Random Matrices, Black holes, and the Sachdev-Ye-Kitaev model

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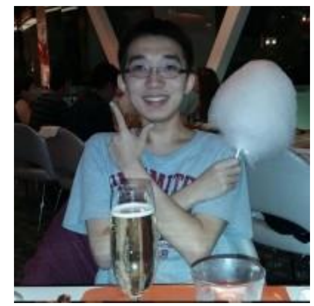
PhD Students needed!







Verbaarschot
Stony Brook

Bermúdez
Leiden

Tezuka
Kyoto

Loureiro
Cambridge

Yiyang Jia
Stony Brook

arXiv:1801.02696
Phys. Rev. D 94, 126010 (2016)
Phys. Rev. D 96, 066012 (2017)
arXiv:1801.03204
arXiv:1801.01071
arXiv:1707.02197

Kitaev 2015 Also: 1711.0847

"A simple model of quantum holography"

http://online.kitp.ucsb.edu/online/entangled15/kitaev/

$$H = J_{ijkl}\psi_i\psi_j\psi_k\psi_l$$

Strong coupling

$$\{\psi_i, \psi_j\} = \delta_{ij}$$

$$\beta J \gg 1$$

$$\tau J \gg 1$$

$$\langle J_{ijkl}^2 \rangle = \mathrm{J}^2/\mathrm{N}^3$$

AdS2

Quantum gravity?

SYK = Sachdev-Ye-**Kitaev**

Outline

1. Models with infinite range interactions before SYK, random matrix theory and quantum chaos

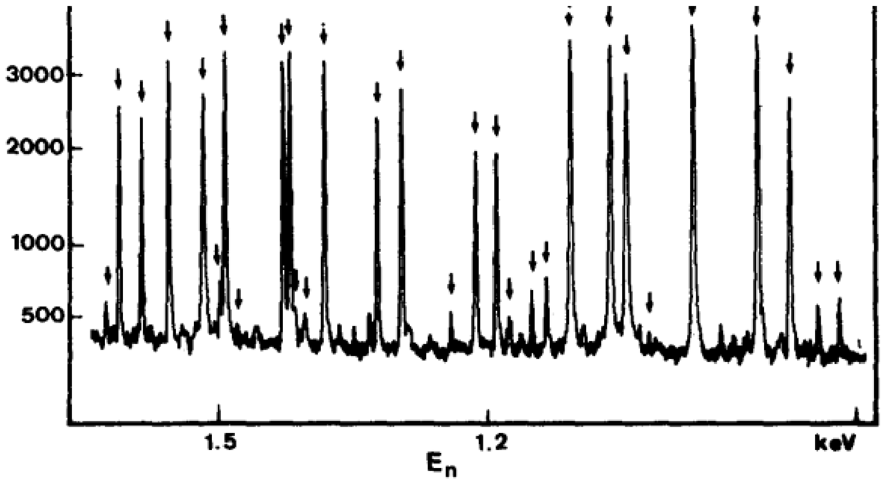
2. An introduction to the SYK model

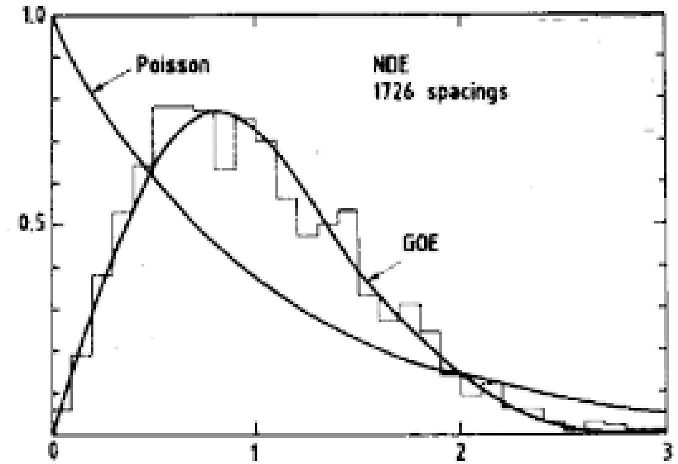
3. SYK model, black holes, random matrices and chaotic-integrable transitions

Nuclear Physics 60's:

The ultimate approximation "A random matrix as an effective nuclear Hamiltonian"

Fermionic quantum dot with N-body random interactions of infinite range





O. Bohigas, R.U. Haq, and A. Pandey, in Nuclear Data for Science and Technology, (1983)

Coceva and Stefanon, Nuclear Physics A, 1979

Flores, Bohigas, French 1970

Random Matrix

Dyson-Mehta

$$\begin{pmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{N1} & \cdots & a_{NN} \end{pmatrix}$$

$\beta = 1$ GOE

$\beta = 2$ GUE

$\beta = 4$ GSE

Semicircle law $\rho(E) \sim \sqrt{E_0^2 - E^2}$ No universal

Level Repulsion $P(s) \sim s^{\beta} e^{-As^2}$ $s = (E_{i+1} - E_i)/\Delta$

Spectral rigidity

$$\Sigma^2(N) = \langle n(N)^2 \rangle - \langle n(N) \rangle^2 \sim \log(N)$$

Universality: Quantum Chaos, Mesoscopic physics....

k-random body ensembles

$$H = \sum_k \varepsilon_k a_k^\dagger a_k + \lambda \sum_{k\leq l, p\leq q} \langle pq| V |kl\rangle a_p^\dagger a_q^\dagger a_l a_k$$

N fermions
m levels
2-body

Bohigas, Flores, French, Mon, 70's

$$m \gg N \quad \langle H^p \rangle \quad \rightarrow \quad \rho(E) \propto e^{-E^2/\sigma^2}$$

French, Mon, Annals of Physics 95, 90 (1975).

Two-level correlation function ⟺ Random Matrix

Verbaarschot, Zirnbauer 85

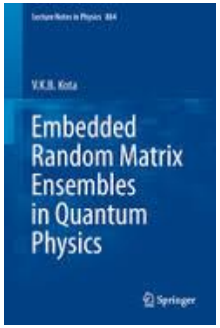
Quantum Chaos 80's 90's:

Level statistics

Thermalisation

Metal-insulator transitions

Many-body localisation



Kota

Quantum spin glasses

Heisenberg Spin-Chain

$$H = \frac{1}{\sqrt{\mathcal{N}N}} \sum_{i<j} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

Infinite Range

Replica Trick

Large N

Stability of magnetic order

Quantum criticality

Cuprates, spin-liquids ....

Sachdev, Ye, PRL. 70, 3339 (1993)

A. Georges, O. Parcollet, S. Sachdev PRB 63, 134406 (2001)

S. Sachdev PRL 83, 74408 (2010)

Finite zero T entropy

Infinite Range - Holography

Classical and Quantum Chaos

Butterfly effect

Classical chaos

Hadamard 1898

Lyapunov 1892

$$\|\delta x(t)\| = e^{\lambda t}\|\delta x(0)\|$$

$$\lambda > 0$$

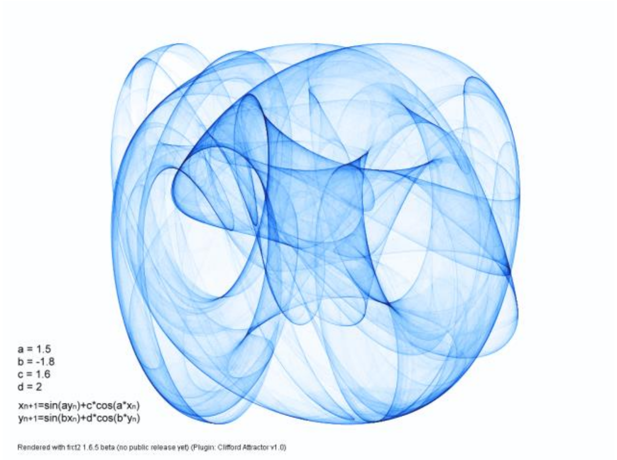
$$h_{KS} > 0$$

Pesin theorem

Difficult to compute!

Lorenz 60's

Meteorology



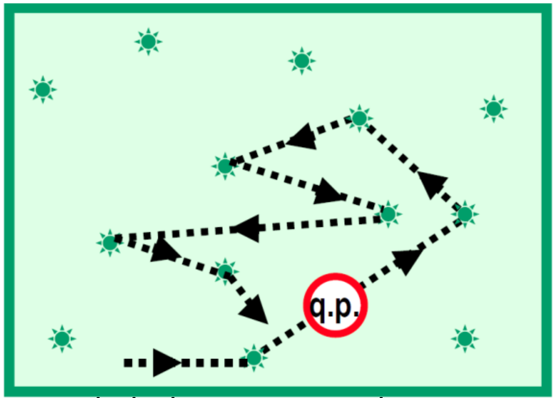
Quantum chaos?

Role of classical chaos in the ***semiclassical*** limit

Quantum butterfly effect?

Disordered system

Larkin, Ovchinnikov,
Soviet Physics JETP 28, 1200 (1969)



Altshuler, Lancaster lectures

$$\langle p_z(t) p_z(0)\rangle \propto e^{-t/\tau}$$

τ Relaxation time

$$\langle [p_z(t), p_z(0)]^2\rangle \approx \hbar^2 \langle \{p_z(t), p_z(0)\}^2\rangle \propto \hbar^2 \exp(\lambda t)$$

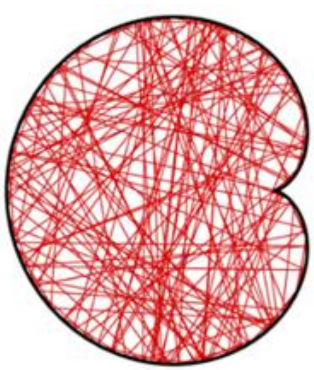
$\tau \ll t < t_E \sim \log \hbar^{-1}/\lambda$ Chaotic

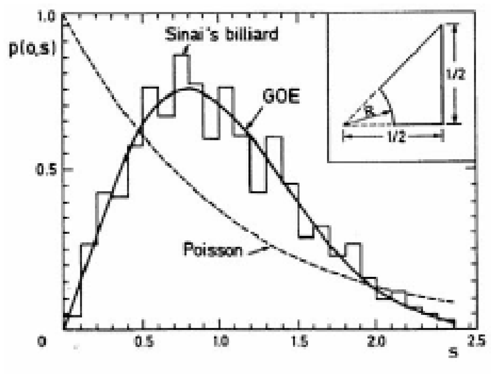
$t_E \propto \hbar^{\alpha}\ \alpha < 0$ Integrable

