Fate of the Higgs mode near quantum criticality

and

Modulated Floquet topological insulators

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September 23, 2013

Fate of the Higgs mode near quantum criticality

Snir Gazit

Assa Auerbach

Dan Arovas
circa 1981

Subir Sachdev

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D.P., Auerbach, Arovas, *Phys. Rev. B* **84**, 174522 (2011)
D.P. and Sachdev, *Phys. Rev. B* **86**, 054508 (2012)
Gazit, D.P., Auerbach, *Phys. Rev. Lett.* **110**, 140401 (2013)
Gazit, D.P., Auerbach, Arovas, arXiv:1309.1765 (2013)

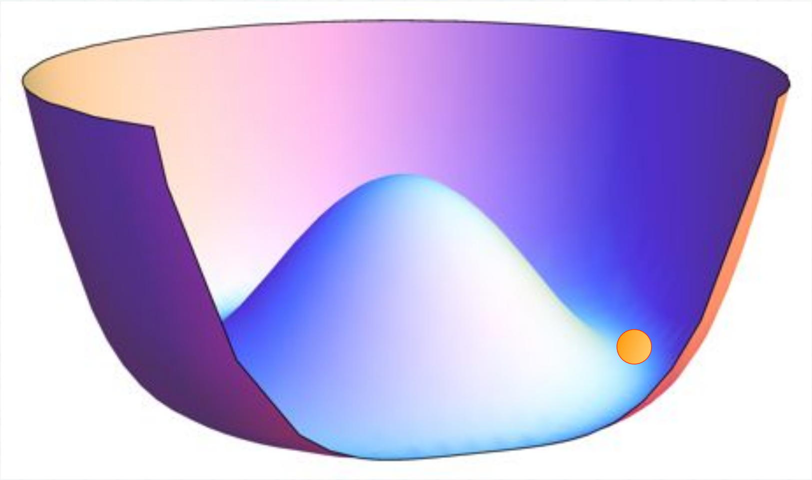
Spontaneous symmetry breaking

N-component order parameter:

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \ldots \\ \phi_N \end{pmatrix}$$

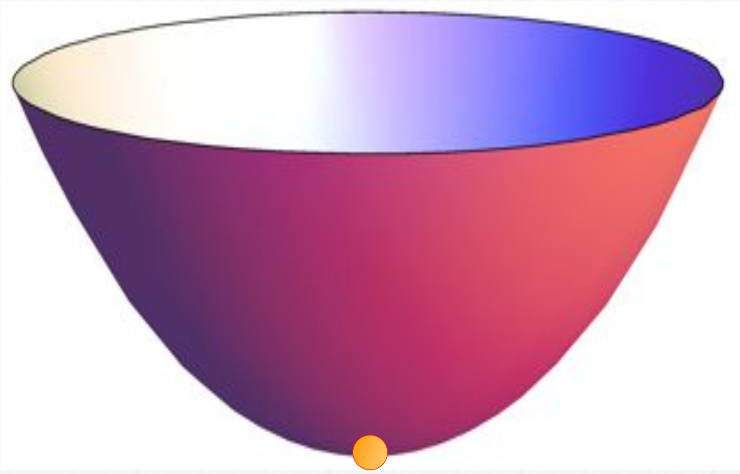
$$V(\boldsymbol{\phi}) = g\boldsymbol{\phi}^2 + u\left(\boldsymbol{\phi}^2\right)^2$$

$g < 0$



$\langle \boldsymbol{\phi} \rangle \neq 0$

$g > 0$



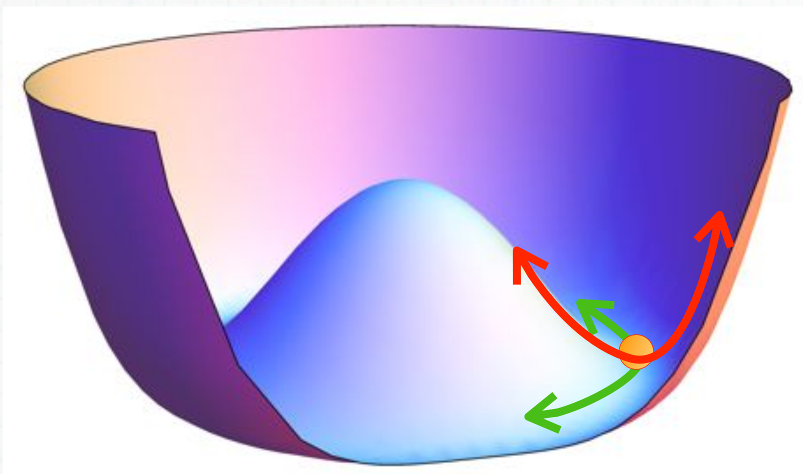
$\langle \boldsymbol{\phi} \rangle = 0$

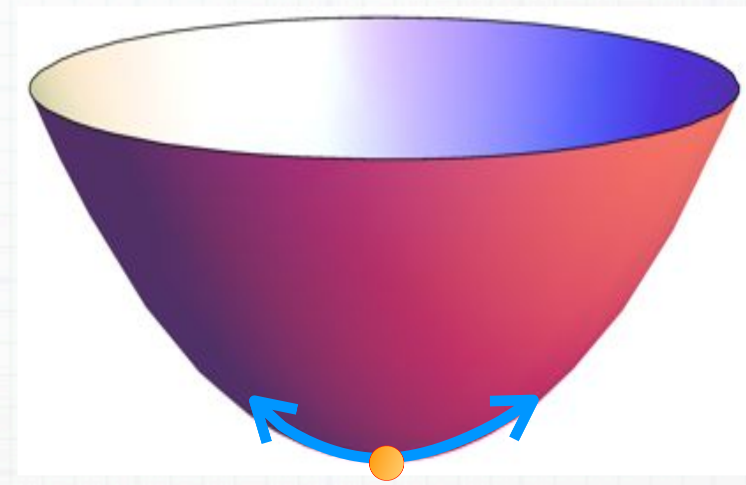
$g_c = 0$

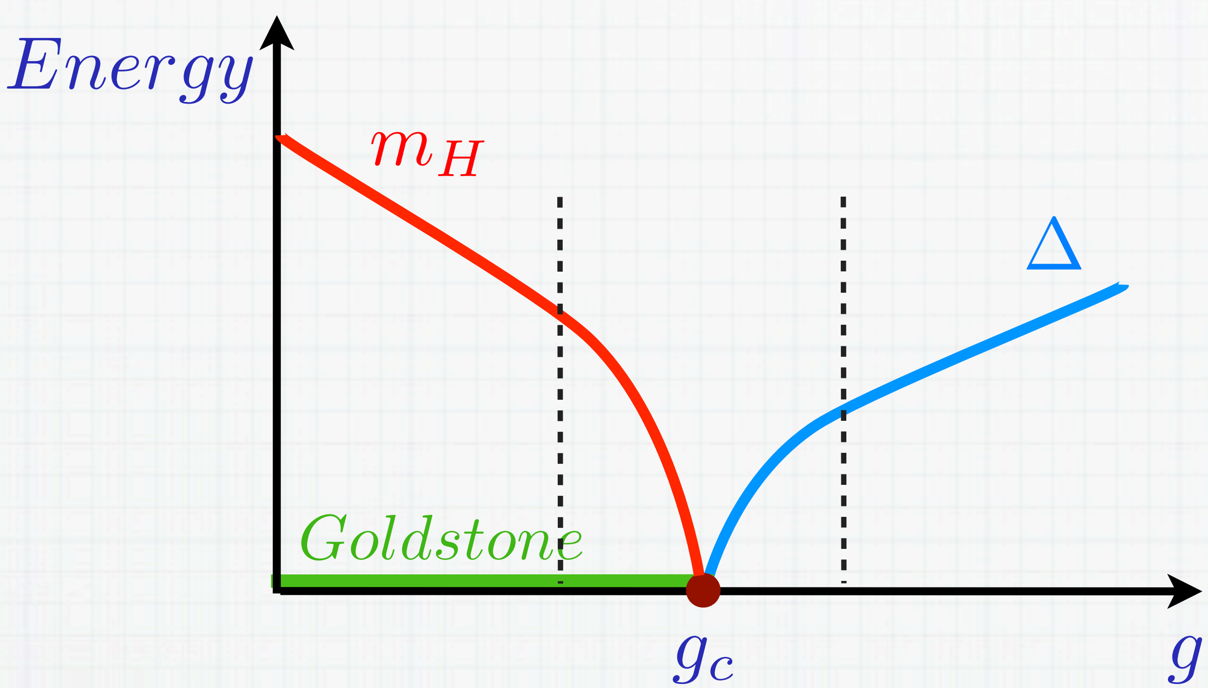
g

Collective excitations

Relativistic dynamics: $S = \int d^d x\, dt\, \left[(\partial_t \phi)^2 - (\nabla \phi)^2 - V(\phi) \right]$







Mean field:

$$\Delta \sim A|g - g_c|^{0.5}$$

$$m_H \sim \sqrt{2} A |g - g_c|^{0.5}$$

$$\frac{m_H}{\Delta} = \sqrt{2}$$

Bosons in an optical lattice

Bose-Hubbard model $$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + U \sum_i n_i^2 - \mu \sum_i n_i$$

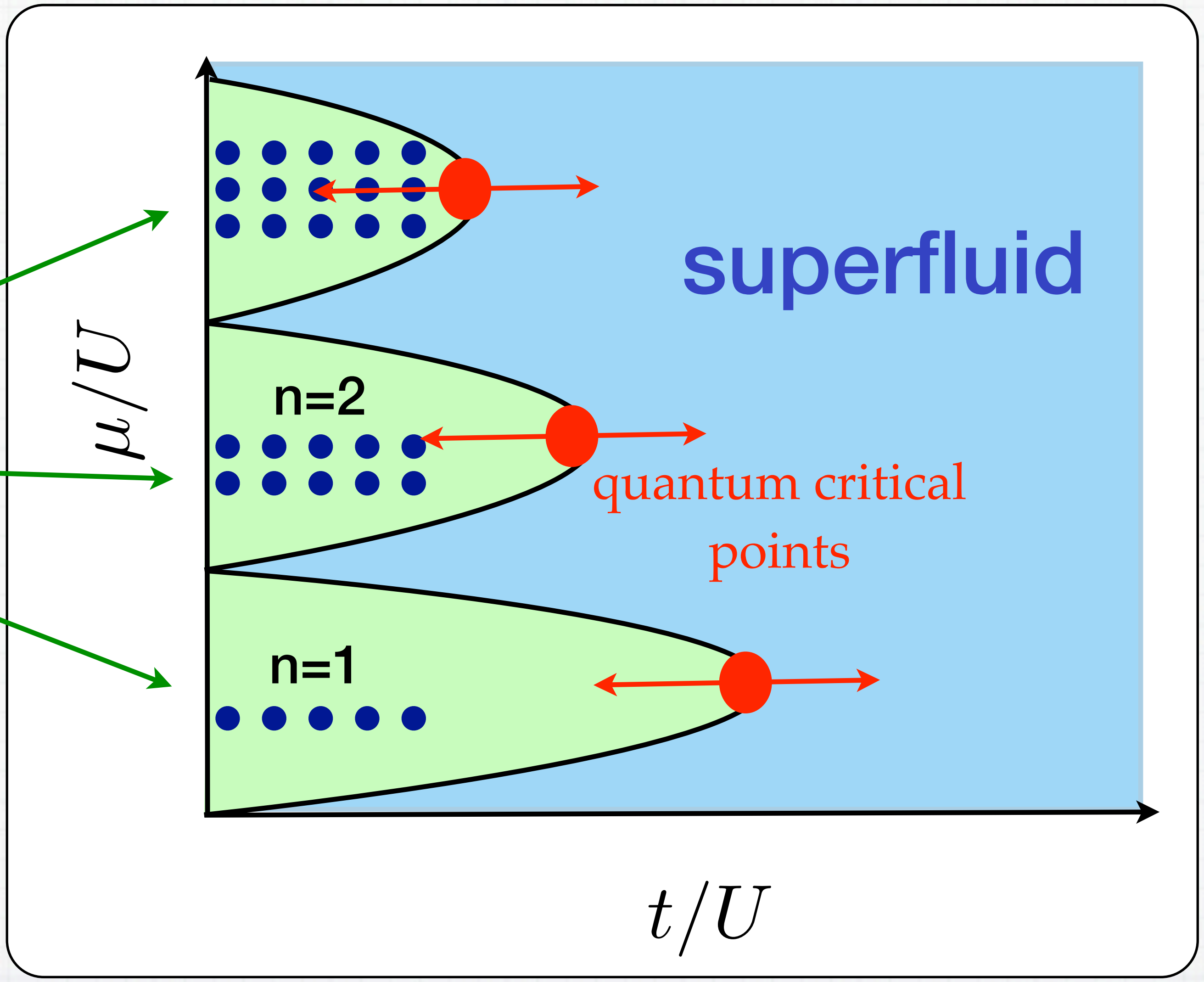
Large t/U : system is a superfluid (Bose condensate).

