Two-fluid model and emergent states in heavy electron materials

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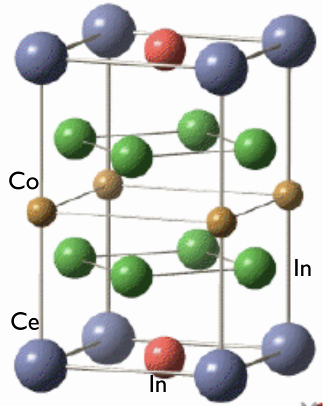
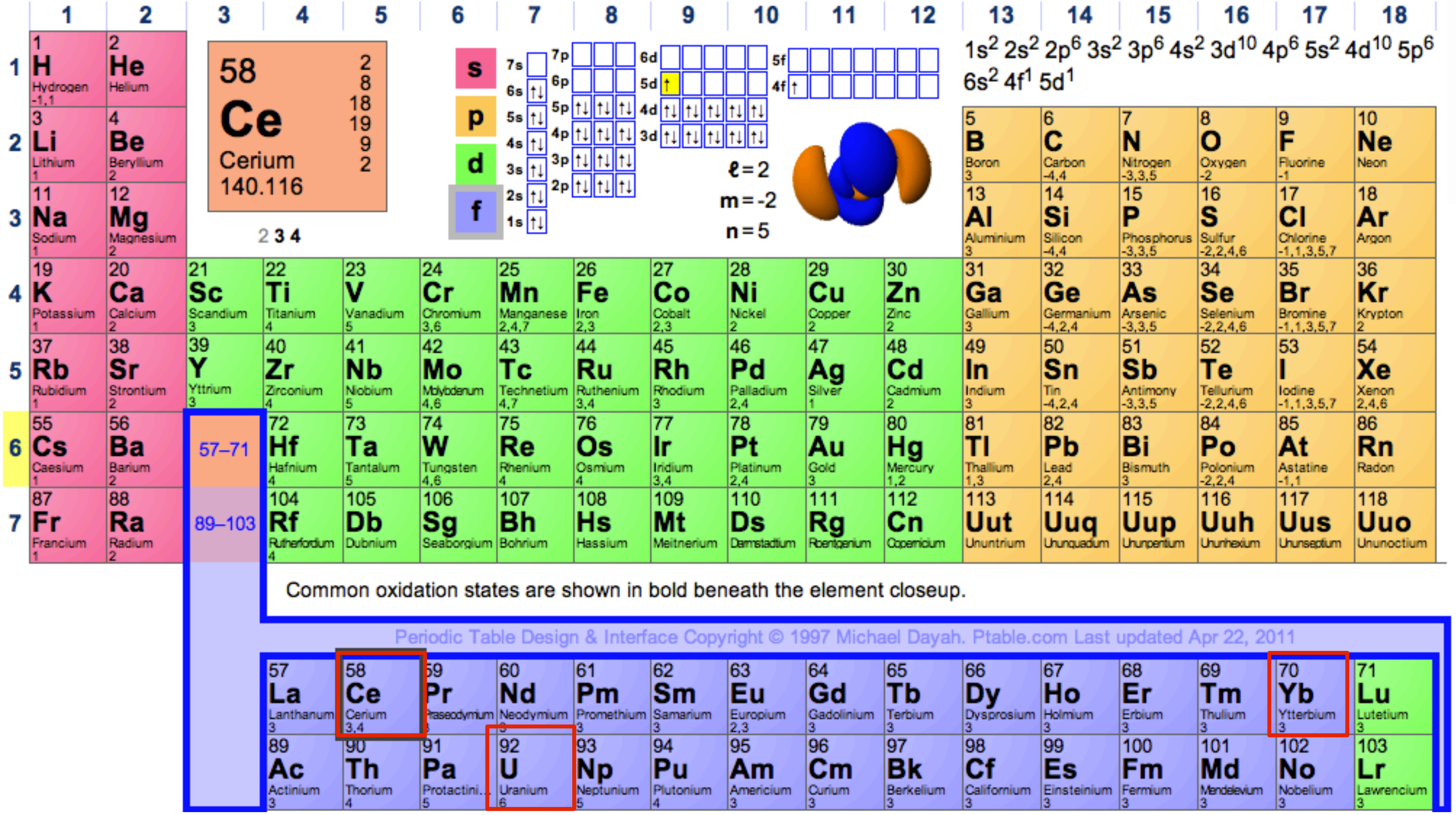
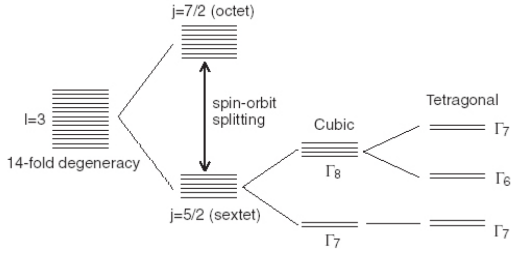
Outline

- Introduction to heavy fermion physics
- What is the two-fluid model?
- Heavy fermion physics revisited
- A new theoretical framework

Heavy electron materials

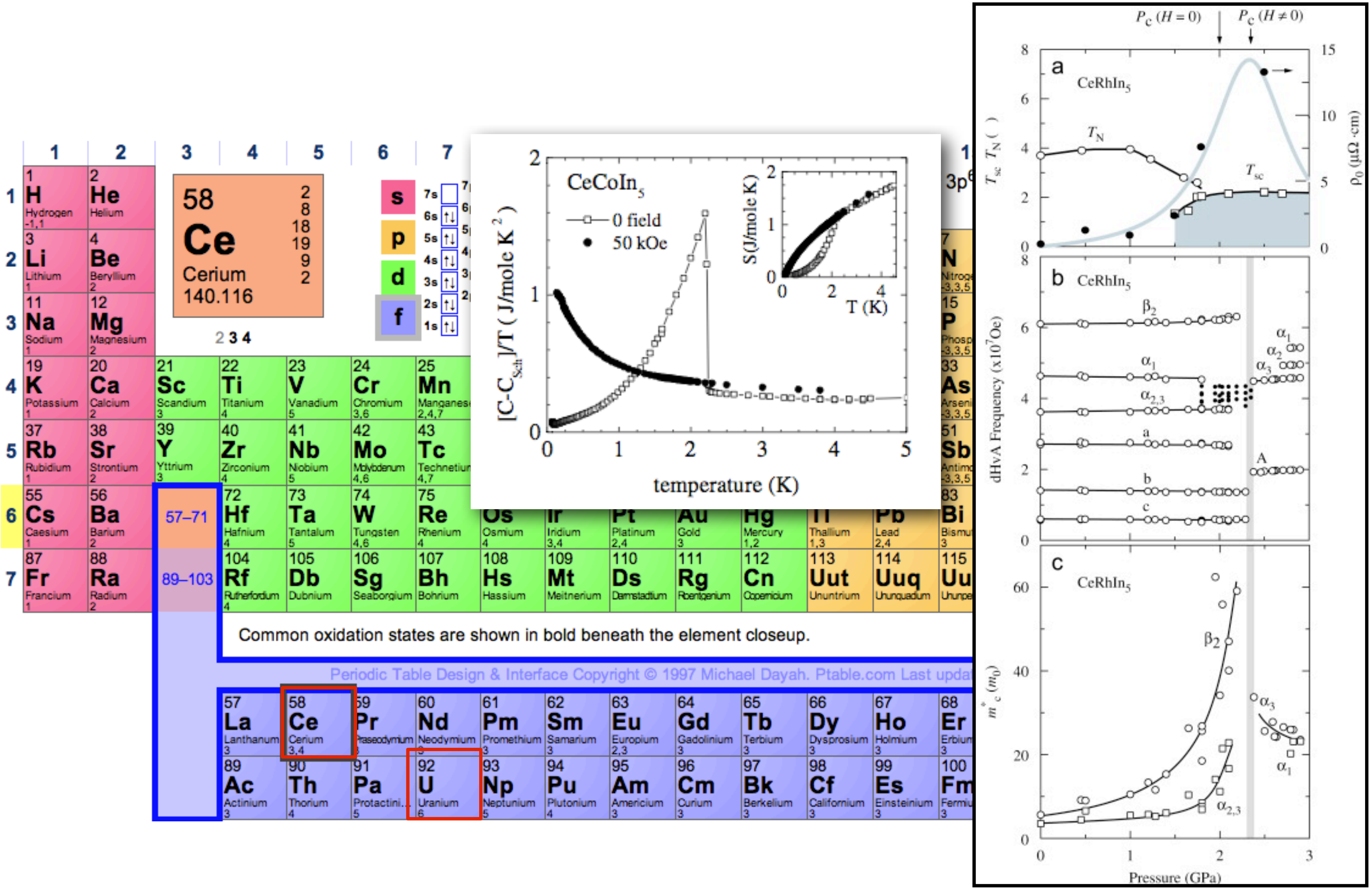






- intermetallics of Ce, U, Yb
- some other f and d-electron systems
- $m_{eff}/m_{bare} \sim 10^2\text{-}10^3$

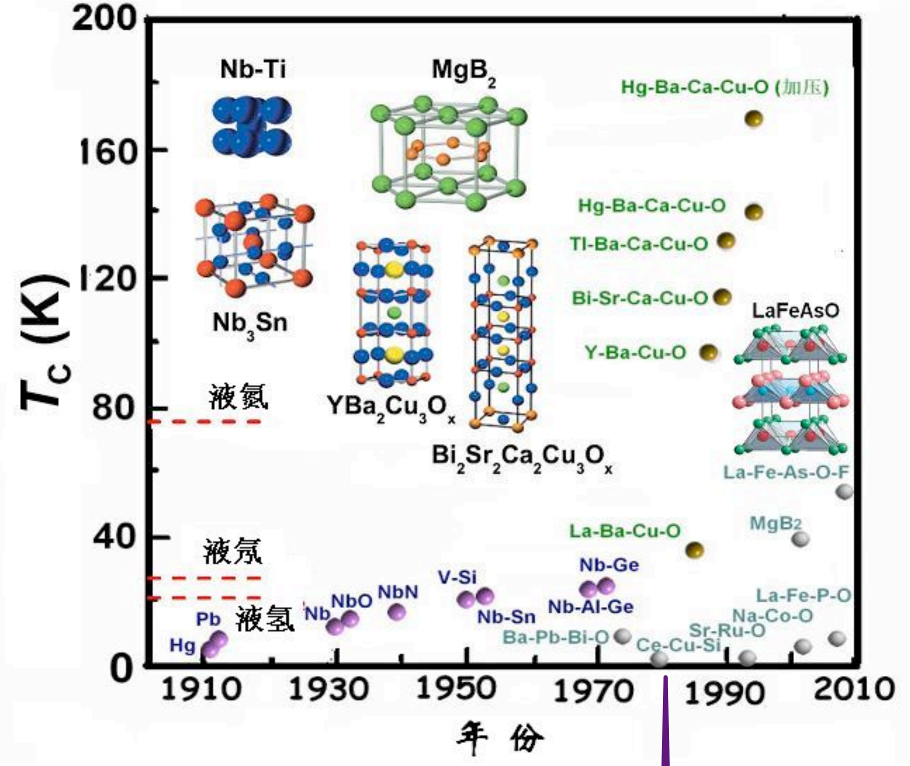
Heavy electron materials



- intermetallics of Ce, U, Yb
- some other f and d-electron systems
- $m_{eff}/m_{bare} \sim 10^2$-$10^3$

- Non-Fermi liquid, unconventional superconductivity, unconventional quantum criticality

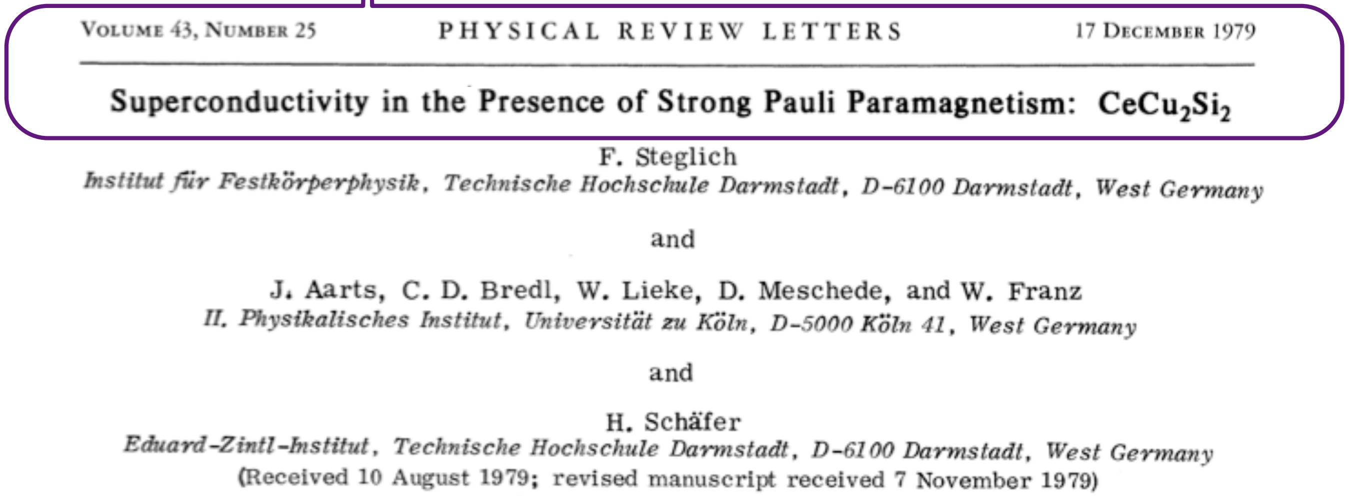
Heavy fermion superconductors



	T_c(K)	
	Ce-based	
CeCu$_2$Si$_2$	0.7	('79 DA/K)
[p = 2.9 GPa:	2.3	('84 GE/GR)]
CeNi$_2$Ge$_2$	0.2	('97 DA, '98 CA/GR)
CeIrIn$_5$	0.4	('00 LANL)
CeCoIn$_5$	2.3	('00 LANL)
Ce$_2$CoIn$_8$	0.4	('02 NA)
Ce$_2$PdIn$_8$	0.7	('09 WR)
CePt$_3$Si	0.7	('03 VI)
p > 0		
CeCu$_2$Ge$_2$	0.6	('92 GE)
CePd$_2$Si$_2$	0.4	('94 CA)
CeRh$_2$Si$_2$	0.4	('95 LANL)
CeCu$_2$	0.15	('97 GE/KA)
CeIn$_3$	0.2	('98 CA)
CeRhIn$_5$	2.1	('00 LANL)
Ce$_2$RhIn$_8$	1.1	('03 LANL)
CeRhSi$_3$	0.8	('05 SE)
CeIrSi$_3$	1.6	('06 OS)
CeCoGe$_3$	0.7	('06 OS)
Ce$_2$Ni$_3$Ge$_5$	0.26	('06 OS)
CeNiGe$_3$	0.4	('06 OS)
CeRhGe$_2$	0.4	('09 OS)
CePt$_2$In$_7$	1.5	('09 LANL)

	T_c(K)	
	Pr-based	
PrOs$_4$Sb$_{12}$	1.85	('01 UCSD)
	Yb-based	
β-YbAlB$_4$	0.08	('08 TO/IR)
	U-based	
UBe$_{13}$	0.9	('83 Z/LANL)
UPt$_3$	0.5	('84 LANL)
URu$_2$Si$_2$	1.4	('84 K/DA)
UNi$_2$Al$_3$	1.2	('91 DA)
UPd$_2$Al$_3$	2.0	('91 DA)
URhGe	0.3	('01 GR)
UCoGe	3.0	('07 AM/KA)
p > 0		
UGe$_2$	0.7	('00 CA/GR)
UIr	0.14	('04 OS)
	Np-based	
NpPd$_5$Al$_2$	5.0	('07 OS)
	Pu-based	
PuCoGa$_5$	18.5	('02 LANL)
PuRhGa5	8.7	('03 KA)
	Am-based	
p > 0		
Am metal	2.2	('05 KA)

(source: F. Steglich)



VOLUME 43, NUMBER 25 PHYSICAL REVIEW LETTERS 17 DECEMBER 1979

Superconductivity in the Presence of Strong Pauli Paramagnetism: CeCu$_2$Si$_2$

F. Steglich
Institut für Festkörperphysik, Technische Hochschule Darmstadt, D-6100 Darmstadt, West Germany

and

J. Aarts, C. D. Bredl, W. Lieke, D. Meschede, and W. Franz
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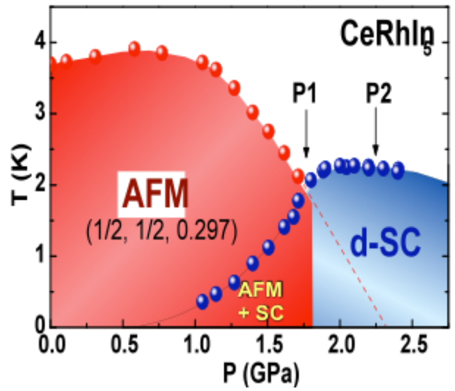
and

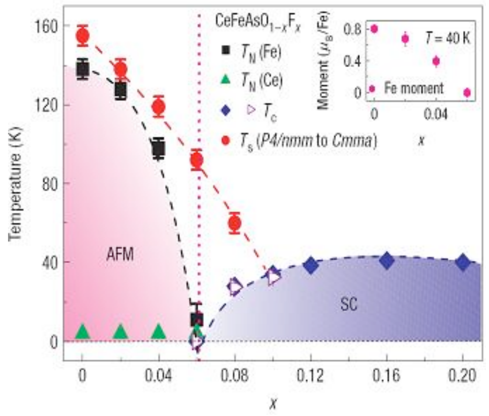
H. Schäfer
Eduard-Zintl-Institut, Technische Hochschule Darmstadt, D-6100 Darmstadt, West Germany

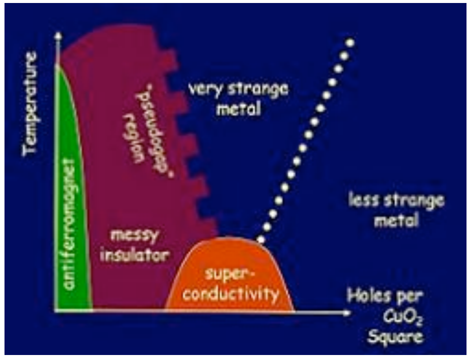
(Received 10 August 1979; revised manuscript received 7 November 1979)

Other exotic phenomena

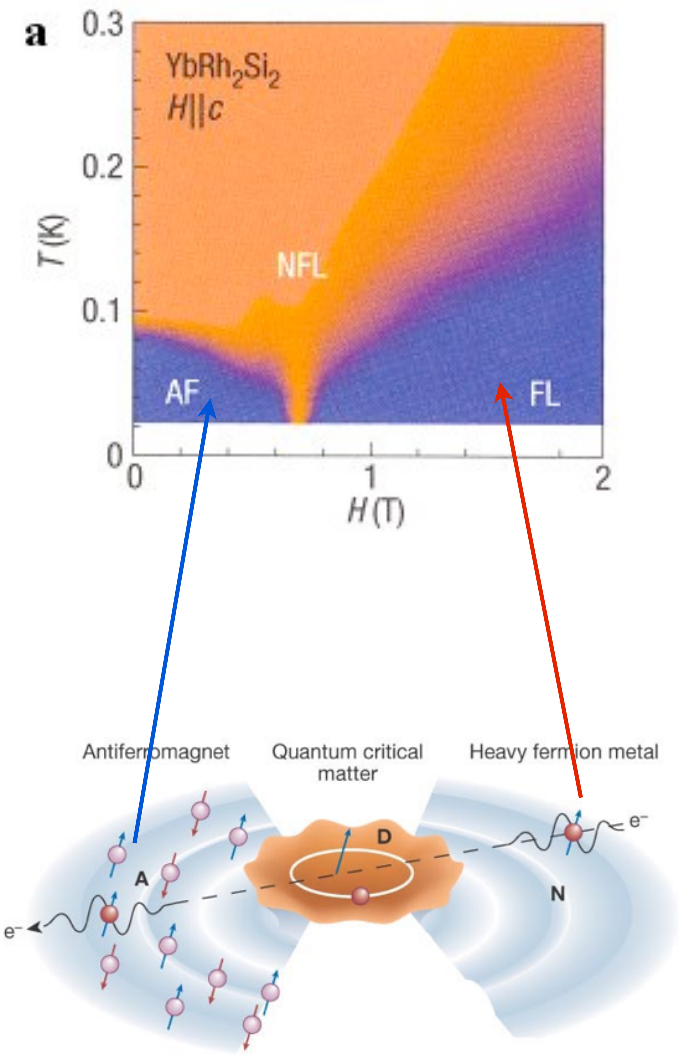
Coexistence of SC&AFM





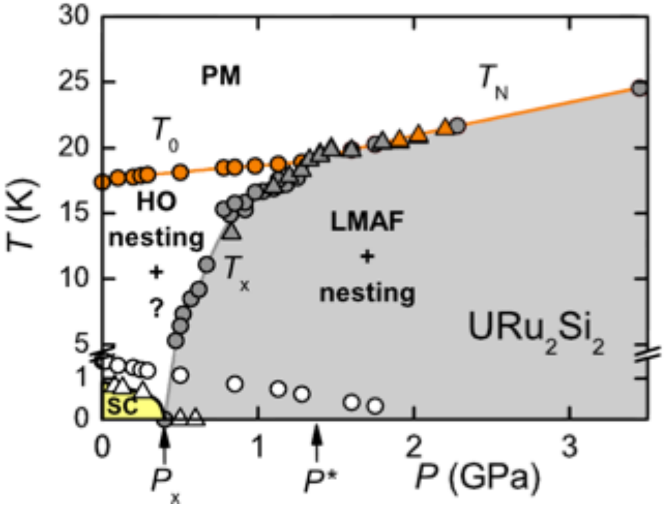


Non-Fermi liquid behavior



Local quantum criticality vs
spin density wave quantum criticality

Hidden order

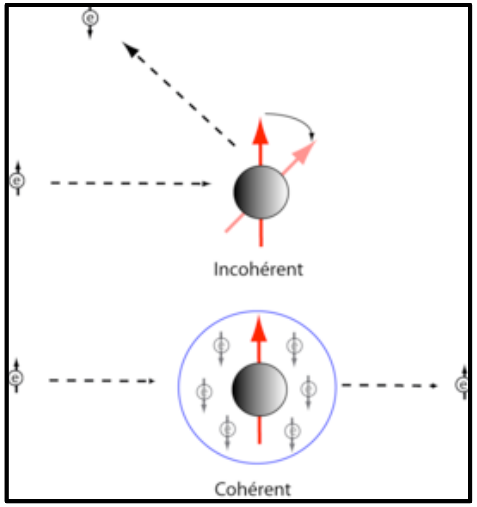


- Lev. P. Gorkov: 1996:
  - *-Mixed valency, coupling to lattice degrees of freedom.*
- Chandra *et al*., Nature'02
  - *- Incommensurate Orbital Antiferromagnetism (based on "old" NMR)*
- Mineev & Zhitomirsky, PRB '05
  - *- SDW (with tiny moment… problem with entropy)*
- Varma & Zhu, PRL'06
  - *- Helical Order, Pomeranchuk instability of the Fermi surface ?*
- Elgazaar, & Oppeneer, Nature Materials'08
  - *- DFT: antiferromagnetic order parameter, but weak AFM moment (can not explain large entropy loss, stress, adiabatic continuity, moment in z dir….)*
- Santini and Amoretti PRL 04
  - *-Quadrupolar ordering.*
- Fazekas and Kiss PRB 07
  - *-Octupolar ordering. [Many Many more , even recently*

From Haule

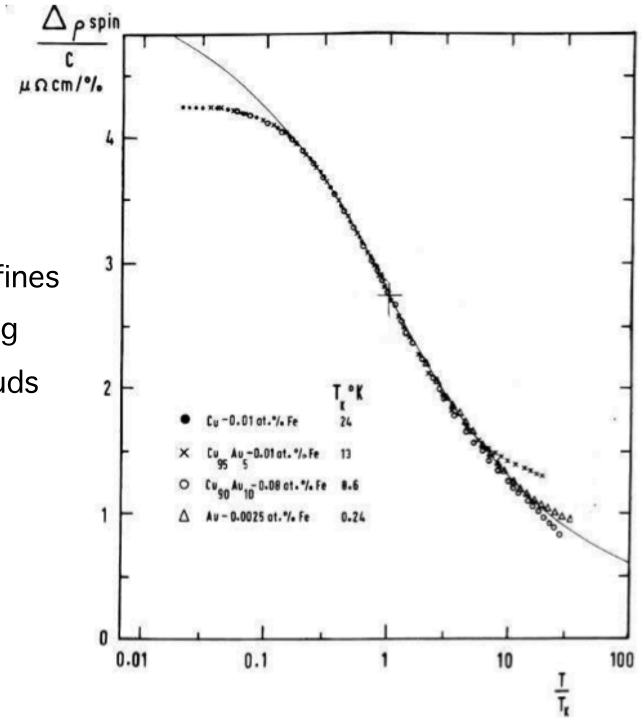
- **Haule and Kotliar 2010**
  - - **Hexadecapolar** order
- **Balatsky 2010**
  - **- Hybridization wave**
- **Pepin 2011**
  - **- Modulated spin liquid**

From the Kondo physics to the Kondo lattice physics



Crossover to the low temperature singlet defines

- ➡ Kondo temperature T_K and universal scaling
- ➡ Kondo screening and collective Kondo clouds
- ➡ Large Fermi surface containing local spin



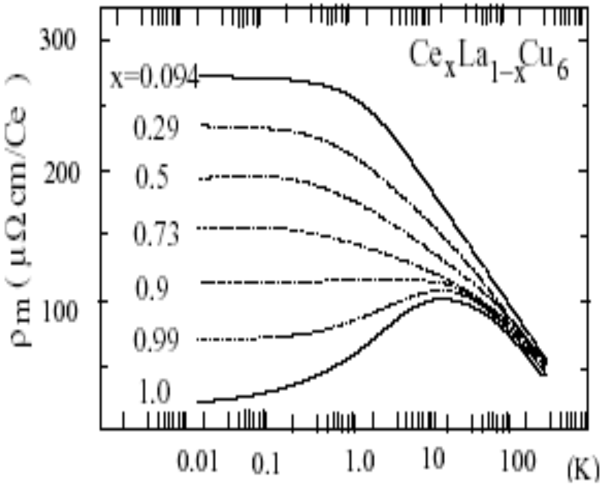


FIGURE 15. Development of coherence in heavy fermion systems. Resistance in $Ce_{1-x}La_xCu_6$ after Onuki and Komatsubara[35]

- ➡ Is there a characteristic temperature “T_K”?
- ➡ Is there universality related to T_K?
- ➡ What is the collective motion?
- ➡ How do f-electrons enter the Fermi surface?

