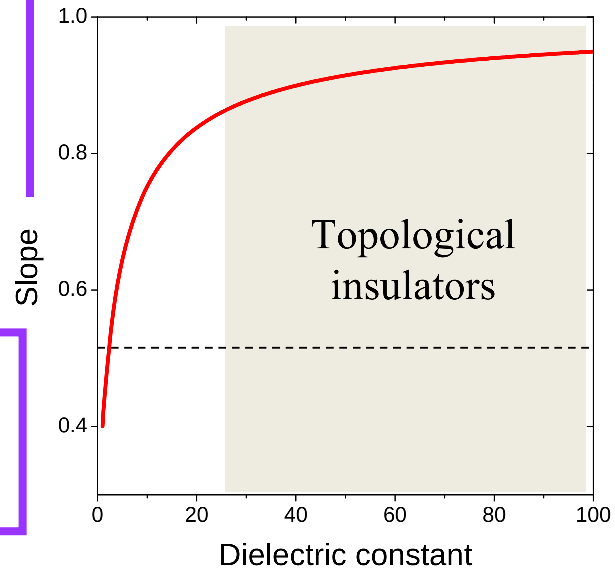
Conductivity vs. Temperature





$$\kappa^{ee} \equiv \frac{\pi h}{e^2} \frac{\partial \sigma^{ee}}{\partial \ln T}$$

Slope from interaction depends on Dielectric constant



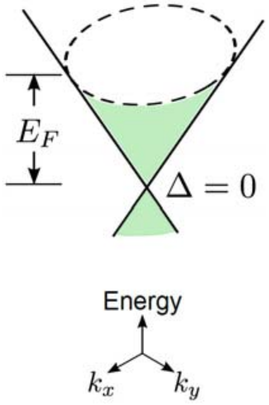
$\varepsilon_r = 113$ (Bi2Se3)

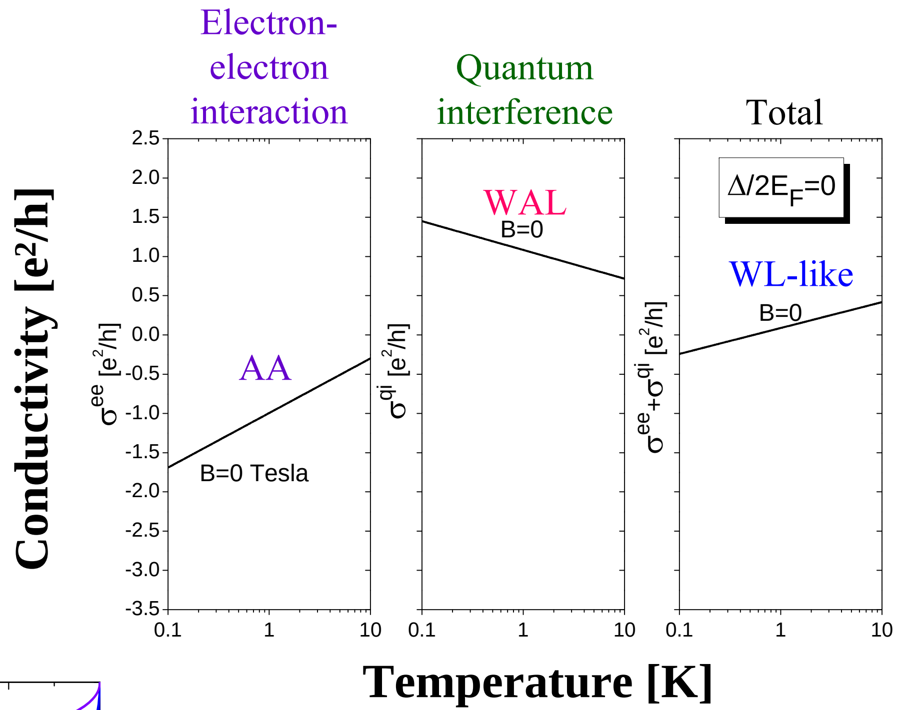
75 (Bi2Te3)