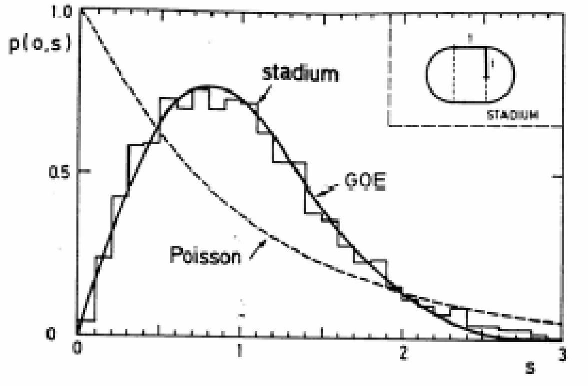
Chaos, disorder and random matrix theory

Bohigas-Giannoni-Schmit conjecture

PRL 52, 1 (1984)







Quantum Chaos ⟷ RMT correlations

$$\hat{G}_{\mathrm{R,A}} = (E - \hat{H} \pm i\eta)^{-1}$$

$$(E - \hat{H} + i\eta)^{-1}_{kl} = -i \frac{\int [\mathrm{d}\phi^* \mathrm{d}\phi] \phi_k \phi_l^* \exp\{i \sum_{ij} \phi_i^* [(E + i\eta)\delta_{ij} - H_{ij}] \phi_j\}}{\int [\mathrm{d}\phi^* \mathrm{d}\phi] \exp\{i \sum_{ij} \phi_i^* [(E + i\eta)\delta_{ij} - H_{ij}] \phi_j\}}$$

1982-84: Grassmannian variables can help

Efetov

$$\chi_k \chi_l = -\chi_l \chi_k$$

$$I = \int \exp\left(-\chi^+ A \chi\right) \prod_{i=1}^{n} d\chi_i^* d\chi_i = Det A$$

$$(E - \hat{H})^{-1}_{kl} = -i \int [\mathrm{d}\Phi^* \mathrm{d}\Phi] S_k S_l^* \exp\{i \sum_{ij} \Phi_i^\dagger [E\delta_{ij} - H_{ij}] \Phi_j\}$$

$$\Phi^\dagger = (S_1^*, \ldots, S_n^*, \chi_1^*, \ldots, \chi_n^*)$$

$$\left\langle \exp(i \sum_{ij} \Phi_i^\dagger H_{ij} \Phi_j) \right\rangle = \exp\left\{ -\frac{1}{2N} \sum_{ij} (\Phi_i^\dagger \Phi_j)(\Phi_j^\dagger \Phi_i) \right\}$$

Disorder
Is integrated!!

Disordered metals in d > 2 ⟷ RMT correlations

Quantum Chaos in holography

Relation to black-hole physics

Fast Scramblers

t_E?

Sekino, Susskind,JHEP 0810:065,2008

P. Hayden, J. Preskill, JHEP 0709 (2007) 120

1. Most rapid scramblers (black holes) take a time logarithmic in N
2. Thermalization in black holes is as fast as possible

Membrane paradigm hypothesis

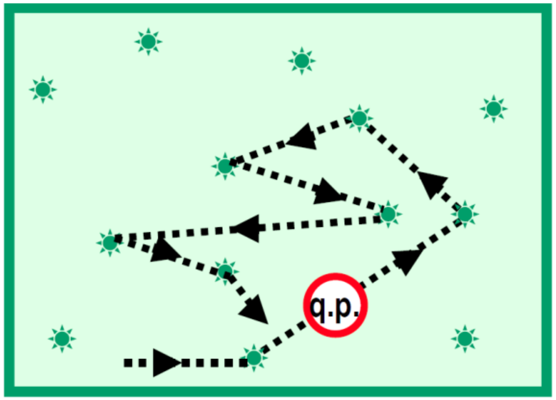
(Quantum) black hole physics

AdS/CFT

Strongly coupled (quantum) QFT

Quantum butterfly effect in gravity/holography?

Maldacena, Shenker, Stanford arXiv:1503.01409



$\tau \ll t < t_E \sim \log \hbar^{-1}/\lambda$ Chaotic

$$\langle [p_z(t), p_z(0)]^2 \rangle \approx \hbar^2 \langle \{p_z(t), p_z(0)\}^2 \rangle \propto \hbar^2 \exp(\lambda t)$$

Field theory?

Black holes?

λ?

A bound on chaos arXiv:1503.01409

Maldacena, Shenker, Stanford

$$y^4 = \frac{1}{Z} e^{-\beta H} \qquad F(t) = \mathrm{tr}[yVyW(t)yVyW(t)]$$

$$t_* = \frac{\beta}{2\pi} \log N^2 \qquad F_d \equiv \mathrm{tr}[y^2 V y^2 V] \mathrm{tr}[y^2 W(t) y^2 W(t)]$$

$$t_d \ll t < t_* \qquad F_d - F(t) = \epsilon \exp \lambda_L t + \cdots \qquad \epsilon \sim 1/N^2$$

$$F(t) = f_0 - \frac{f_1}{N^2} \exp \frac{2\pi}{\beta} t + \mathcal{O}(N^{-4})$$

$$\lambda \le 2\pi T/\hbar$$

Black holes and its field theory dual saturate the bound

Causality constraints
+
Uncertainty relations

Berenstein,AGG
arXiv:1510.08870

$$p \leq e^{t/4MG}$$

How is this related to quantum information?

$$S \sim t/\tau$$

How universal?

$$\tau \geq \hbar/2\pi k_B T$$

QM induces entanglement but also limits its growth

Introduction to the SYK model

Kitaev 2015

"A simple model of quantum holography"

http://online.kitp.ucsb.edu/online/entangled15/kitaev/

No kinetic term $H = J_{ijkl}\psi_i\psi_j\psi_k\psi_l$

Majoranas $\{\psi_i, \psi_j\} = \delta_{ij}$

Gaussian $\langle J_{ijkl}^2 \rangle = \mathrm{J}^2/\mathrm{N}^3$

Strong coupling $\beta J \gg 1 \quad \tau J \gg 1$

A solvable finite model of quantum gravity

SYK = Sachdev-Ye-**Kitaev**

Correlation functions

Disorder average by replica trick

$$\langle Z(\beta)\rangle_J = \int DGD\Sigma\, e^{-N\,I(G,\Sigma)}$$

$$I(G,\Sigma) = -\frac{1}{2}\log\det(\partial_\tau - \Sigma) + \frac{1}{2}\int_0^\beta d\tau_1 d\tau_2 \left[\Sigma(\tau_1,\tau_2)G(\tau_1,\tau_2) - \frac{J^2}{q}G(\tau_1,\tau_2)^q\right]$$

q = q/2-body interaction

Σ is a Lagrange multiplier

$$G(\tau_1,\tau_2) \sim \langle\psi(\tau_1)\psi(\tau_2)\rangle$$

Zero Temperature

$N \to \infty$ Self-consistent Schwinger-Dyson equations

$$\Sigma_* = J^2 G_*^{q-1}$$

$$G_* = \frac{1}{\partial_\tau - \Sigma_*}$$

$$J\tau, J\beta \gg 1 \quad \partial_\tau \to 0$$

Conformal in the IR limit

$$G_*(0,\tau) \to \frac{const.}{(\sin \frac{\pi\tau}{\beta})^{2\Delta}}, \qquad \Delta = \frac{1}{q} \qquad q=4$$

Finite Zero Temperature entropy

$$\frac{S_0}{N} = \frac{1}{2}\log 2 - \int_0^\Delta dx\pi\left(\frac{1}{2} - x\right)\tan \pi x$$

As in some AdS_2 background

Kitaev

Corrections

Classical

$$\frac{1}{N} \ll 1$$

Conformal

$$\frac{1}{J\beta} \ll 1$$

Why?

Thermodynamic properties

(Quantum) chaos bound

In the conformal limit:

$$G \longrightarrow G_f(\tau,\tau') = [f'(\tau)f'(\tau')]^{\Delta} G(f(\tau), f(\tau'))$$

Conformal symmetry spontaneously broken

f Goldstone modes

Low temperature: Correction to conformal

$$S = -N\frac{\alpha_S}{\mathcal{J}}\int d\tau\{f,\tau\}$$

$$\{f,\tau\} \equiv \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2$$

Schwarzian action

Same as in AdS_2

$$f \to \frac{af+b}{cf+d}$$

SL(2,R)

Finite T=1/β saddle

$$f(\tau) = \tan\left(\frac{\pi\tau}{\beta}\right)$$

$$-\beta F \supset \frac{N\alpha_S}{\mathcal{J}}\int_0^\beta d\tau\left\{\tan\frac{\pi\tau}{\beta},\tau\right\} = 2\pi^2\alpha_S\frac{N}{\beta\mathcal{J}}$$

Linear specific heat

J. Maldacena, D. Stanford, Phys. Rev. D 94, 106002 (2016)

1/N Quantum corrections

$$\frac{S}{N} = \frac{J^2(q-1)}{4} g \cdot (\tilde{K}^{-1} - 1) g$$

$$-\beta F \supset -\frac{1}{2} \sum_{h,n} \log[1 - k(h,n)] \qquad k(2,n) = 1 - \frac{\alpha_K}{\beta \mathcal{J}} |n| + \cdots$$

$$-\beta F \supset -\sum_{n=2}^{\infty} \log \frac{n}{\beta J} + \text{const} \to \#\beta J - \frac{3}{2} \log \beta J + \text{const}$$

$$\frac{\langle \psi_i(0) \psi_j(\tau) \psi_i(0) \psi_j(\tau) \rangle}{\langle \psi_i(0) \psi_i(0) \rangle \langle \psi_j(\tau) \psi_j(\tau) \rangle} \propto 1 + i \frac{\beta J}{N} e^{\frac{2\pi\tau}{\beta}}$$

SYK saturates Maldacena bound

Gravity dual: Quantum AdS2

Jackiw-Teitelboim
AdS2 background

$$I_{JT} = -\frac{1}{16\pi G}\left[\int d^2x \phi\sqrt{g}(R+2) + 2\int_{bdy}\phi_b K\right]$$

Maldacena, Stanford, Yang, 1606.01857

Almheiri, Polchinski, 1402.6334

Quantum Chaos

$$\langle V(a) W_3(b+\hat{u}) V(0) W(\hat{u})\rangle \sim \frac{\beta\Delta^2}{C} e^{\frac{2\pi\hat{u}}{\beta}}$$

Same pattern of symmetry breaking

Schwarzian action

SYK dual to a quantum AdS2

Results

A feature of quantum chaos is level statistics given by RMT

Density is doable analytically

Phys. Rev. D 94, 126010 (2016)

Phys. Rev. D 96, 066012 (2017)

SYK, Quantum Chaos

=

RMT?