The Kondo lattice model

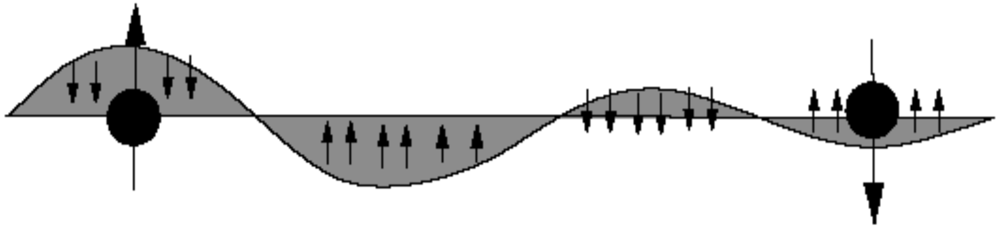
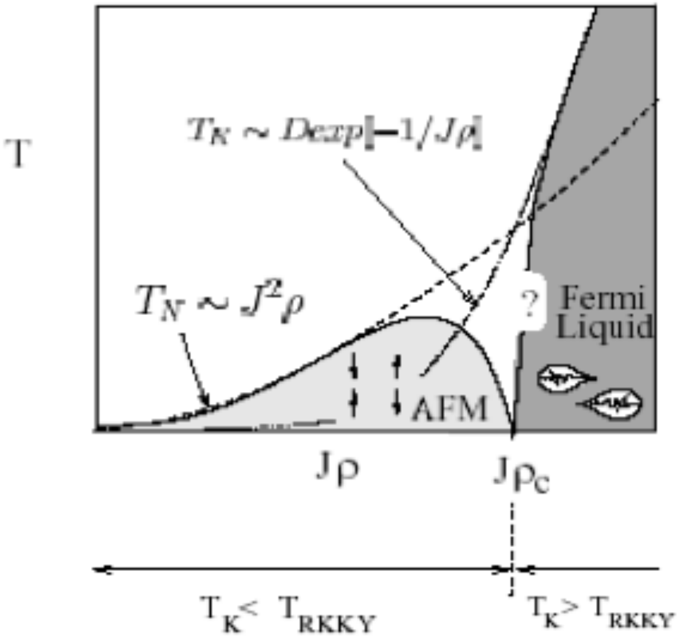
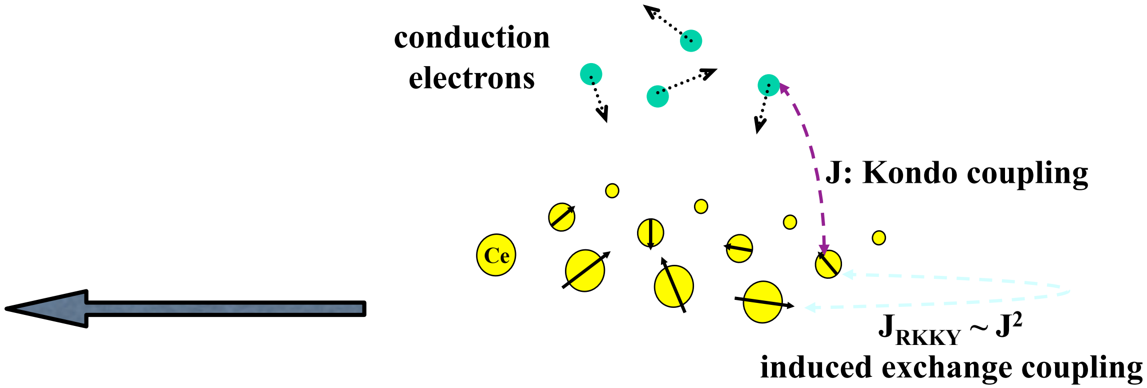


FIGURE 12. Illustrating how the polarization of spin around a magnetic impurity gives rise to Friedel oscillations and induces an RKKY interaction between the spins





Kondo lattice model

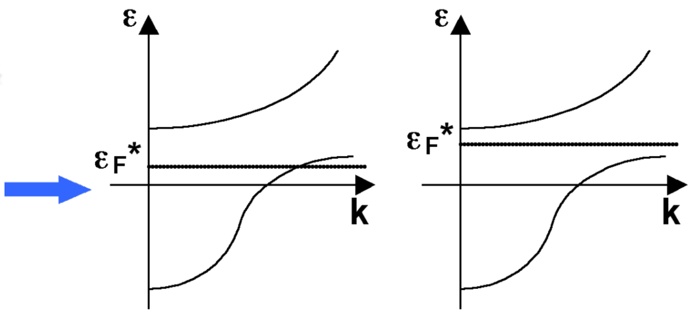
$$H_{KLM} = \sum_{k\sigma} c^{\dagger}_{k\sigma} c_{k\sigma} + J \sum_{i} \mathbf{S}_i \cdot \mathbf{s}_i + J_{RKKY} \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

Competition between Kondo and RKKY

The hybridization picture

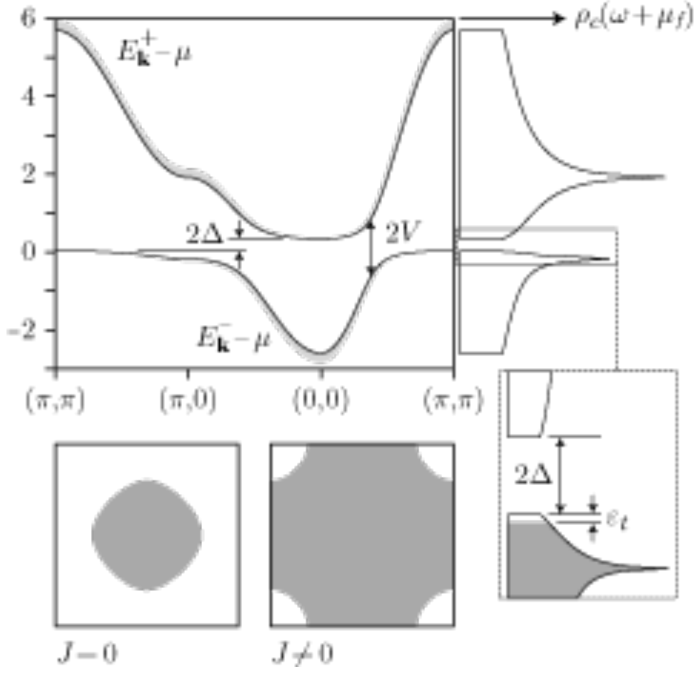
periodic Anderson model

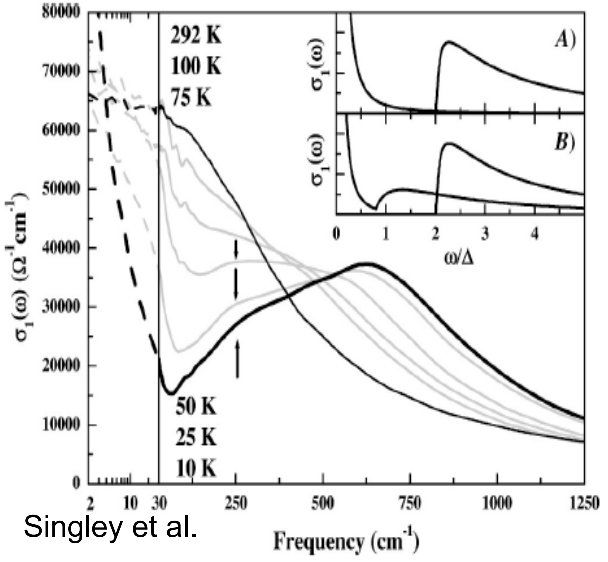
$$\mathcal{H}_0 = \sum_{k,\sigma} \varepsilon_k c^\dagger_{k,\sigma} c_{k,\sigma} + \varepsilon_f \sum_{k,\sigma} f^\dagger_{k,\sigma} f_{k,\sigma} + V \sum_{k,\sigma} (f^\dagger_{k,\sigma} c_{k,\sigma} + c^\dagger_{k,\sigma} f_{k,\sigma}) + (U/2) \sum_{i,\sigma} n^f_{i,\sigma} n^f_{i,-\sigma}.$$



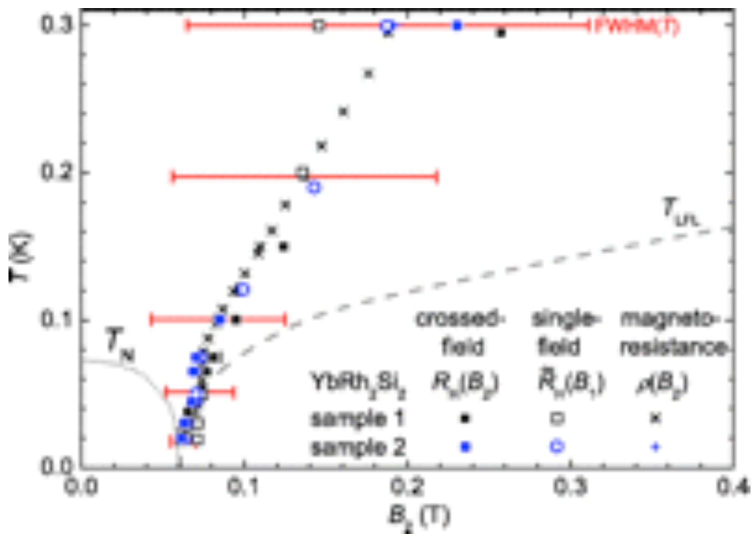
Unhybridized f-electrons **Heavy fermion** **Kondo insulator**

f-electrons become part of the Fermi Surface via the Kondo coupling.





Singley et al.



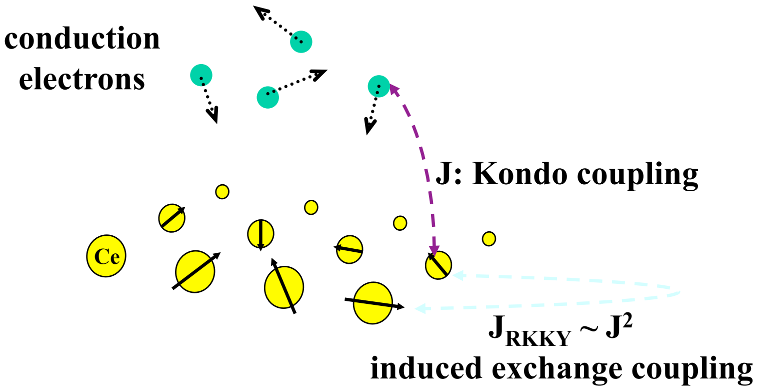
Numerical Methods: EDMFT, DMFT(OCA), DMFT(CTQMC), etc

Fermi surface reconstruction

What’s the problem?

It seems that we’ve had a pretty good theory, but after 30 years of research

- No systematic experimental determination of T_K and T_{RKKY}
- No exact solution of the model due to RKKY and 14 f-states
- A number of exotic behaviors unexplained
- Little is known about the temperature evolution



Most work focus on the quantum critical behavior.
Can quantum criticality explain everything?
Don’t we need to understand the normal state physics first?

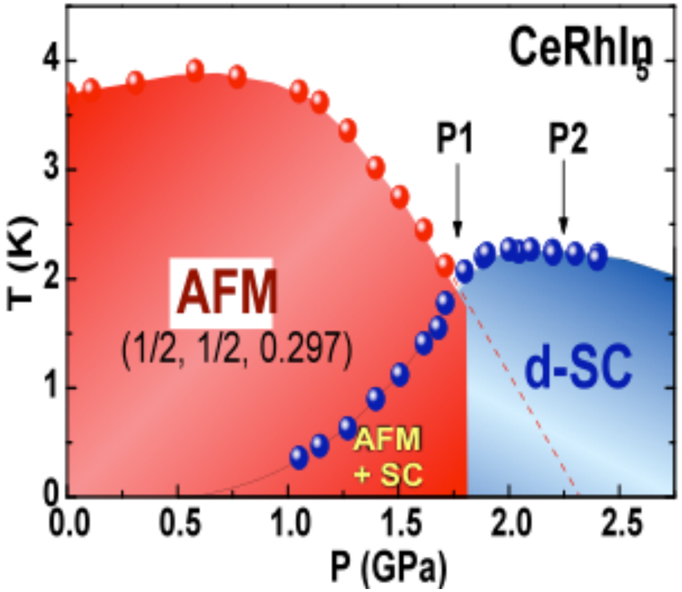
Kondo lattice model

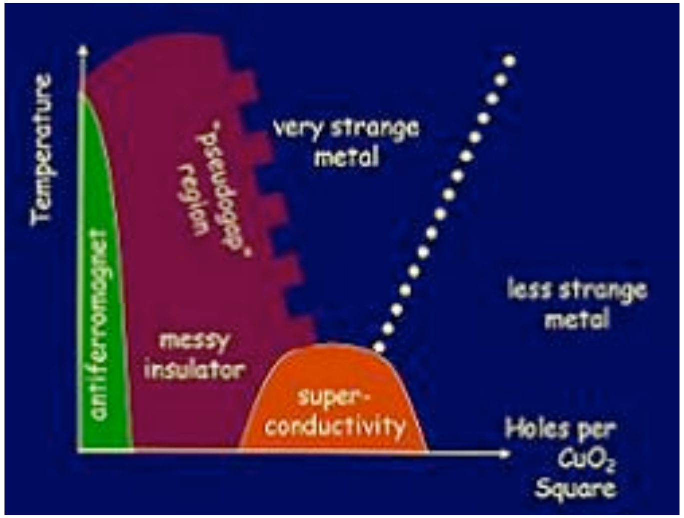
What do experiments tell us?

$$H_{KLM} = \sum_{k\sigma} c^{\dagger}_{k\sigma} c_{k\sigma} + J \sum_{i} \mathbf{S}_i \cdot \mathbf{s}_i + J_{RKKY} \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

The two fluid model

Heavy fermions vs cuprates





Similarity

- An antiferromagnetic parent state, AFM&SC closely related
- A quantum critical point beneath the superconducting dome?
- Non-Fermi liquid behavior in the normal state
- Change of Fermi surface with pressure (doping)

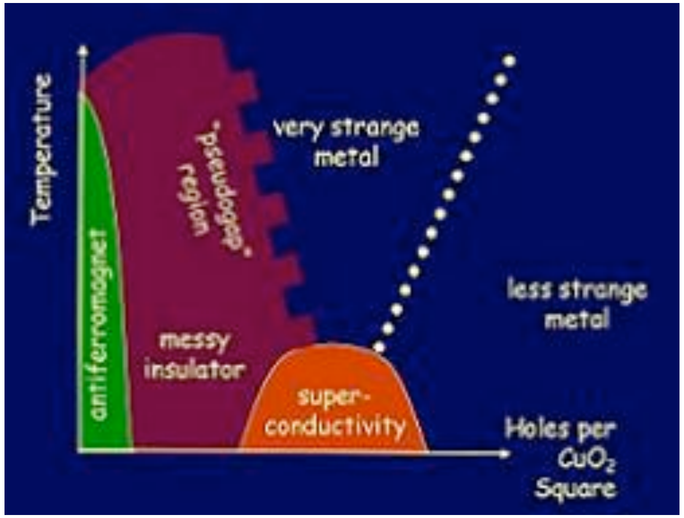
Difference

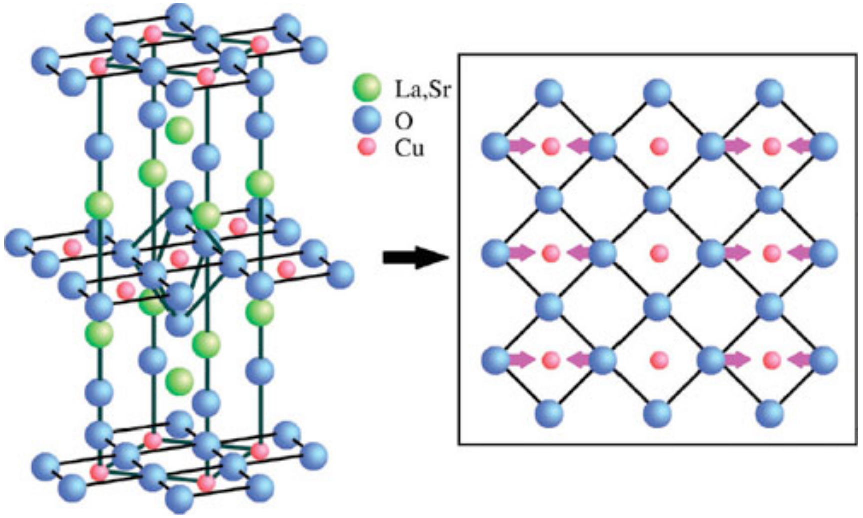
- Inhomogeneity (cuprates)
- Pseudo gap (cuprates)
- Rich variety in critical behaviors
- Microscopic coexistence of AFM&SC

Superconductivities are both mediated by spin fluctuations !

On the other hand, we may need first to understand the normal state physics !

Cuprates





One band Hubbard model

$$H = -t \sum_{\langle i,j \rangle, \sigma} \left(c^\dagger_{i\sigma} c_{j\sigma} + h.c. \right) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

$$\longrightarrow$$

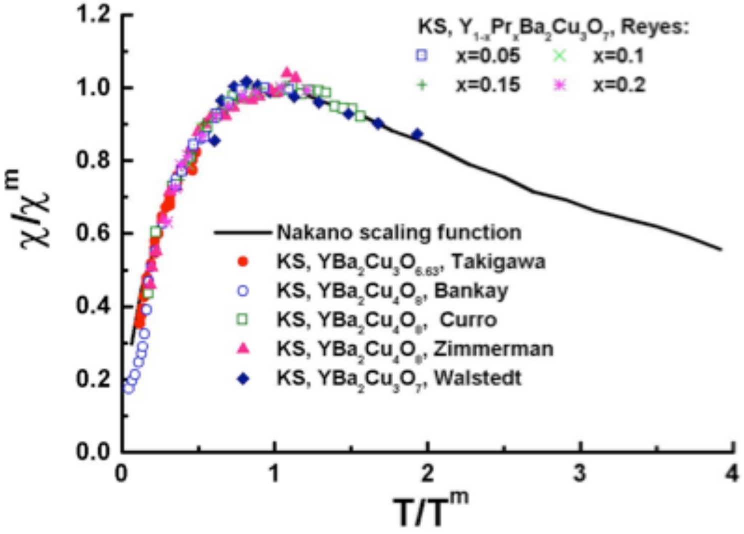
t-J model

$$H = -t \sum_{\langle i,j \rangle, \sigma} \left(\tilde{c}^\dagger_{i\sigma} \tilde{c}_{j\sigma} + h.c. \right) + J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

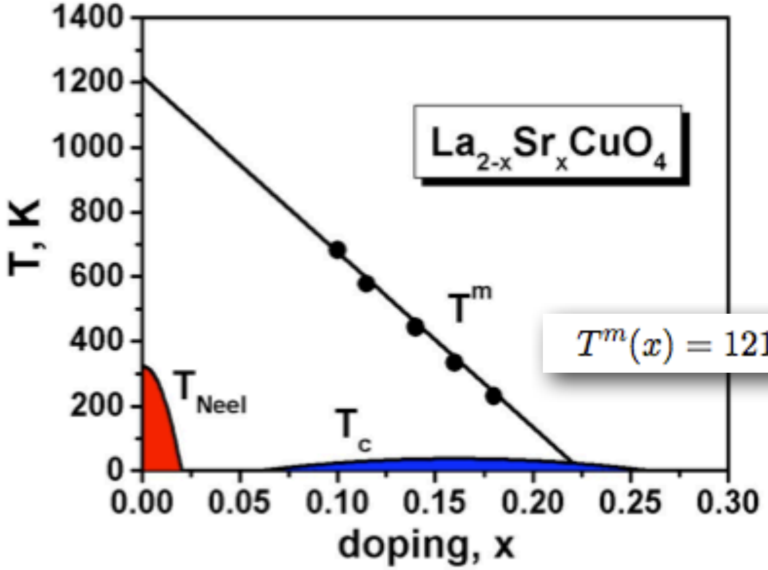
It may therefore be possible to approximate the physics of doped system as an effective spin system plus some additional hole excitations.

Nakano scaling and the reduced exchange coupling

Nakano scaling

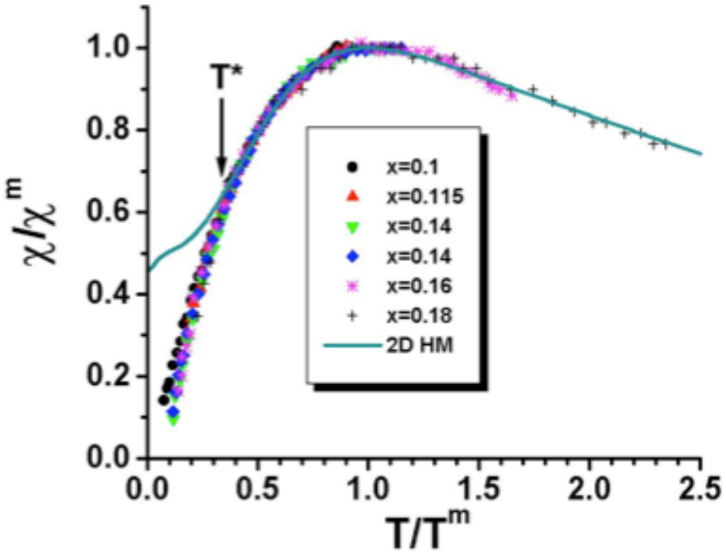


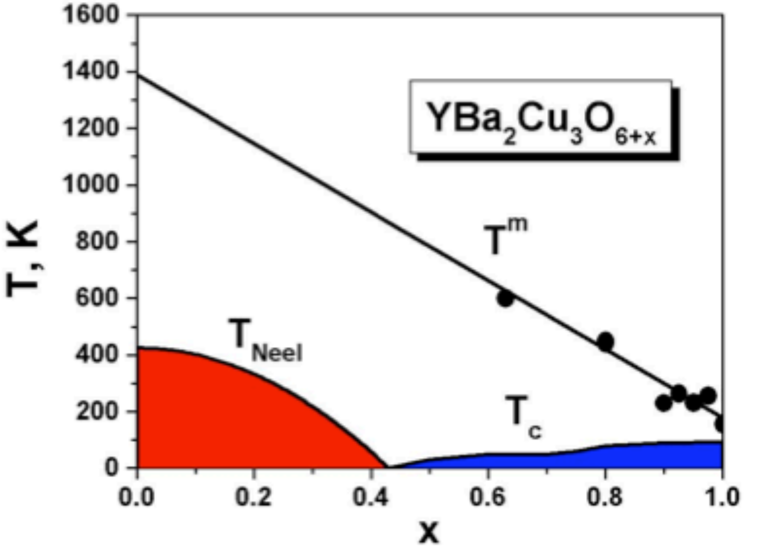
A reduced effective coupling



$$T^m(x) = 1218K(1 - 4.45x).$$

2D Heisenberg model



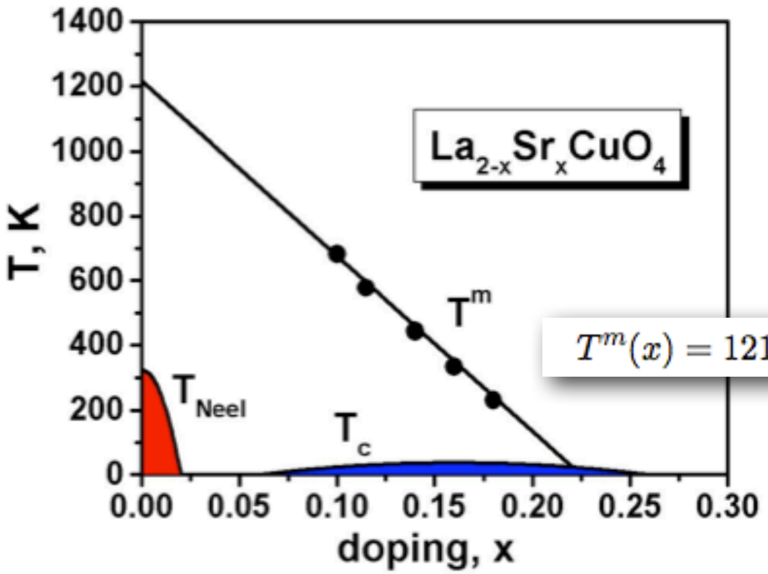


Nakano's formula may be understood from a 2D Heisenberg lattice with a reduced exchange coupling.

Barzykin & Pines, 2009.