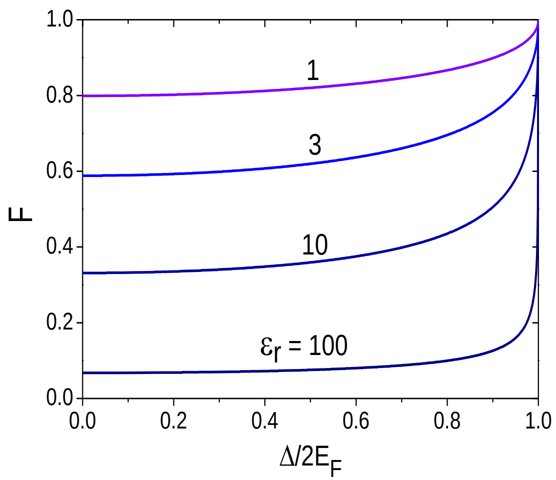
Richter, Köhler, Becker, Phys. Status Solidi (b) 84, 619 (1977).

Lu & Shen, PRL 112, 146601 (2014)

Conductivity: massless







Slope $$\kappa \equiv \frac{\pi h}{e^2}\frac{\partial \sigma}{\partial \ln T} = \frac{1}{2} - \frac{3}{4}F$$

Screening factor $$F \equiv \frac{\langle V(k-k') \rangle}{V(0)} \in [0,\ \ 1]$$

No screening — Fully screened

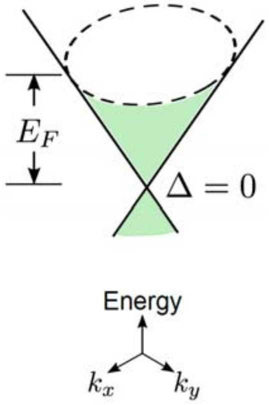
Screening length $\xi \in [\infty,\ \ 0]$

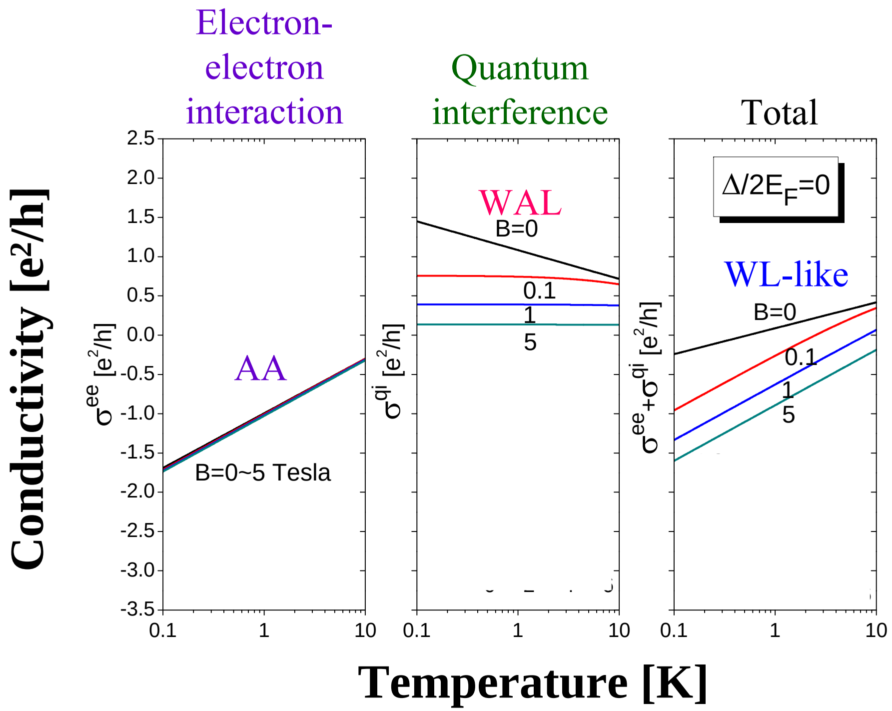
$$V(r) = \frac{e^2}{r}\exp(-r/\xi)$$

Dielectric constant: ε_r =113 (Bi2Se3) 75 (Bi2Te3)

Richter, Köhler, Becker, Phys. Status Solidi (b) 84 (1977) 619.