Small t/U : system is a Mott insulator (gap for charge fluctuations).

Mott insulator
incompressible

$$\langle b \rangle = 0$$

$$\frac{\partial \langle n_i \rangle}{\partial \mu} = 0$$



Dynamics in the superfluid phase

Far from Mott, Gross-Pitaevskii action:

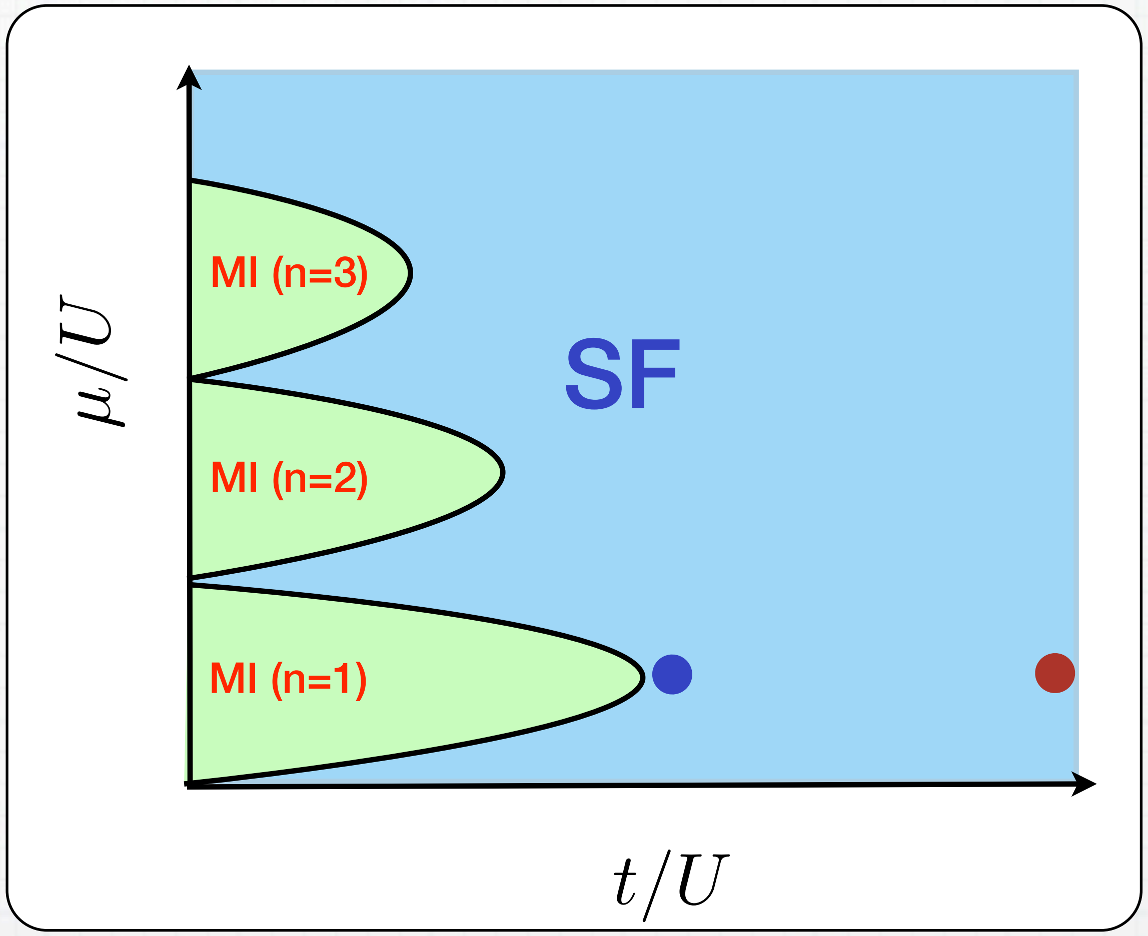
$$S = \int d^3r \left(-i\psi^* \partial_t \psi - \frac{1}{2m^*} |\nabla \psi|^2 + \mu |\psi|^2 - g|\psi|^4 \right)$$

Galilean invariant. Goldstone mode, but **no Higgs**.

Near Mott at integer filling, particle-hole symmetry:

$$S = \int d^3r \left(|\partial_t \psi|^2 - c^2 |\nabla \psi|^2 + r|\psi|^2 - u|\psi|^4 \right)$$

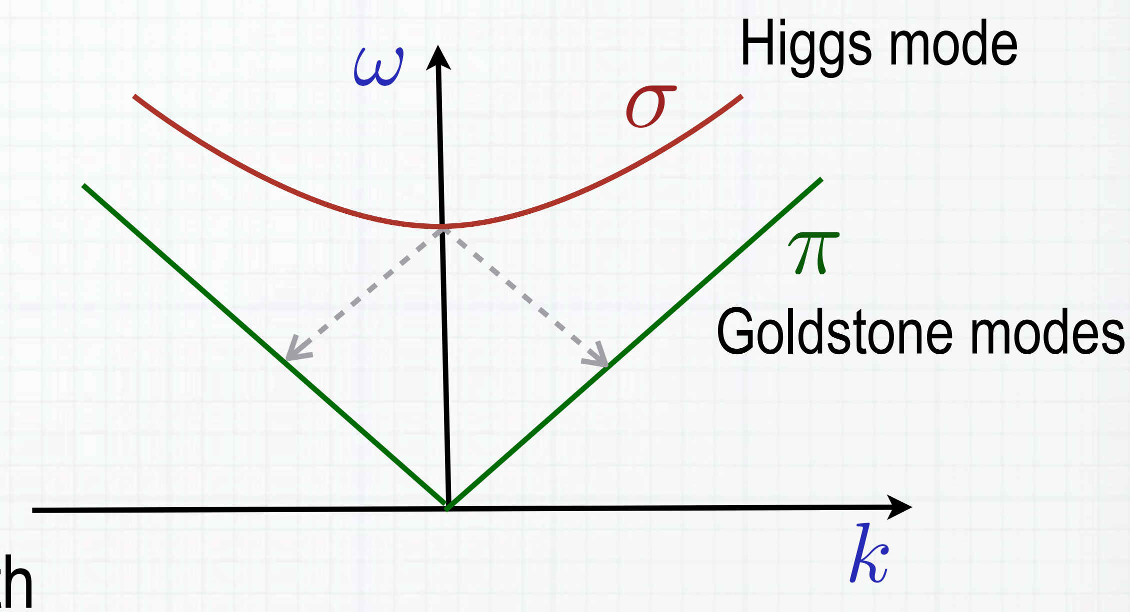
Emergent Lorentz invariance.
Goldstone **and** Higgs mode.



Varma (2002)
Huber, Theiler, Altman, Blatter (2008)

The Higgs decay

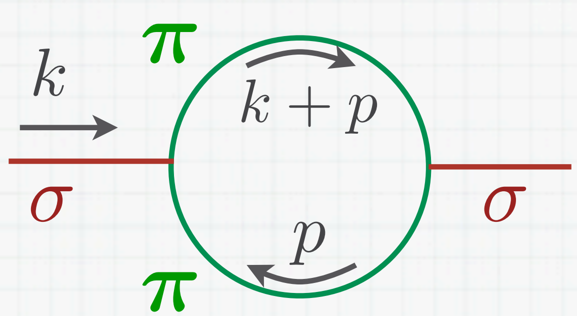
The Higgs mode can decay into a pair of Goldstone bosons:



***d*=3** Higgs decay rate is bounded by coupling strength

***d*=2** self-energy diverges at low frequency, even at weak coupling :

$$\Sigma_\sigma(k) =$$



$$\propto \int \frac{d^3p}{(2\pi)^3} \frac{1}{p^2(p+k)^2} = \frac{1}{8|k|}$$

$$\mathrm{Im}\Sigma(\omega) \propto \frac{1}{|\omega|}$$

infrared divergent!

(Nepomnyaschii)^2 (1978)
Sachdev (1999), Zwerger (2004)

Different behavior of different response functions

longitudinal susceptibility

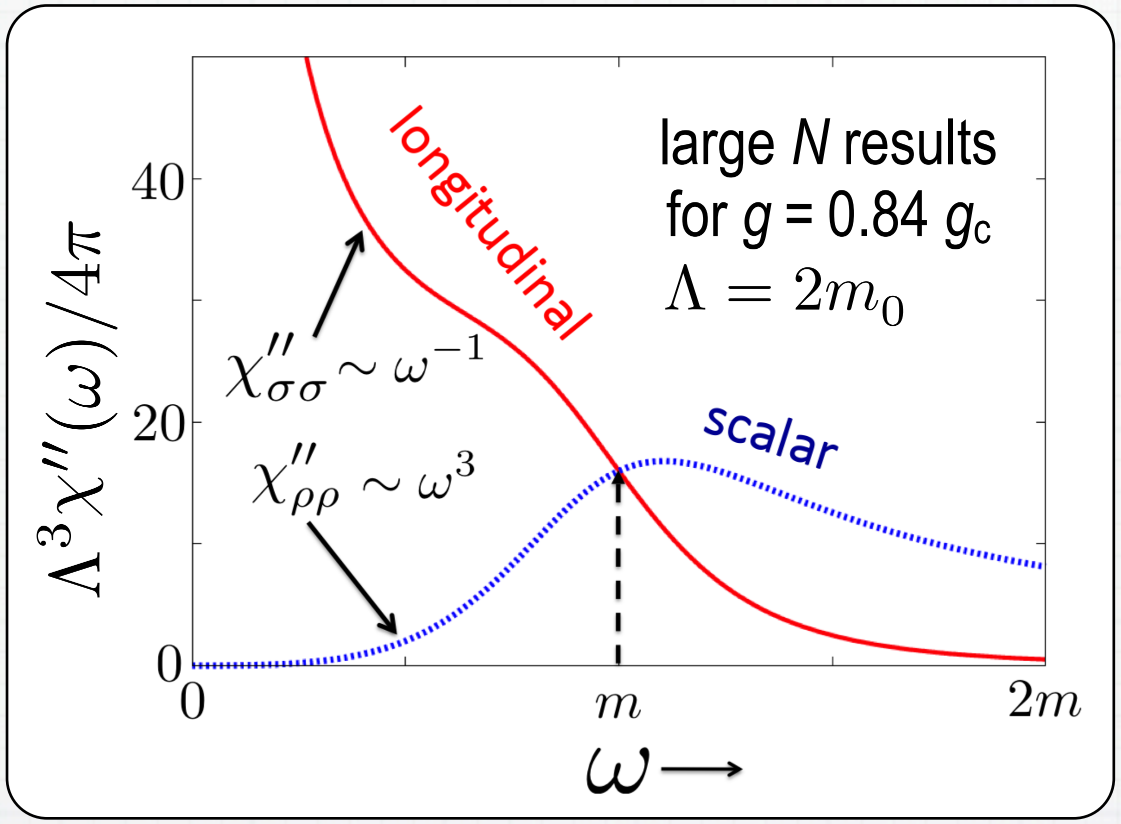
$$\chi_{\text{long}}(\omega) = \langle \phi_1(\omega)\phi_1(-\omega) \rangle \sim \omega^{-1}$$