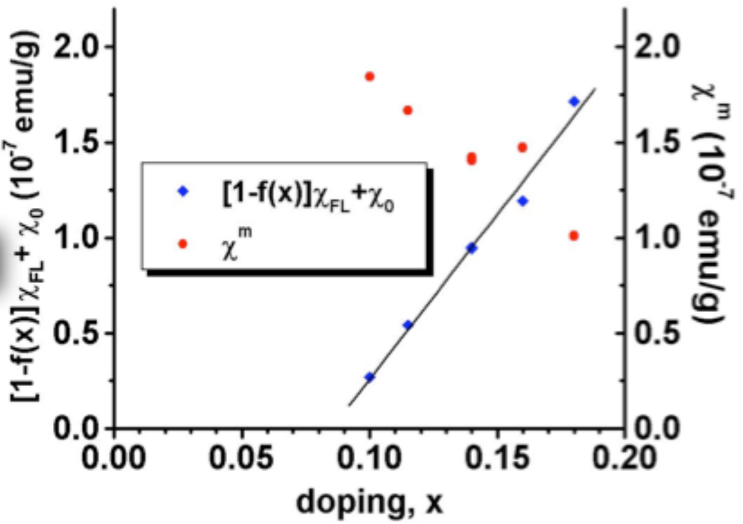
The two-fluid scenario in cuprates

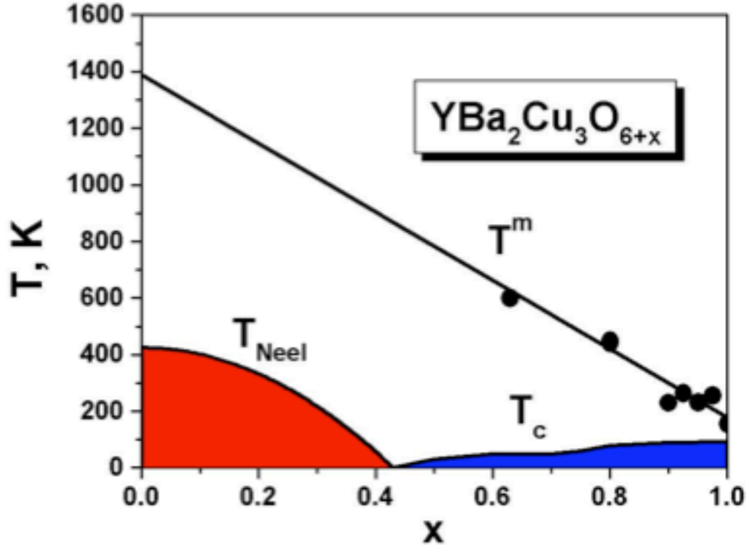
A reduced effective coupling

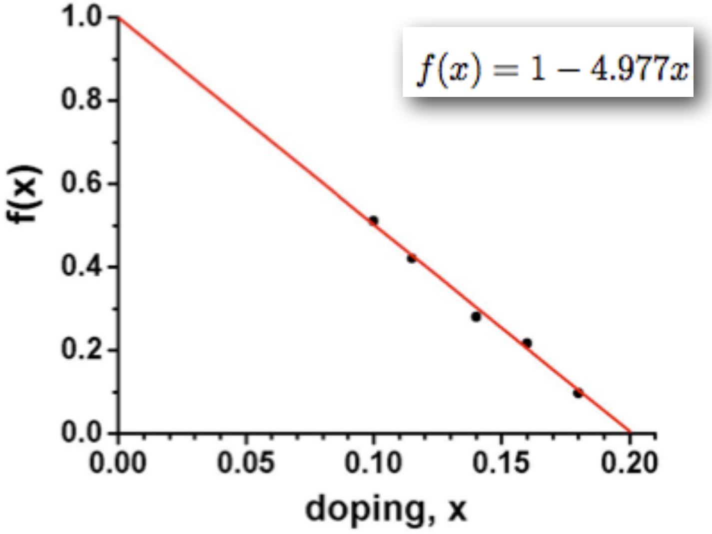


$$T^m(x) = 1218K(1 - 4.45x).$$

$$\chi(T) = f(x)\chi_{SL}(T/T^m(x)) + (1 - f(x))\chi_{FL} + \chi_0$$







$$f(x) = 1 - 4.977x$$

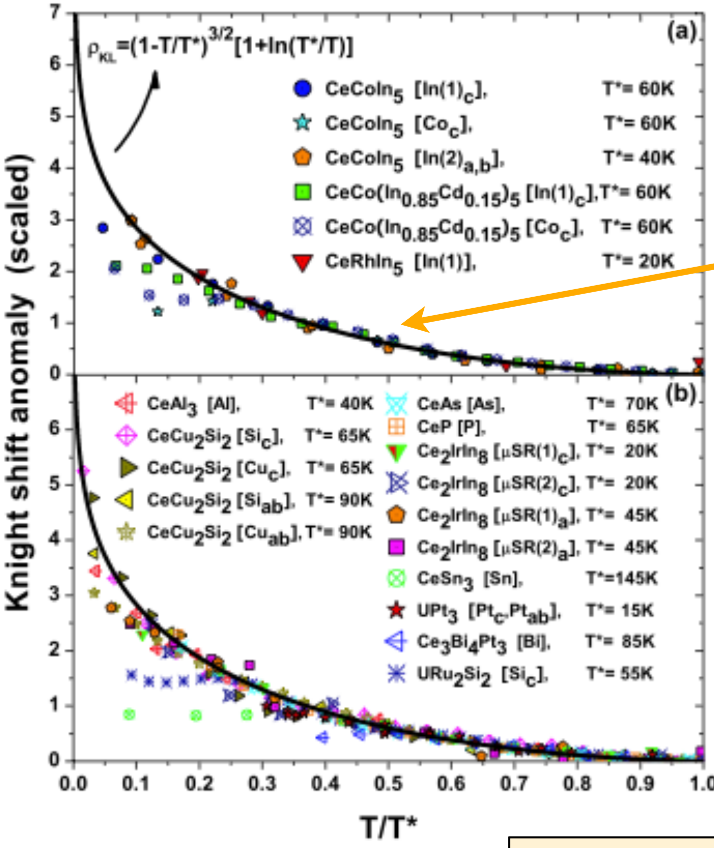
More Knight shift experiments argue against a single fluid picture.
In a two-fluid picture, the second component increases with increasing doping.

Barzykin & Pines, 2009.

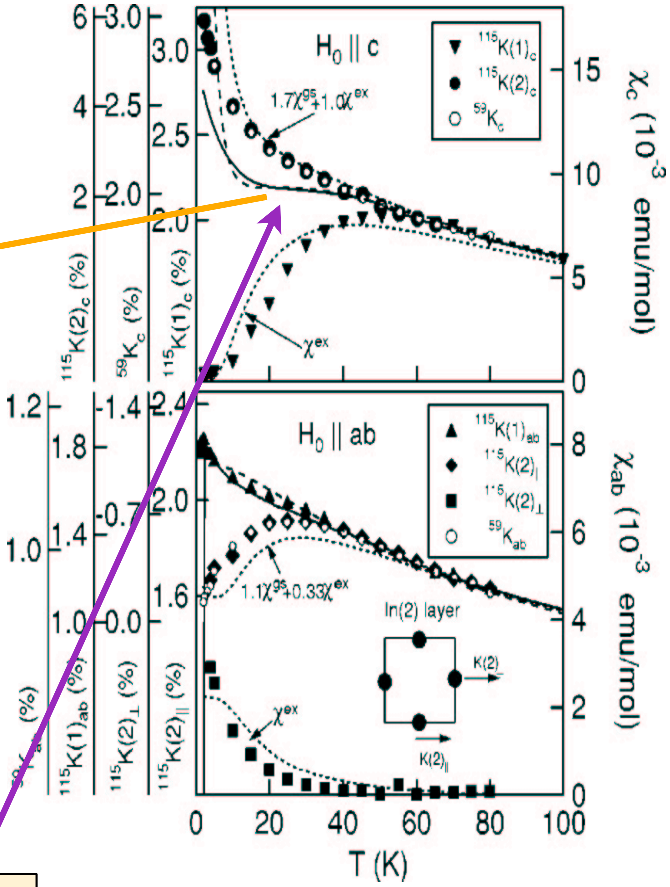
An emergent heavy electron state

The Knight shift anomaly

Knight shift anomaly



CeCoIn5



Curro et al, PRL 90, 227202 (2003)

The Knight shift anomaly is therefore a strong evidence for the universality of an emergent state in heavy electron materials. It shouldn't be ascribed to crystal field effect.

$T > T^* :$ $\chi = \chi_{sl}$

$K = K_0 + A\chi_{sl}$

$T < T^* :$ $\chi = \chi_{sl} + \chi_{kl}$

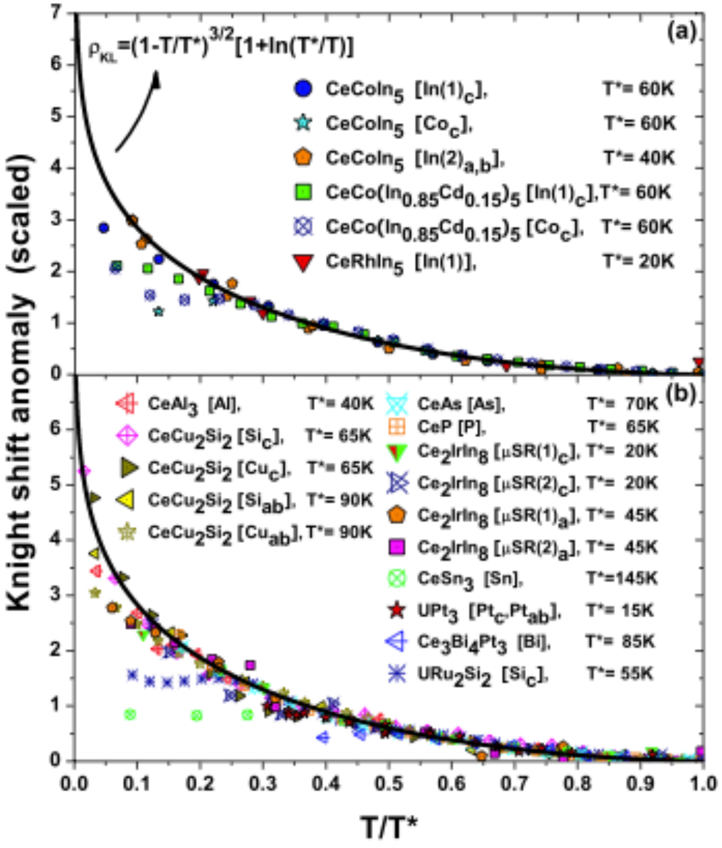
$K = K_0 + A\chi_{sl} + B\chi_{kl}$

$K_a = K - K_0 - A\chi = (B - A)\chi_{kl}$

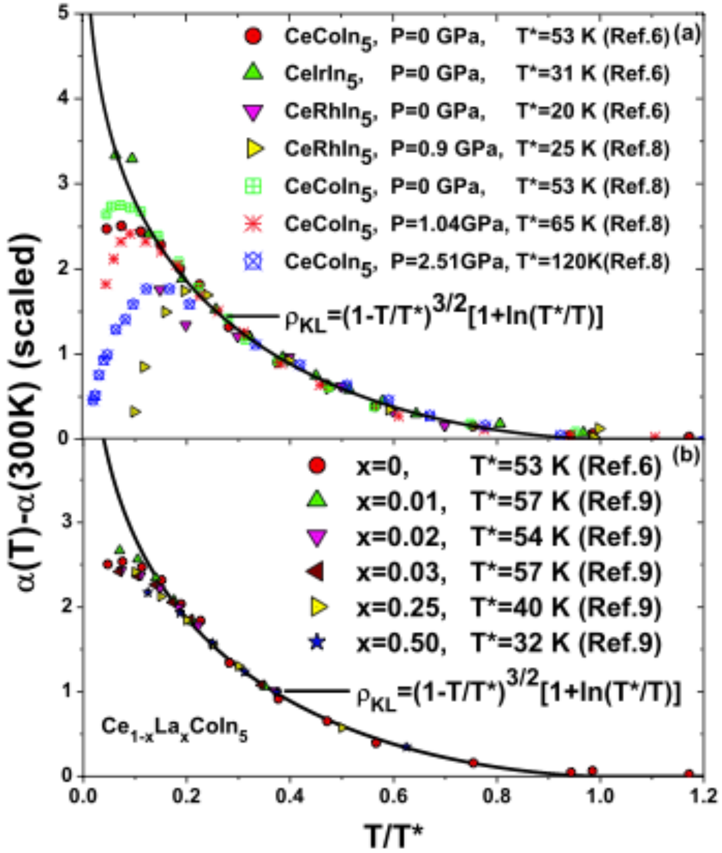
Yang & Pines, PRL 100, 096404 (2008)

More experimental evidences on the emergent state

Knight shift anomaly



Hall coefficient

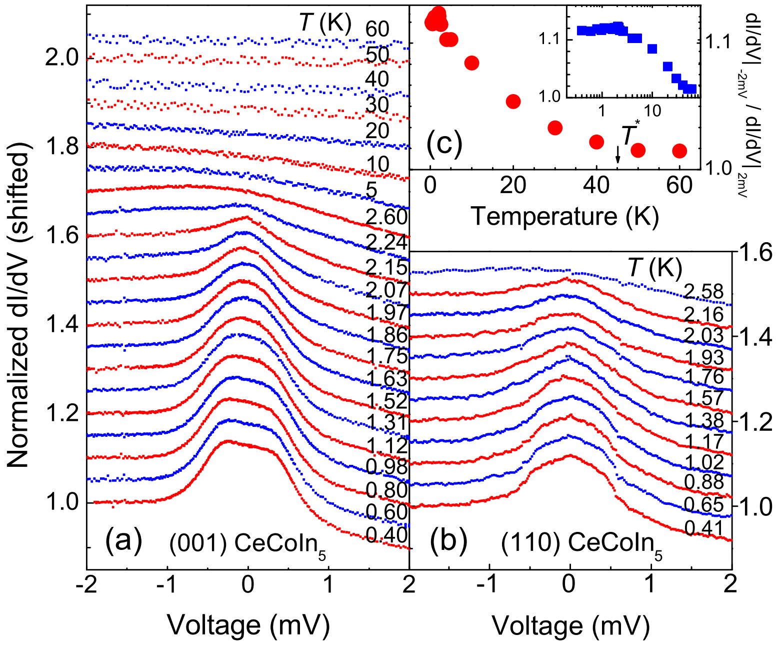


Hundley et al, PRB 70, 035113 (2004)
Nakajima et al, JPSJ 76, 024703 (2007)

Yang & Pines, PRL 100, 096404 (2008)

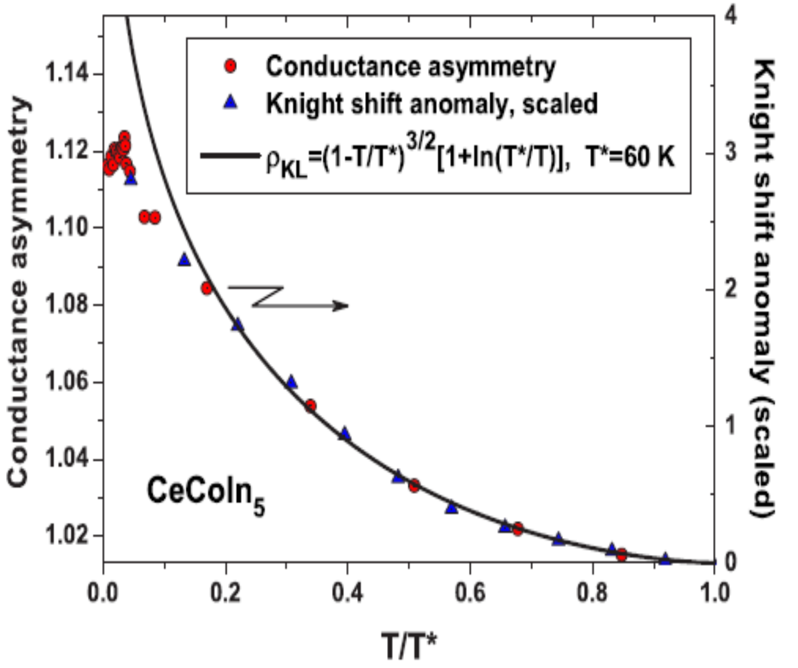
More experimental evidences on the emergent state

Point contact spectroscopy



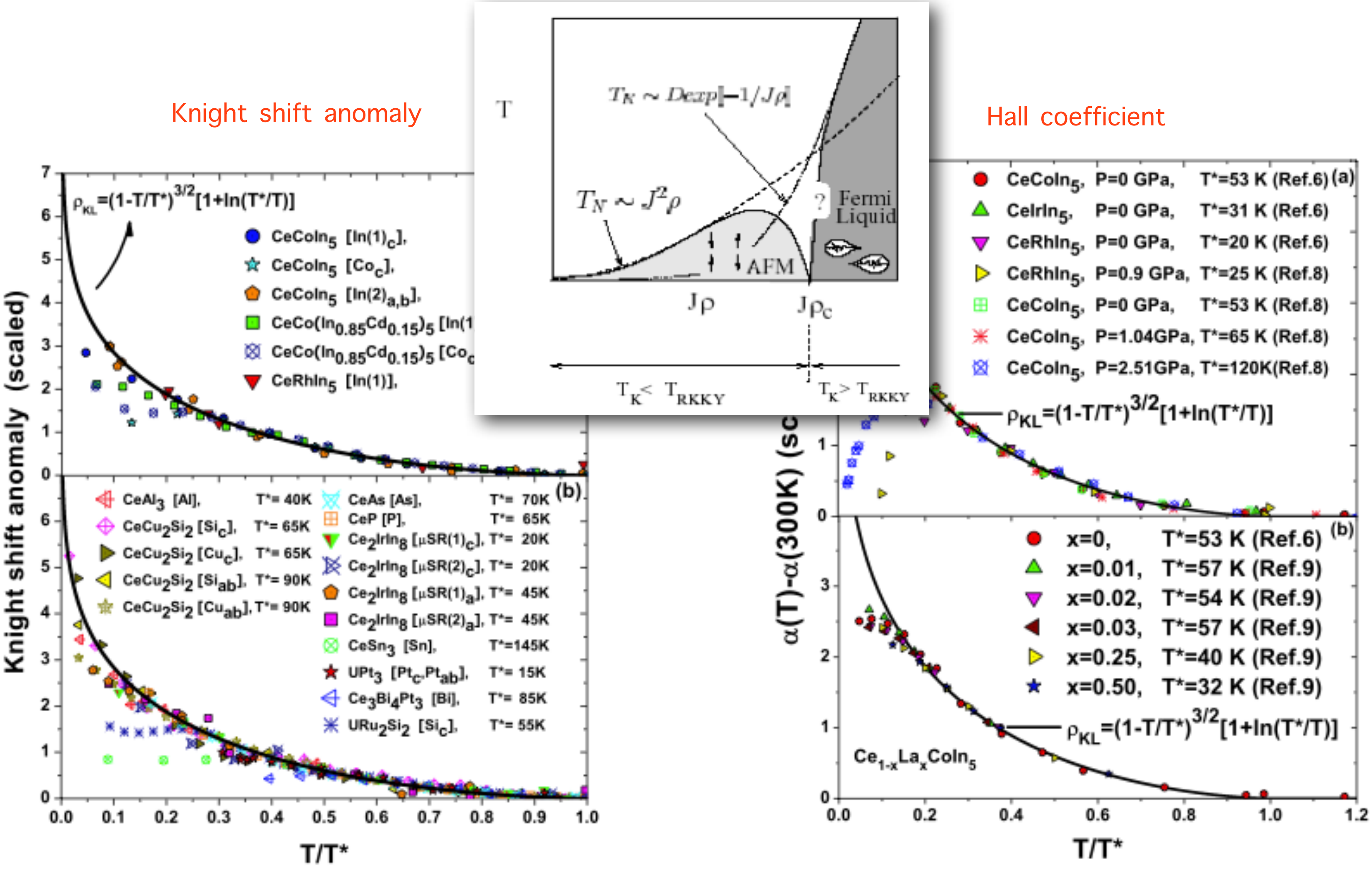
Park et al, PRL 100, 177001 (2008)

A theoretical explanation for the Fano line-shape in
Yang, PRB 79, 241107(R) (2009)



Yang & Pines, PRL 100, 096404 (2008)

Universal scaling vs competing scales



Knight shift anomaly

Hall coefficient

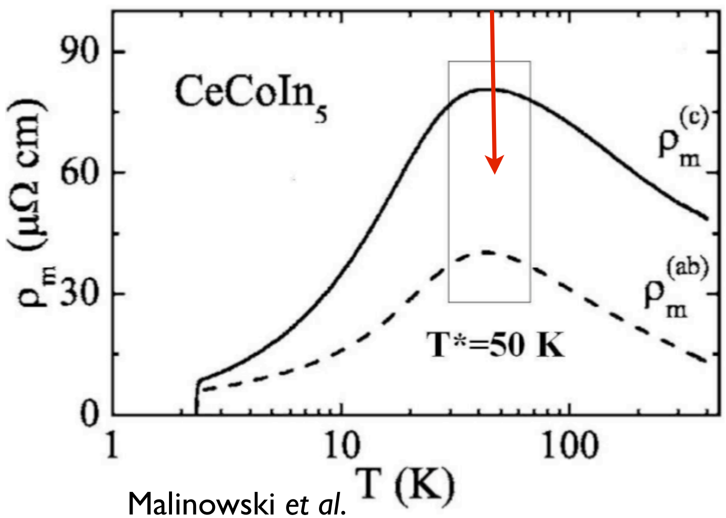
Hundley et al, PRB 70, 035113 (2004)
Nakajima et al, JPSJ 76, 024703 (2007)

Is there any relation between T* and the competing scales?

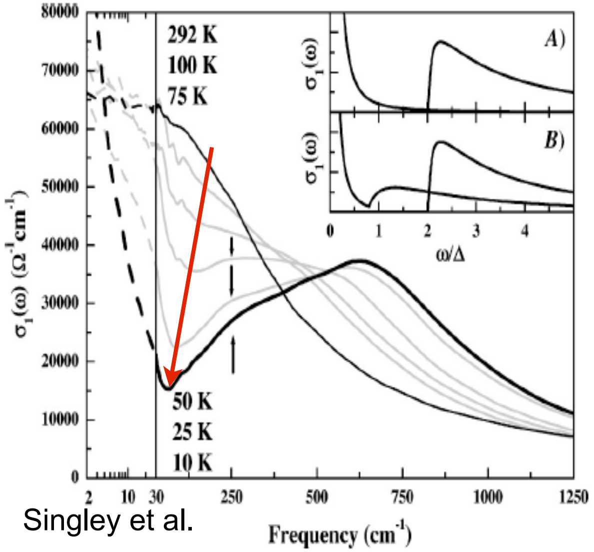
Yang & Pines, PRL 100, 096404 (2008)

A unified temperature scale

Coherence temperature



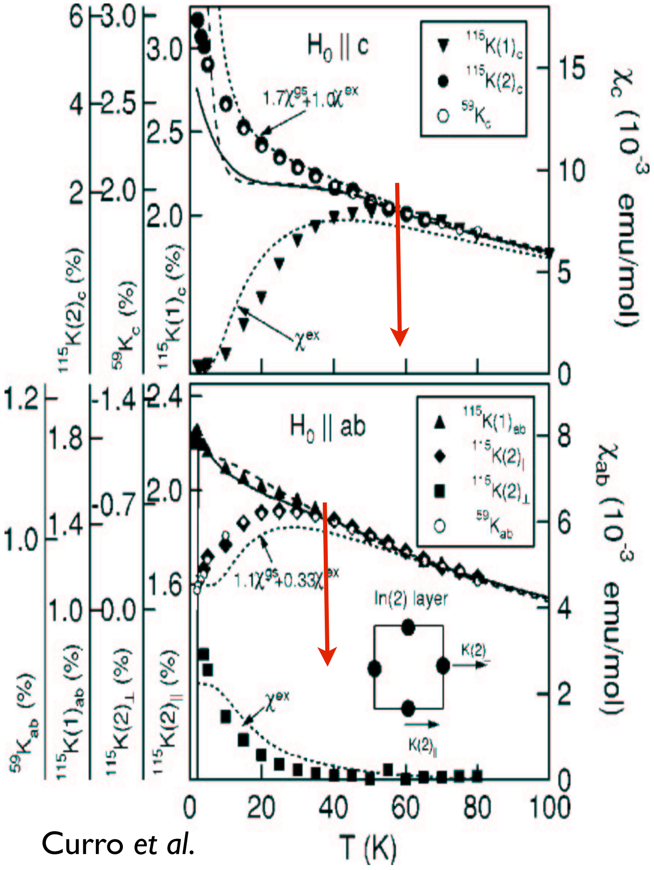
Malinowski *et al.*



Singley et al.

The coherence temperature marks the onset of f-electron band. However, its value was not taken seriously and was often regarded as the Kondo temperature based on the Doniach picture. In many literatures, the coherence temperature also refers to the Fermi liquid temperature.

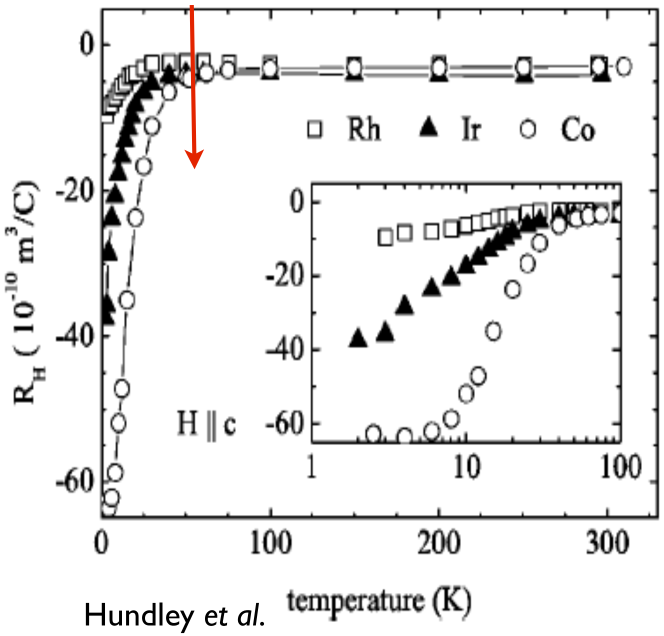
Knight shift and Hall anomalies



Curro *et al.*

Often explained as due to crystal field effect.