A few weeks later
Cotler et al.1611.04650

Exact diagonalization and statistical eigenvalues analysis of the SYK model

Analytical calculation of thermodynamic properties

Further role of chaos in black hole physics

$$H = \frac{1}{4!} \sum_{i,j,k,l=1}^{N} J_{ijkl}\, \chi_i \chi_j \chi_k \chi_l$$

$$q = 4$$

$$N \leq 36$$

$> 5 \times 10^5$ eigenvalues

Defining relation of a Euclidean N-dimensional Clifford algebra

$$\{\chi_i, \chi_j\} = \delta_{ij}$$

$$P(J_{ijkl}) = \sqrt{\frac{N^3}{12\pi J^2}} \exp\left(-\frac{N^3 J_{ijkl}^2}{12 J^2}\right)$$

Spectral density

Partition Function: entropy, specific heat, quantum corrections

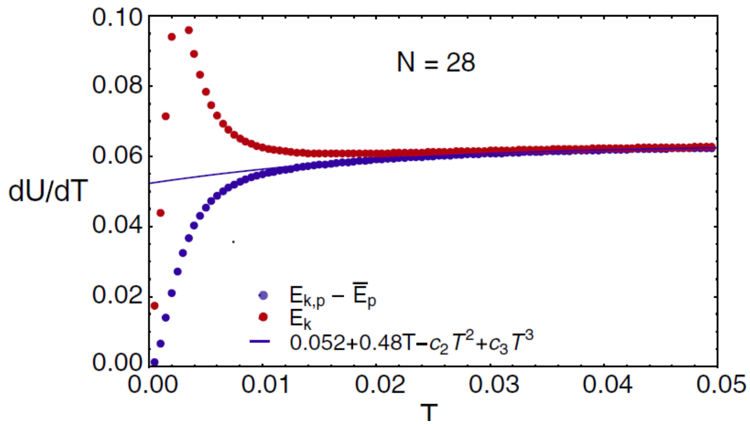
Level statistics

Thermodynamic properties

$$-\beta F \supset -\sum_{n=2}^{\infty} \log \frac{n}{\beta J} + \text{const} \to \#\beta J - \frac{3}{2} \log \beta J + \text{const}$$

$$\frac{dU(T)}{dT} = \frac{q(N)}{N} + c(N)T + c_2(N)T^2 + c_3(N)T^3$$

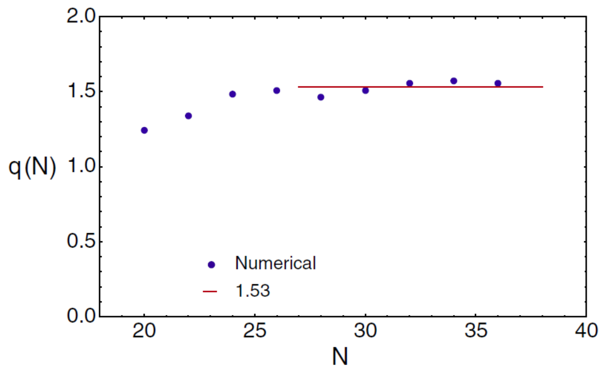
2-body interactions

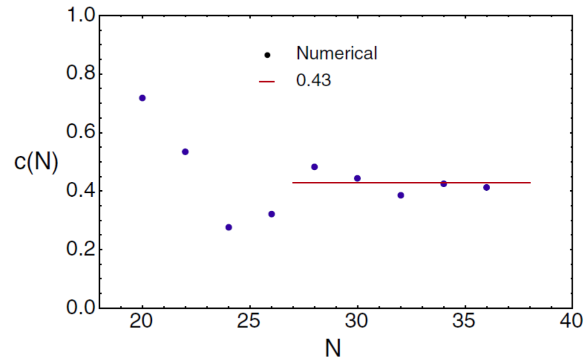


See also:

Jevicki et al., arXiv:1608.07567

Cotler et al. arXiv:1611.04650





$$q = 1.53 \pm 0.2, \qquad c/N = 0.43 \pm 0.10 \qquad S_0 = 0.21N$$

$q = 3/2$ $\quad$ $c/N = 0.40$ $\quad$ $S_0 = 0.23N$

Reasonably good agreement with large N predictions

Spectral density

$$H = \frac{1}{4!} \sum_{i,j,k,l=1}^{N} J_{ijkl}\, \chi_i \chi_j \chi_k \chi_l \qquad \{\chi_i, \chi_j\} = \delta_{ij}$$

Moments

Γ product of 4 χ matrices

$$M_{2p}(N) = \langle \mathrm{Tr}\, H^{2p} \rangle$$

$$M_{2p} = \left\langle \mathrm{Tr} \sum \prod_{k=1}^{2p} J_{\alpha_k} \Gamma_{\alpha_k} \right\rangle$$

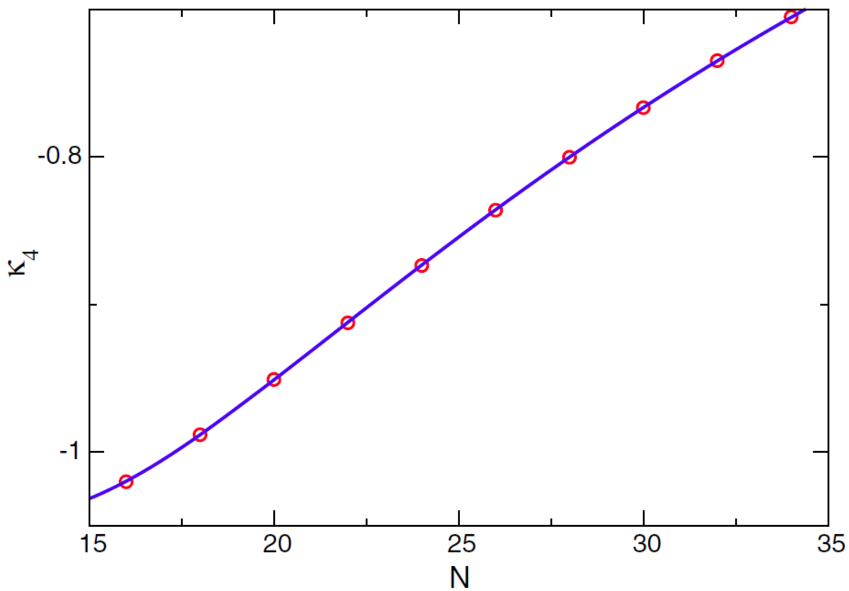
$N \to \infty$ $$M_{2p} = (2p-1)!! \langle J_\alpha^2 \rangle^p 2^{N/2}$$ Gaussian

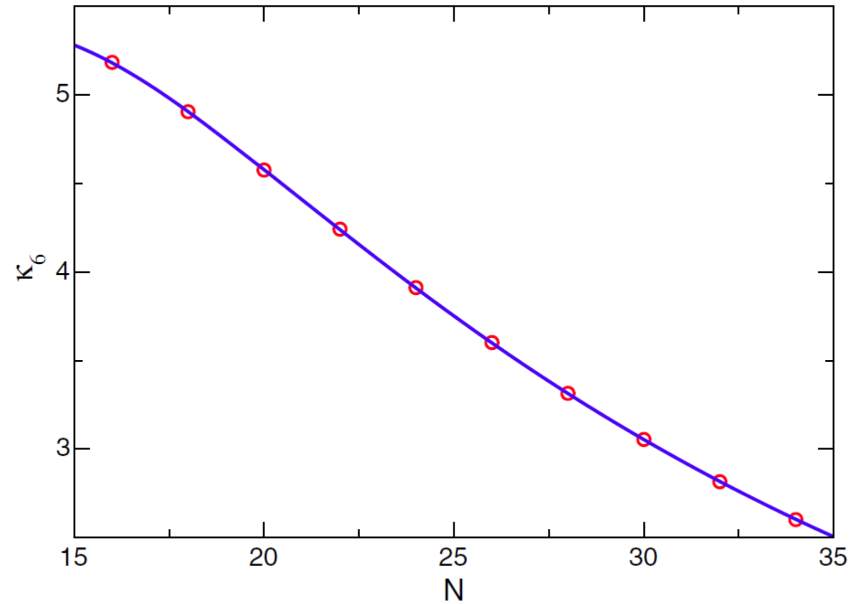
Finite N

$$\kappa_4 = \frac{M_4(N)}{M_2^2(N)} - 3 \qquad \kappa_4(N) = -\frac{32(N-4)(N^2-11N+36)}{N(N-1)(N-2)(N-3)}$$

$$\kappa_6 = \frac{M_6(N)}{M_2^3(N)} - 15 \frac{M_4(N)}{M_2^2(N)} + 30$$

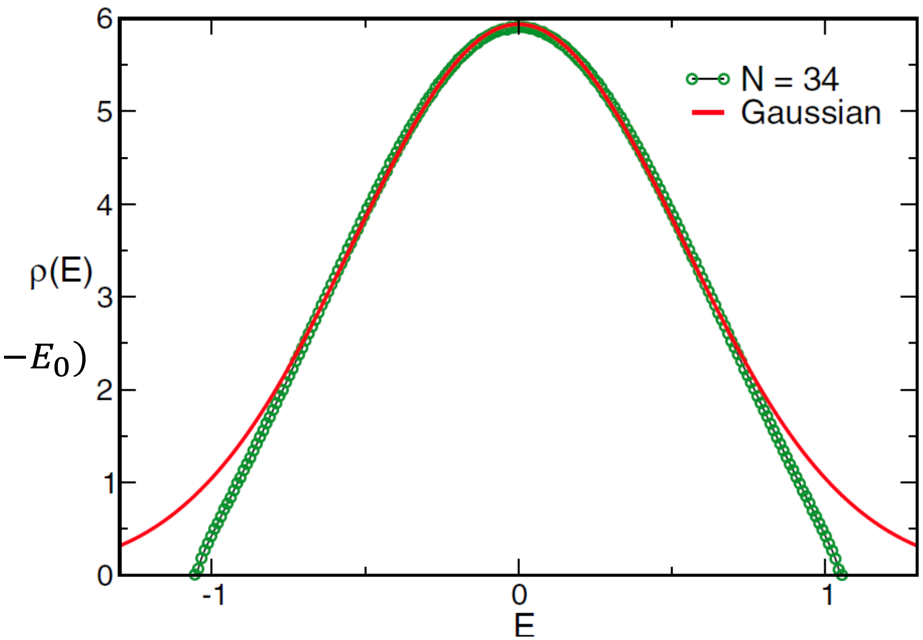
$$\kappa_6(N) = \frac{512(N-4)(11N^5 - 304N^4 + 3535N^3 - 21302N^2 + 65856N - 82656)}{(N-3)^2(N-2)^2(N-1)^2 N^2}$$





$$\rho(E) \sim e^{a\sqrt{(E-E_0)}}$$

?



Can we do better?