infrared divergent in d=2

(Nepomnyaschii)[2] (1978)
Sachdev (1999), Zwerger (2004)

scalar susceptibility

$$\chi_{\text{scalar}}(\omega) = \langle |\vec{\phi}|^2(\omega)|\vec{\phi}|^2(-\omega) \rangle \sim \omega^3$$



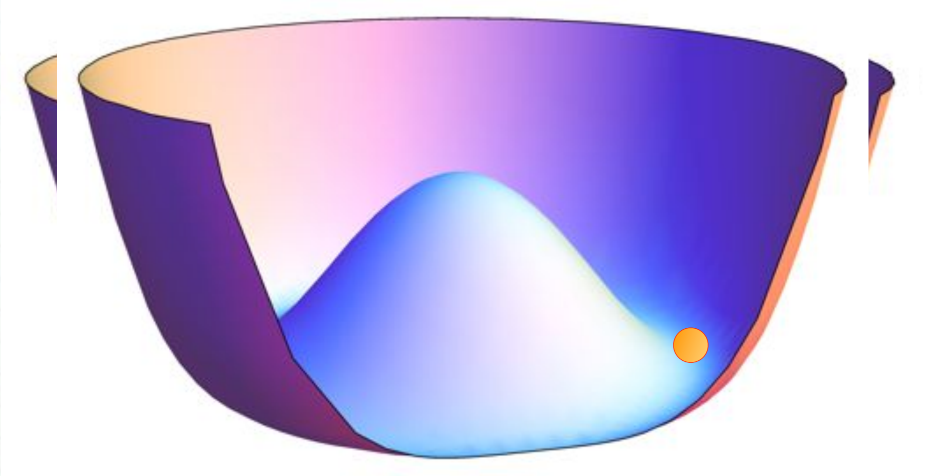
infrared regular in d=2

Podolsky, Auerbach and Arovas, *PRB* (2011)

Longitudinal versus scalar measurements

Longitudinal: couples to order parameter as a vector

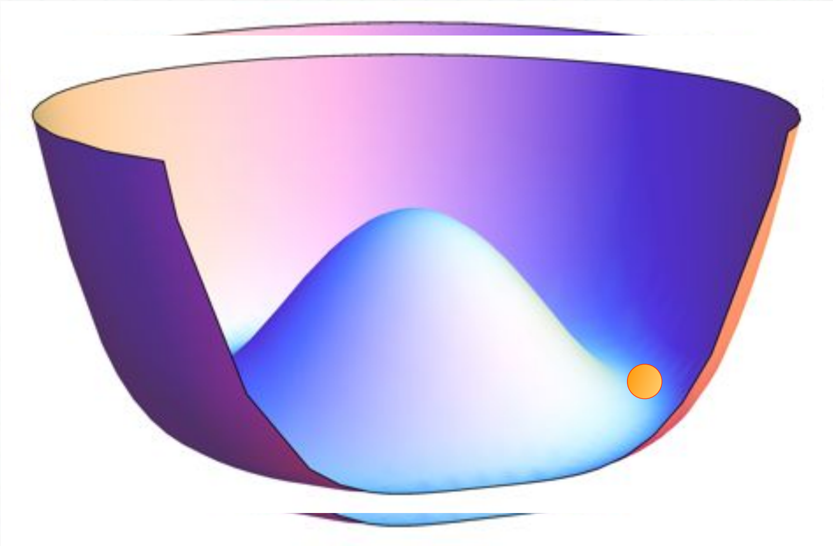
$$\mathcal{H}_{\text{probe}} = \vec{h}_{\text{ext}} \cdot \vec{\phi}$$



Example : neutron scattering in an antiferromagnet.

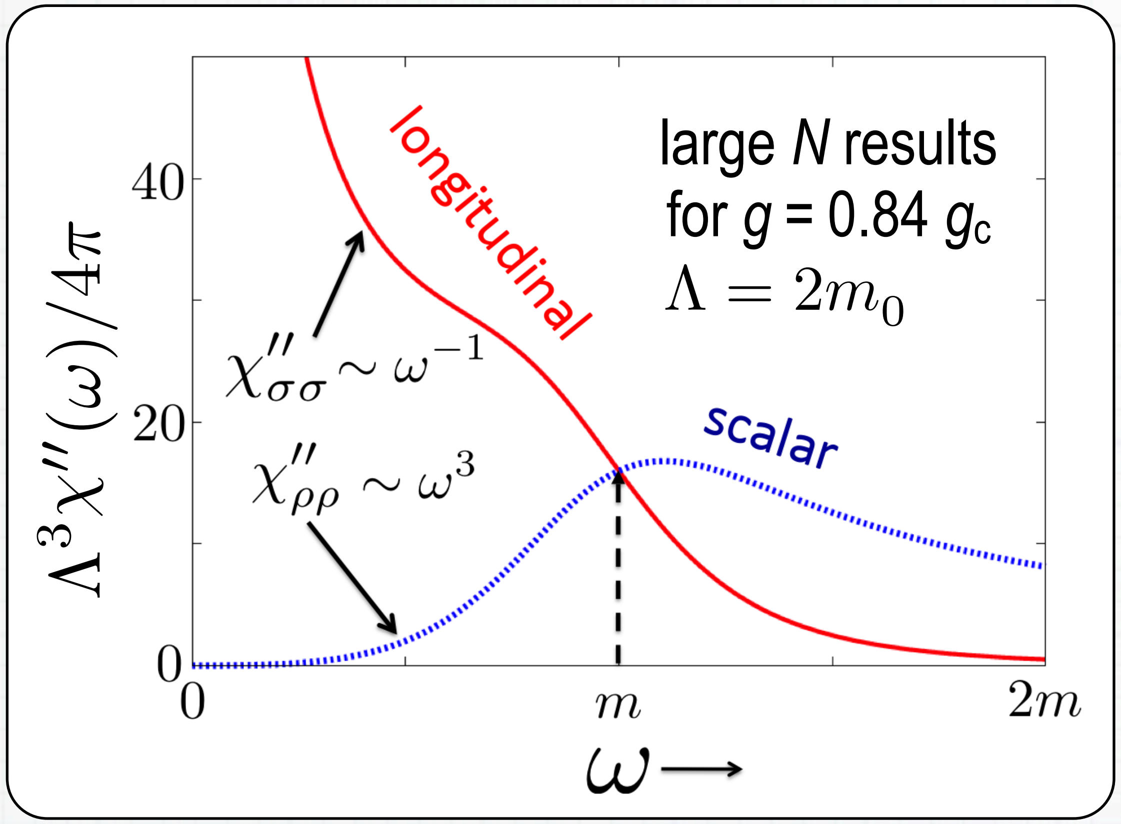
Scalar: couples to the *magnitude* of the order parameter

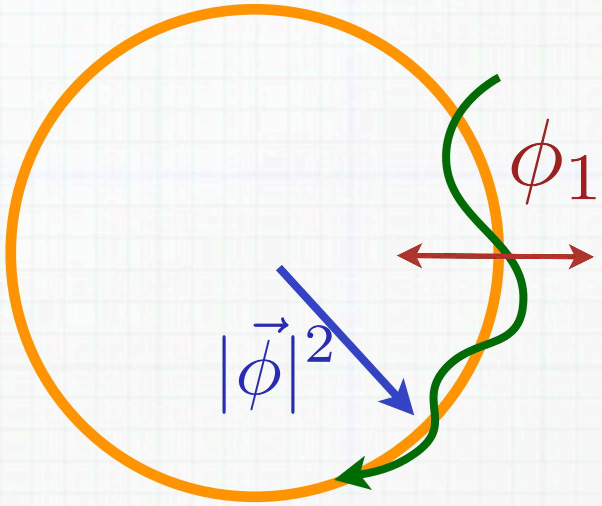
$$\mathcal{H}_{\text{probe}} = u_{\text{ext}} \, |\vec{\phi}|^2$$



Example: Lattice depth modulation of bosons

Why is the scalar response function sharper?





Radial motion is less damped, since it is not effected by azimuthal meandering.