Conductivity vs. Magnetic field

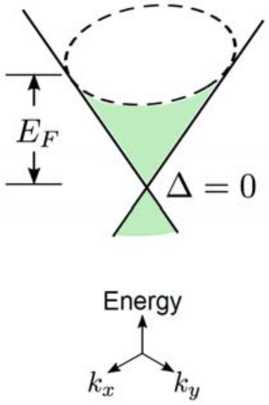


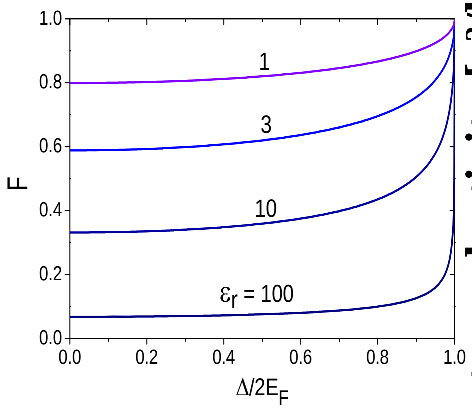


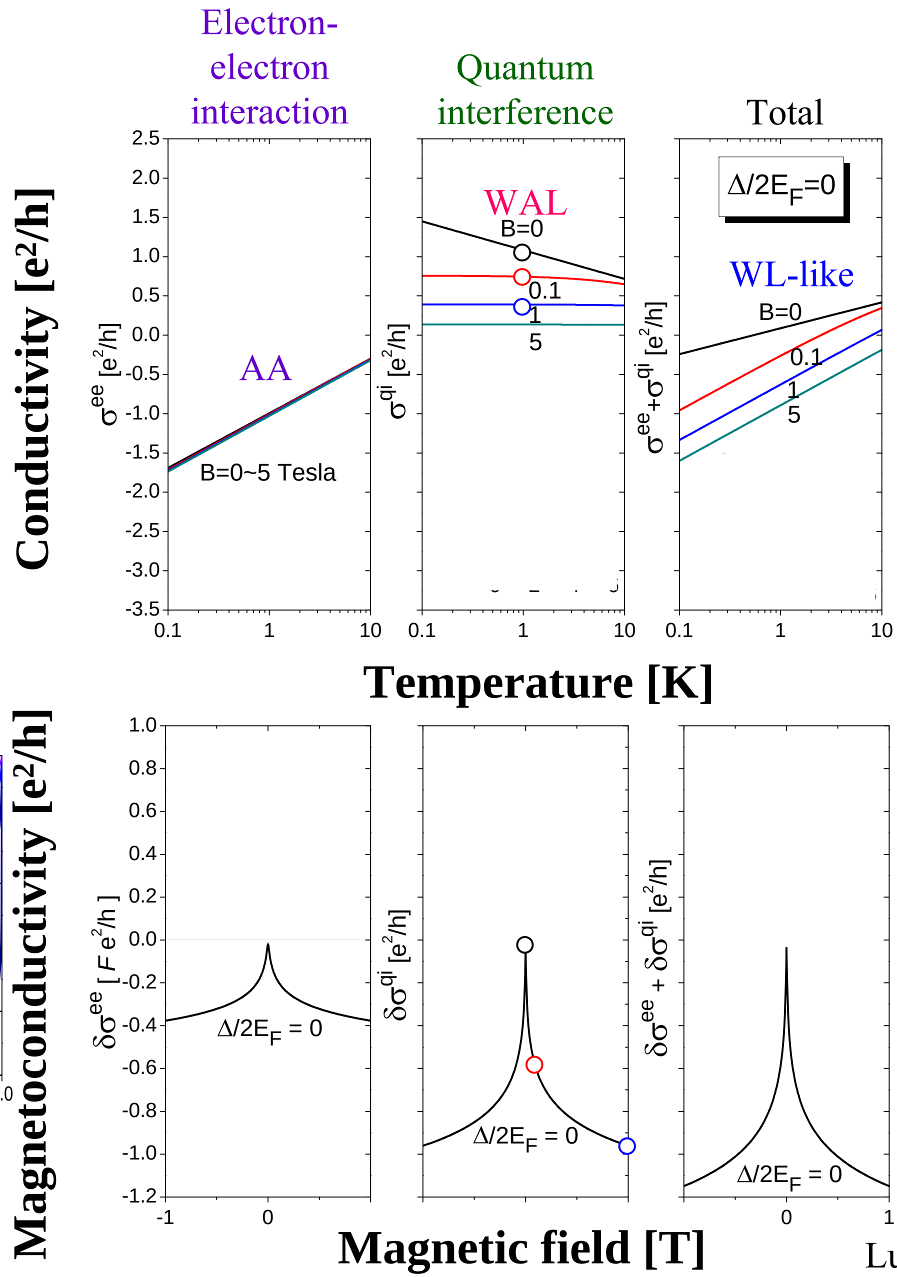
In magnetic field
(1) WAL is suppressed;
(2) Interaction part ? little change