Yes

Finite N

$$\Gamma_\alpha^2 = 1 \qquad \Gamma_\alpha \Gamma_\beta - (-1)^r \Gamma_\beta \Gamma_\alpha = 0$$

Intersecting relative to nested contraction

$$M_{2p}(d) = \langle \mathrm{Tr} H^{2p} \rangle = \left\langle \mathrm{Tr} \left(\sum_\alpha J_\alpha \Gamma_\alpha \right)^{2p} \right\rangle$$

Suppression factor assuming no correlations

$$\eta_{N,q} = \binom{N}{q}^{-1} \sum_{r=0}^{q} (-1)^r \binom{q}{r} \binom{N-q}{q-r}$$

Number of crossings of a diagram

$$\alpha_P$$

$$Tr[\Gamma_\alpha \Gamma_\beta \Gamma_\gamma \Gamma_\alpha \Gamma_\beta \Gamma_\gamma]$$

$$\frac{M_{2p}}{M_2^p} = \sum_{\alpha_p} \eta_{N,q}^{\alpha_p} \quad ?$$

Riordan-Touchard formula!

J. Riordan, Mathematics of Computation 29, 215 (1975) $Exact\ q \propto N^{1/2}$

$$\frac{M_{2p}}{M_2^p} = \sum_{\alpha_p} \eta_{N,q}^{\alpha_p} = \frac{1}{(1-\eta_{N,q})^p} \sum_{k=-p}^{p} (-1)^k \eta_{N,q}^{k(k-1)/2} \binom{2p}{p+k}$$

Spectral density for Q-Hermite polynomials

Erdos, Mathematical Physics, Analysis and Geometry 17, 9164 (2014).

Renjie Feng et al. 1801.10073

$$\rho(E) = \rho_{\mathrm{QH}}(E) = c_N \sqrt{1-(E/E_0)^2} \prod_{k=1}^{\infty} \left[1 - 4\frac{E^2}{E_0^2}\left(\frac{1}{2+\eta^k+\eta^{-k}}\right)\right]$$

$$\sigma^2 = \binom{N}{q} \frac{1}{(q-1)!N^{q-1}} \qquad E_0^2 = \frac{4\sigma^2}{1-\eta} \qquad Q = \eta$$

More precisely……

AGG, Jia, Verbaarschot, arXiv:1801.02696

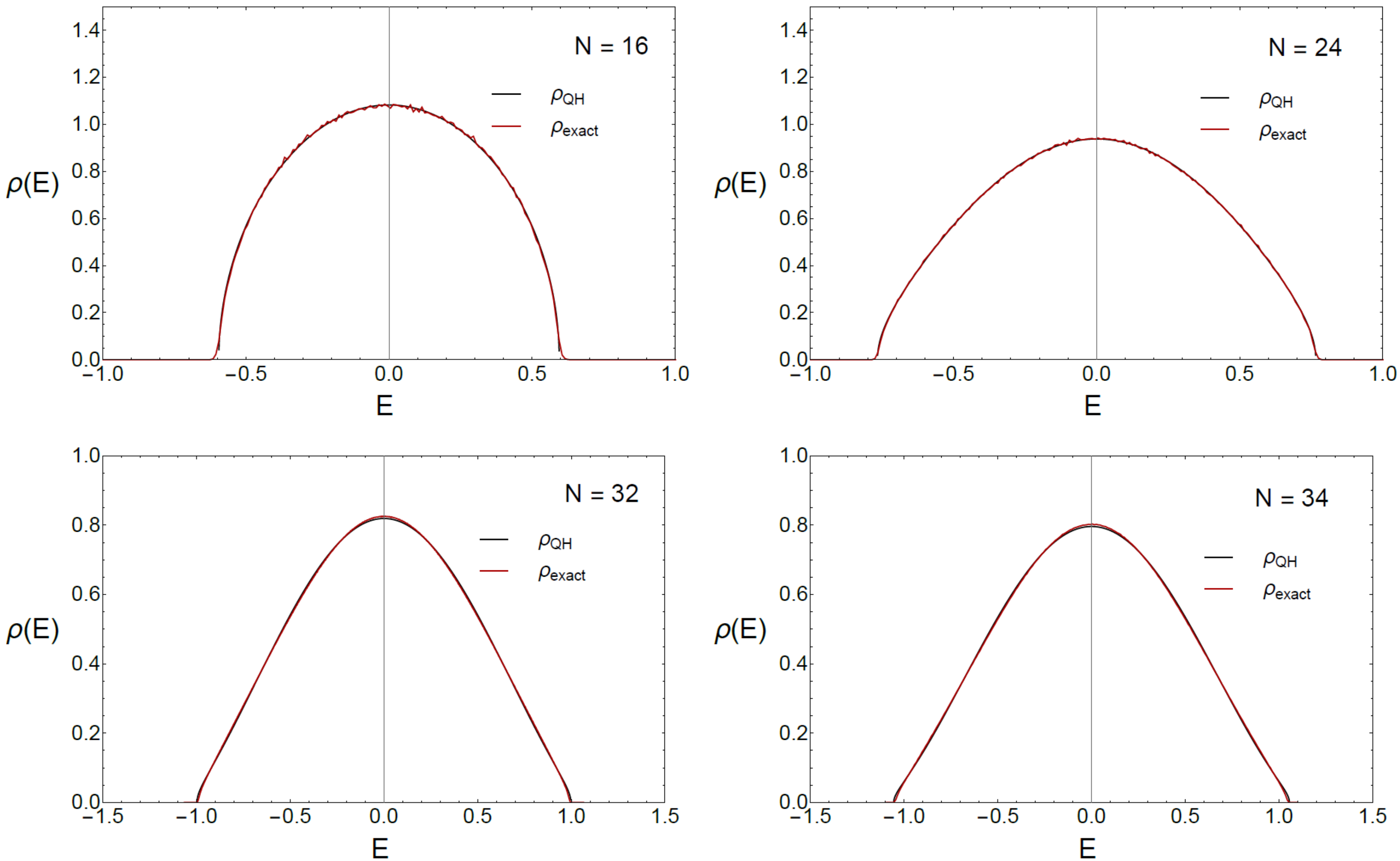
Q-Hermite gives the exact 1/N corrections

$1/N^2$ corrections can be computed explicitly

$$\frac{M_{2p}}{M_2^p} = \frac{1}{(1-\eta)^p} \sum_{k=-p}^{p} (-1)^k \eta^{k(k-1)/2} \binom{2p}{p+k} + \binom{p}{3} \left(\frac{8q^3}{N^2}\right)$$

$1/N^3$ and higher orders are feasible (in progress)

Improvement in thermodynamic properties?



No fitting parameters !!!!

Looking at the tail

$$\rho_{\rm edge}(E) \approx c_N \exp\left[\frac{\pi^2}{2\log\eta} - \frac{2\pi\sqrt{2}\sqrt{1-(E/E_0)}}{\log\eta}\right]\left(1-\exp\left[\frac{4\pi}{\log\eta}\sqrt{2}\sqrt{1-(E/E_0)}\right]\right)$$

Exact?

$$= 2c_N \exp\left[\frac{\pi^2}{2\log\eta}\right]\sinh\left[\frac{2\pi\sqrt{2}\sqrt{1-(E/E_0)}}{-\log\eta}\right]$$

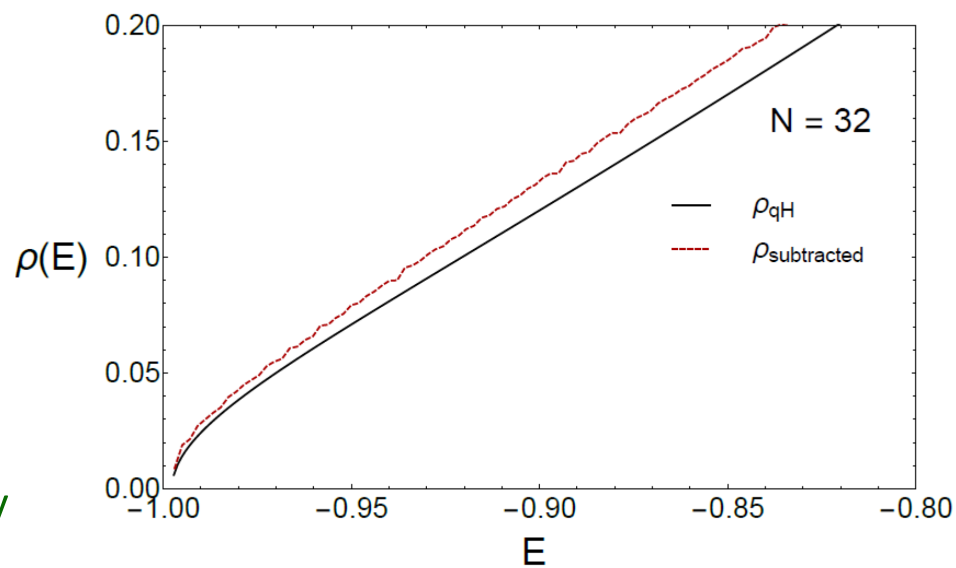
Exact Bagrets, et al., 1702.08902 Stanford, Witten 1703.04612

Exponential increase

Cardy's formula

Bethe's formula

Black holes density



Sqrt edge

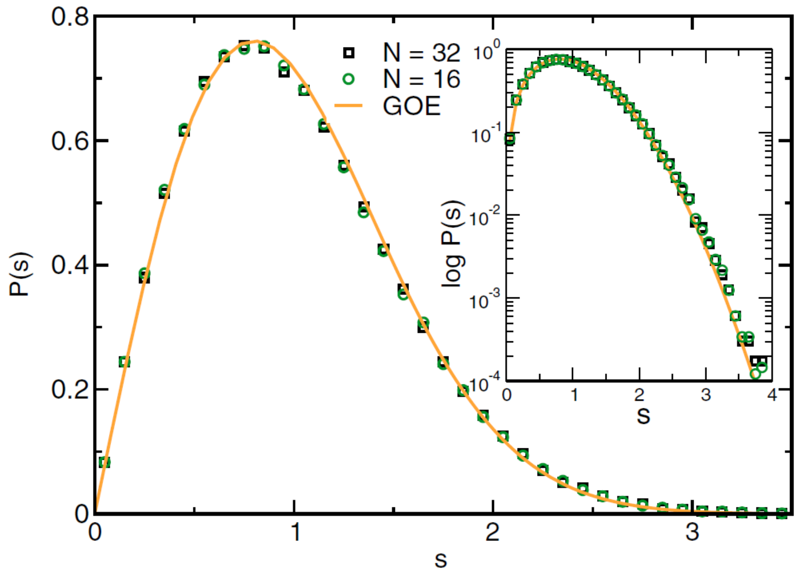
Typical of random matrices

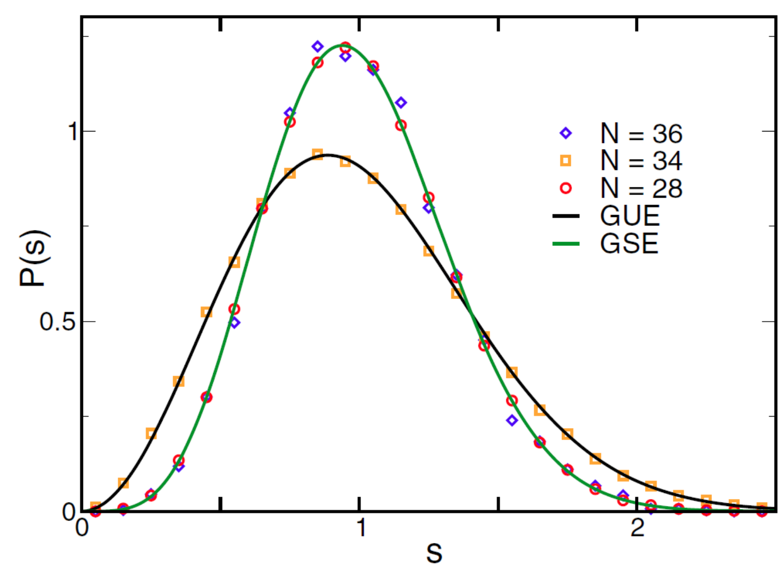
Bulk Level statistics

Level spacing distribution

$$P(s) = \sum_i \langle \delta(s - \epsilon_i + \epsilon_{i+1}) \rangle \quad \epsilon_i = E_i/\Delta$$

$$P(s) \approx a_\beta s^\beta \exp(-b_\beta s^2) \qquad \beta = 1\ GOE \qquad \beta = 2\ GUE \qquad \beta = 4\ GSE$$