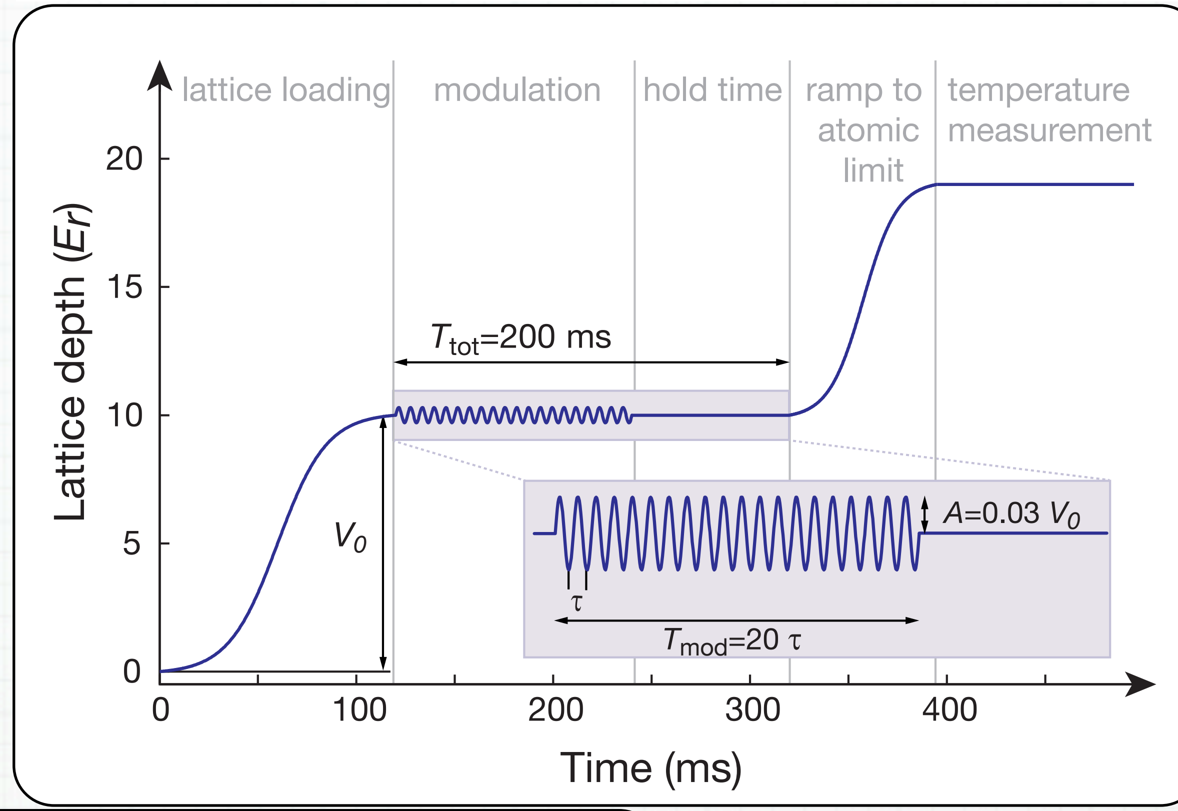
Higgs near criticality: ^{87}Rb

M. Endres *et al.*, *Nature* **487**, 454 (2012)

LETTER

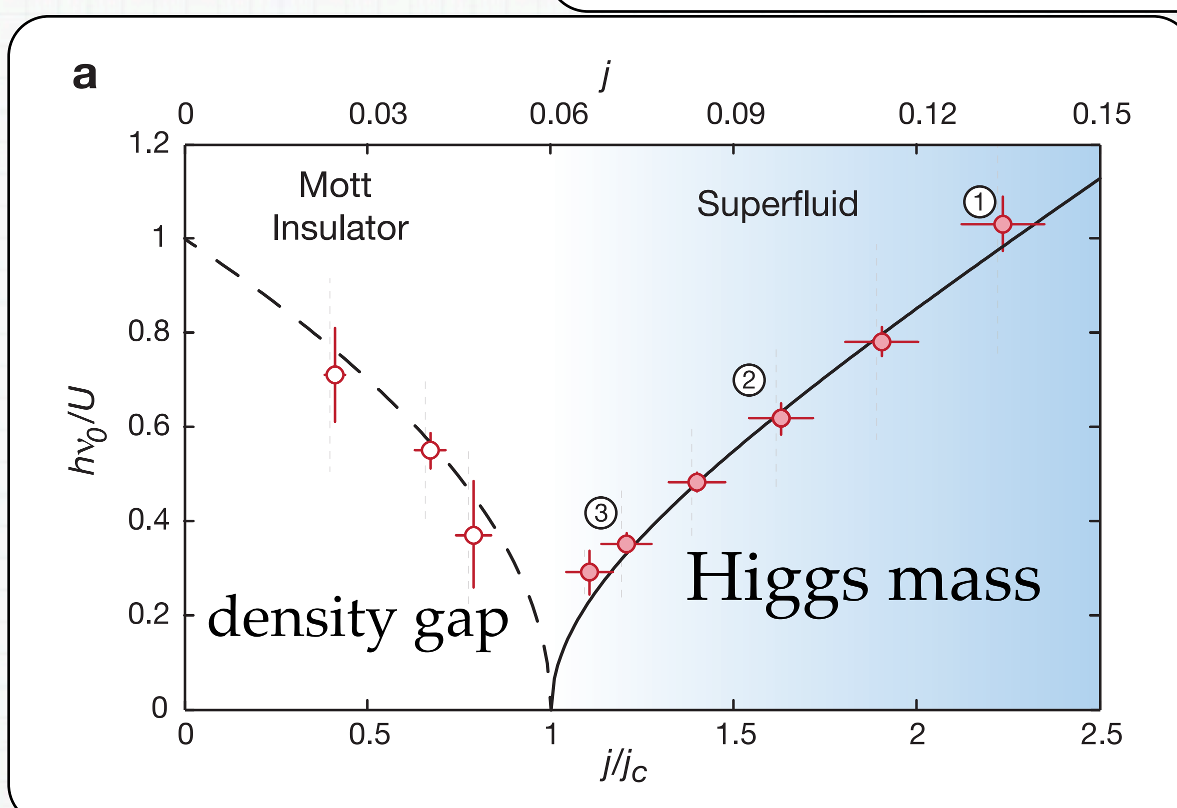
doi:10.1038/nature11255

The 'Higgs' amplitude mode at the two-dimensional superfluid/Mott insulator transition



- Energy absorption rate of periodically modulated lattice $\propto \omega \chi''_{\mathrm{scalar}}(\omega)$

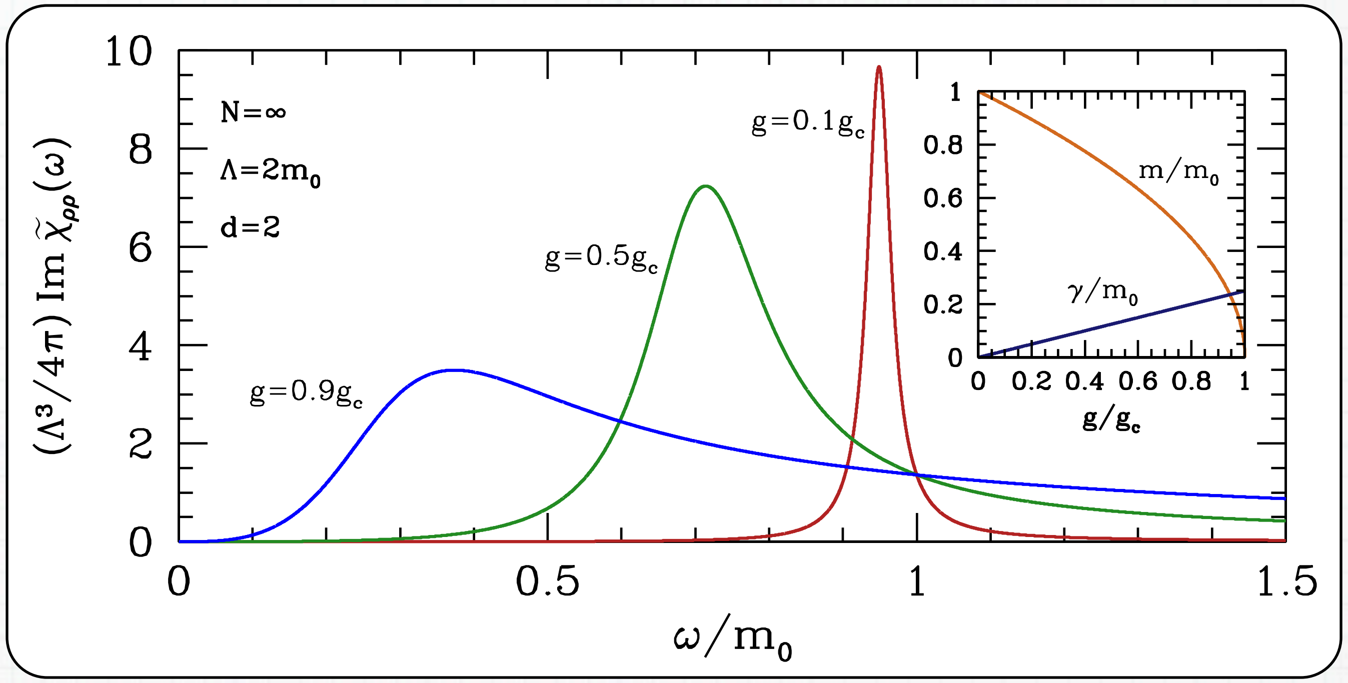
Critical energies:



What happens near the quantum critical point?

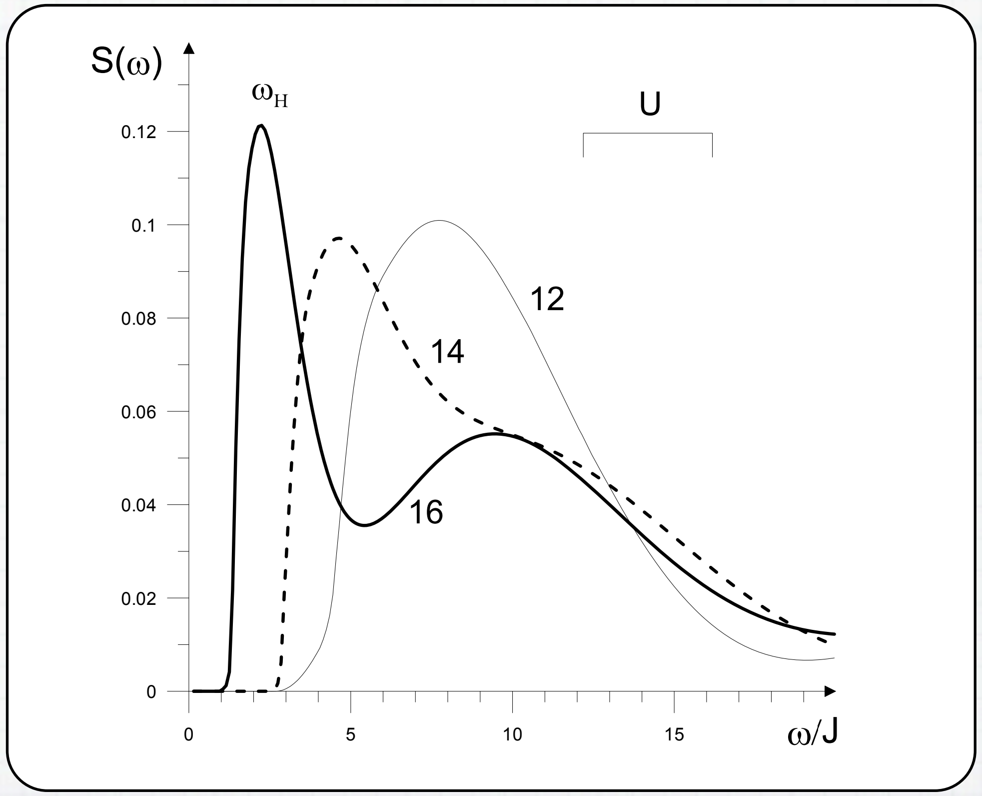
Analytics for $N = \infty$

Podolsky, Auerbach and Arovas, *PRB* (2011)