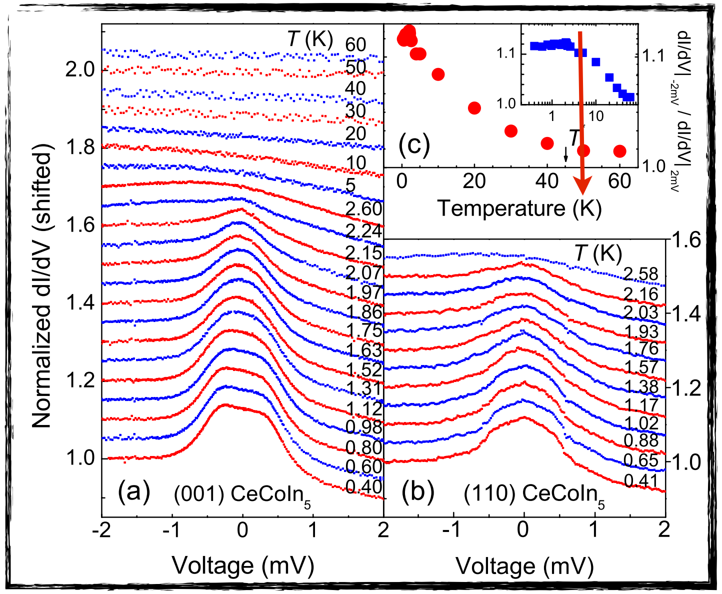
However, the anomaly takes place also at T*.



Hundley *et al.*

Hall measurements point to an emergent component.

Fano line-shape in the point contact spectroscopy



Park *et al.*

Yi-feng Yang, PRB 79, 241107 (2009).

$$H=\sum_{k,m}\left[\epsilon_k c^{\dagger}_{km}c_{km}+\epsilon_0 f^{\dagger}_{km}f_{km}+\tilde{V}(c^{\dagger}_{km}f_{km}+\text{H.c.})\right],$$

$$H_t=\sum_{km}(M_{fkm}f^{\dagger}_{km}t+M_{ckm}c^{\dagger}_{km}t+\text{H.c.}),$$

$$d_{1km}=u_k f_{km}+v_k c_{km},$$

$$d_{2km}=-v_k f_{km}+u_k c_{km},$$

$$|(d_{1km}|H_t|t)|^2=|u_k(f_{km}|H_t|t)+v_k(c_{km}|H_t|t)|^2$$

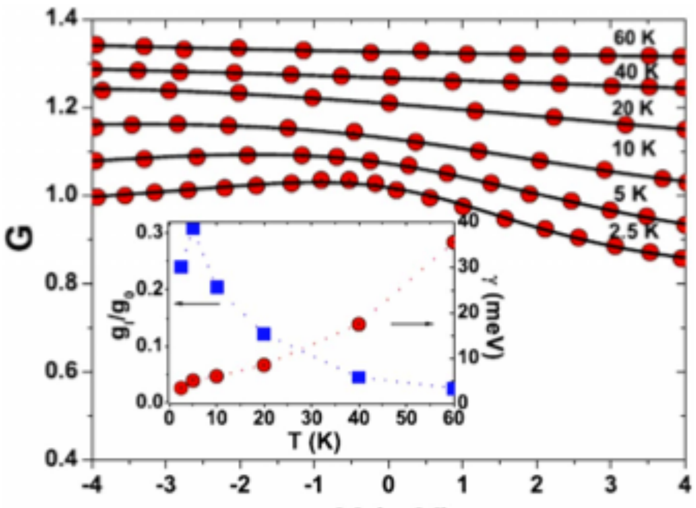
$$=\left|q+\frac{v_k}{u_k}\right|^2|u_k|^2|M_{ckm}|^2=\frac{|q-\tilde{E}_{1k}|^2}{1+\tilde{E}^2_{1k}}|M_{ckm}|^2,$$

$$|(d_{2km}|H_t|t)|^2=|-v_k(f_{km}|H_t|t)+u_k(c_{km}|H_t|t)|^2$$

$$=\left|q-\frac{u_k}{v_k}\right|^2|v_k|^2|M_{ckm}|^2=\frac{|q-\tilde{E}_{2k}|^2}{1+\tilde{E}^2_{2k}}|M_{ckm}|^2,$$

$$G(V,T)=g_0+\int g_I(E)T(E)\frac{df(E-V)}{dV}dE\approx g_0+g_I T(V)$$

$$T(E)=\frac{|q-\tilde{E}|^2}{1+\tilde{E}^2}$$

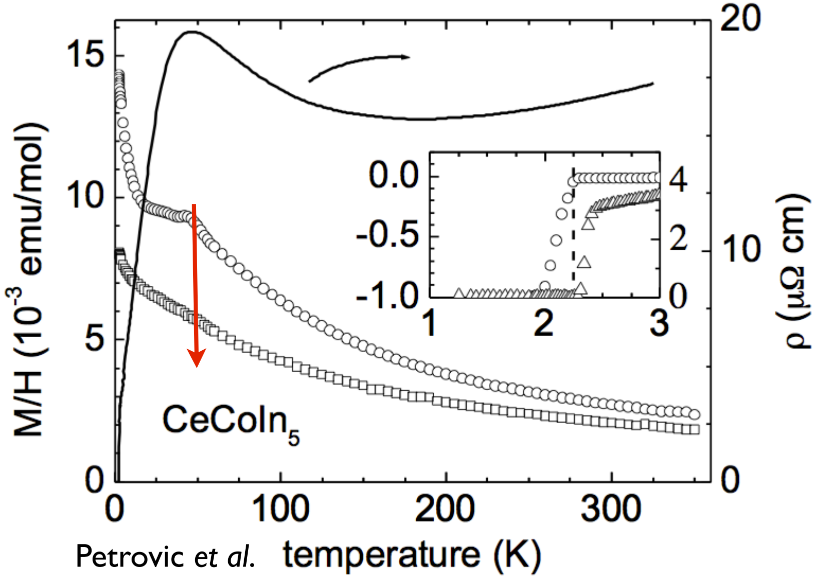


Yang 09

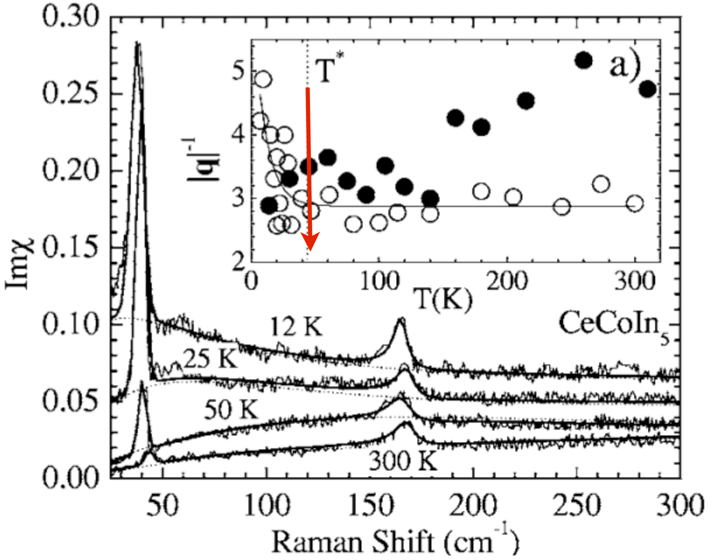
First theoretical explanation of the Fano line-shape in PC/tunneling experiment.
Later also observed in STM/STS measurements.

Yang, PRB 79, 241107(R) (2009)

Susceptibility and Raman shift



Petrovic *et al.*



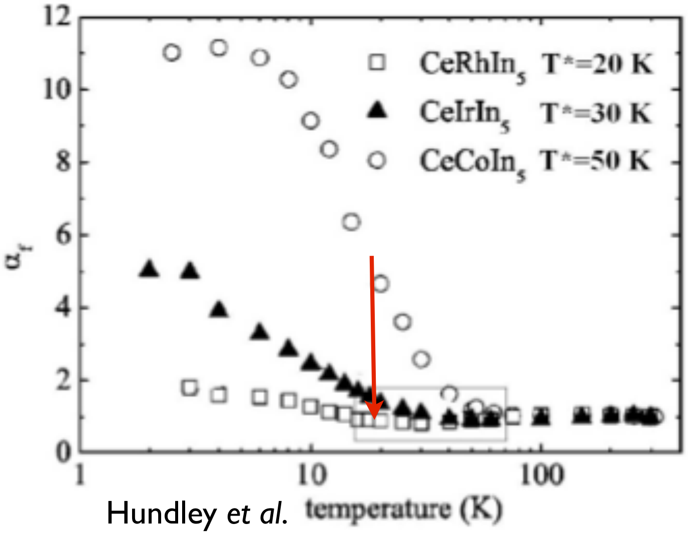
Plateau in the magnetic susceptibility and deviation from Curie-Weiss law

Raman suggests emergent heavy electrons

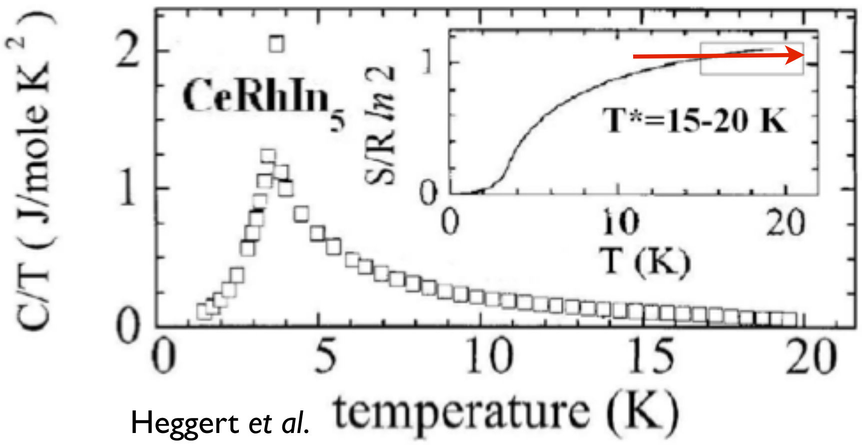
These phenomena were often attributed to different origins.
However, the fact that they all take place at ~T^* suggests a common origin.

This is in contrast to single impurity Kondo physics, where even though we can define a temperature scale T_K, it starts to take effect at very different temperature ranges in different physical quantities.

Entropy quench below T^*



Hundley *et al.*



Heggert *et al.*

Entropy also starts to be quenched at T^*, different from conventional idea of f-electron band formation from local Kondo resonances.

For single impurity, Kondo screening occurs above T_K with $S(T_K)=R\ln 2/2$.

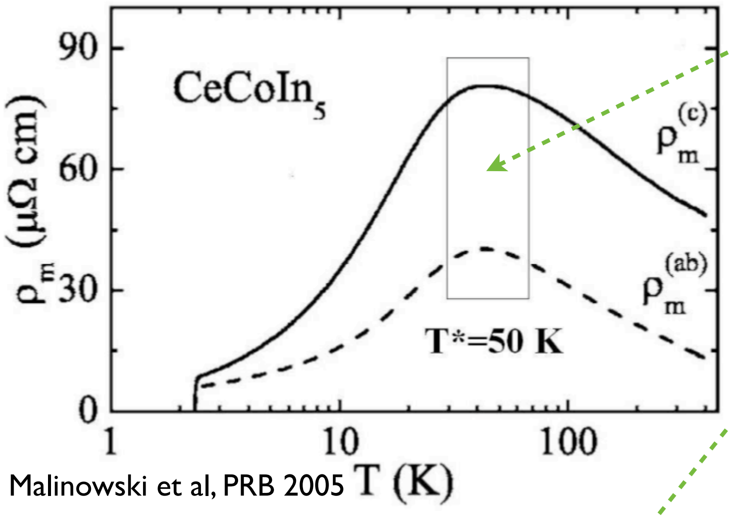
T^* sets the temperature scale for coherence, magnetic correlations and various anomalies.

A unified temperature scale T^*

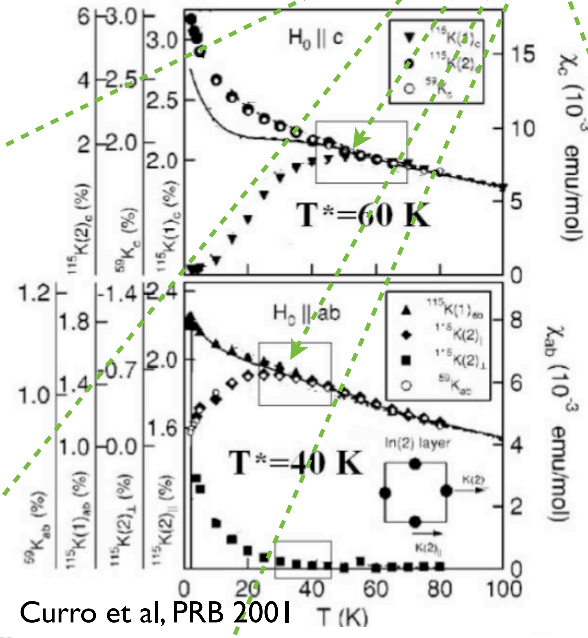
CeCoIn₅

A common $T^* \sim 50$K!

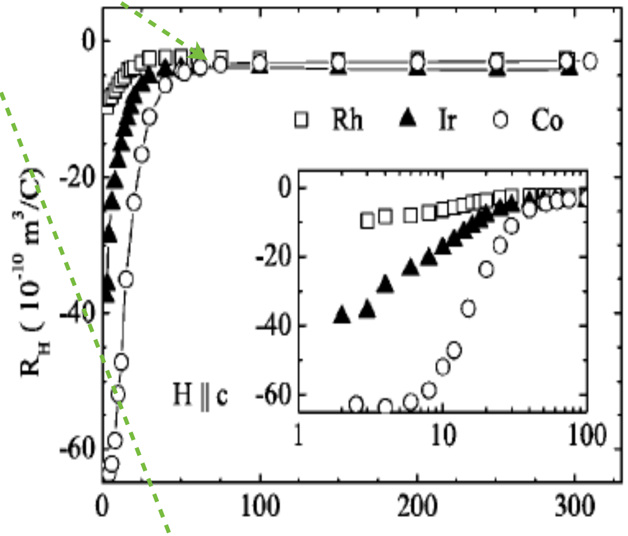
Yi-feng Yang et al, Nature 454, 611 (2008).



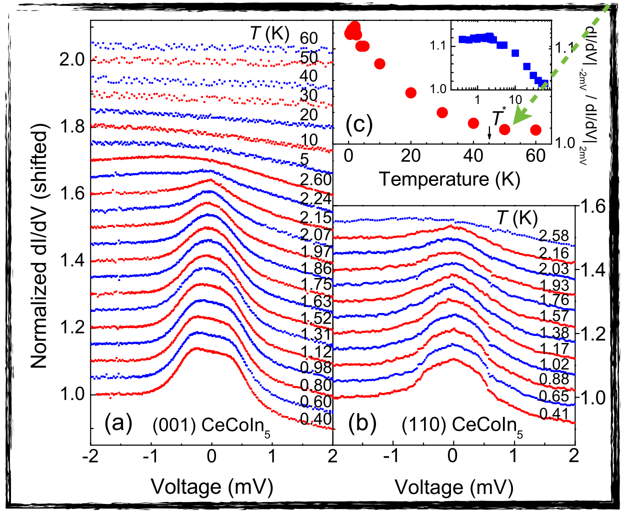
Malinowski et al, PRB 2005



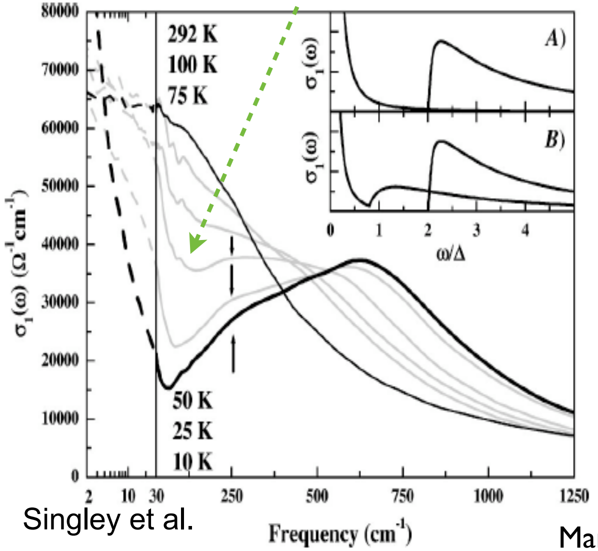
Curro et al, PRB 2001



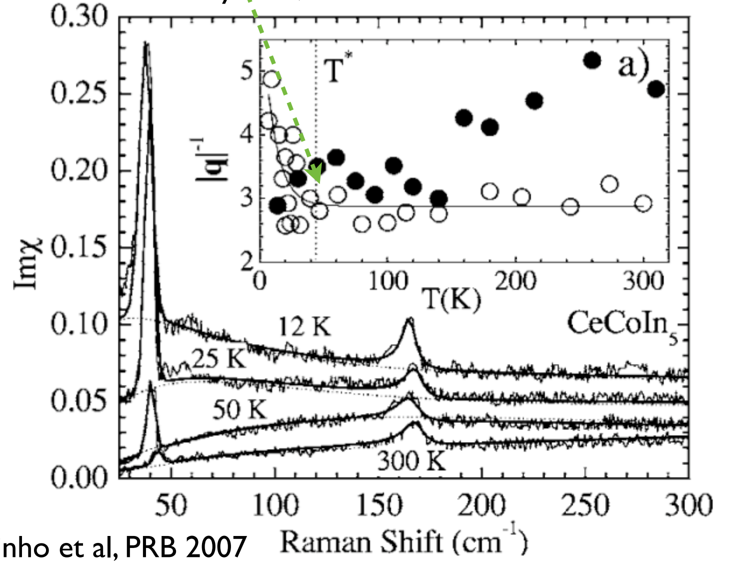
Hundley et al, PRB 2004



Park et al, PRL 2008



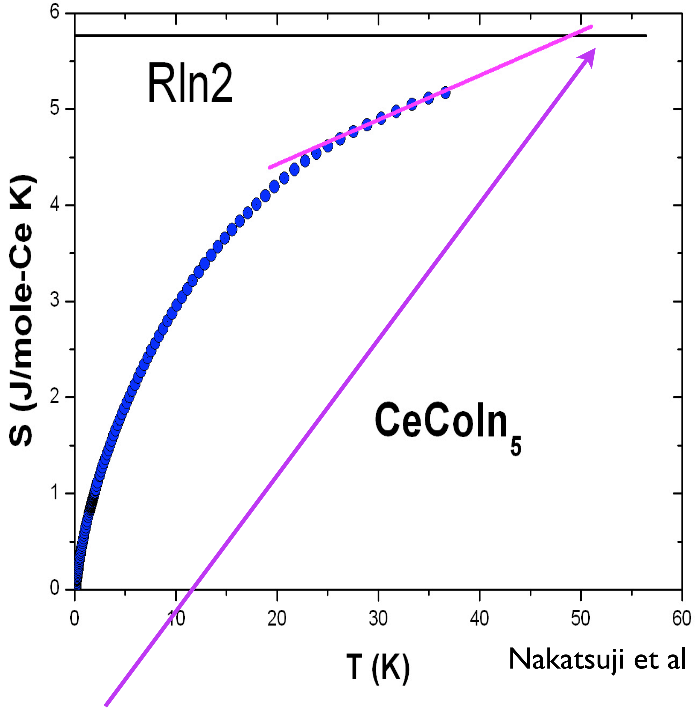
Singley et al.



Martinho et al, PRB 2007

Onset of magnetic correlation at T^*

- Resistivity
- Susceptibility
- Knight shift anomaly
- Hall anomaly
- Optical conductivity
- Magnetic entropy
- Point contact spectroscopy
- Neutron/Raman scattering
- NMR spin-lattice relaxation

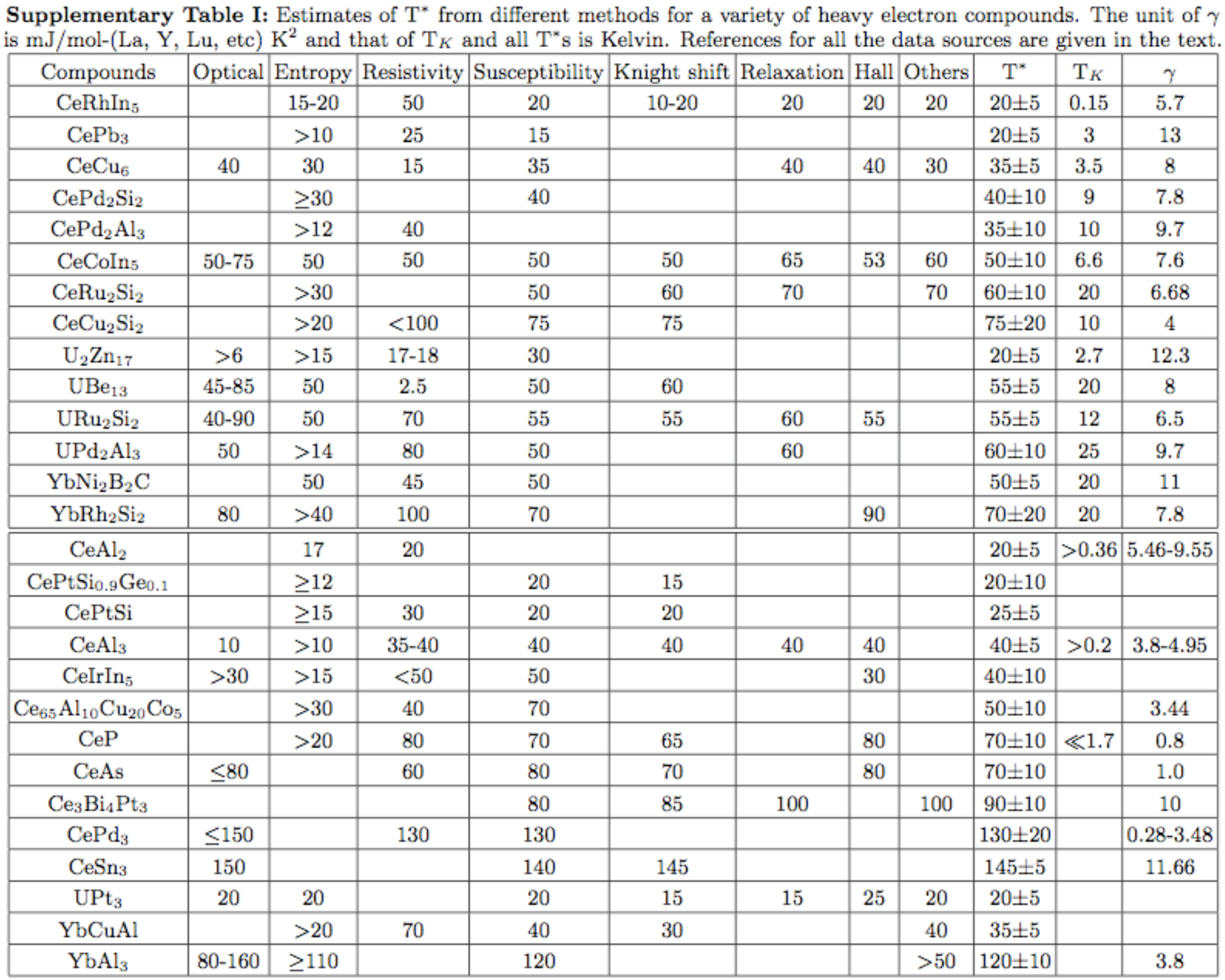


Nakatsuji et al

- T^* cannot be ascribed to the crystal field effect.
- T^* cannot be the Kondo temperature since the entropy is $R\ln2/2$ at T_K.
- At $T=T^*$, the magnetic entropy starts to be quenched. T^* marks the onset of magnetic correlation.
- Another possibility: T^* originates from the spin-correlation between f-ions

Yang et al, Nature 454, 611 (2008).

What is T^*?



Supplementary Table I: Estimates of T^* from different methods for a variety of heavy electron compounds. The unit of γ is mJ/mol-(La, Y, Lu, etc) K^2 and that of T_K and all T^*s is Kelvin. References for all the data sources are given in the text.

Compounds	Optical	Entropy	Resistivity	Susceptibility	Knight shift	Relaxation	Hall	Others	T^*	T_K	γ
$CeRhIn_5$		15-20	50	20	10-20	20	20	20	20±5	0.15	5.7
$CePb_3$		>10	25	15					20±5	3	13
$CeCu_6$	40	30	15	35		40	40	30	35±5	3.5	8
$CePd_2Si_2$		≥30		40					40±10	9	7.8
$CePd_2Al_3$		>12	40						35±10	10	9.7
$CeCoIn_5$	50-75	50	50	50	50	65	53	60	50±10	6.6	7.6
$CeRu_2Si_2$		>30		50	60	70		70	60±10	20	6.68
$CeCu_2Si_2$		>20	<100	75	75				75±20	10	4
U_2Zn_{17}	>6	>15	17-18	30					20±5	2.7	12.3
UBe_{13}	45-85	50	2.5	50	60				55±5	20	8
URu_2Si_2	40-90	50	70	55	55	60	55		55±5	12	6.5
UPd_2Al_3	50	>14	80	50		60			60±10	25	9.7
$YbNi_2B_2C$		50	45	50					50±5	20	11
$YbRh_2Si_2$	80	>40	100	70			90		70±20	20	7.8
$CeAl_2$		17	20						20±5	>0.36	5.46-9.55
$CePtSi_{0.9}Ge_{0.1}$		≥12		20	15				20±10		
$CePtSi$		≥15	30	20	20				25±5		
$CeAl_3$	10	>10	35-40	40	40	40	40		40±5	>0.2	3.8-4.95
$CeIrIn_5$	>30	>15	<50	50			30		40±10		
$Ce_{65}Al_{10}Cu_{20}Co_5$		>30	40	70					50±10		3.44
CeP		>20	80	70	65		80		70±10	≪1.7	0.8
$CeAs$	≤80		60	80	70		80		70±10		1.0
$Ce_3Bi_4Pt_3$				80	85	100		100	90±10		10
$CePd_3$	≤150		130	130					130±20		0.28-3.48
$CeSn_3$	150			140	145				145±5		11.66
UPt_3	20	20		20	15	15	25	20	20±5		
$YbCuAl$		>20	70	40	30			40	35±5		
$YbAl_3$	80-160	≥110		120				>50	120±10		3.8

Yang et al, Nature 454, 611 (2008).