Universality class depends on N

N	$(C_1K)^2$	$(C_2K)^2$	C_1KC_2K	RMT
2	1	-1	$-i\Gamma_5$	GUE
4	-1	-1	$-\Gamma_5$	GSE
6	-1	1	$-i\Gamma_5$	GUE
8	1	1	Γ_5	GOE
10	1	-1	$-i\Gamma_5$	GUE
12	-1	-1	Γ_5	GSE

Why? Clifford algebra representations in N dimensions

You, Ludwig, Xu
1604.06964

Bulk level statistics is well described by random matrix theory

Weak N dependence of short-range spectral correlators

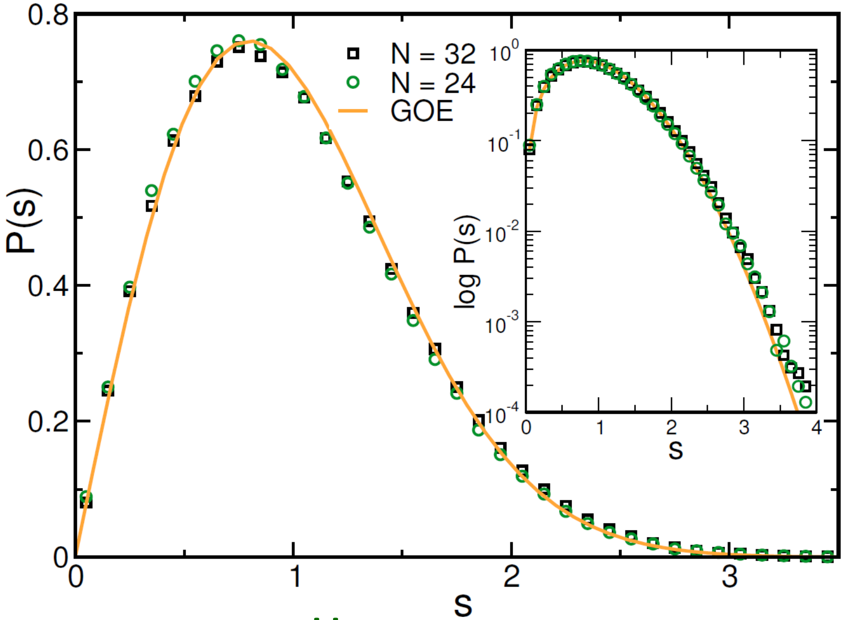
SYK is ergodic and always thermalizes for high energy initial states

Correction to random matrix, low energy?

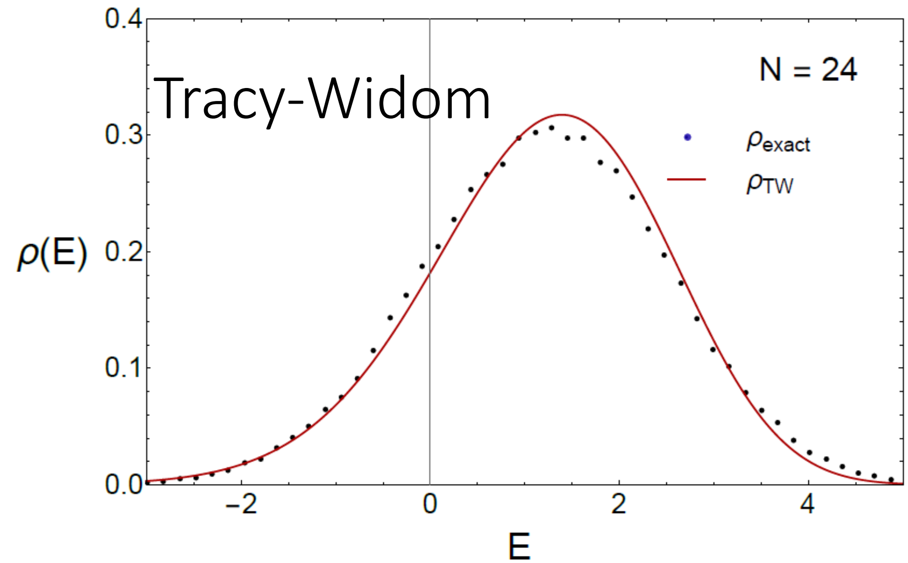
Level statistics close to the edge

Exponential increase of the density

Level spacing distribution



Distribution lowest eigenvalue



Still agreement with random matrix theory !!

Random matrix correlations characterize quantum black holes

Tenfold way in black hole physics?

Yes!

Universality and Thouless energy in the supersymmetric SYK Model

AGG, Jia, Verbaarschot, 1801.01071

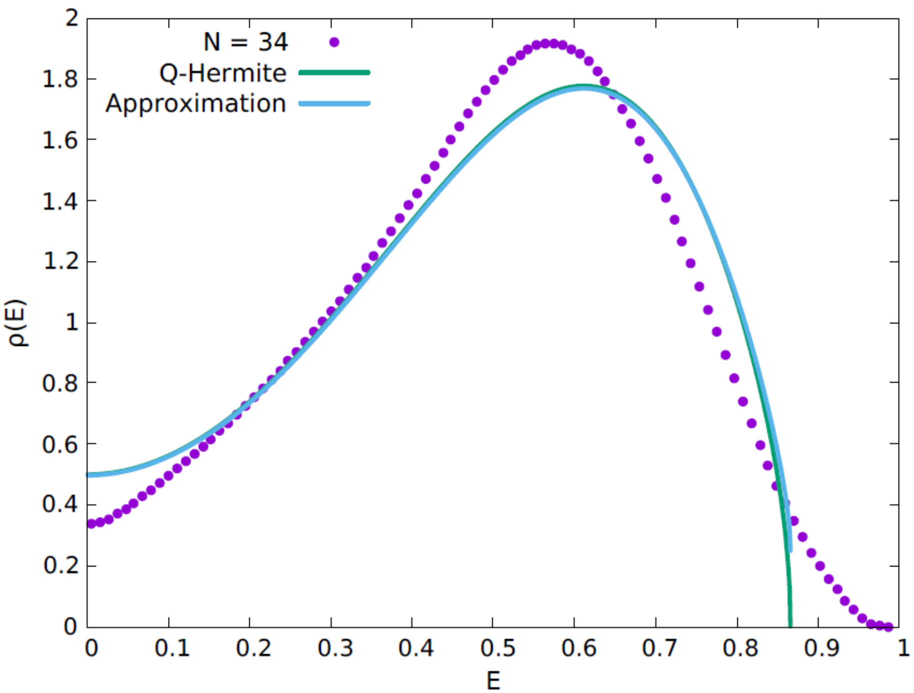
1610.08917

Fu, Gaiotto, Maldacena, Sachdev

1702.01738

Li, Liu, Xin, Zhou

$$H = Q^2 \qquad Q = i \sum_{i,j,k=1}^{N} J_{ijk}\,\gamma_i\,\gamma_j\,\gamma_k$$



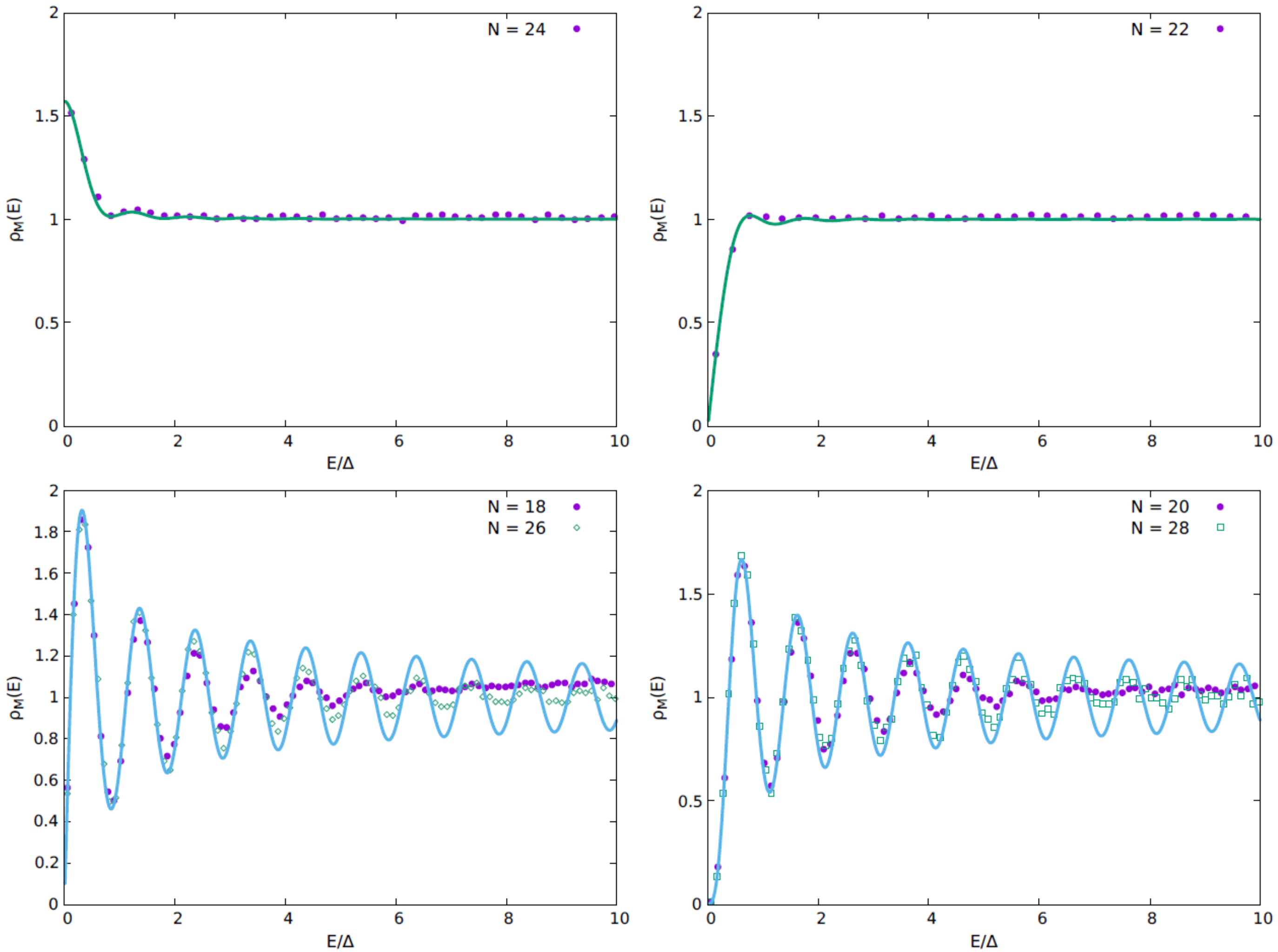
$$\rho_{\text{asym}}(E) = c_N \cosh\left(\frac{\pi \arcsin(E/E_0)}{\log|\eta|}\right) \exp\left[2\frac{\arcsin^2(E/E_0)}{\log|\eta|}\right]$$