Lu & Shen, PRL 112, 146601 (2014)

Magnetoconductivity





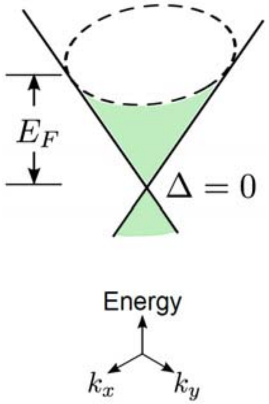


$$\delta\sigma \equiv \sigma(B) - \sigma(0)$$

(1) Magnetoconductivity mainly from WAL
(2) Interaction part? little contribution

Lu & Shen, PRL 112, 146601 (2014)

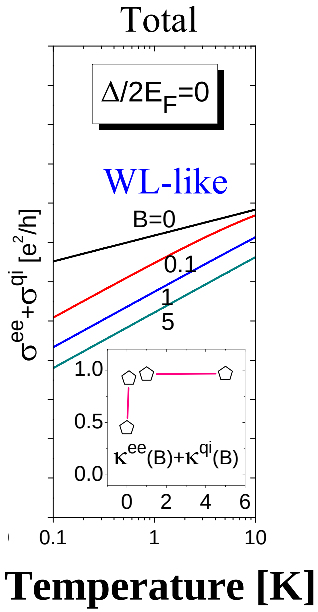
Theory & Experiments

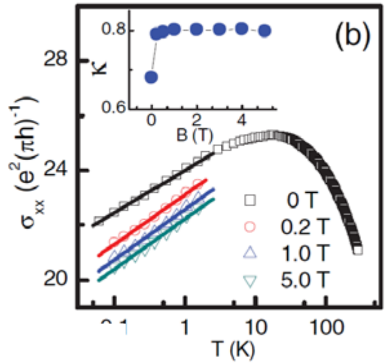


Theory & experiments:

Conductivity, temperatures and magnetic fields,

All of same orders

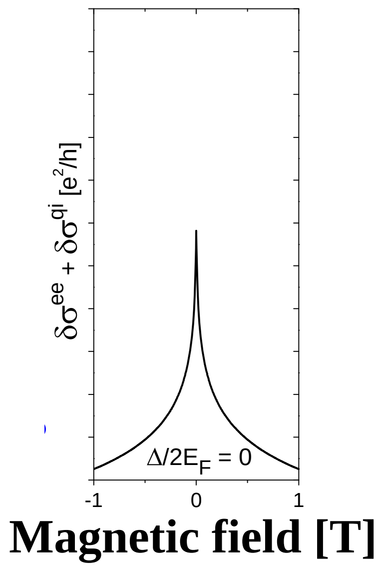


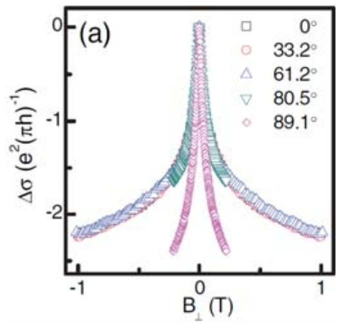


The higher bound of the temperature is given by

$$\ell << \ell_T = \sqrt{\frac{D\hbar}{2\pi k_B T}} \approx \sqrt{\frac{\ell \hbar v}{2\pi k_B T}}$$

The theory is valid for temperatures well below T~55K when the mean free path is about 10nm, and $\hbar v = 3\mathrm{eV.A}$

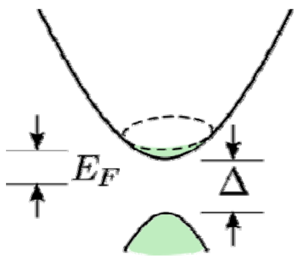




Chen, Y.Q. Li, K. H. Wu, L. Lu, et al (IOP), PRB 83, 241304 (R) (2011)

Lu & Shen, PRL 112, 146601 (2014)

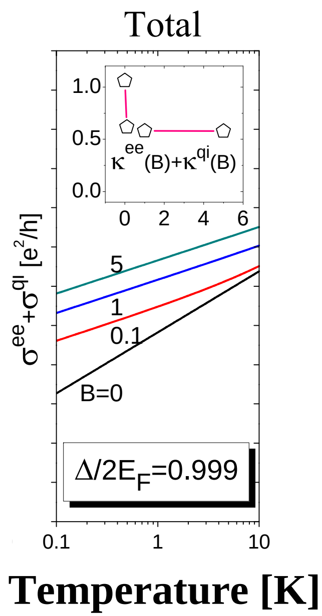
Theory & Experiments

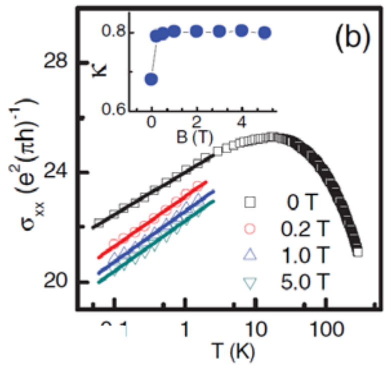


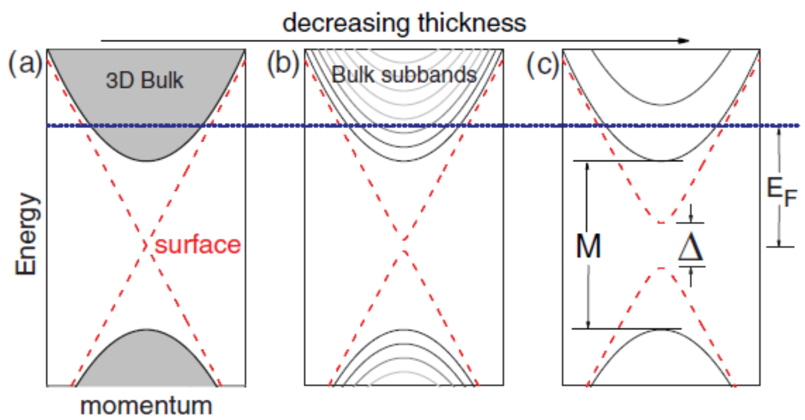
Large-gap

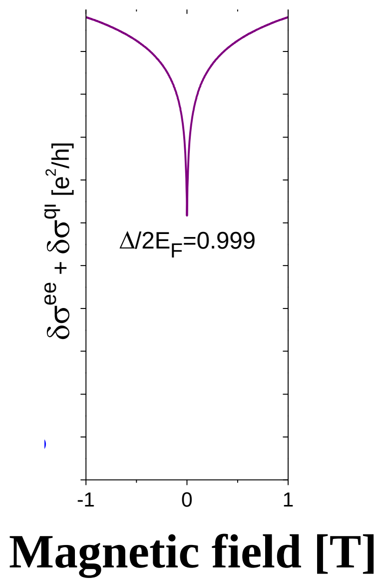
Not consitent with large mass limit.