RKKY origin of T^*

Diluted compounds: $T_K \rho = \exp(-1/J\rho)$

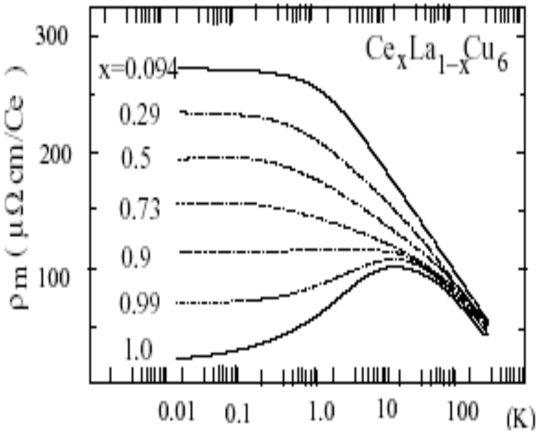


FIGURE 15. Development of coherence in heavy fermion systems. Resistance in $Ce_{1-x}La_xCu_6$ after Onuki and Komatsubara[35]

Table 1 | Experimental T^*, T_K and γ values for a variety of Kondo lattice compounds

Compound	T^* (K)	T_K (K)	γ (mJ mol^{-1} K^2)	$J\rho$	J (meV)	c	Reference
$CeRhIn_5$	20 ± 5	0.15	5.7	0.10	40	0.45	6, 8, H.-O.L.*
$CeCu_6$	35 ± 5	3.5	8	0.15	43	0.49	9, 10
$CeCu_2Si_2$	75 ± 20	10	4	0.15	90	0.47	6, 11, 12
$CePb_3$	20 ± 5	3	13	0.15	28	0.41	13, 14
$CeCoIn_5$	50 ± 10	6.6	7.6	0.16	49	0.55	4, 6, 7
$CePd_2Si_2$	40 ± 10	9	7.8	0.17	51	0.41	15, 16
$CePd_2Al_3$	35 ± 10	10	9.7	0.18	43	0.40	17, 18, 19
$CeRu_2Si_2$	60 ± 10	20	6.68	0.19	66	0.42	20, 21
U_2Zn_{17}	20 ± 5	2.7	12.3	0.15	29	0.41	22, 23
URu_2Si_2	55 ± 5	12	6.5	0.17	62	0.45	6, 24, 25
UBe_{13}	55 ± 5	20	8	0.19	57	0.43	26, 27
UPd_2Al_3	60 ± 10	25	9.7	0.21	51	0.48	19, 28
$YbRh_2Si_2$	70 ± 20	20	7.8	0.19	58	0.53	Z.F.†
$YbNi_2B_2C$	50 ± 5	20	11	0.21	44	0.47	29

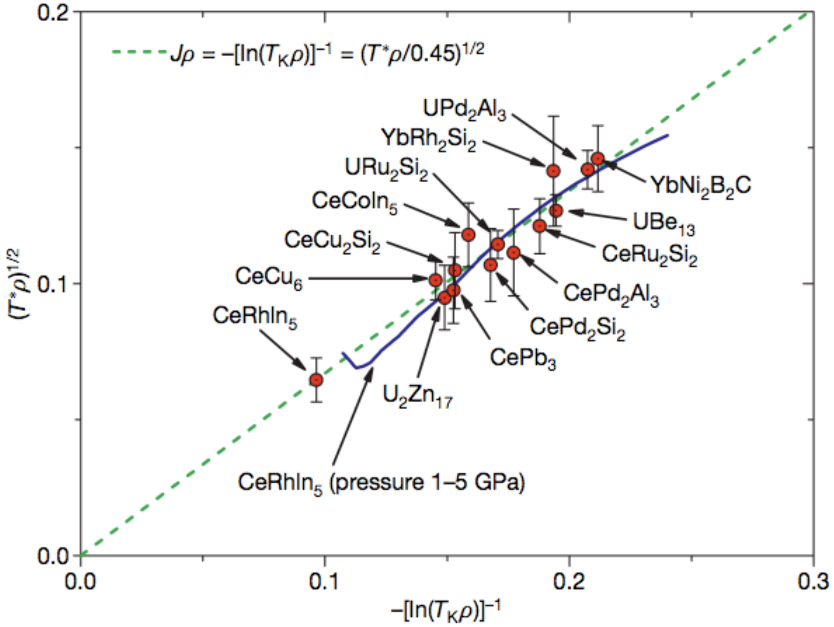


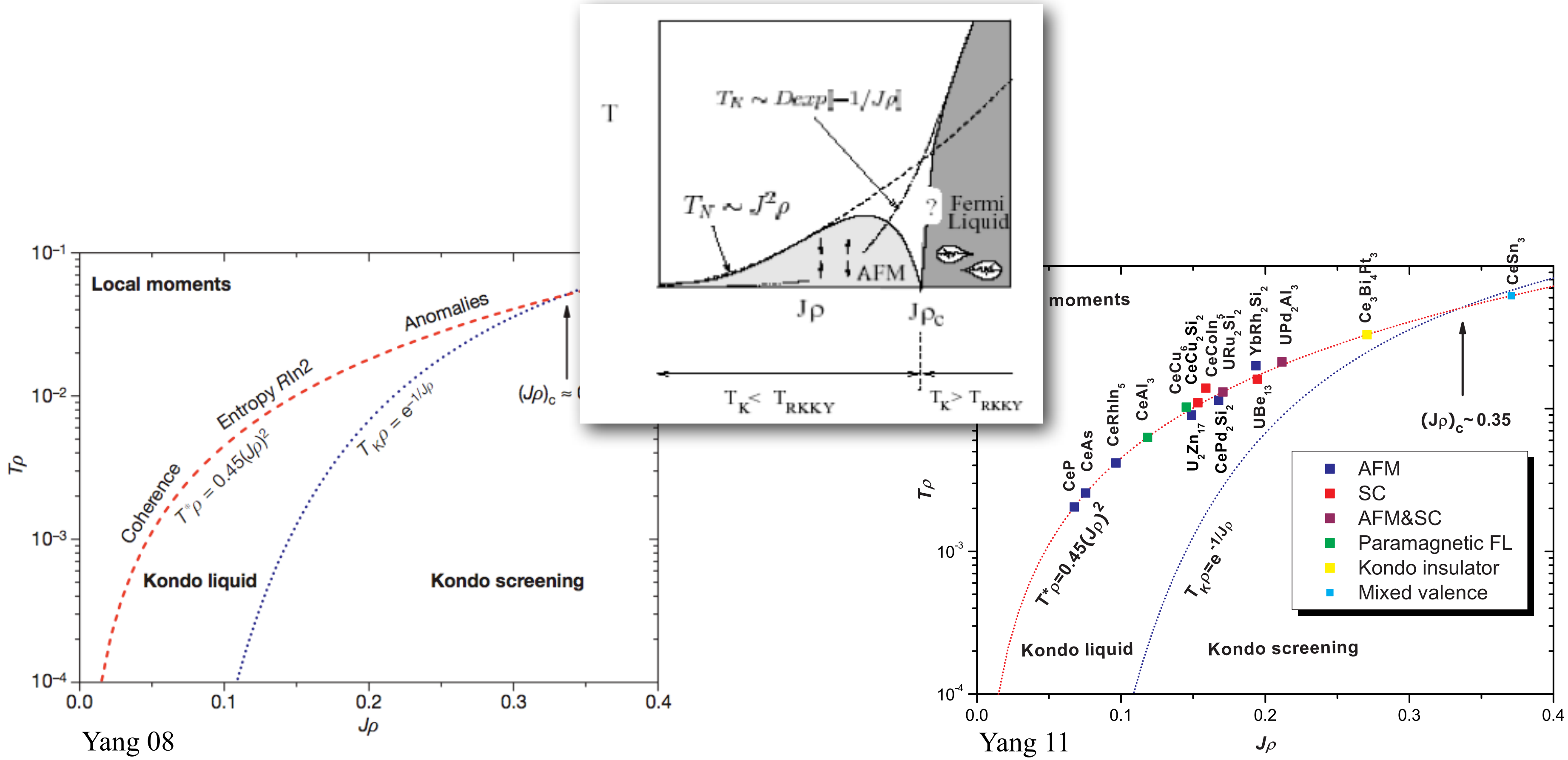
Figure 1 | Confirmation of T^* given by the intersite RKKY interaction for a variety of Kondo lattice materials. The solid line shows T^* (resistivity peak) of $CeRhIn_5$ under pressure from 1 GPa (lower left) to 5 GPa (upper right).

T^* has a form of RKKY coupling for all heavy electron materials with AFM/SC ground state or near QCP.

A possible contradiction with conventional scenario suggesting competition with Kondo screening.

Yang et al, Nature 454, 611 (2008).

A new phase diagram



Yang 08

Yang 11

A temperature scale unifies emergence of coherence, magnetic correlations and all anomalies.

Superconductors cluster around Jρ~0.15, much smaller than the “critical” coupling.

Yang et al, Nature 454, 611 (2008).

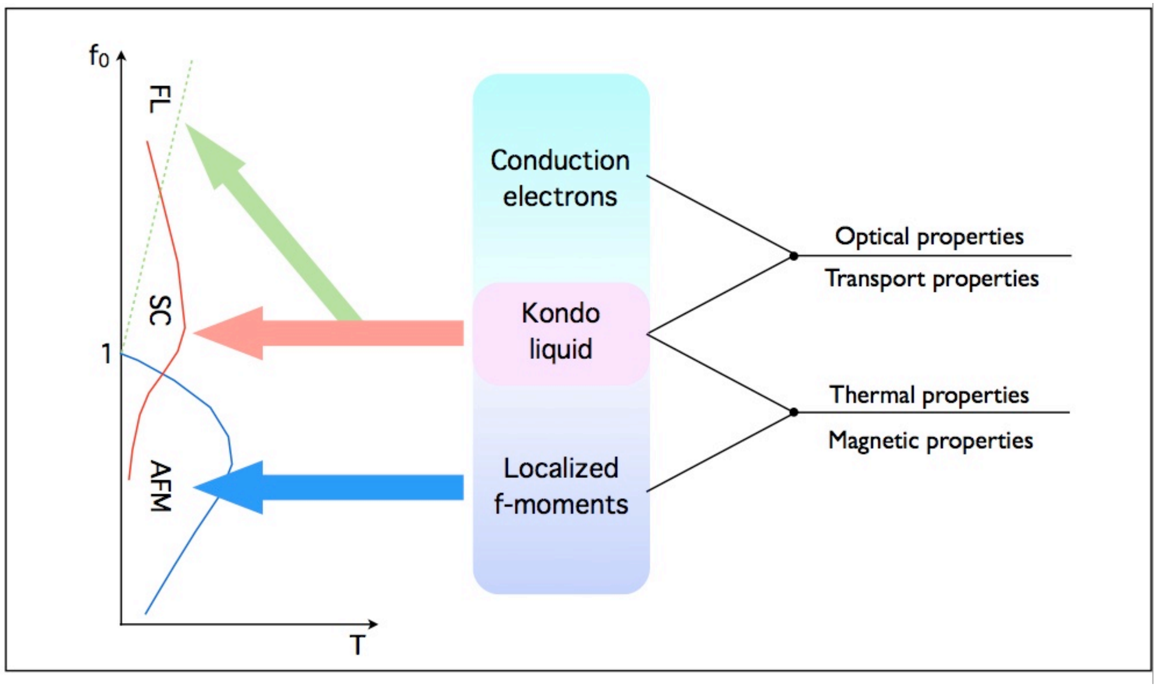
The two-fluid scenario for the ground states

$$f_l(T) = 1 - f_0\left(1 - \frac{T}{T^*}\right)^{1.5}$$

A hybridization effectiveness that varies with temperature and pressure determines the properties of both normal and ground states

$f_0>1$, no f-moments

$f_0<1$, f-moments exist



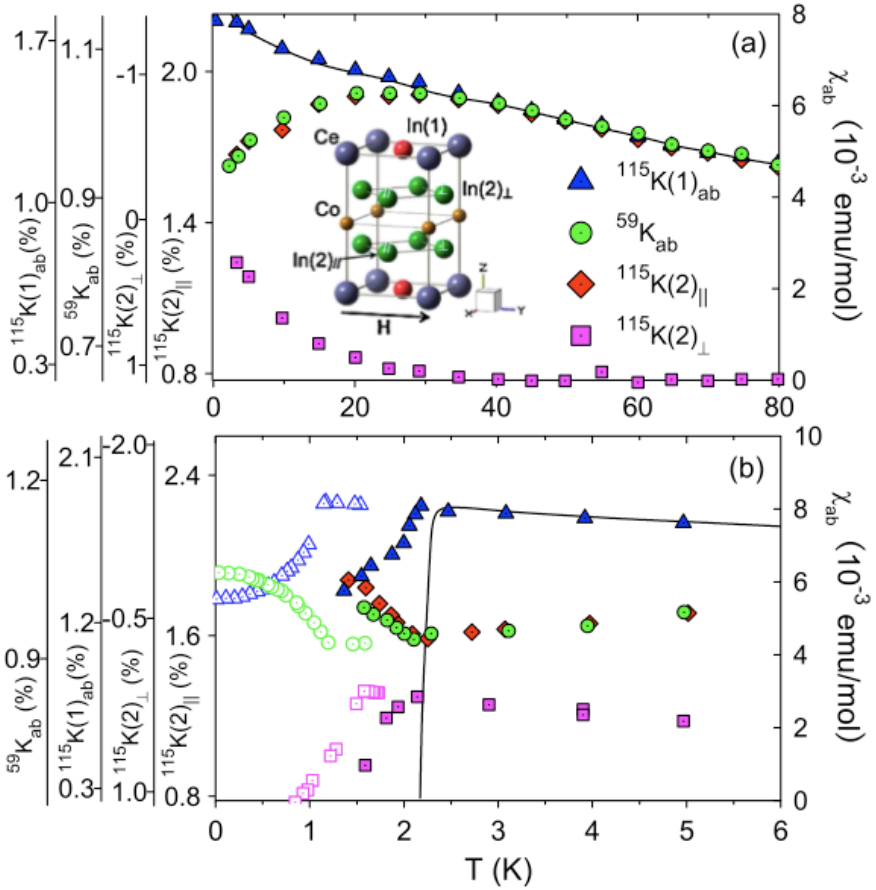
This illustrates the whole idea of the two fluid model. Each physical property is determined by a background contribution from the localized f-moments and a universal contribution from the Kondo liquid.

The two components are also responsible for the low temperature emergent ordered states.

- Antiferromagnetic ordering
- Superconducting condensation

Down to lower temperatures

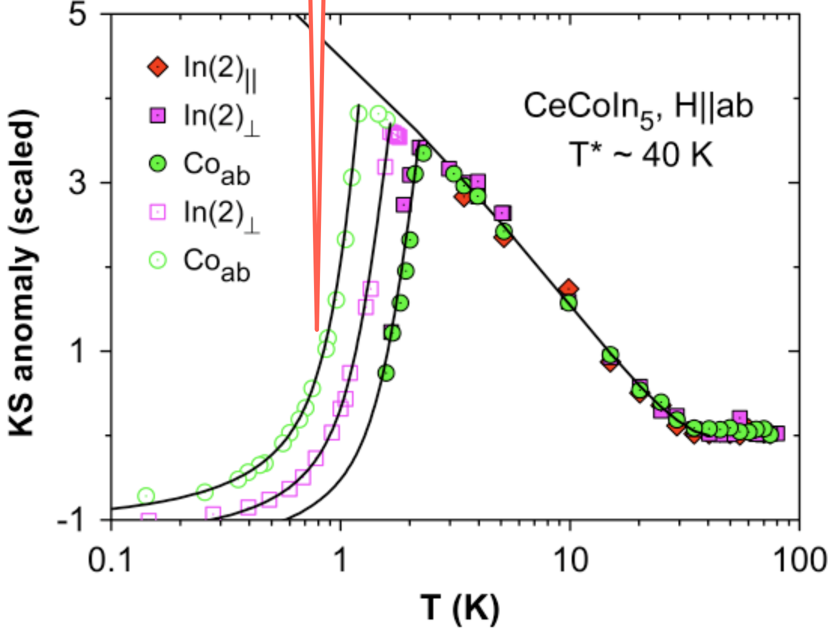
Knight shift anomaly



$$K_{\rm anom}(T) - K_{\rm anom}(0) \propto \int dE\left(-\frac{\partial f(E)}{\partial E}\right)N(E),$$

$$N(E) \propto \langle |E|/\sqrt{E^2-\Delta_k(T)^2}\rangle_{\rm FS}$$

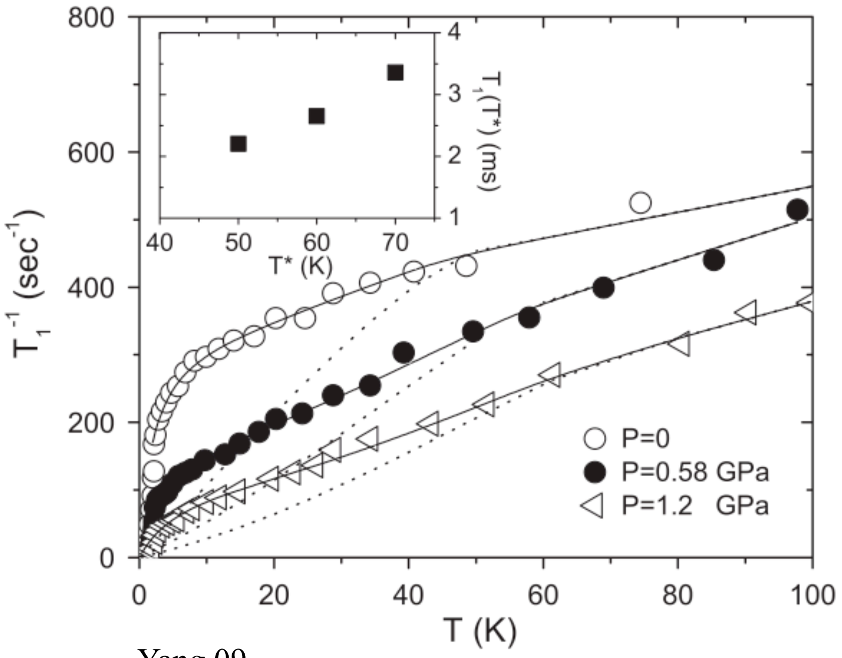
$$\Delta_k(T) = g_k\Delta(0)\tanh\left[\sqrt{\left|\frac{\partial\Delta^2}{\partial T}\right|_{T_c}\frac{T_c}{\Delta(0)^2}\left(\frac{T_c}{T}-1\right)}\right],$$



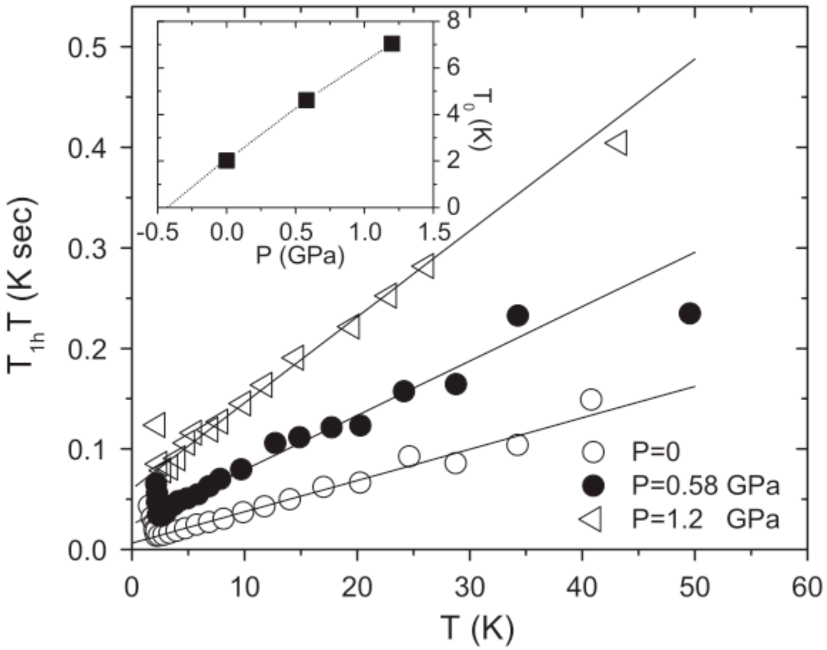
Kondo liquid is responsible for superconductivity.

Yang et al, PRL 103, 197004 (2009)

Superconductivity



Yang 09

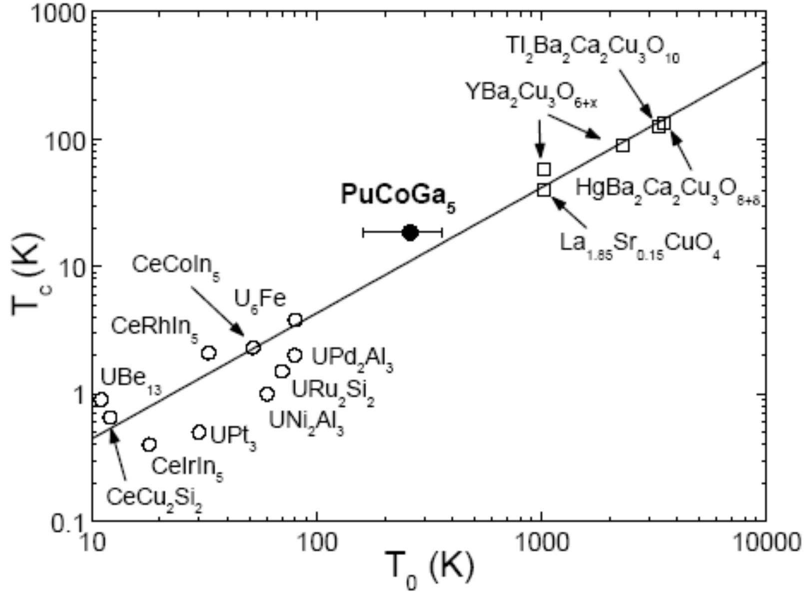


$$\boxed{\frac{1}{T_1} = \frac{1-f(T)}{T_{1l}} + \frac{f(T)}{T_{1h}}}$$

$$\boxed{T_{1h}T \propto (T + T_0)}$$

Kondo liquid exhibits critical fluctuations.

What determines T_c?



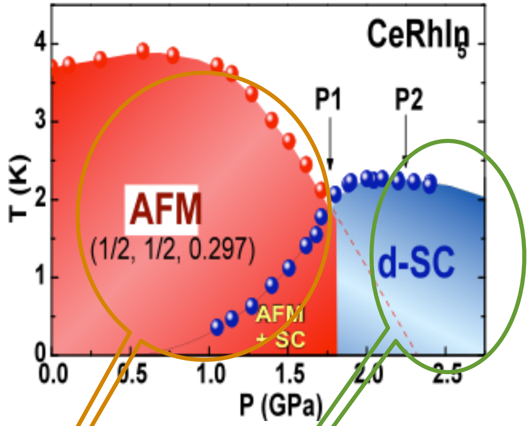
Supplementary Figure 1. The superconducting transition temperature, T_c, versus the characteristic spin fluctuation temperature, T_0. Data are shown for the heavy-fermion compounds (open circles), high- T_c cuprates (open squares), and $PuCoGa_5$ (solid circle). The line is a guide to the eye with $T_c \sim T_0$. Such a proportionality over three orders of magnitude implies that a single energy scale governs both the superconducting transition temperature and the spin-lattice relaxation in the normal state, leading to the scaling relation of $1/T_1$ shown in Figure 3(b). The data are taken from Refs.[18,28], except that for $CeMIn_5$ (M=Co, Rh, Ir)[29] and $PuCoGa_5$[17].

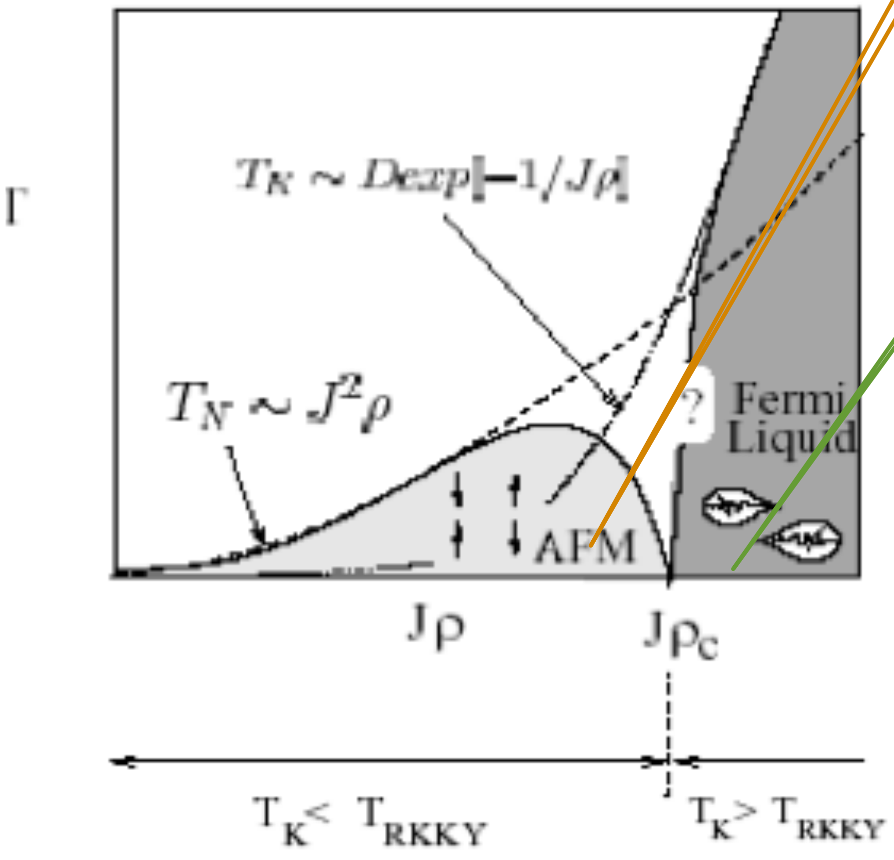
- Antiferromagnetic ordering
- Superconducting condensation

Start from the magnetic regime

$$H_{KLM} = \sum_{k\sigma} c^\dagger_{k\sigma} c_{k\sigma} + J \sum_i \mathbf{S}_i \cdot \mathbf{s}_i + J_{RKKY} \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

No good solution to this model with nonlocal RKKY correlations.