Numerics on Bose-Hubbard model

L. Pollet and N. Prokof'ev, PRL (2012)



Scaling near criticality

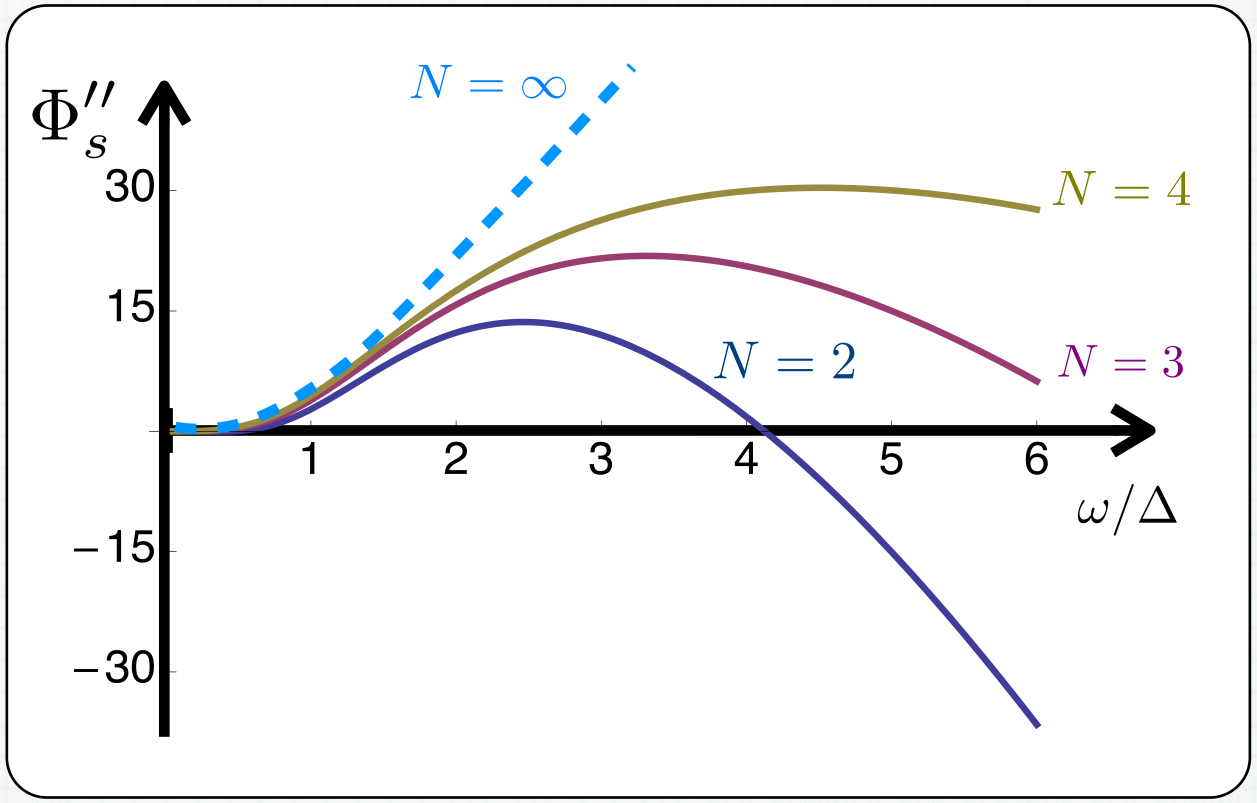
critical gap: $\Delta \sim |g - g_c|^\nu$ $\nu = 0.6717(1) \quad (N = 2)$

$$\chi_{scalar}(\omega) = \Delta^{3-2/\nu} \Phi_s\left(\frac{\omega}{\Delta}\right) + \ldots$$

universal function

Does it have a peak?

Scaling function
to O(1/N)



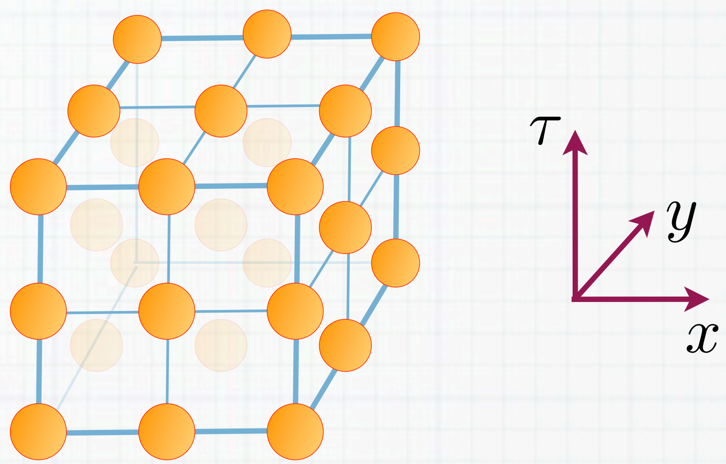
Podolsky and Sachdev, PRB (2012)

Monte Carlo Simulations

Lattice model:

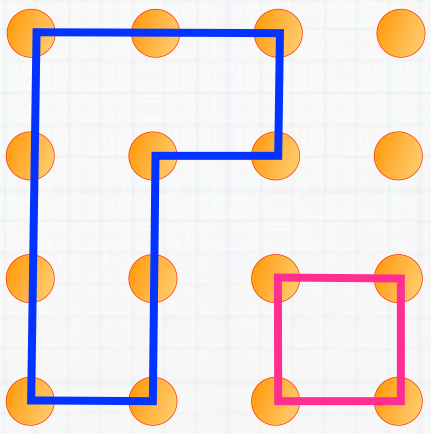
$$\mathcal{Z} = \int \mathcal{D}\vec{\phi}\, e^{-S[\vec{\phi}]}$$

$$S = -\sum_{\langle ij \rangle} \vec{\phi}_i \cdot \vec{\phi}_j + \mu \sum_i |\vec{\phi}_i|^2 + g \sum_i |\vec{\phi}_i|^4$$



Worm algorithm:

Dual loop model
with N flavors:

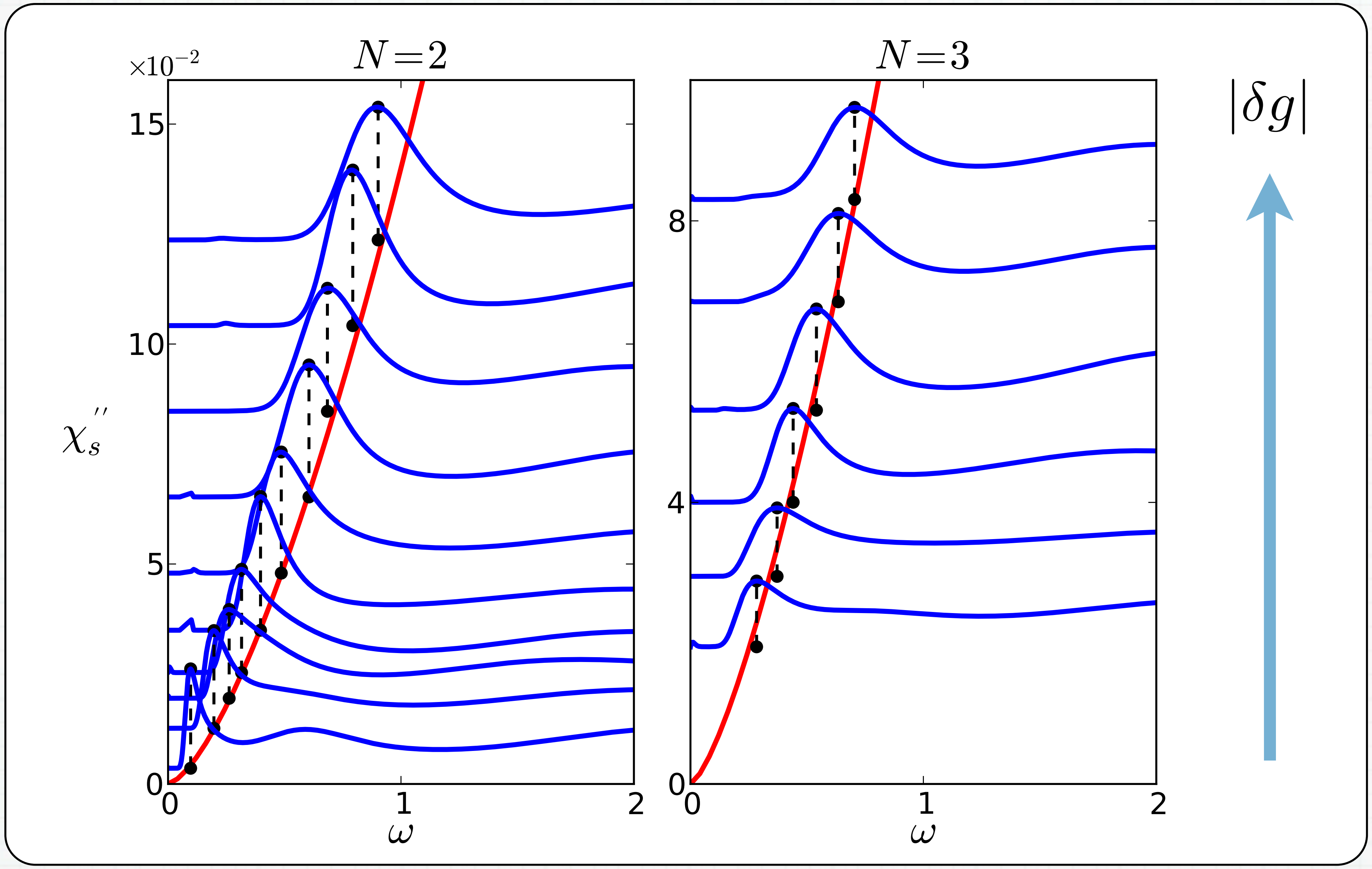


System size: $1 \ll \xi \ll L \qquad (1 \ll 30 \ll 200)$

Numerical analytical continuation from Matsubara to real frequencies

Tracking the Higgs peak

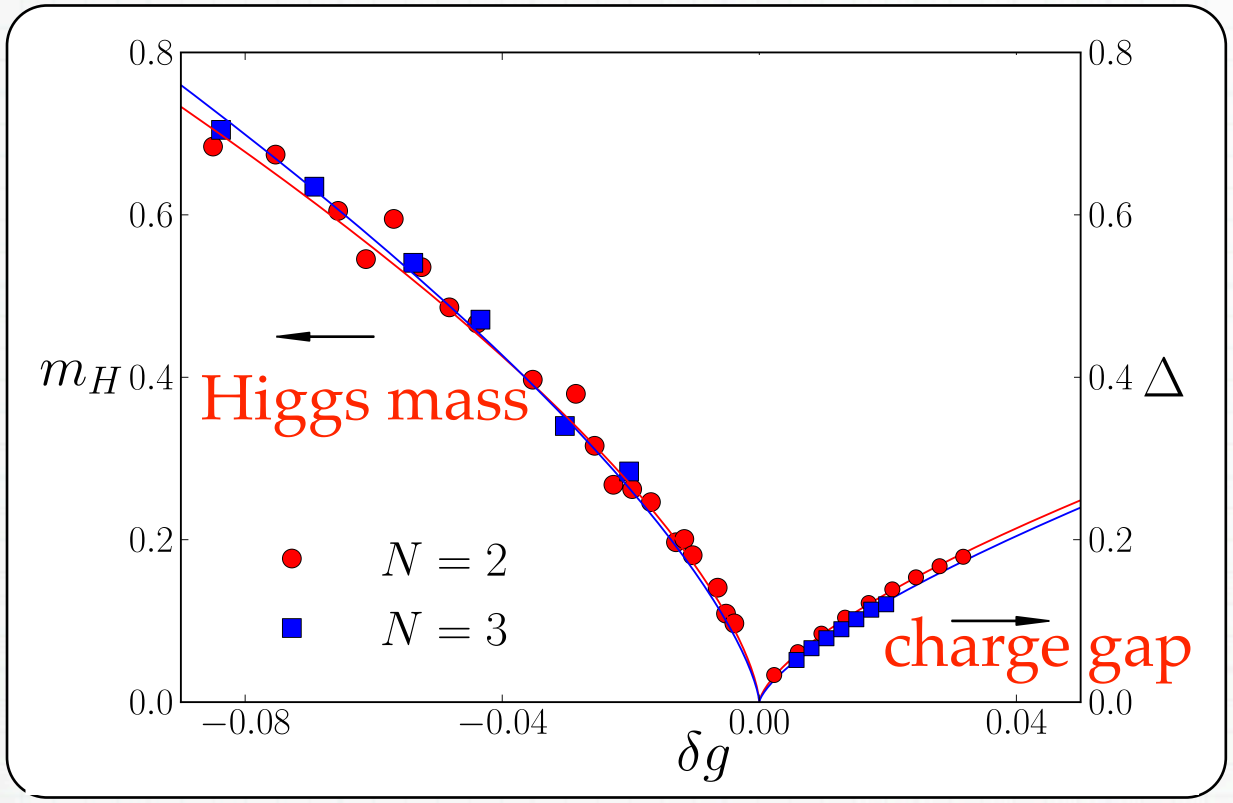
Scalar susceptibility in ordered phase:

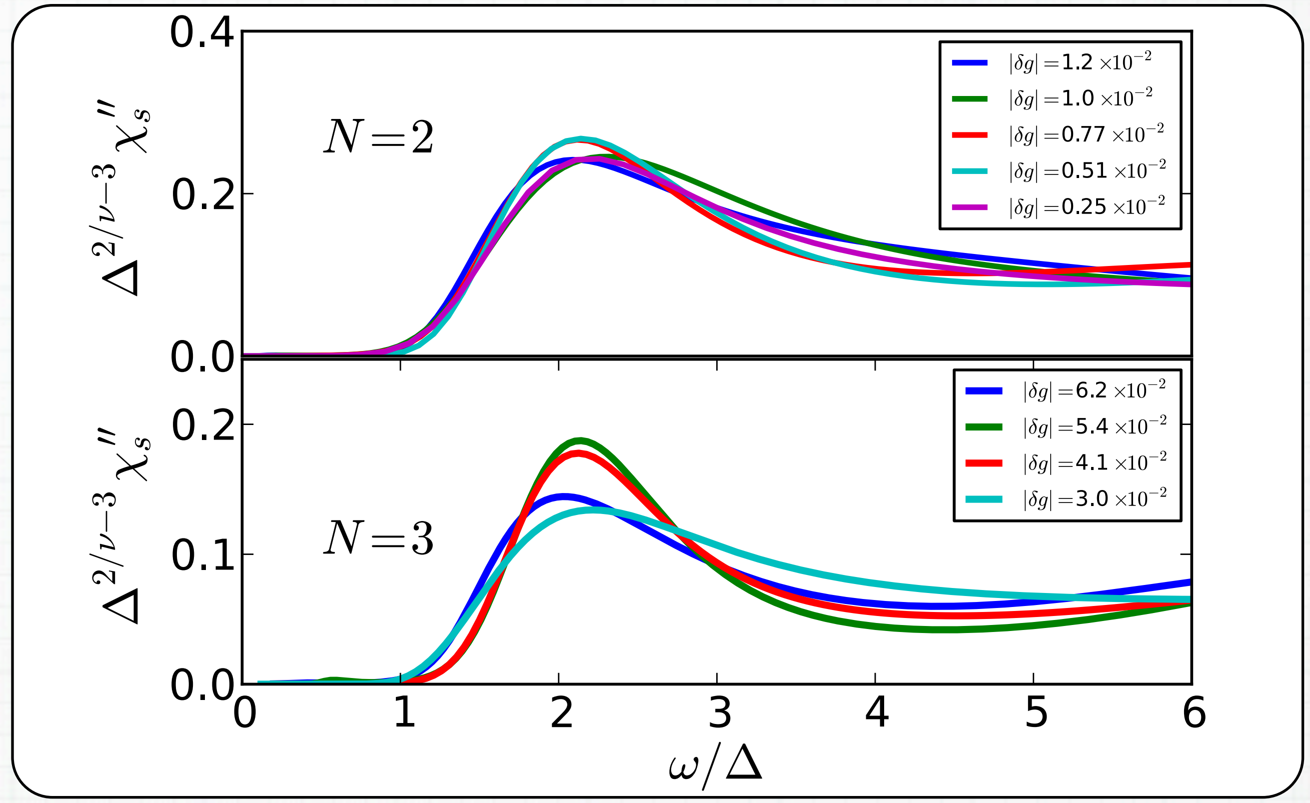


$$m_H \sim B|\delta g|^{\nu}$$

$$\frac{12\%}{0.25\%} = 48$$

Numerical simulations





universal Higgs spectral function

*Conclusion: Higgs resonance **survives** close to criticality in d=2*

Prediction: $\frac{m_H}{\Delta} = 2.1(3)$

Chen *et al*, PRL (2013): $\frac{m_H}{\Delta} = 3.3(8)$

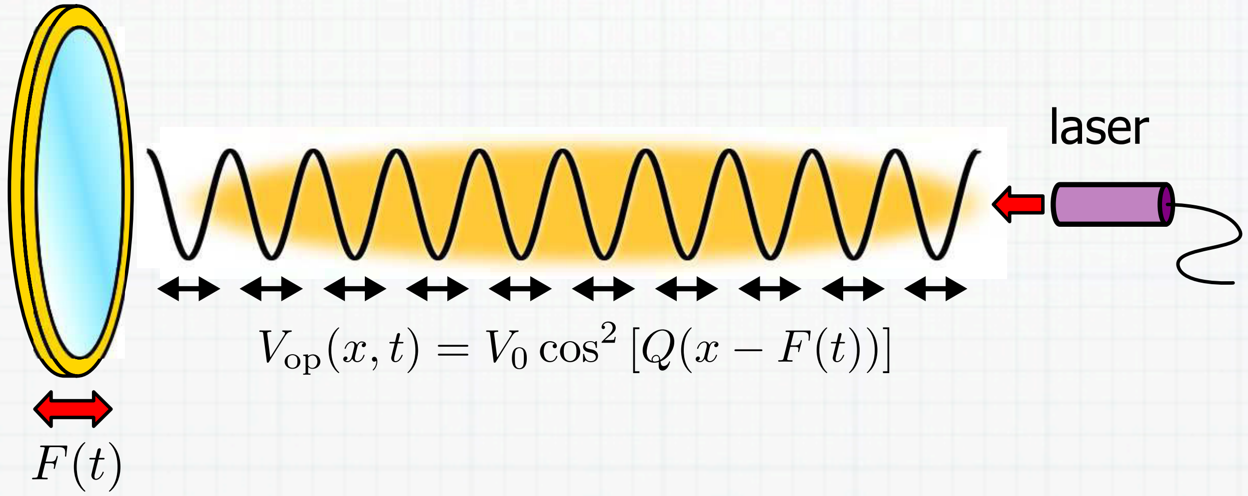
Gazit, Podolsky, Auerbach PRL (2013), Gazit, Podolsky, Auerbach, Arovas arXiv:1309.1765

Optical conductivity

Higgs peak can be seen in optical conductivity of charged bosons

Lindner and Auerbach, PRB (2010)

Can be measured in cold atoms in a phase-fluctuating optical lattice:

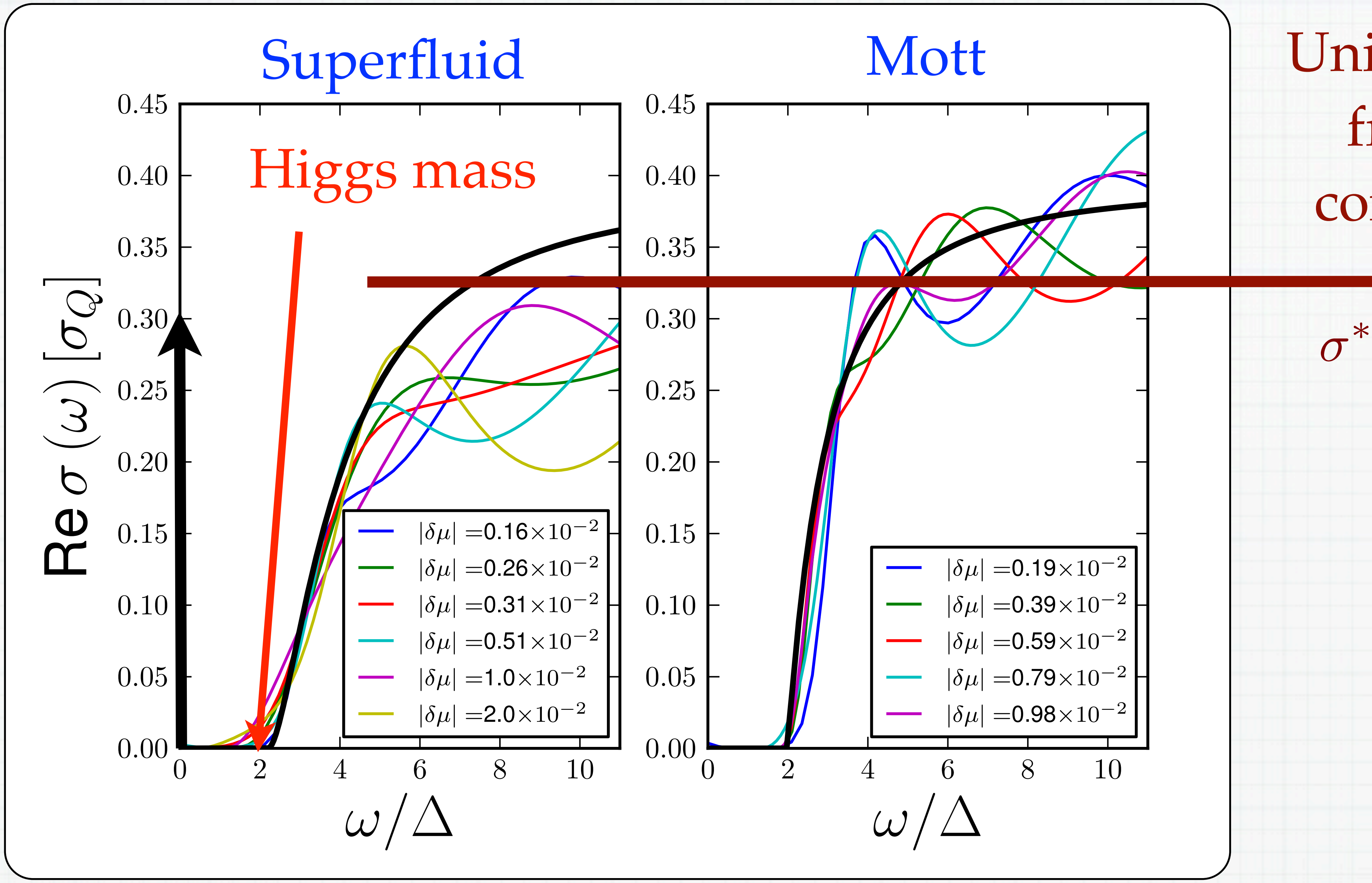


For $F(t) = F_0 \cos(\omega t)$ the energy absorption rate is $\propto \sigma(\omega)$

Tokuno and Giamarchi, PRL (2011)