Q-Hermite analytical prediction just OK

Why?

Microscopic spectral density

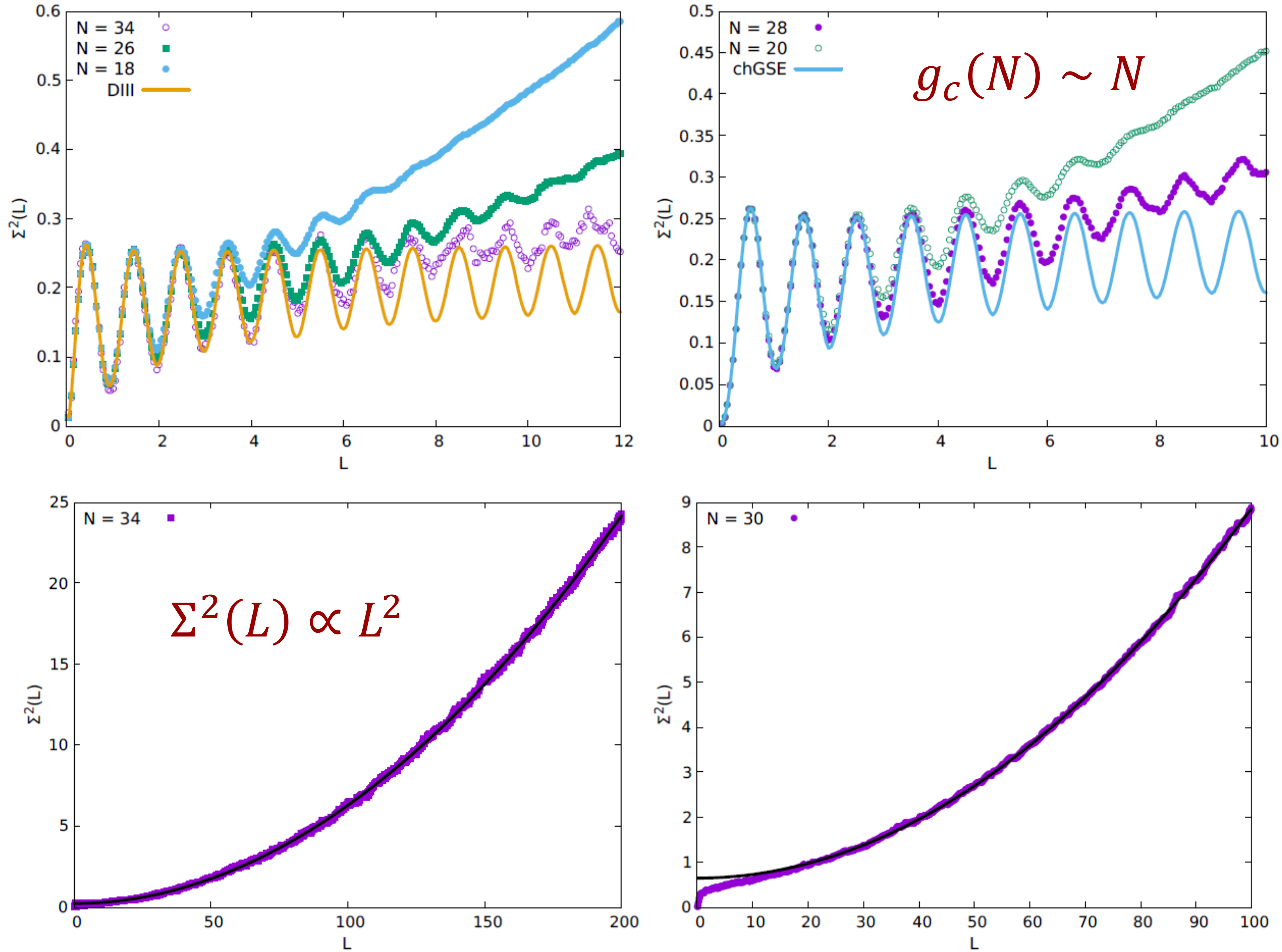
$$\rho_M(E) = \Delta\rho\left(\frac{E}{\Delta}\right)$$



Agreement with random matrix theory

(quantum) Gravity dual interpretation?

Number Variance & Thouless Energy

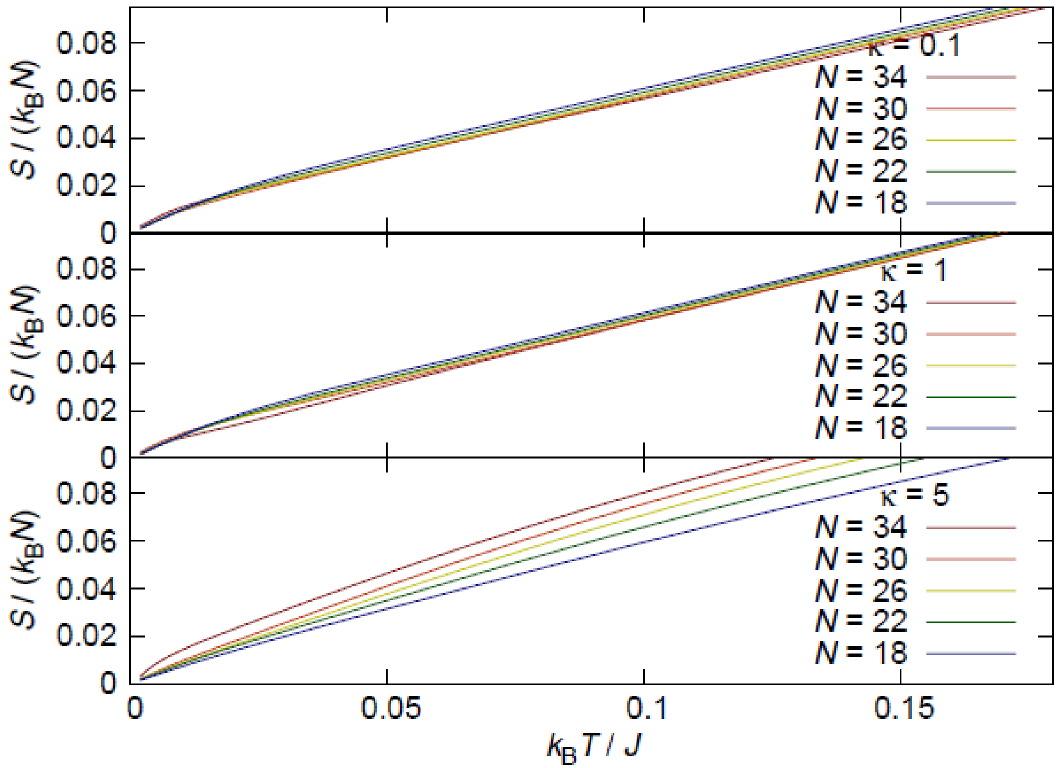


Chaotic-Integrable transition in the SYK model

A. Bermudez, AGG, B. Loureiro, M. Tezuka, arXiv:1707.02197

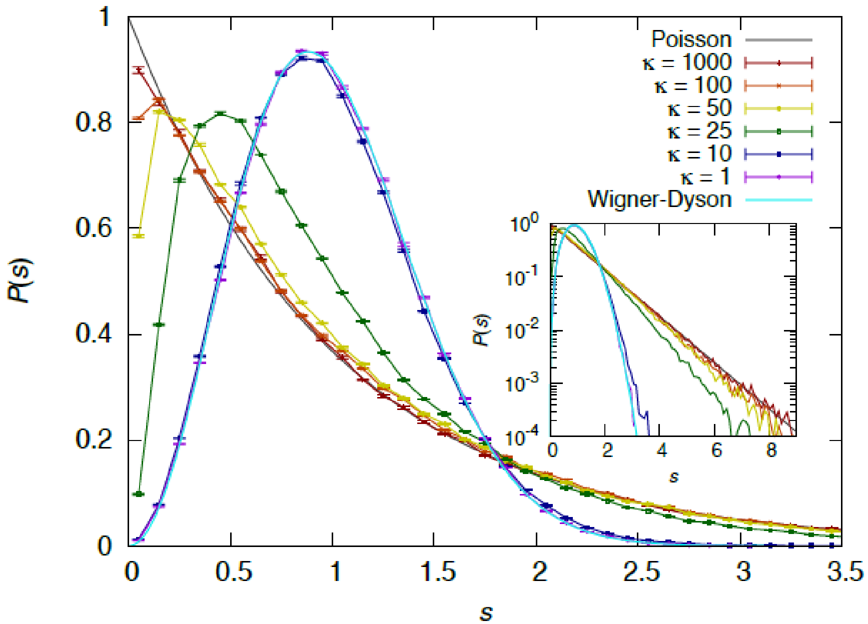
$$H = \frac{\kappa}{4!}\sum_{i,j,k,l=1}^{N} J_{ijkl}\chi_i\chi_j\chi_k\chi_l + \frac{i}{2!}\sum_{i,j=1}^{N} K_{ij}\chi_i\chi_j$$