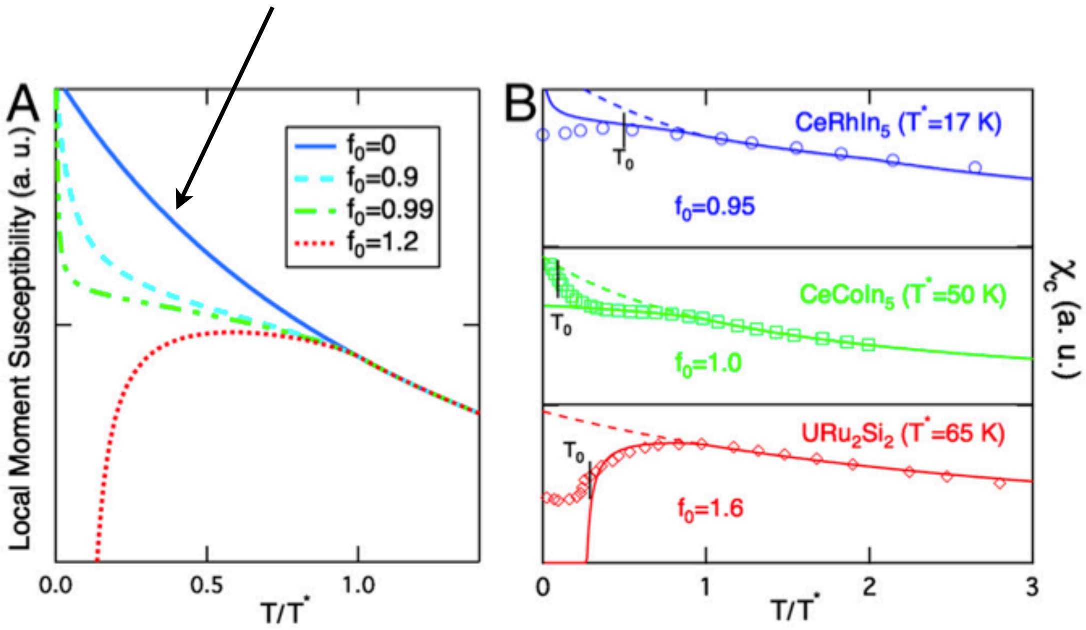


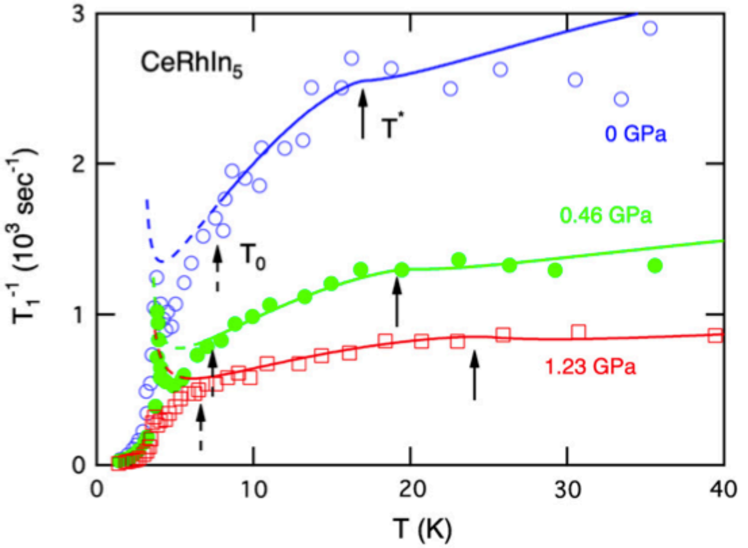
1. Is the Fermi liquid really described by Kondo singlets?
2. What is the role of the RKKY interaction?
3. How can we determine these scales experimentally?

We may also start from the magnetic regime where the system can be well described by a spin lattice and see what may happen if we introduce the hybridization.

$$J \to \tilde{J} = J f_l(T)$$

A hybridized spin liquid


Curie-Weiss law



$$\chi_l(q,\omega) = \frac{f_l\chi_0}{1 - zJ_q f_l\chi_0 - i\omega/\gamma_l}$$

$$f_h(T) = f_0\left(1-\frac{T}{T^*}\right)^{3/2}$$

$$\frac{1}{T_1} = \gamma^2 T\lim_{\omega\to 0}\sum_q F(q)^2\,\frac{\mathrm{Im}\chi_l(q,\omega)}{\omega}$$

We may also start from the magnetic regime where the system can be well described by a spin lattice and see what may happen if we introduce the hybridization.

$$J \to \tilde{J} = J f_l(T)$$

Yang and Pines, PNAS 109, 18241 (2012)

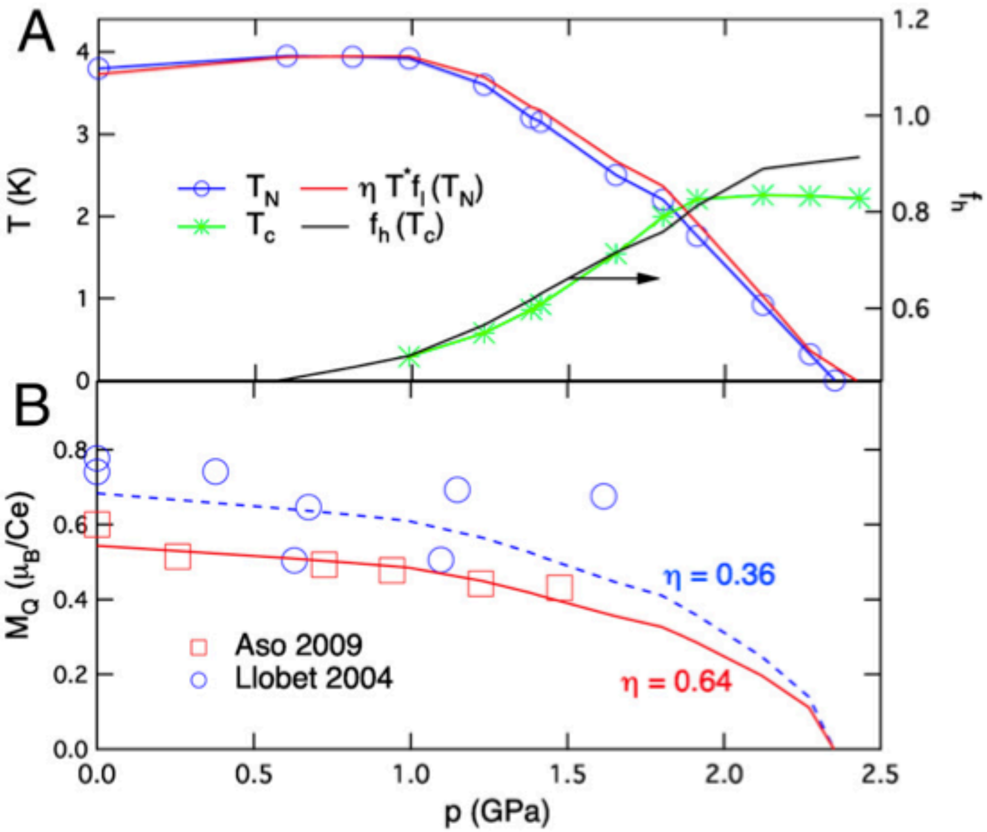
Antiferromagnetism: suppression of T_N

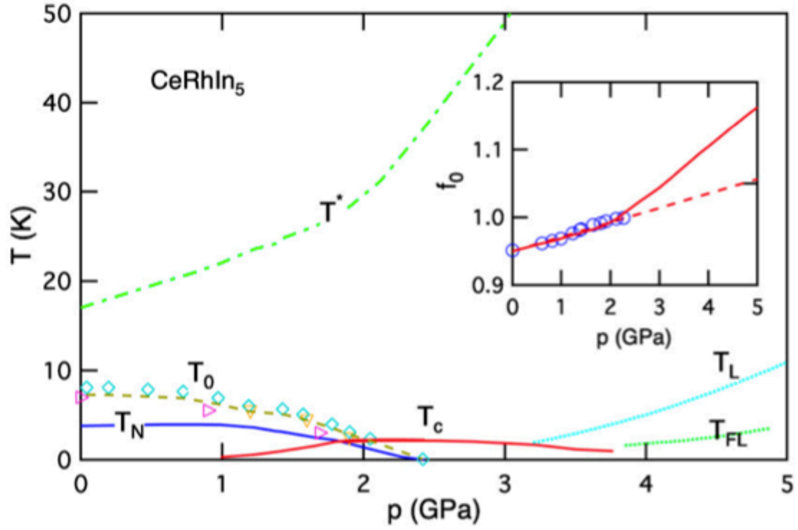
$$\chi_l(q,\omega) = \frac{f_l \chi_0}{1 - zJ_q f_l \chi_0 - i\omega/\gamma_l}$$

$$zJ_Q f_l(T_N)\chi_0(T_N) = 1.$$

$$\frac{T_N}{T^*} = \eta f_l(T_N),$$

$$\frac{\mu^2}{\mu_0^2} = f_l(T_N)$$





Yang and Pines, PNAS 109, 18241 (2012)

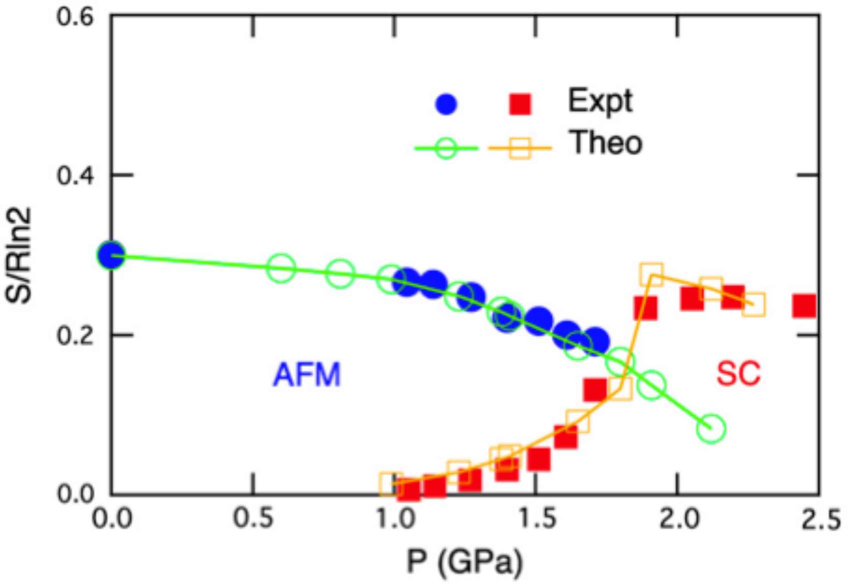
Thermodynamics: entropy and specific heat

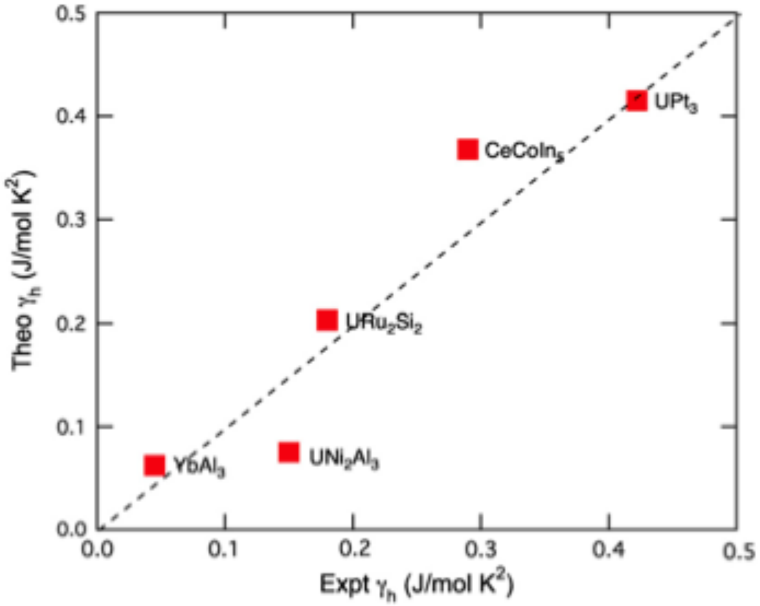
$$S(T) = R\ln 2\left[f_l(T) + f_h(T)\frac{T}{2T^*}\left(2 + \ln\frac{T^*}{T}\right)\right].$$

$$S(T_c) = r_N f_l(T_N)[1 - a(T_c)]R\ln 2 + r_N f_h(T_N) S_h(T_N)\frac{T_c}{T_N}$$

$$S_h(T_x) = R\ln 2\,\frac{T_x}{2T^*}\left[2 + \ln\left(\frac{T^*}{T_x}\right)\right].$$

$$\gamma_h \sim \frac{S_h(T_L)}{T_L} = \frac{R\ln 2}{2T^*}\left[2 - \ln\left(1 - f_0^{-2/3}\right)\right].$$





Yang and Pines, PNAS 109, 18241 (2012)

A new framework

transfer of spectral weight from localized f-moments to itinerant f-electrons due to collective hybridization

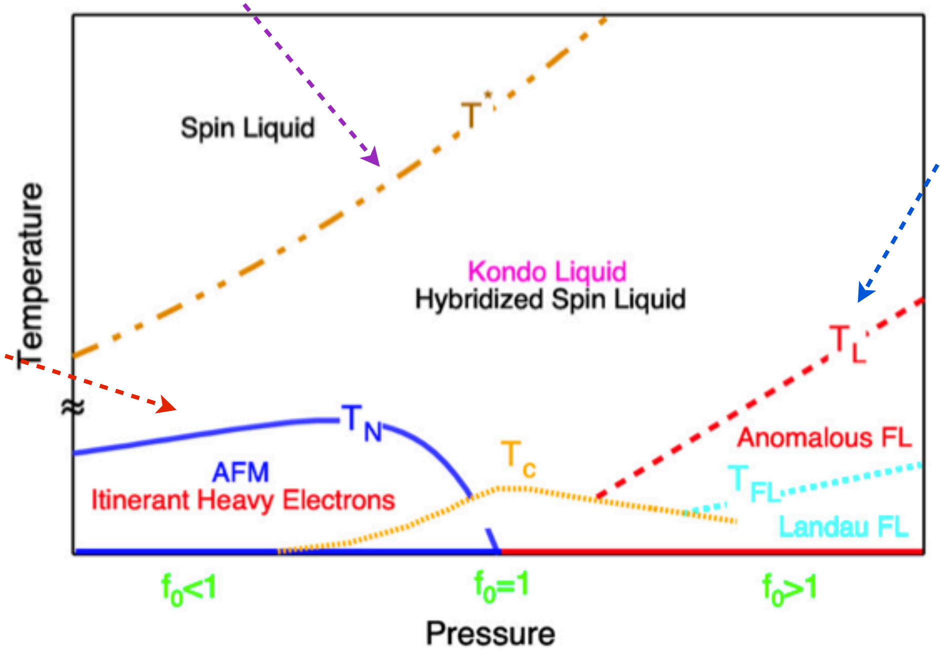
$$\chi_l(\mathbf{q},\omega)=\frac{f_l\chi_l}{1-J_{\mathbf{q}}f_l\chi_l-i\omega/\gamma_l}$$

Coherence, magnetic correlations, anomalies

$$\boxed{J_{\mathbf{Q}}f_l(T_N)\chi_l(T_N)=1}$$

$$\boxed{1-\frac{T_N}{\eta T^*}=f_0\left(1-\frac{T_N}{T^*}\right)^{3/2}}$$

$$\boxed{f_0\left(1-\frac{T_L}{T_*}\right)^{1.5}=1}$$

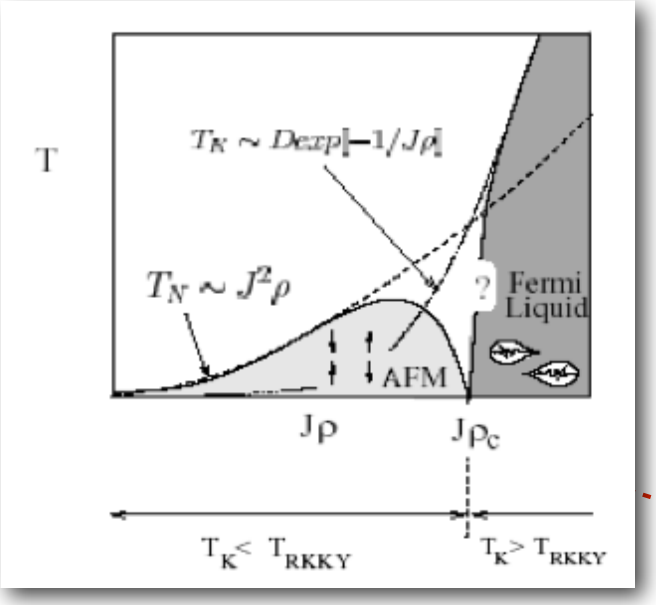


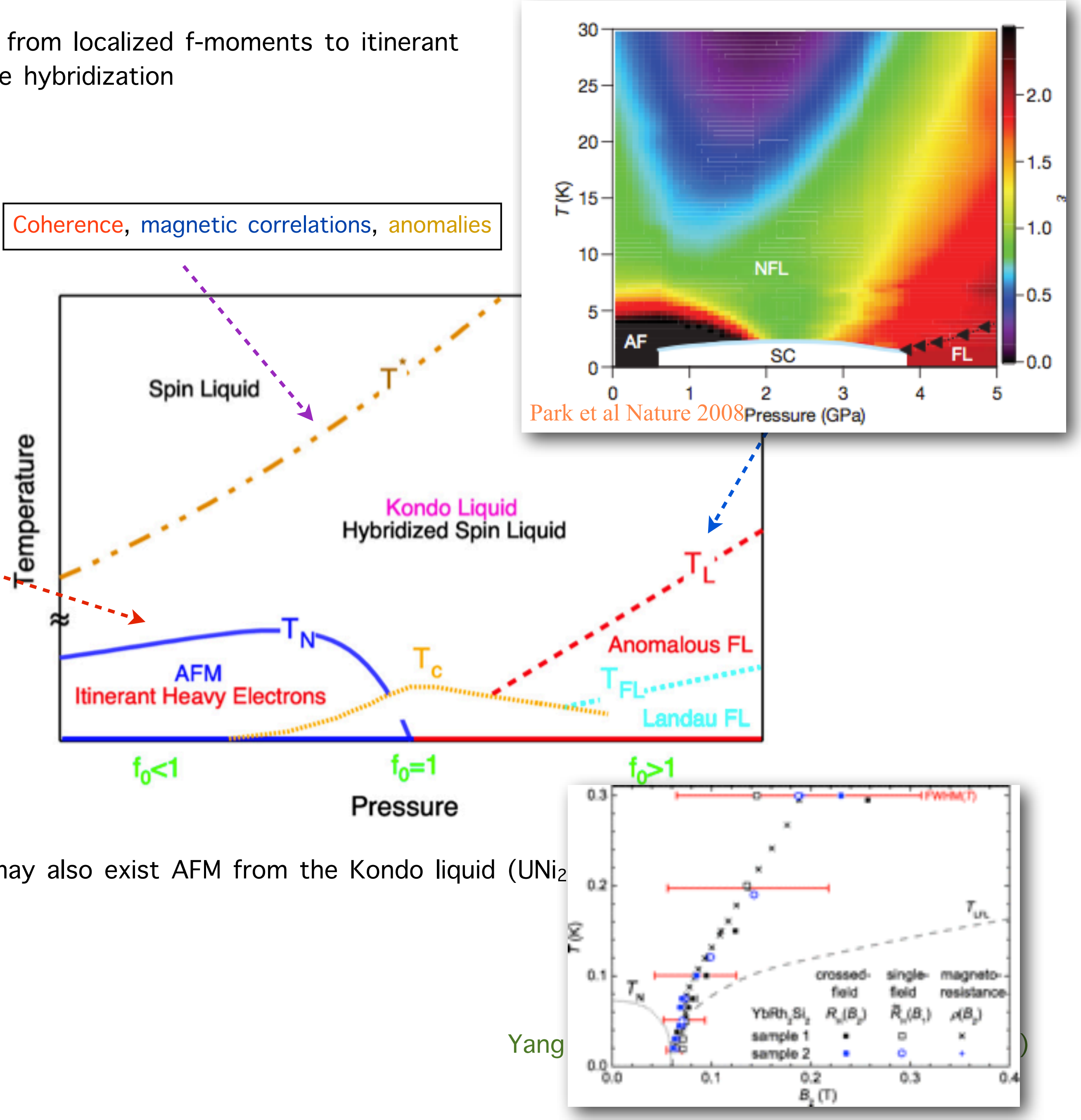
There may also exist AFM from the Kondo liquid (UNi_2Al_3 compared to UPd_2Al_3)

Yang and Pines, PNAS 109, 18241 (2012)

A new framework

transfer of spectral weight from localized f-moments to itinerant f-electrons due to collective hybridization





Coherence, magnetic correlations, anomalies

Park et al Nature 2008

There may also exist AFM from the Kondo liquid (UNi$_2$

Yang)

The anomalous Hall effect

Transport: the anomalous Hall effect

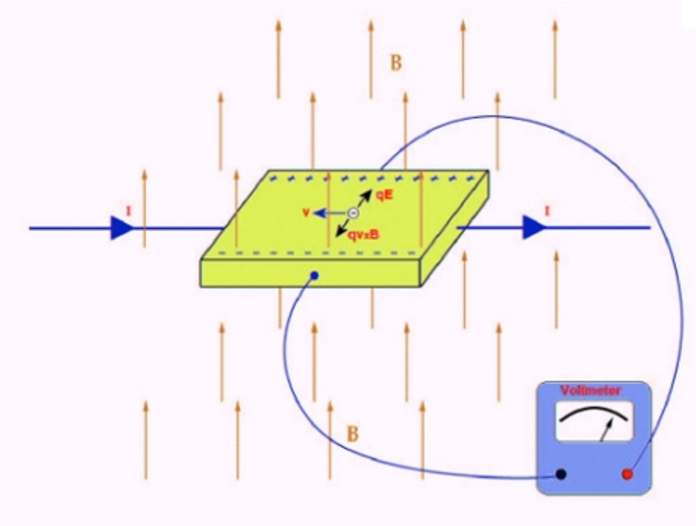
$$R_H = R_0 + R_s$$

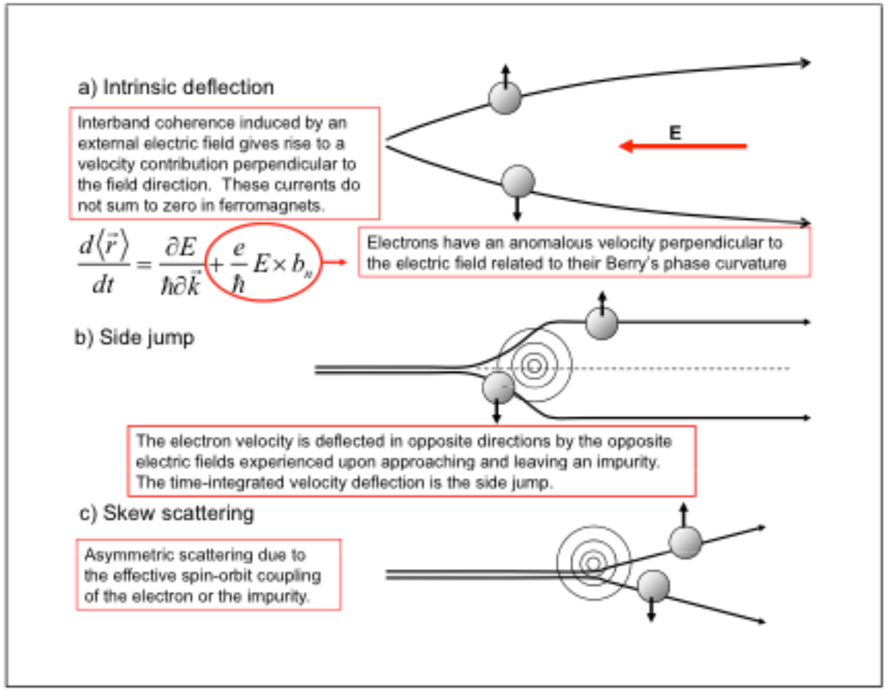
Ordinary Hall coefficient

Anomalous Hall coefficient

intrinsic
skew scattering
side-jump

$$\sigma_{xy}^{AH} = \sigma_{xy}^{AH-int} + \sigma_{xy}^{AH-skew} + \sigma_{xy}^{AH-sj}$$



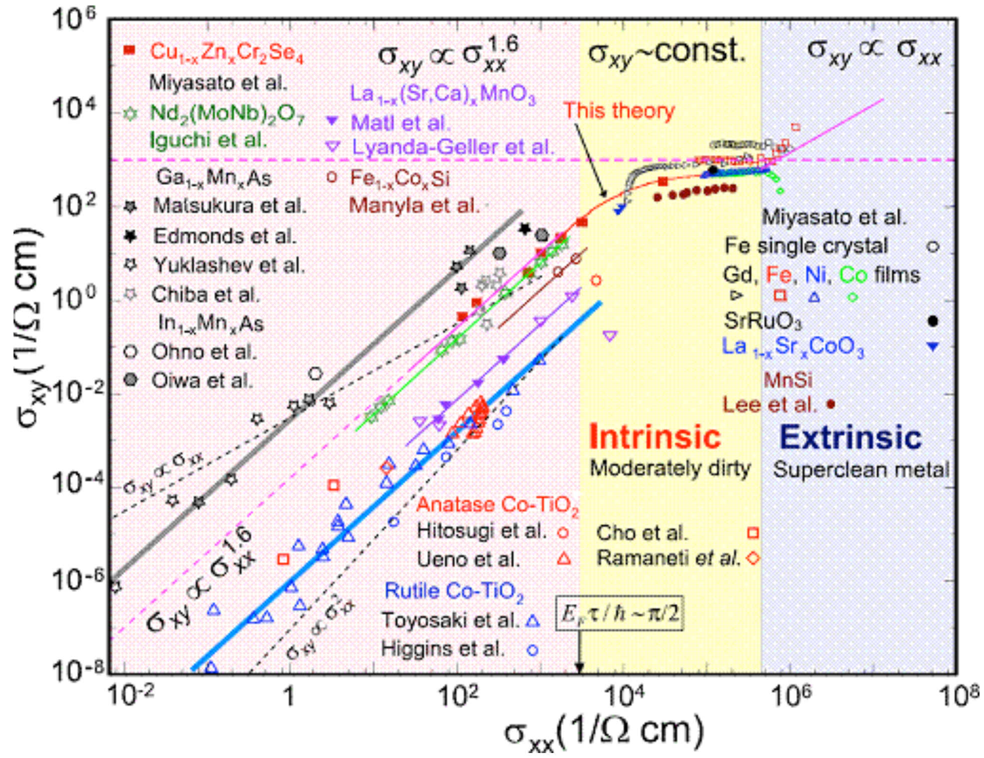


Interband coherence induced by an external electric field gives rise to a velocity contribution perpendicular to the field direction. These currents do not sum to zero in ferromagnets.