Optical conductivity

Gazit, Podolsky, Auerbach, Arovas arXiv:1309.1765



Universal high frequency conductivity

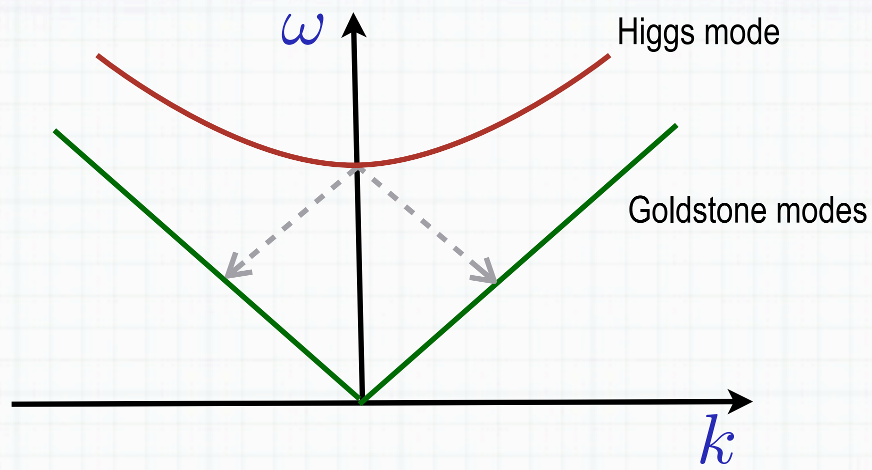
$\sigma^* = 0.33(7)\sigma_Q$

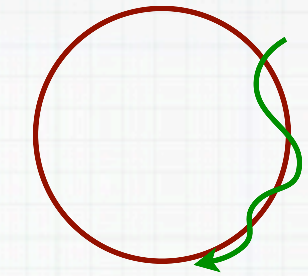
$$\sigma_{\rm SF}(\omega) = 2\pi\sigma_Q \left(\frac{\omega^2 - m_H^2}{4\omega^2}\right)^2 \Theta(\omega - m_H)$$

$$\sigma_{\rm Mott}(\omega) = 2\pi\sigma_Q \left(\frac{\omega^2 - 4\Delta^2}{16\omega^2}\right) \Theta(\omega - 2\Delta)$$

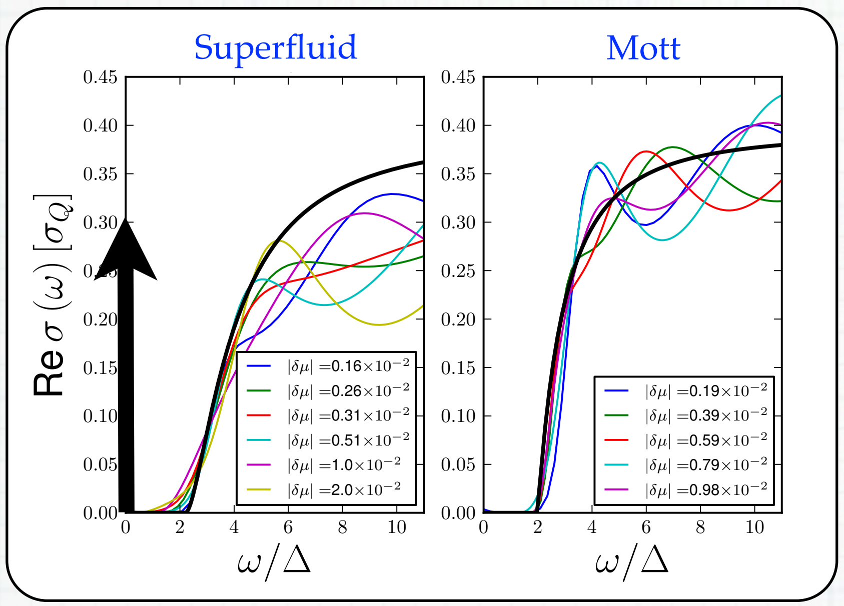
Summary part 1

Higgs decays into Goldstone bosons



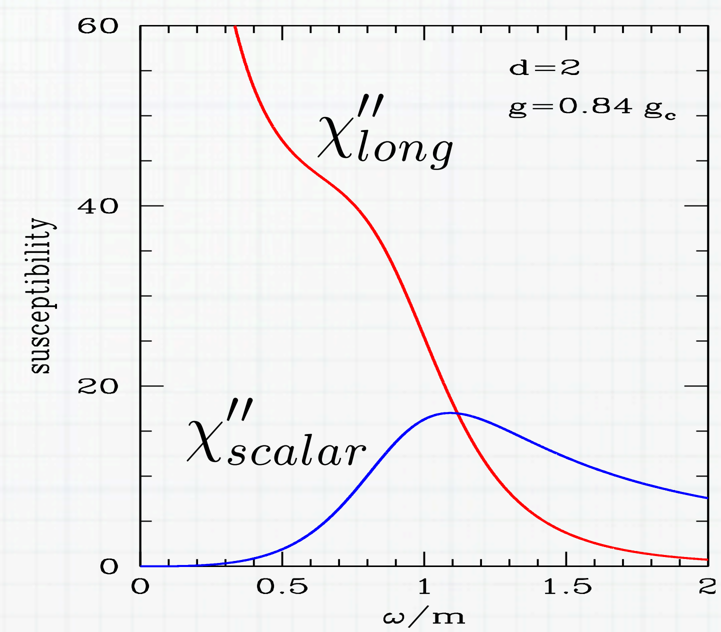


scalar is sharper than longitudinal

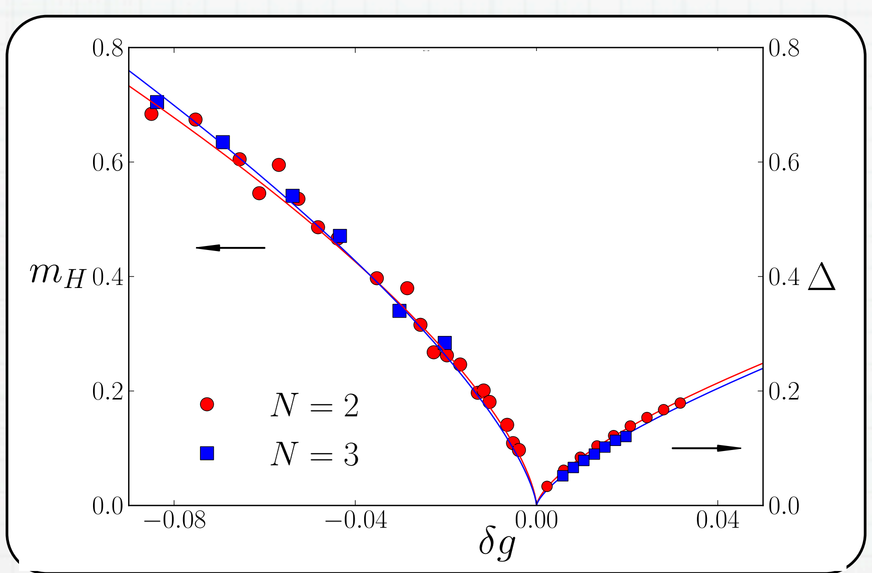


Optical conductivity

$\sigma^* = 0.33(7)\sigma_Q$

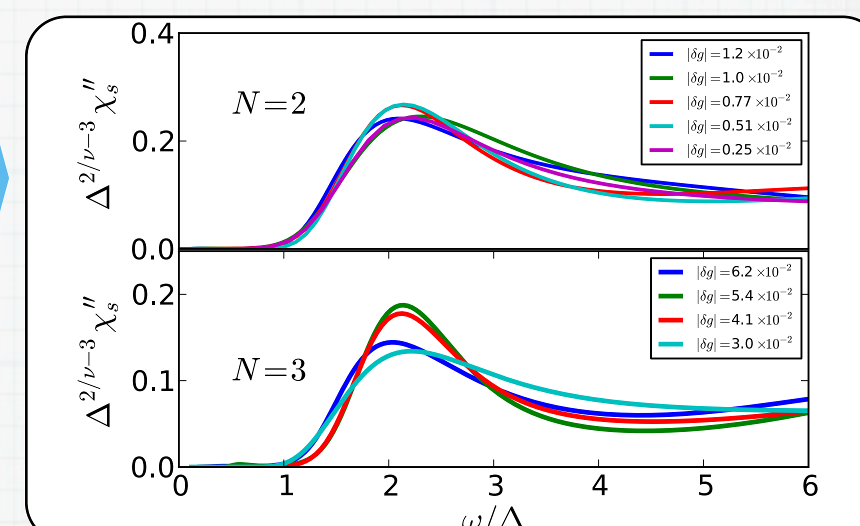


$$\frac{m_H}{\Delta} = 2.1(3)$$



scaling, O(1/N) and numerical simulations

$$\chi_{scalar}(\omega) = \Delta^{3-2/\nu}\Phi_s\left(\frac{\omega}{\Delta}\right)$$



Modulated Floquet Topological Insulators

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Tenenbaum Katan & D.P., Phys. Rev. Lett. **110**, 016802 (2013)
Tenenbaum Katan & D.P., arXiv:1309.0203 (2013)
Rechtsman *et al.*, Nature **496**, 196 (2013)