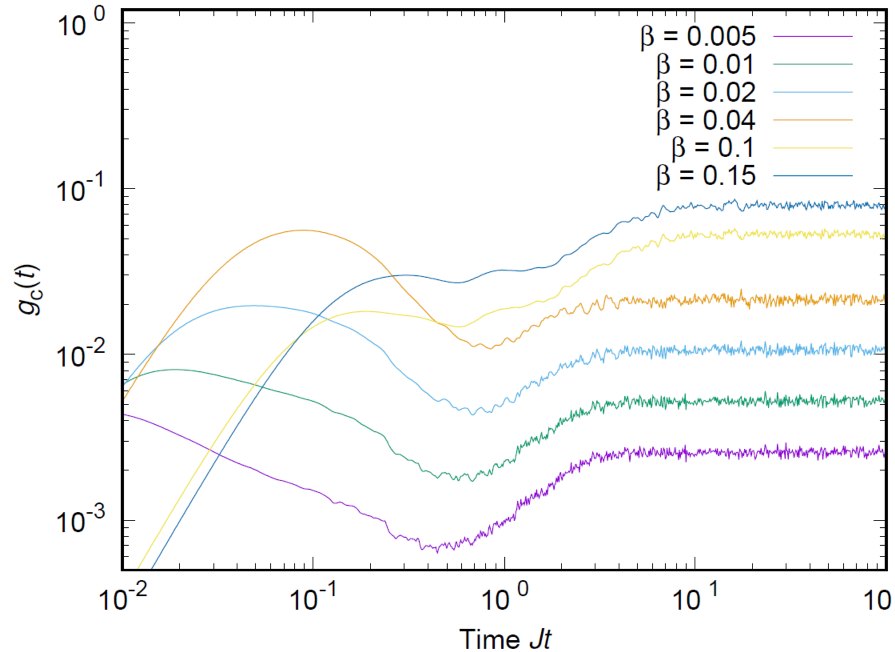
See also, Chen et al., PRL119, 207603 (2017)

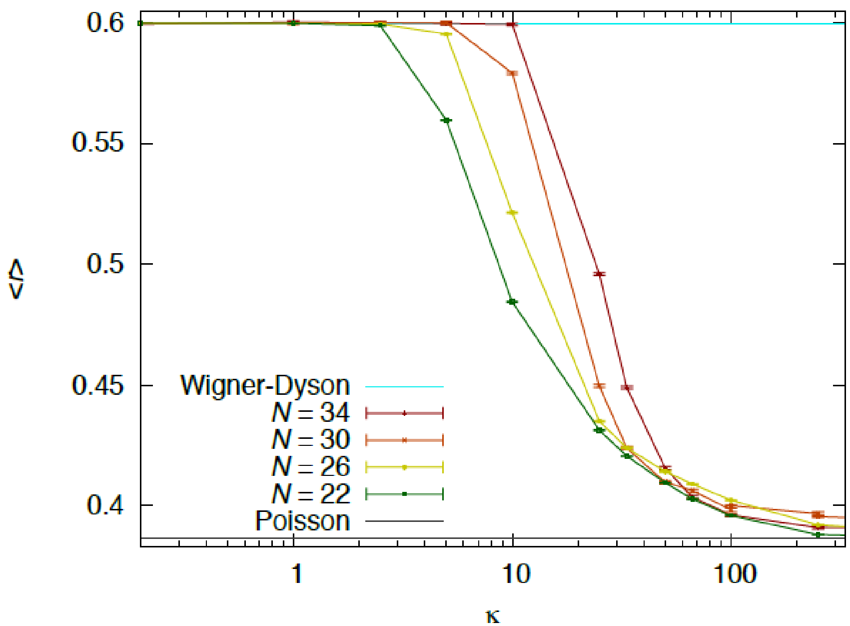


$$S_0 = 0$$

$$C_v = \mathrm{c}T$$
$$\mathrm{c} \propto N$$



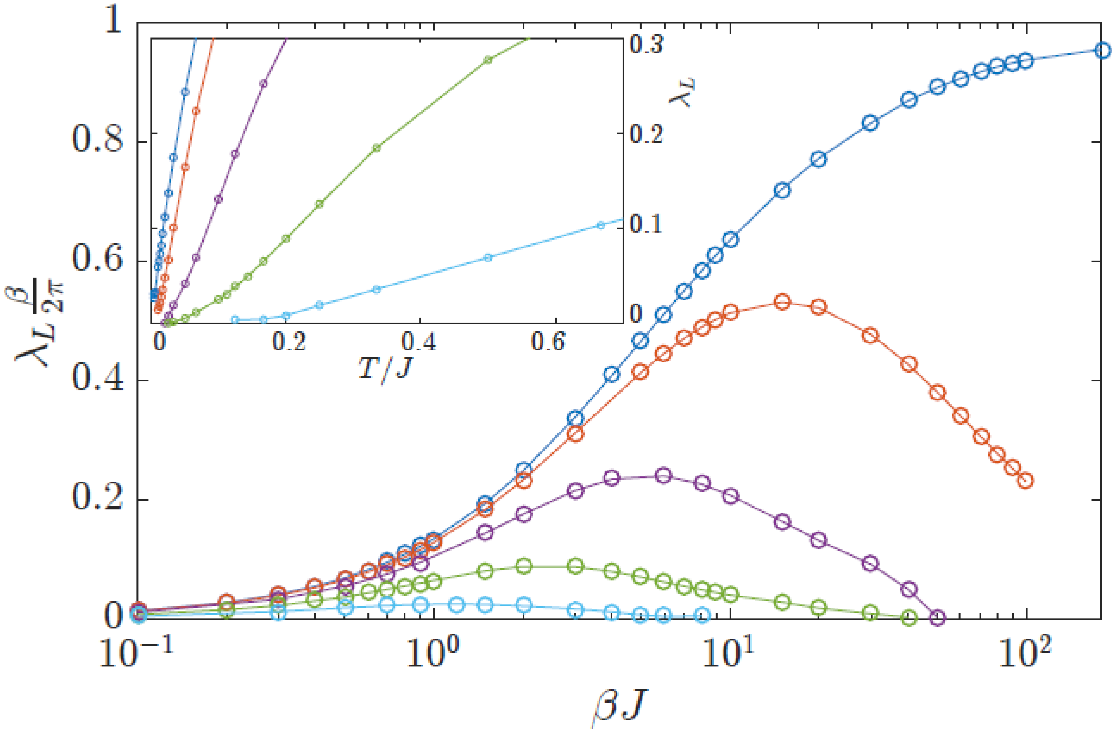




$$g(t,\beta) \equiv \left\langle \frac{Z(t,\beta)Z^*(t,\beta)}{Z(0,\beta)^2} \right\rangle$$

$$Z(t,\beta) = \mathrm{Tr} e^{-\beta H - iHt}$$

Chaotic – Integrable transition at $\kappa = \kappa_c$



Finite Lyapunov exponent only for high temperature

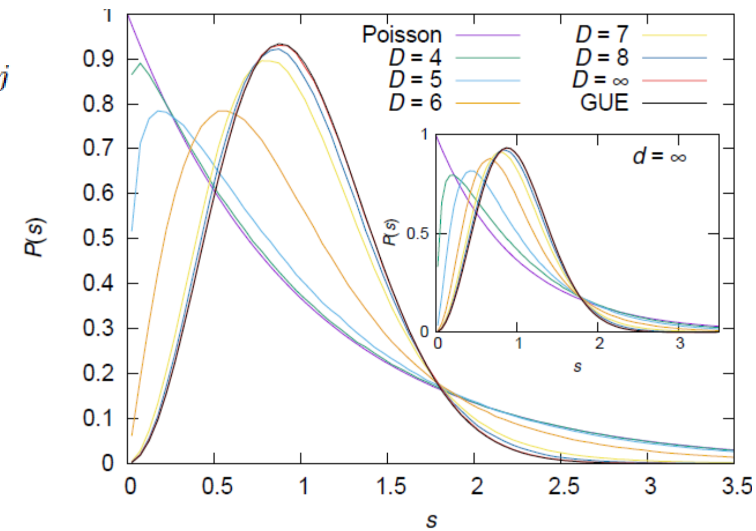
Chaos only for not too low T or not too strong coupling

Gravity dual?

Many body localization in the SYK model

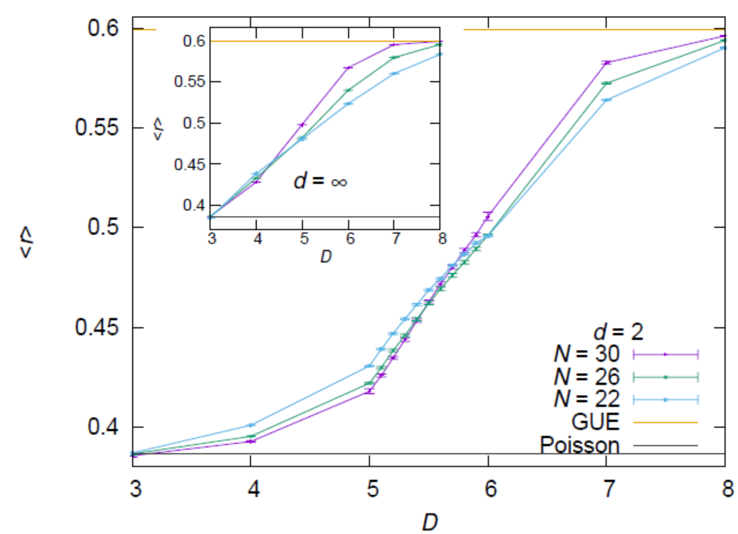
AGG, Tezuka1801.03204

$$H = \sum_{1=i<j<k<l}^{N} \tilde{J}_{ijkl}(D)\, \chi_i \chi_j \chi_k \chi_l + i\kappa \sum_{1=i<j}^{N} \tilde{K}_{ij}(d)\, \chi_i \chi_j$$



Reduction of the range interaction D

Many body metal-insulator transition



Different from Jian, Yao PRL 119, 206602 (2017)

What type of transition?

$$P(s) \sim e^{-As} \; A > 1 \; s \gg 1$$

$$\Sigma^2(L) \sim \chi L \;\; \chi < 1 \; L \gg 1$$

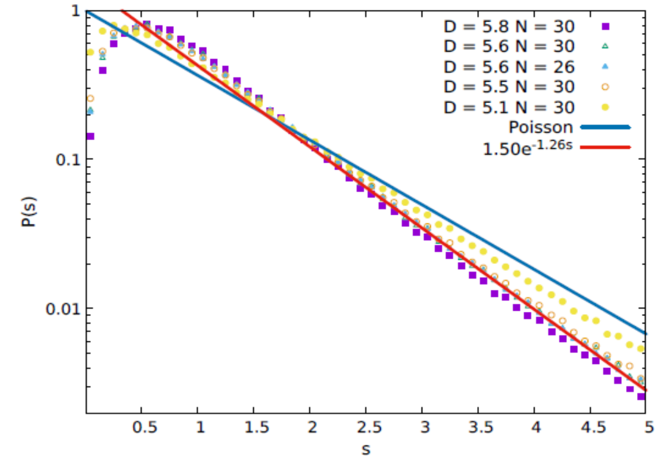
No correlation hole (dip)

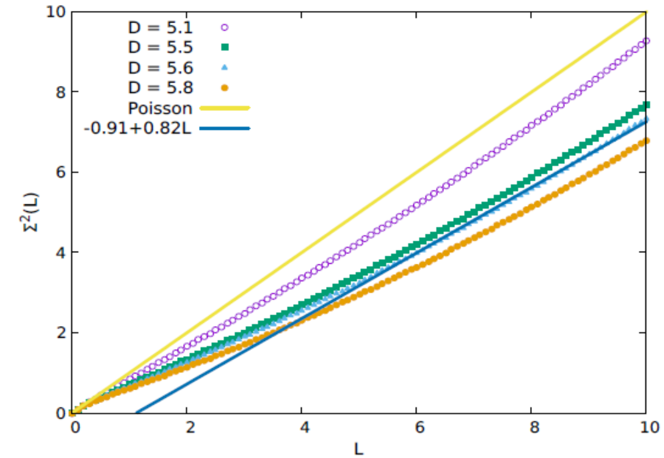
Coherence and interactions both important!