Electrons have an anomalous velocity perpendicular to the electric field related to their Berry's phase curvature

The electron velocity is deflected in opposite directions by the opposite electric fields experienced upon approaching and leaving an impurity. The time-integrated velocity deflection is the side jump.

Asymmetric scattering due to the effective spin-orbit coupling of the electron or the impurity.



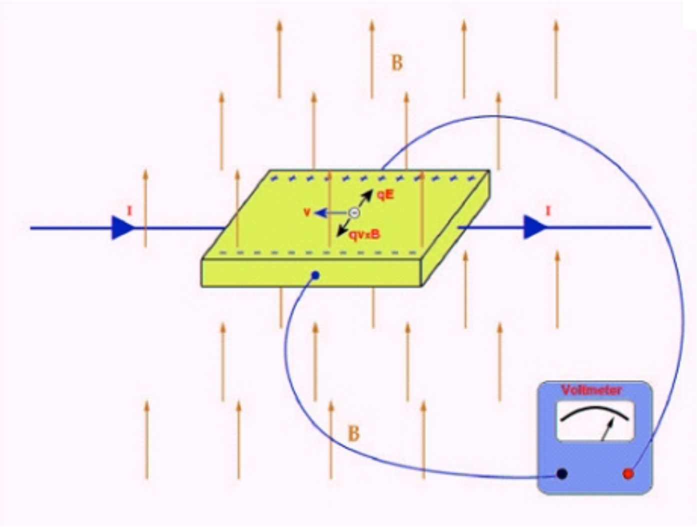
Nagaosa et al., RMP 82, 1539 (2010)

The anomalous Hall effect: skew scattering

$$R_H = R_0 + R_s$$

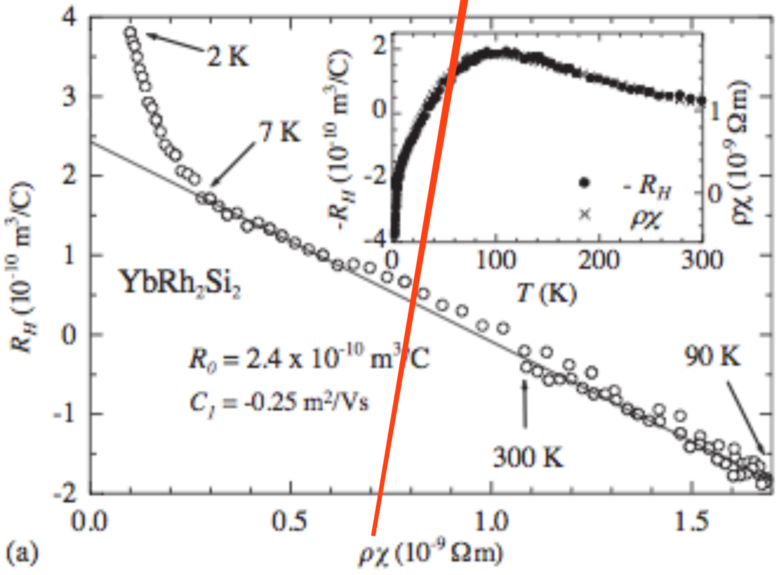
Ordinary Hall coefficient

Anomalous Hall coefficient

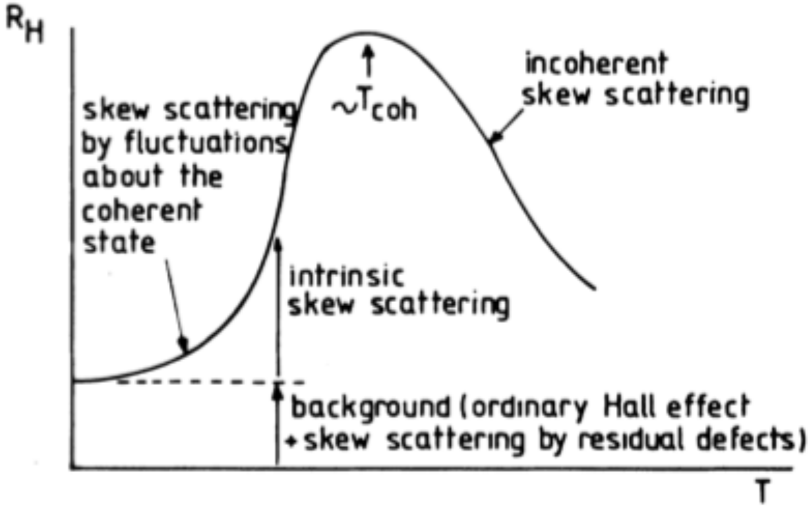


intrinsic
skew scattering
side-jump

independent local f-moments



Paschen et al, Physica B 359-361, 44 (2005)



Fert & Levy, PRB 36, 1907 (1987)

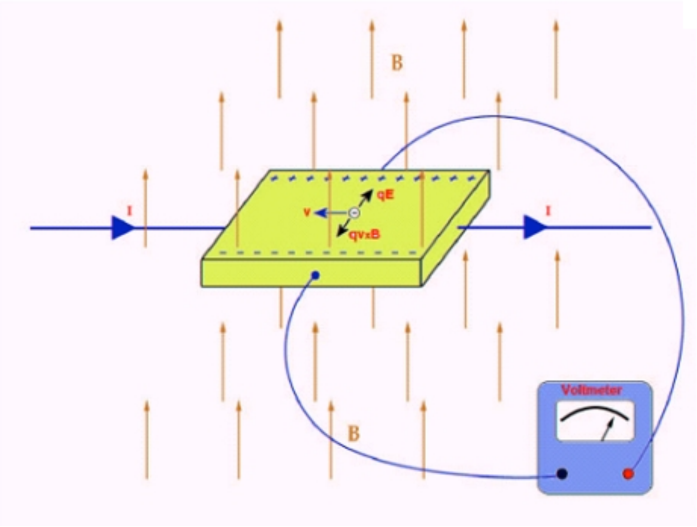
The anomalous hall effect: coherent contribution

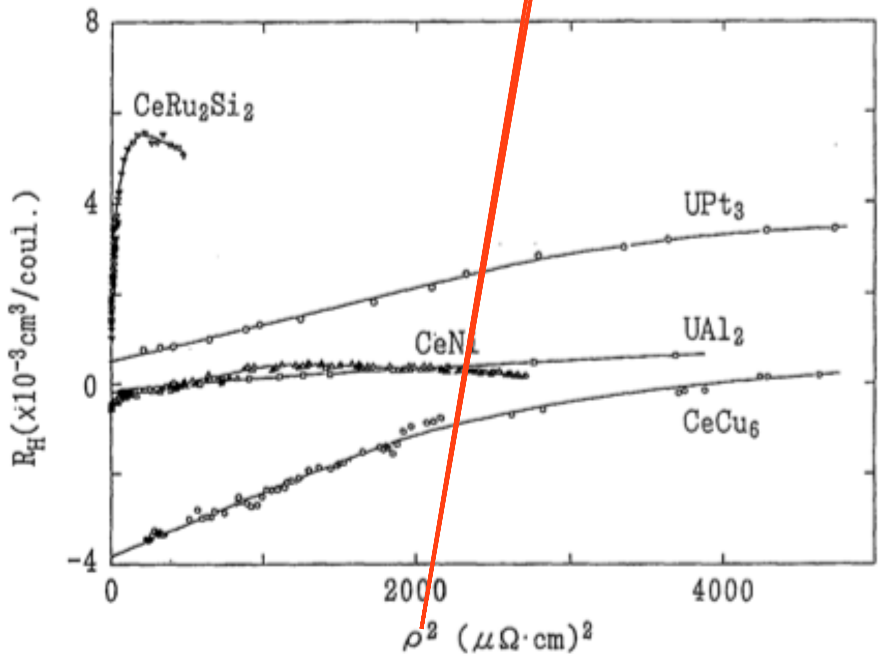
$$R_H = R_0 + R_s$$

Ordinary Hall coefficient

Anomalous Hall coefficient

- intrinsic
- skew scattering
- side-jump





Yamada et al, Prog Theor Phys, 89, 1155 (1993)

- in contradiction with Fert & Levy
- seldom observed

$R_H^a \propto \rho^2$ $\qquad$ $R_H^a \propto \chi_k \cdot \rho^0$

$T \sim T_M$ $\qquad$ T

$|\mu - E_M^f| \gg \Delta_{kM}$, $\qquad$ $|\mu - E_M^f| \lesssim \Delta_{kM}$

resonant level

broadening, the imaginary part of the self-energy

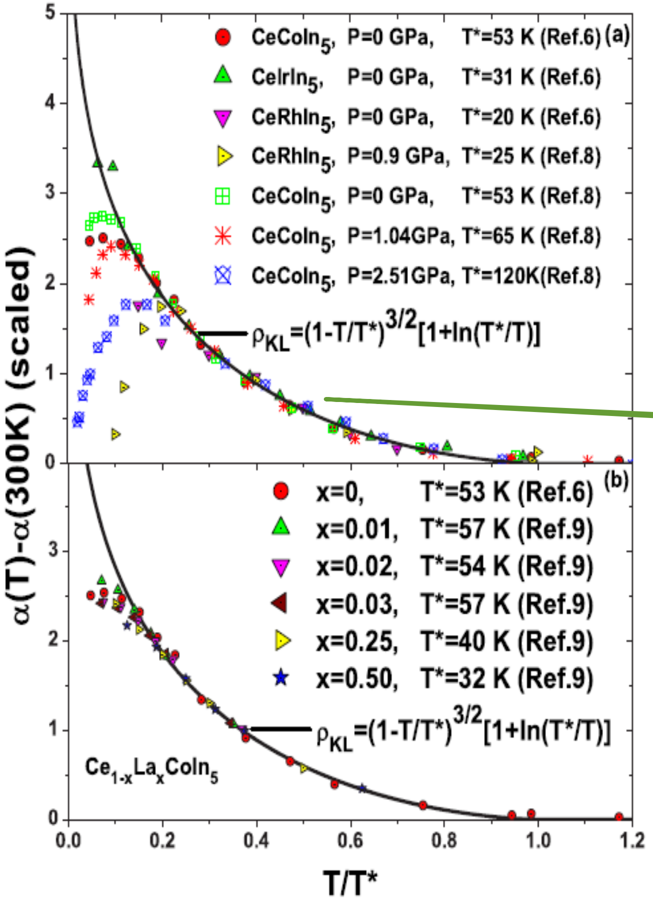
Kontani & Yamada, JPSJ 63, 2627 (1994)

The anomalous hall effect: coherent contribution

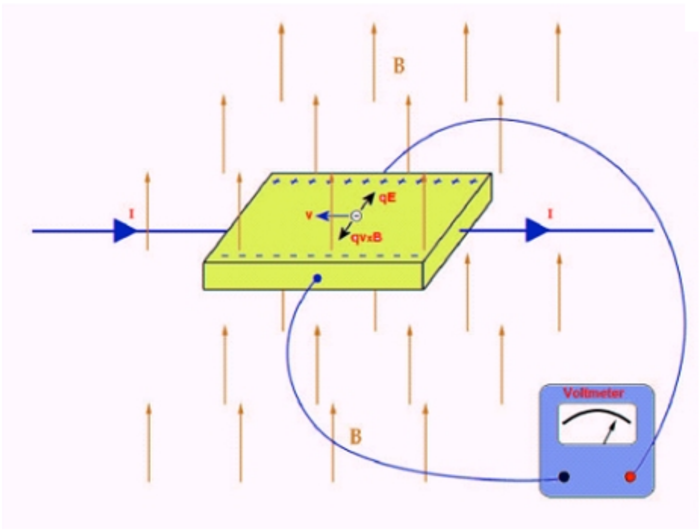
$$R_H = R_0 + R_s$$

Ordinary Hall coefficient

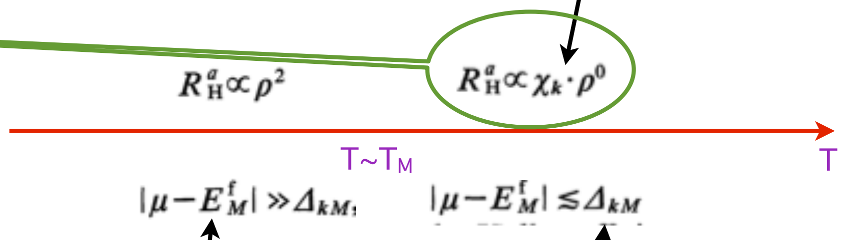
Anomalous Hall coefficient



Yang & Pines, PRL 100, 096404 (2008)



- in contradiction with Fert & Levy
- seldom observed



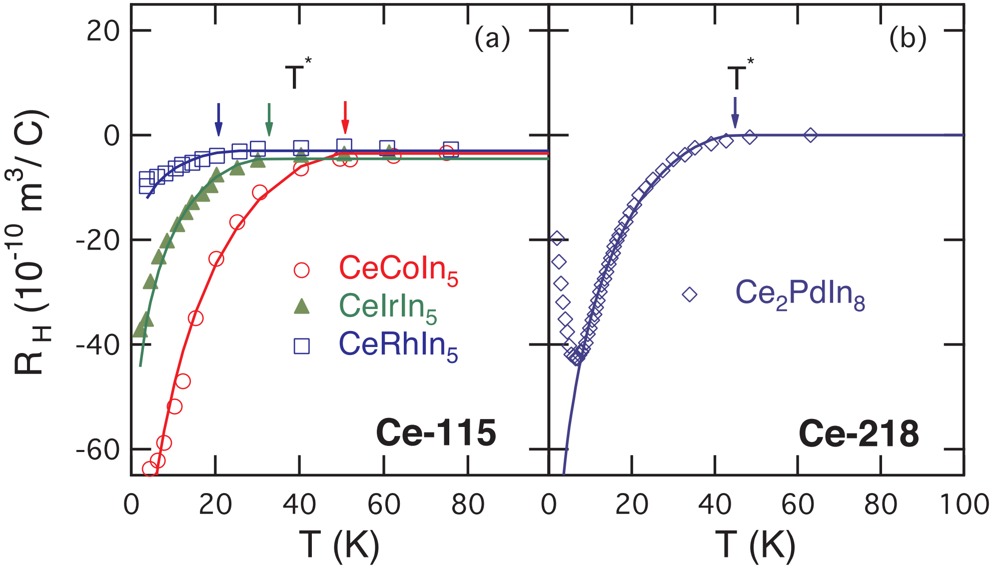
resonant level

broadening, the imaginary part of the self-energy

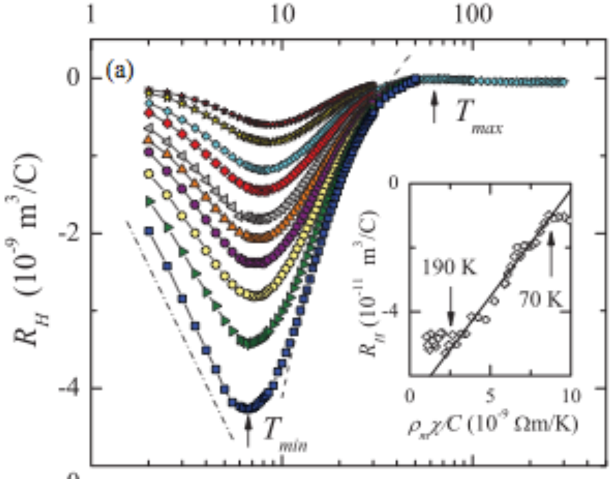
Kontani & Yamada, JPSJ 63, 2627 (1994)

The anomalous Hall effect: scaling

$$\chi_h = \min\left\{\chi, \chi_0\left(1-\frac{T}{T^*}\right)^{3/2}\left(1+\ln\frac{T^*}{T}\right)\right\}$$



Yang, PRB 87, 045102 (2013).



Gnida et al, PRB 85, 060508 (2012).

$$\sigma_{\alpha\beta} = \lim_{\omega\to 0}\frac{1}{\hbar\omega}\int_0^\infty dt\, e^{i\omega t}\langle[J_\alpha(t), J_\beta(0)]\rangle,$$

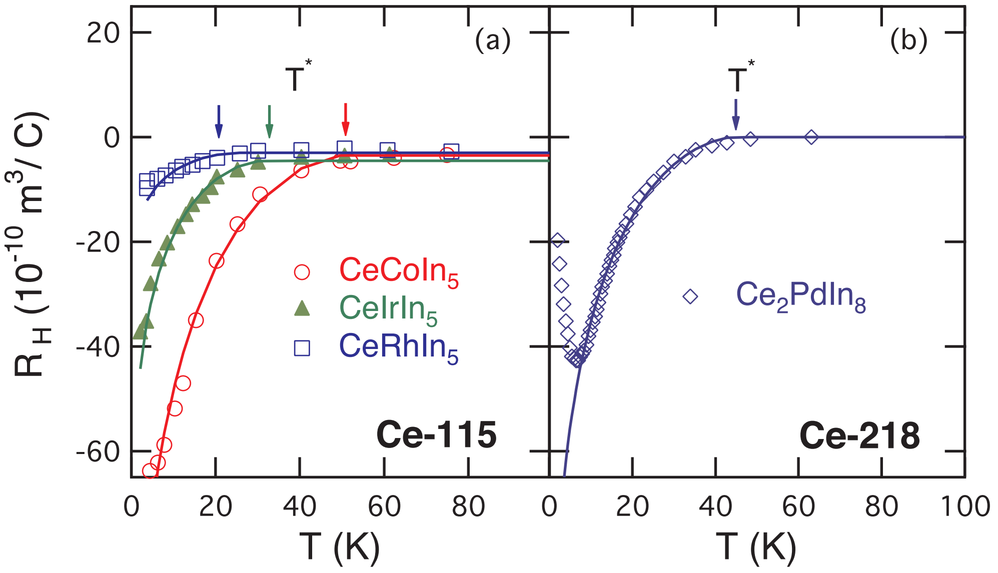
$$J_\alpha = J_\alpha^c + J_\alpha^f,$$

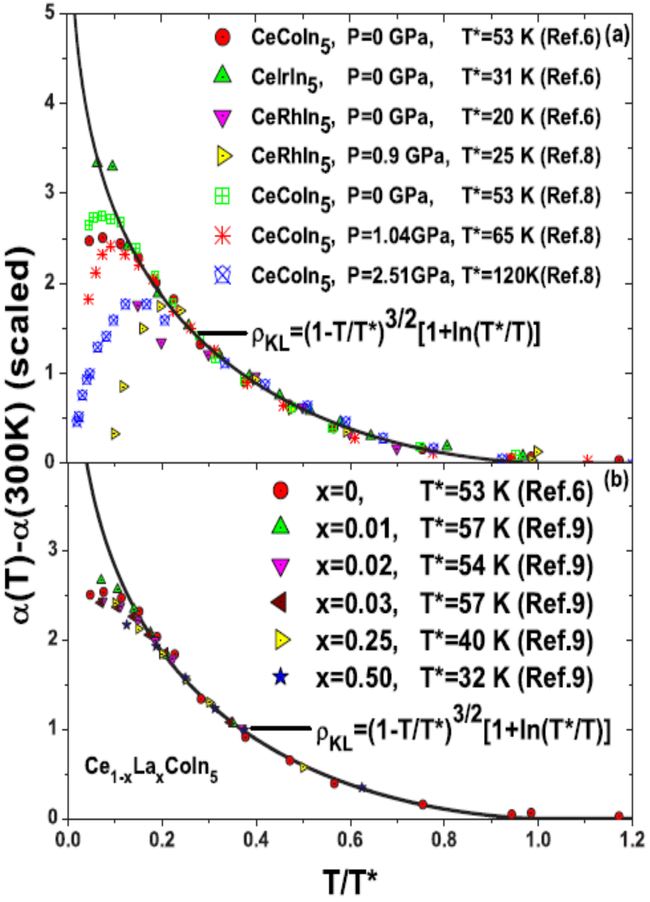
$$\sigma_{\alpha\beta} = \sigma^c_{\alpha\beta}+\sigma^f_{\alpha\beta}+\sigma^{cf}_{\alpha\beta},$$

$$R_s = \rho^2\sigma_{xy}/H = \left(\frac{\sigma^l_{xx}}{\sigma_{xx}}\right)^2 R^l_s + \left(\frac{\sigma^h_{xx}}{\sigma_{xx}}\right)^2 R^h_s$$

The anomalous Hall effect: scaling

$$\chi_h = \min\left\{\chi, \chi_0 \left(1-\frac{T}{T^*}\right)^{3/2}\left(1+\ln\frac{T^*}{T}\right)\right\}$$





$$R_H = R_0 + r_l \rho \chi_l + r_h \chi_h$$

incoherent skew scattering from f-moments

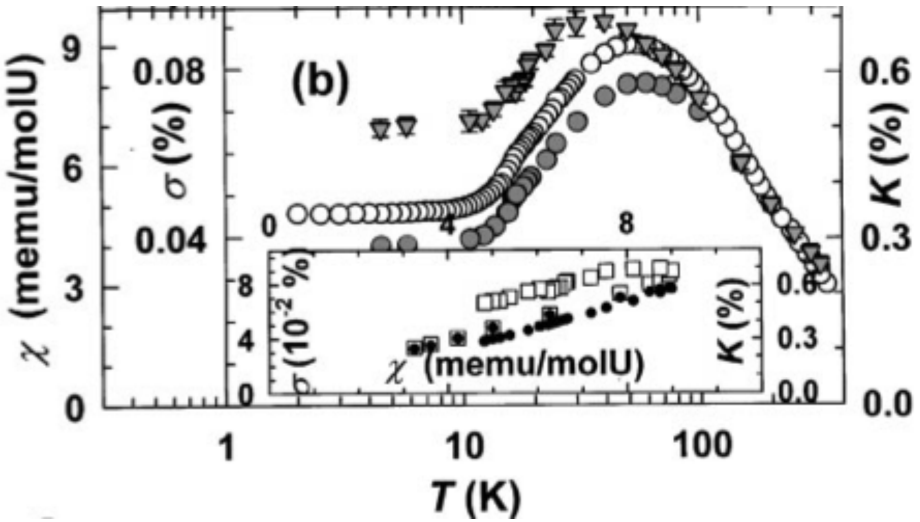
coherent skew scattering from itinerant heavy f-electrons

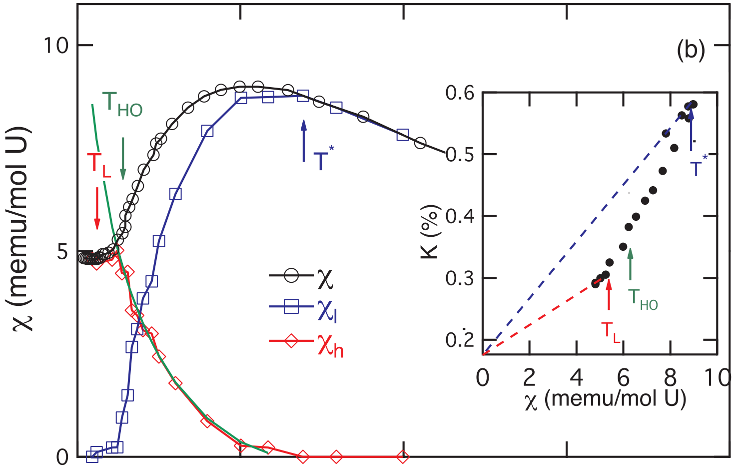
Yang, PRB 87, 045102 (2013).

To prove this formula, we need to separate the two components.