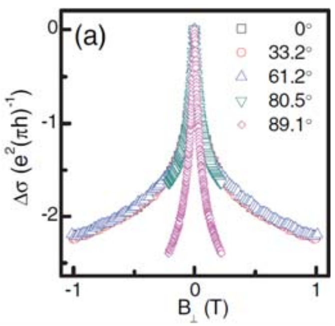
Massless Dirac fermions dominate







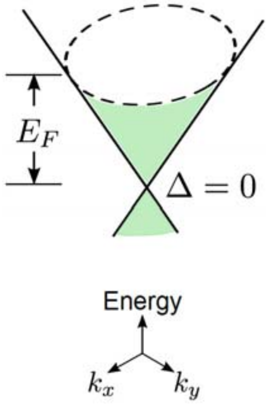


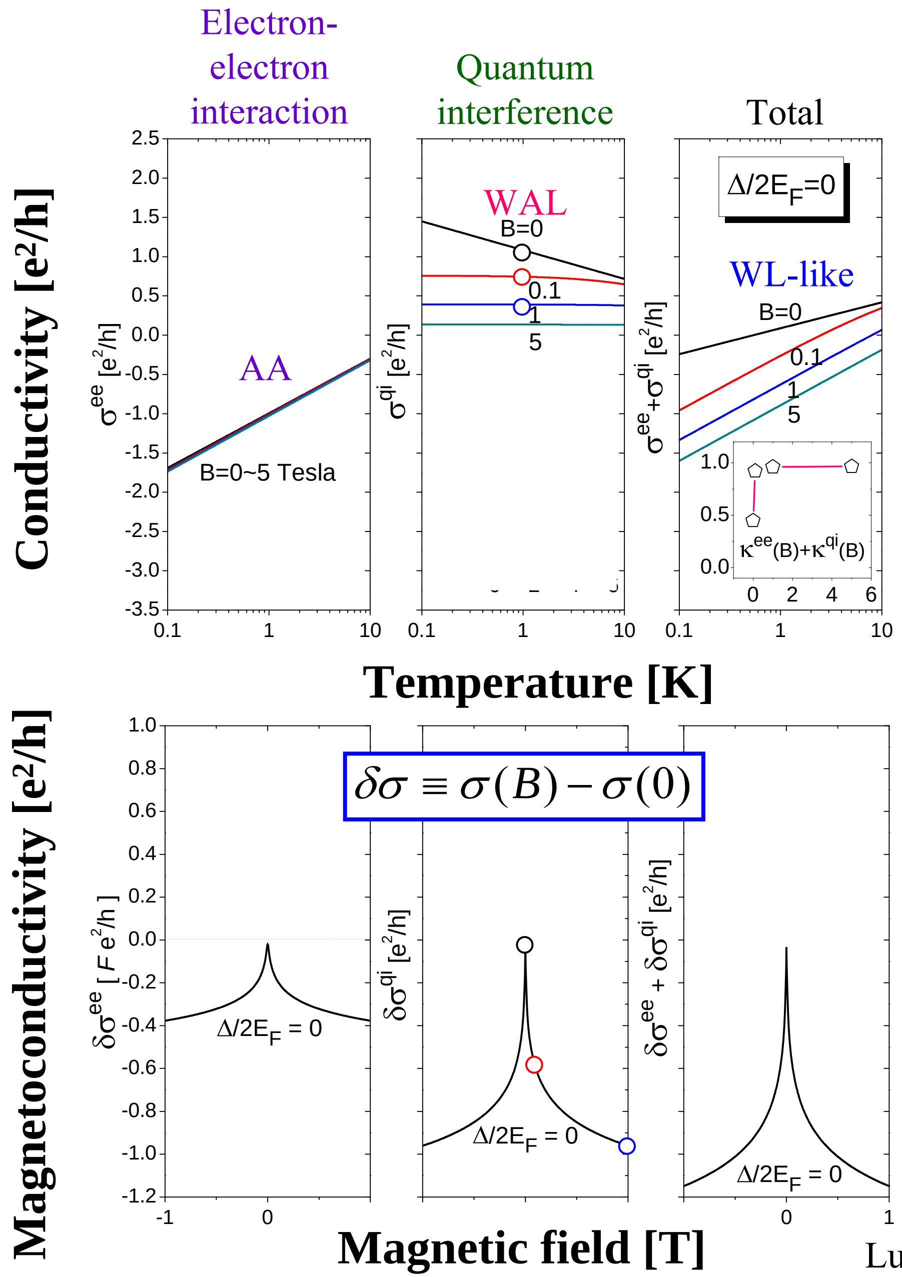


Lu & Shen, PRL 112, 146601 (2014)

Chen, Y.Q. Li, K. H. Wu, L. Lu, et al (IOP), PRB 83, 241304 (R) (2011)

Conductivity & Magnetoconductivity





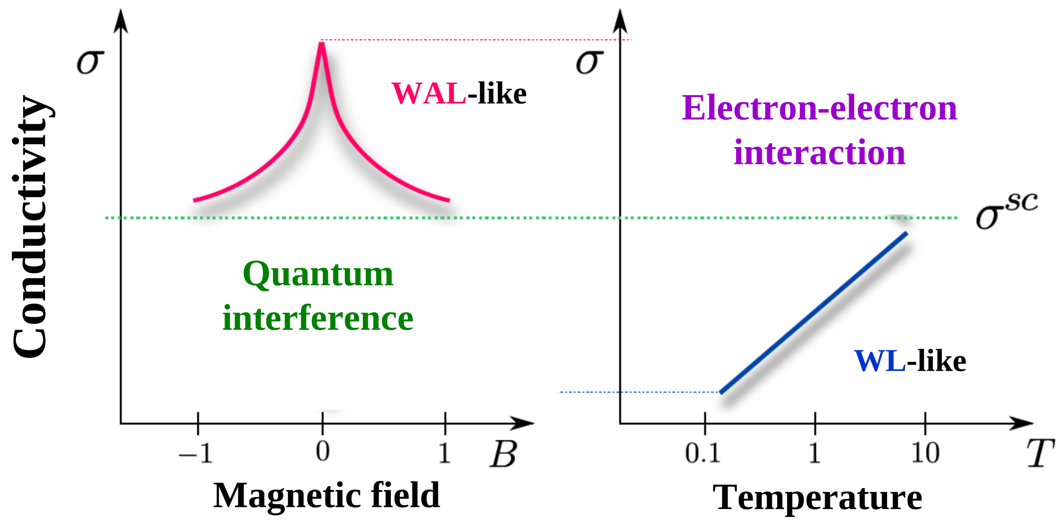
(1) Interaction
Temperature dependence of conductivity

(2) Quantum interference
Magnetoconductivity

Lu & Shen, PRL 112, 146601 (2014)

Summary

Quantum interference + **electron-electron interaction**
explains
conductivity and magnetoconductivity of TIs
[Lu & Shen, PRL 112, 146601 (2014)].



Generalization to other Dirac systems: MoS_2, silicene, etc
[Lu, Xiao, Yao & Shen, PRL, 110, 016806 (2013);
Iwasa group (Tokyo) Science 2012; Nat. Phys. 2013;
Peide Ye group (Purdue) ACS Nano 2013]

Thanks a lot!!!

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Acknowledgment: Prof. Michael Ma (Cincinnati & HKU)