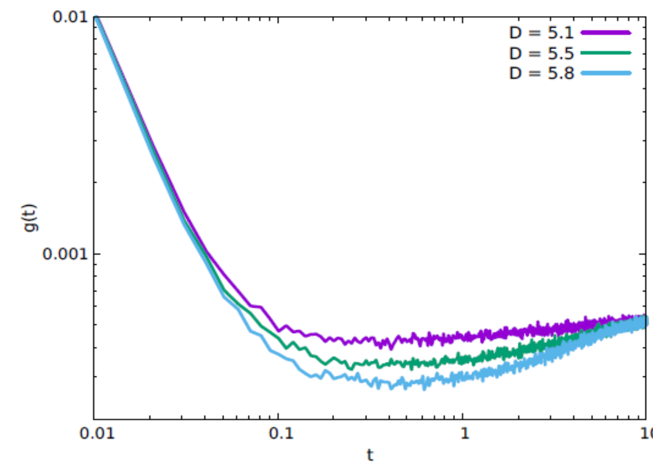
MBL transition in SYK

Gravity dual?

Analytical MBL transition?







Conclusions

Ergodicity and random matrix behaviour seems to be distinctive features of quantum black holes and their field theory duals

Quantum black holes may be classified according to random matrix theory

Generalized SYK models undergoing metal-insulator and chaotic-integrable transitions open new research avenues in both condensed matter and high energy

Thanks!

Low Temperature
Strong coupling

$$S = -N\frac{\alpha_S}{\mathcal{J}}\int d\tau\{f,\tau\}$$

$$-\beta F \supset \frac{N\alpha_S}{\mathcal{J}}\int_0^\beta d\tau\left\{\tan\frac{\pi\tau}{\beta},\tau\right\} = 2\pi^2\alpha_S\frac{N}{\beta\mathcal{J}}$$

OTOC:

$$\frac{\langle\psi_i(0)\psi_j(\tau)\psi_i(0)\psi_j(\tau)\rangle}{\langle\psi_i(0)\psi_i(0)\rangle\langle\psi_j(\tau)\psi_j(\tau)\rangle} \propto 1 + i\frac{\beta J}{N}e^{\frac{2\pi\tau}{\beta}}$$

Linear Specific Heat

Exponential Growth of OTOC

Same pattern of symmetry breaking

SYK
dual
Quantum AdS2

Why is SYK interesting?

Toy model of quantum gravity

"Solvable" for large but finite N

Explicit 2-pt, 4-pt calculations

Emergent conformal symmetry in the IR

Explicitly and spontaneously broken but weakly
Same as in AdS_2 gravity backgrounds
Exponential growth of the spectral density

Maximally chaotic

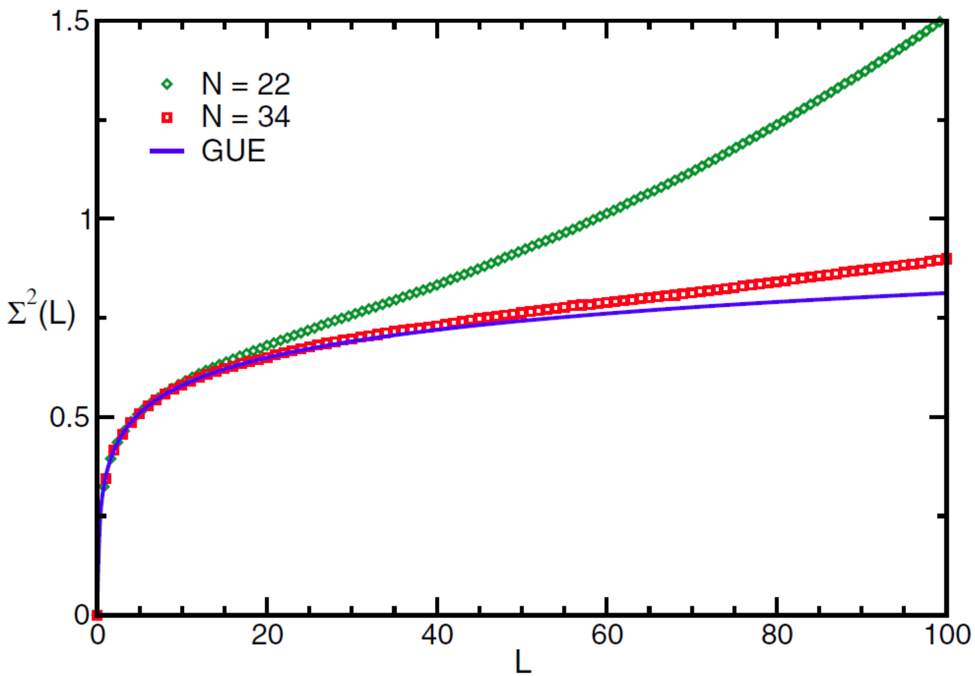
Lyapunov exponent as in black-holes that saturates the Maldacena-Shenker-Stanford bound on chaos

Remarks on the Sachdev-Ye-Kitaev model
J. Maldacena, D. Stanford, Phys. Rev. D 94, 106002 (2016)

Thouless Energy in the SYK model

Number variance $$\Sigma^2(L) = \langle N^2(L)\rangle - \langle N(L)\rangle^2$$

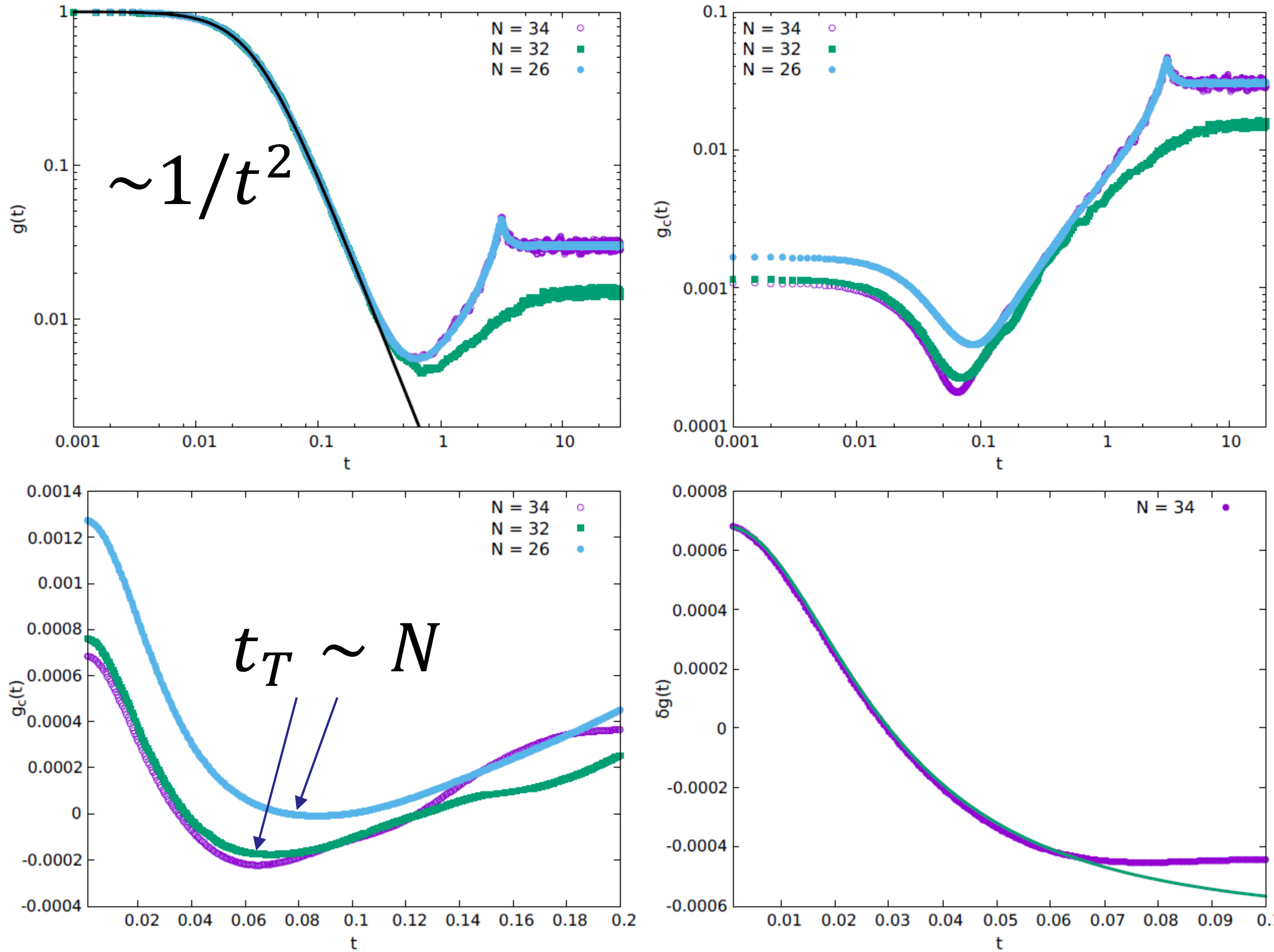
GUE $$\Sigma^2(L) \approx c_\beta(\log(d_\beta \pi L) + \gamma + 1 + e_\beta \ldots)$$



$$\Sigma^2(L) \propto L^2$$
$$L \gg 1$$

$$g_c(N)?$$

Spectral form factor



$$\delta g(t) = g(0)\frac{\beta^2 - t^2}{\beta^2 + t^2}$$