URu_2Si_2: separation of the two components

Bernal et al, Physica B, 281&282, 236 (2000)





$T > T^*$: $\chi = \chi_{sl}$

$K = K_0 + A\chi_{sl}$

$T < T^*$: $\chi = \chi_{sl} + \chi_{kl}$

$K = K_0 + A\chi_{sl} + B\chi_{kl}$

$K_a = K - K_0 - A\chi = (B - A)\chi_{kl}$

$T < T_L$: $\chi = \chi_{kl}$

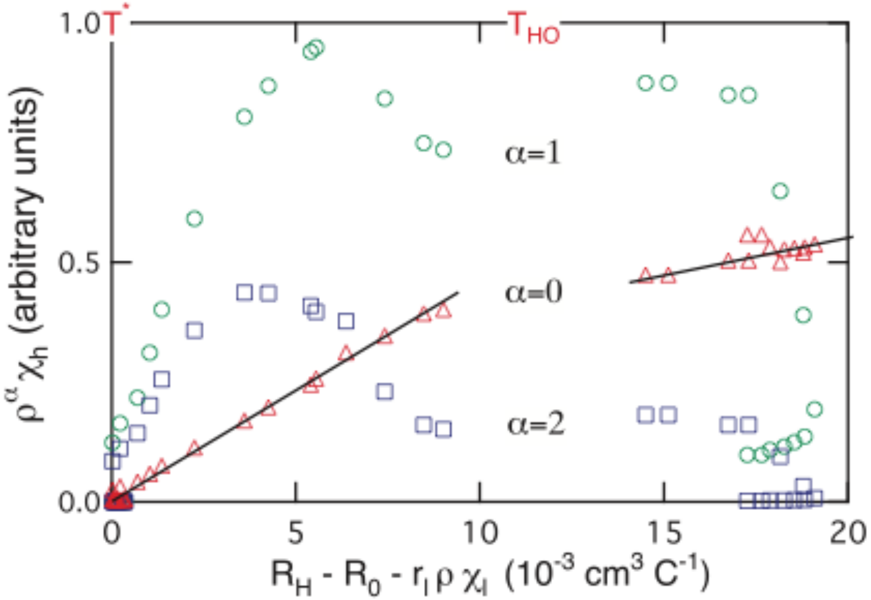
$K = K_0 + B\chi_{kl}$

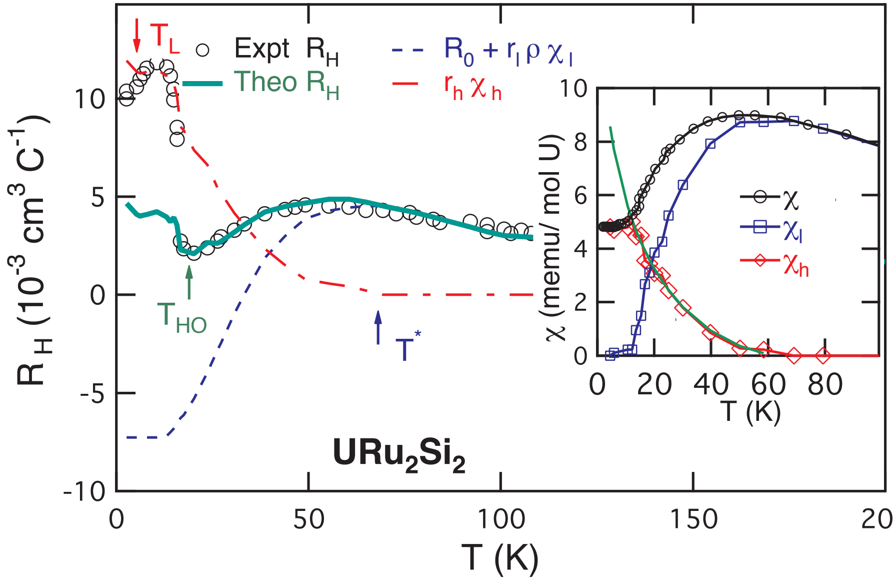
$K_b = K - K_0 - B\chi = (A - B)\chi_{sl}$

$$\chi_{sl} = (K - K_0 - B\chi)/(A - B)$$
$$\chi_{kl} = (K - K_0 - A\chi)/(B - A)$$

Yang, PRB 87, 045102 (2013).
Shirer et al, PNAS 109, 18249 (2012).

The anomalous Hall effect: URu_2Si_2





$$R_H = R_0 + r_l \rho \chi_l + r_h \chi_h$$

incoherent skew scattering from f-moments

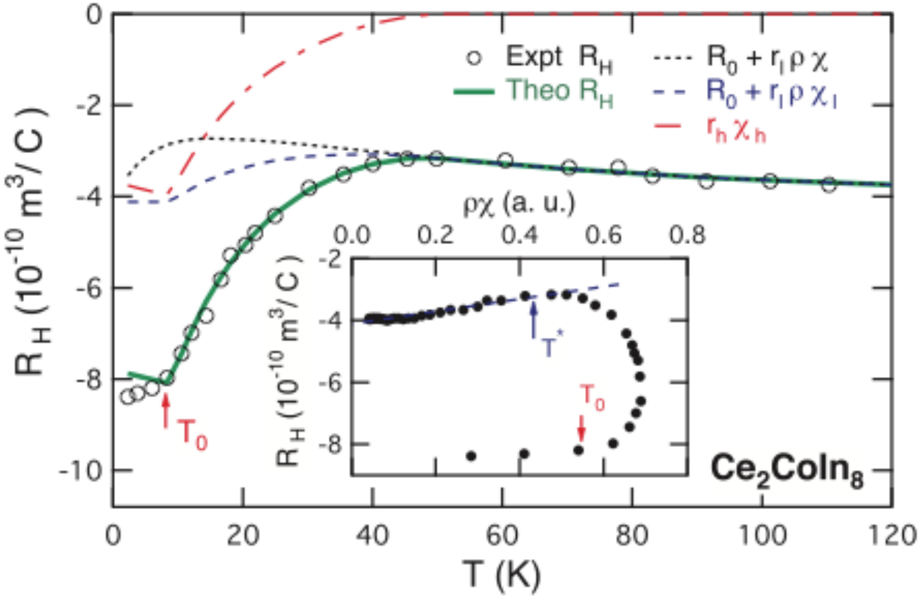
coherent skew scattering from itinerant heavy f-electrons

Yang, PRB 87, 045102 (2013).

The anomalous Hall effect: Ce_2CoIn_8

If a separation is not available from other experiment,

$$\chi_h = \min\left\{\chi, \chi_0\left(1-\frac{T}{T^*}\right)^{3/2}\left(1+\ln\frac{T^*}{T}\right)\right\}$$



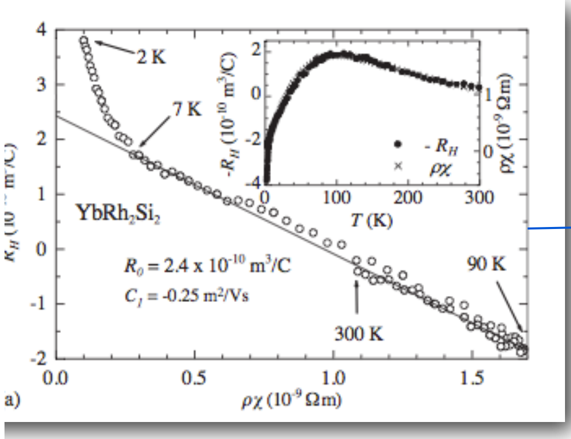
$$R_H = R_0 + r_l\rho\chi_l + r_h\chi_h$$

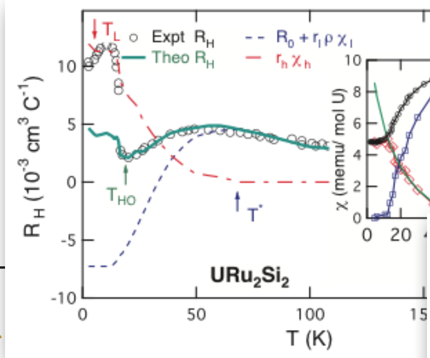
incoherent skew scattering from f-moments

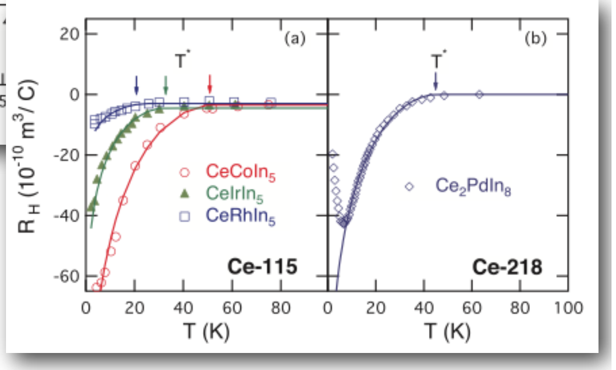
coherent skew scattering from itinerant heavy f-electrons

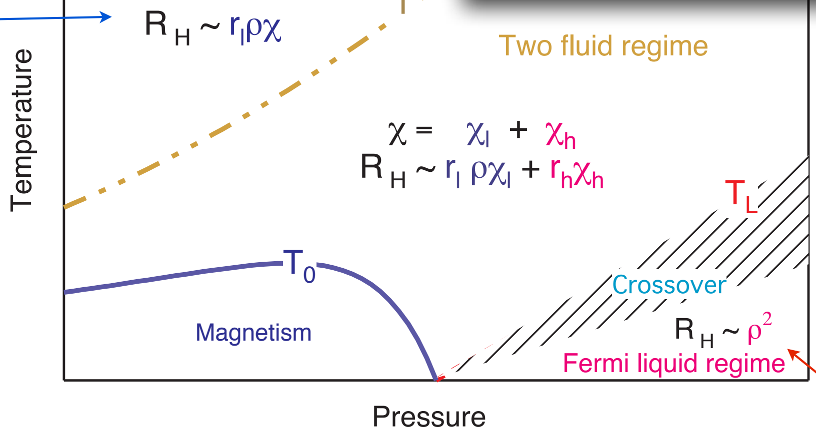
Yang, PRB 87, 045102 (2013).

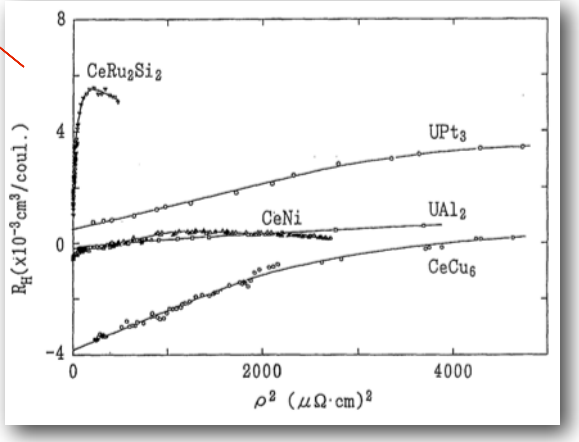
The anomalous Hall effect: a new scenario











$$R_H = R_0 + r_l \rho \chi_l + r_h \chi_h$$

incoherent skew scattering from f-moments

coherent skew scattering from itinerant heavy f-electrons

Yang, PRB 87, 045102 (2013).

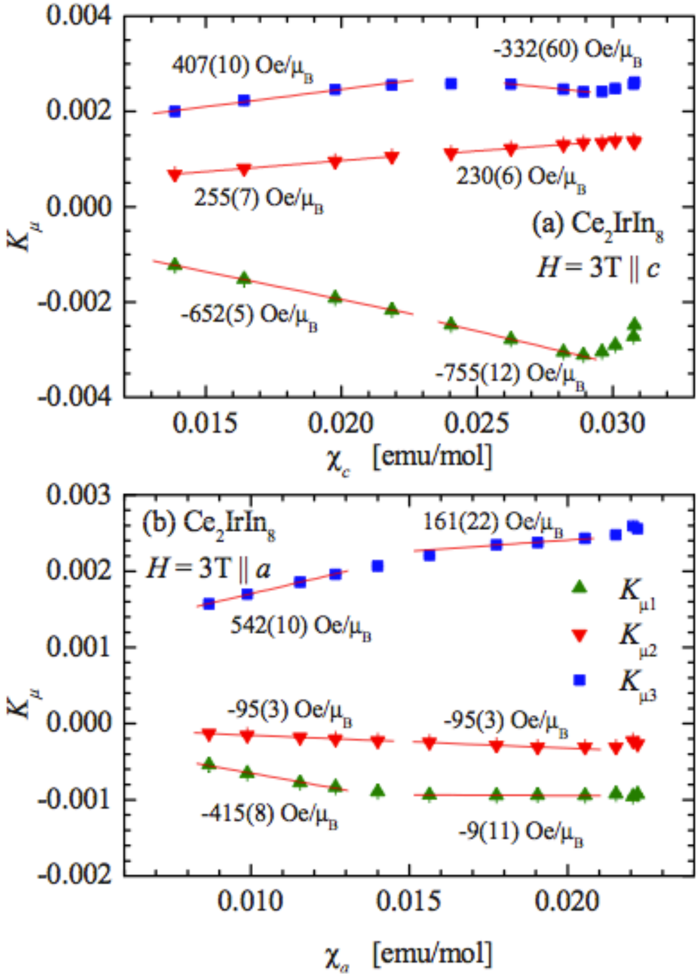
The Hall effect is therefore another evidence for the emergent state.

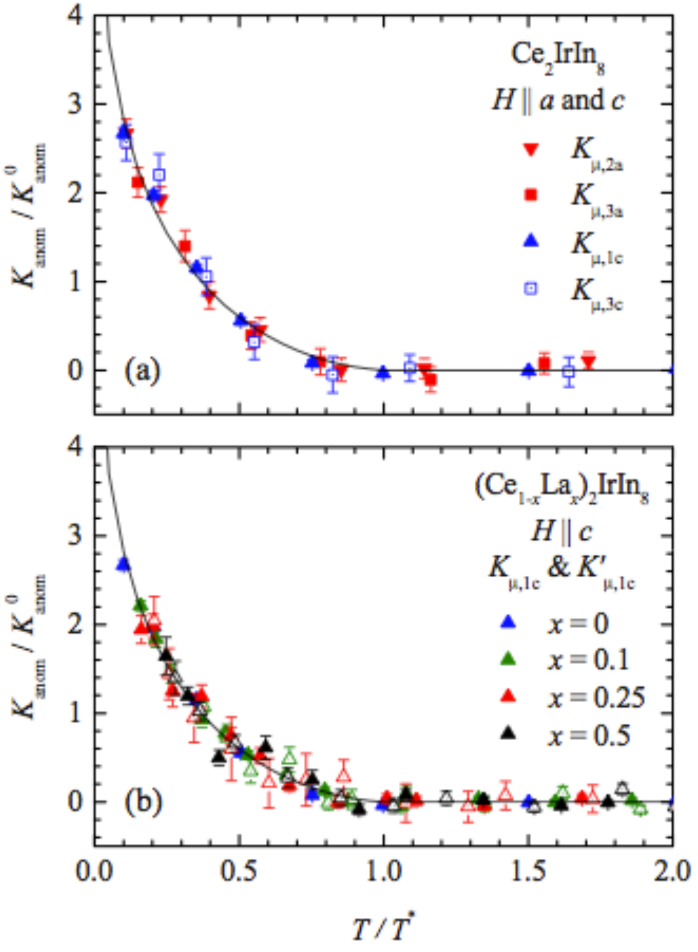
More experiments to do

- Further examination of the emergent state in NMR, Hall etc
- Comparing T^* and T_K with pressure experiment like in La-doped $CeRhIn_5$
- Detecting two coexisting fluids. How? (Neutron, ESR ...)
- Measurement of Fermi surface evolution at T_L
- Relation between Kondo liquid scaling and quantum critical scaling

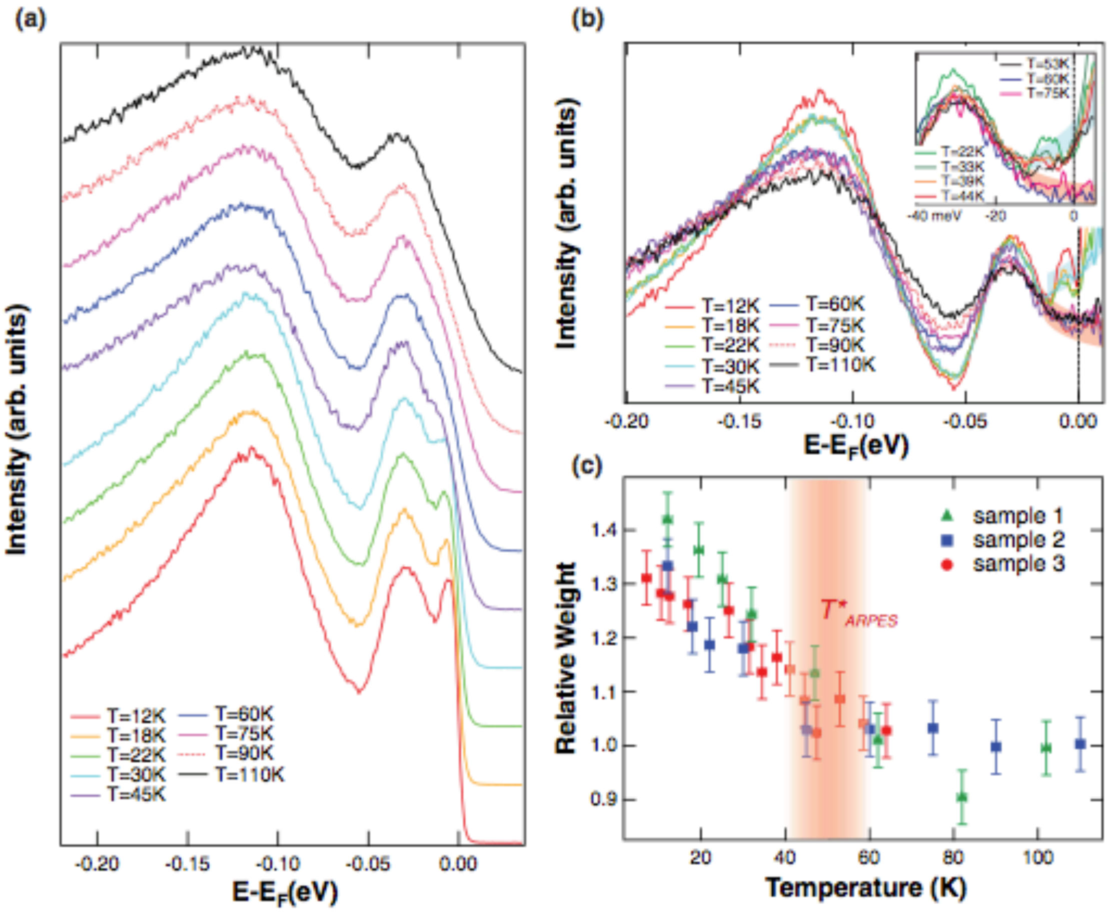
Just a few examples ...

Experiment: μSR on $(Ce_{1-x}La_x)_2IrIn_8$



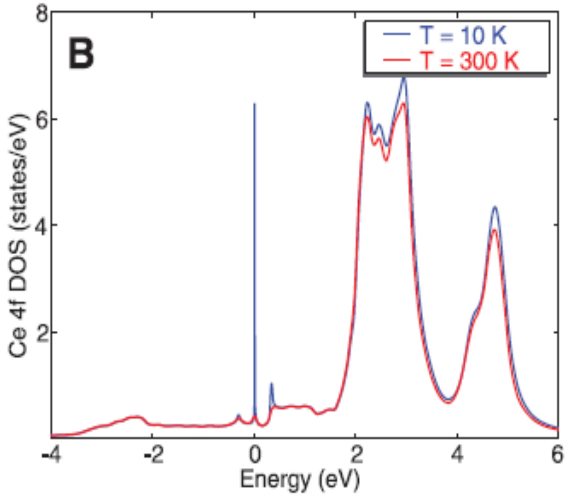


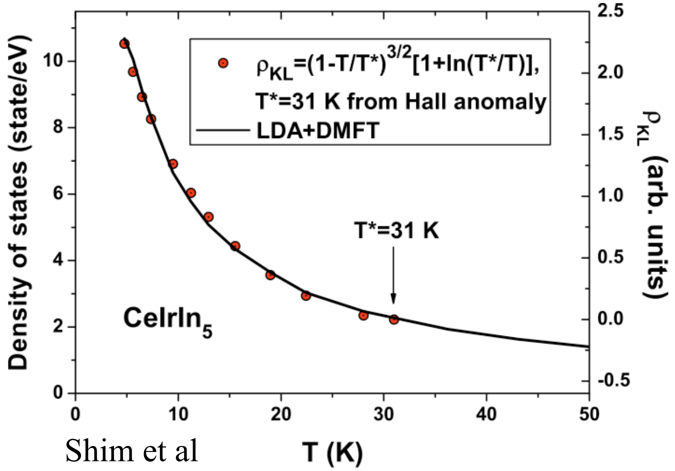
Experiment: ARPES on $YbRh_2Si_2$



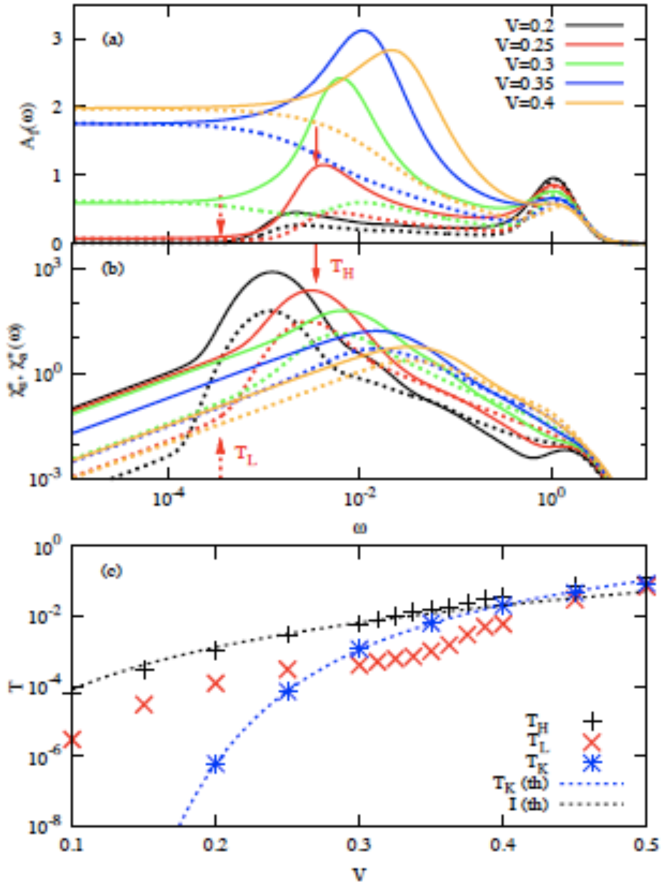
Mo *et al.*, 2012

Theoretical progresses





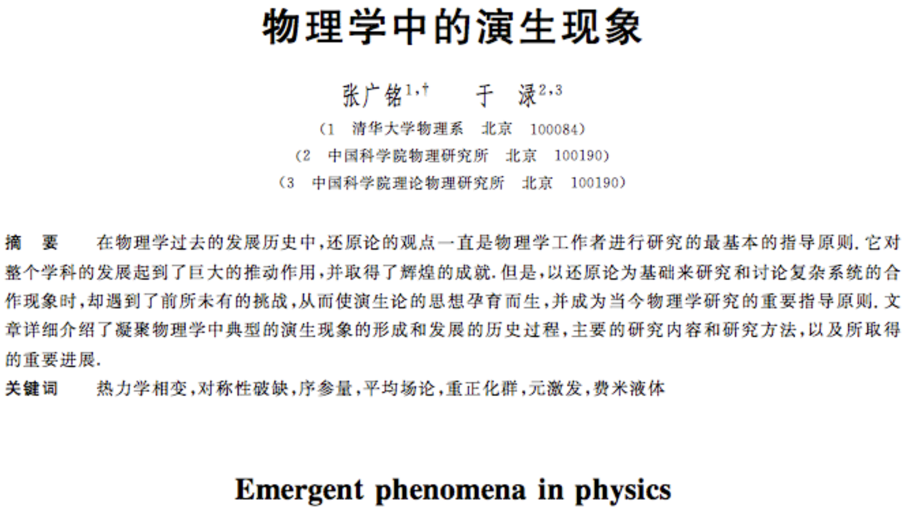
Shim et al



Zhu et al

- LDA+DMFT for CeIrIn₅ obtains similar scaling.
- DMFT+NRG supports dominant RKKY scale.

Emergent phenomenon



物理学中的演生现象

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（3　中国科学院理论物理研究所　北京　100190）

摘　要　　在物理学过去的发展历史中,还原论的观点一直是物理学工作者进行研究的最基本的指导原则.它对整个学科的发展起到了巨大的推动作用,并取得了辉煌的成就.但是,以还原论为基础来研究和讨论复杂系统的合作现象时,却遇到了前所未有的挑战,从而使演生论的思想孕育而生,并成为当今物理学研究的重要指导原则.文章详细介绍了凝聚物理学中典型的演生现象的形成和发展的历史过程,主要的研究内容和研究方法,以及所取得的重要进展.

关键词　　热力学相变,对称性破缺,序参量,平均场论,重正化群,元激发,费米液体

Emergent phenomena in physics

More is different !

- universal
- protected

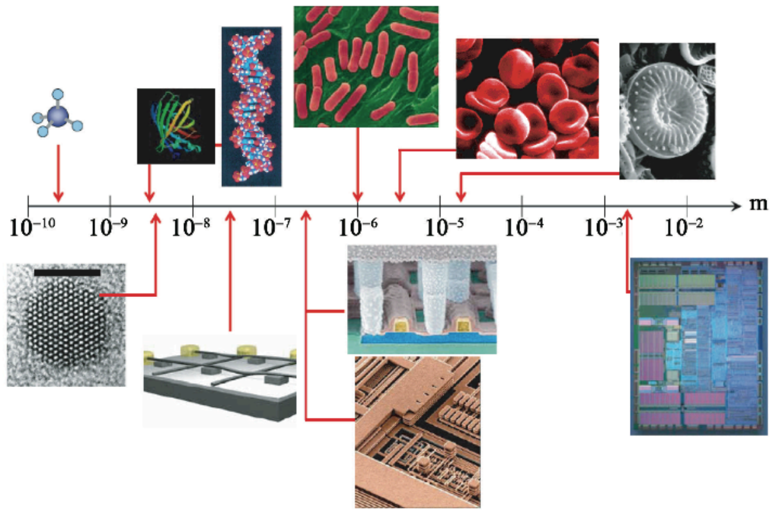

图 2　物质结构被划分为一系列的层次,各层次有其组成的“基本”粒子以及其特征长度和特征能量,每个层次还存在自己特有的基本规律

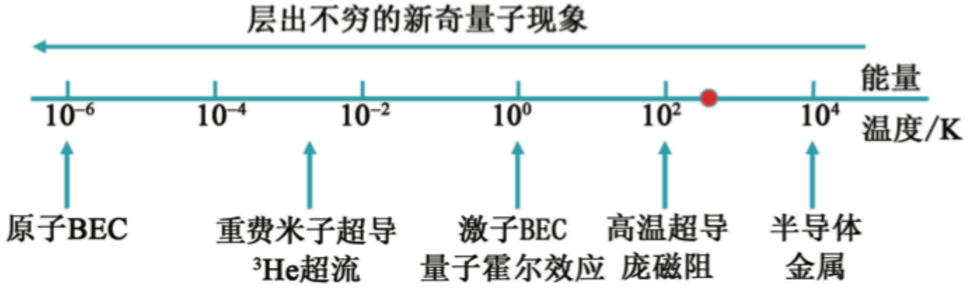

图 3　随着特征能量尺度或温度的不断降低,凝聚态物理体系不断呈现出新奇的量子现象

Work in progress

- How can we understand the emergent states?
- Is this in any way related to the quantum criticality?
- Can we design a decisive experiment on the debate?

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- 2011 - Yang et al. - A new phenomenological framework
- 2012 - Yang & Pines - A new phase diagram & hybridization effectiveness for GS
- 2012 - Yang - Anomalous Hall effect in heavy electron materials

work in progress ...

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