Outline Part 2

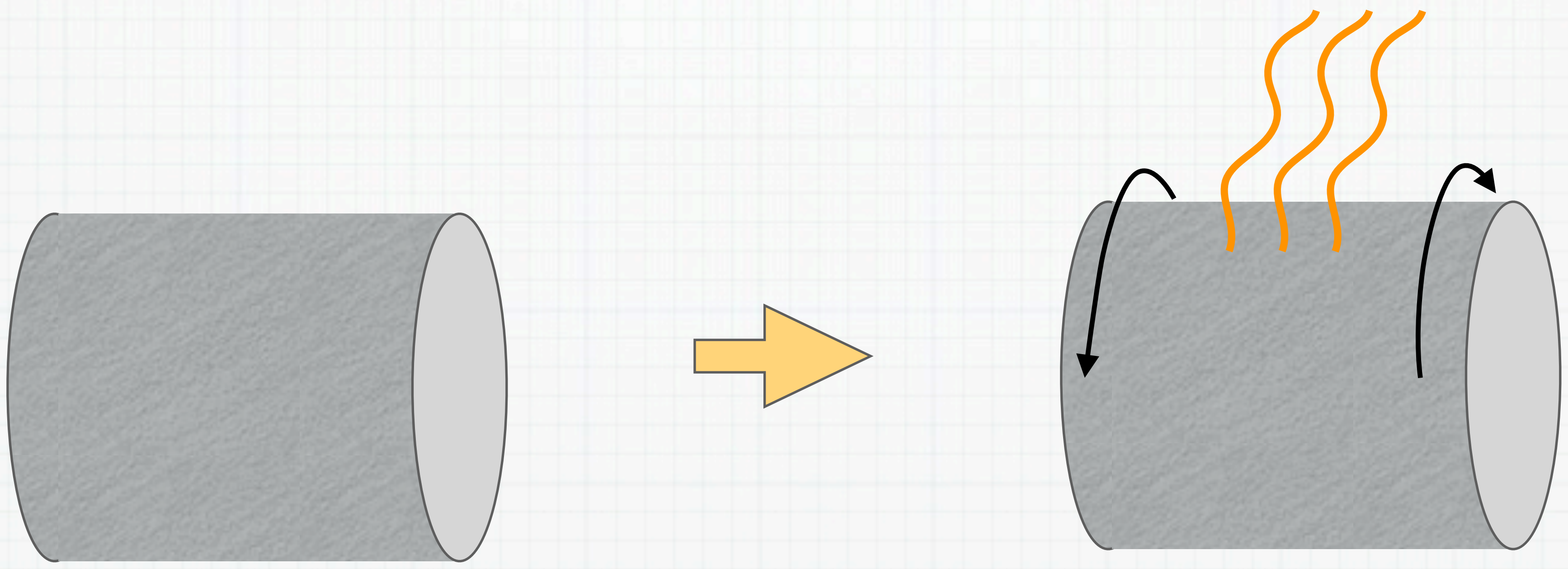
* Floquet topological insulators

* Spatial modulation: domain walls and vortices

* Photogalvanic effect

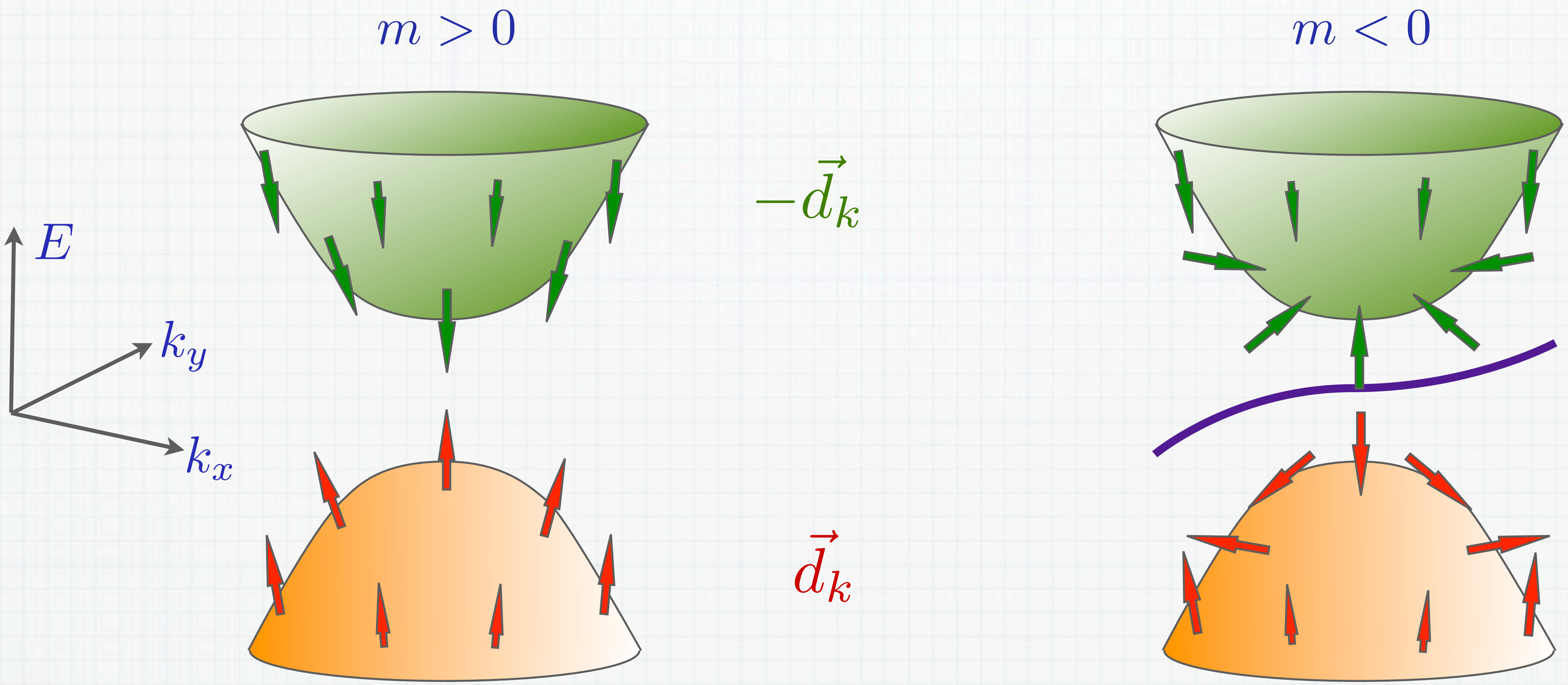
Floquet Topological Insulators

* Light can induce topological behavior



Trivial vs topological

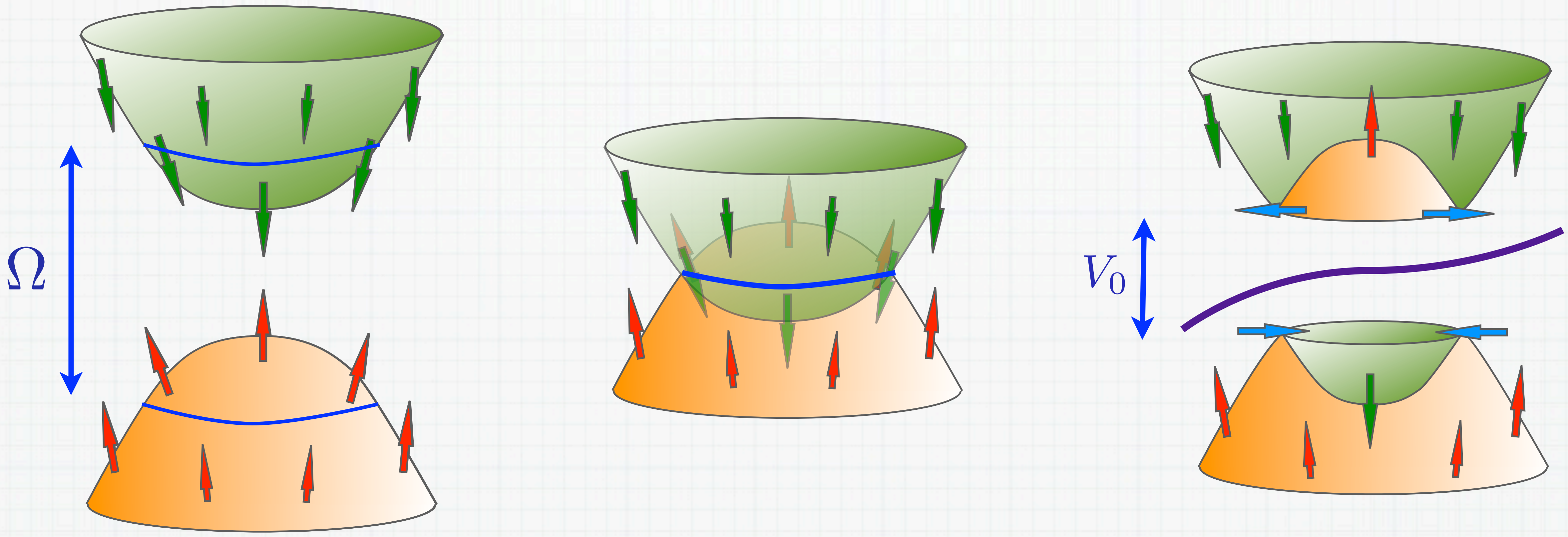
- Two-band system $H = \vec{d}_k \cdot \vec{\sigma}$ $\vec{d}_k = \begin{pmatrix} k_x \\ k_y \\ k^2 + m \end{pmatrix}$



"Trivial" "Topological"

Inducing topology

* Two-band system, with light $H = \vec{d}_k \cdot \vec{\sigma} + V_0 \sigma_z \cos \Omega t$



Floquet Theorem

- Time-periodic Hamiltonian $H(t) = H(t+\tau)$
- Solutions to Schrödinger equation:

$$\psi(t) = \sum_n a_n \, e^{-i\varepsilon_n t} \varphi_n(t)$$

$$\varphi_n(t+\tau) = \varphi_n(t)$$

$$\varepsilon_n \sim \varepsilon_n \bmod (2\pi/\tau)$$

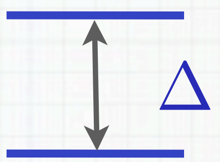
- "Floquet Hamiltonian"

$$H_F \varphi_n = \varepsilon_n \varphi_n$$

$$U(\tau) = \mathcal{T} e^{-i \int_0^\tau dt' \, H(t')} \equiv e^{-i\tau H_F}$$

NMR

- Two-level problem (NMR)

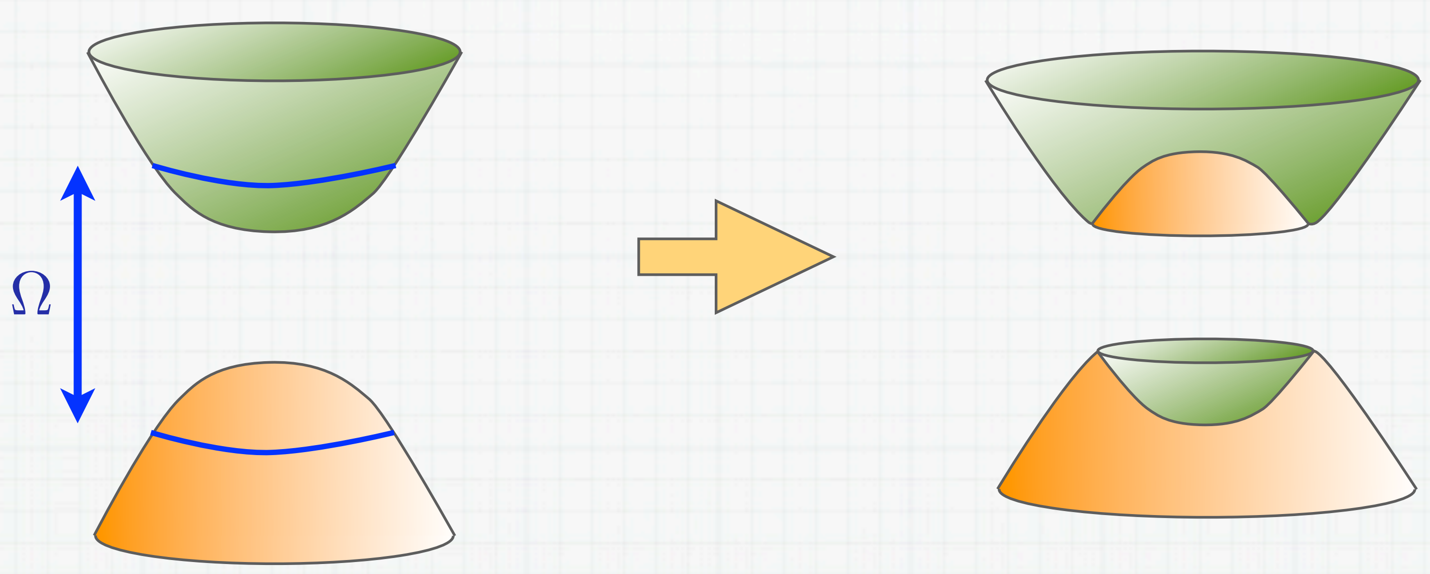


$$H(t) = \sigma_z\,\Delta/2 + \sigma_x\,B\cos(\Omega t)$$

- Rotating wave approximation

$$H_F \approx \sigma_z(\Delta - \Omega)/2 + \sigma_x B/2$$

- In context of solids, bands get “sewn together”



Example: HgTe

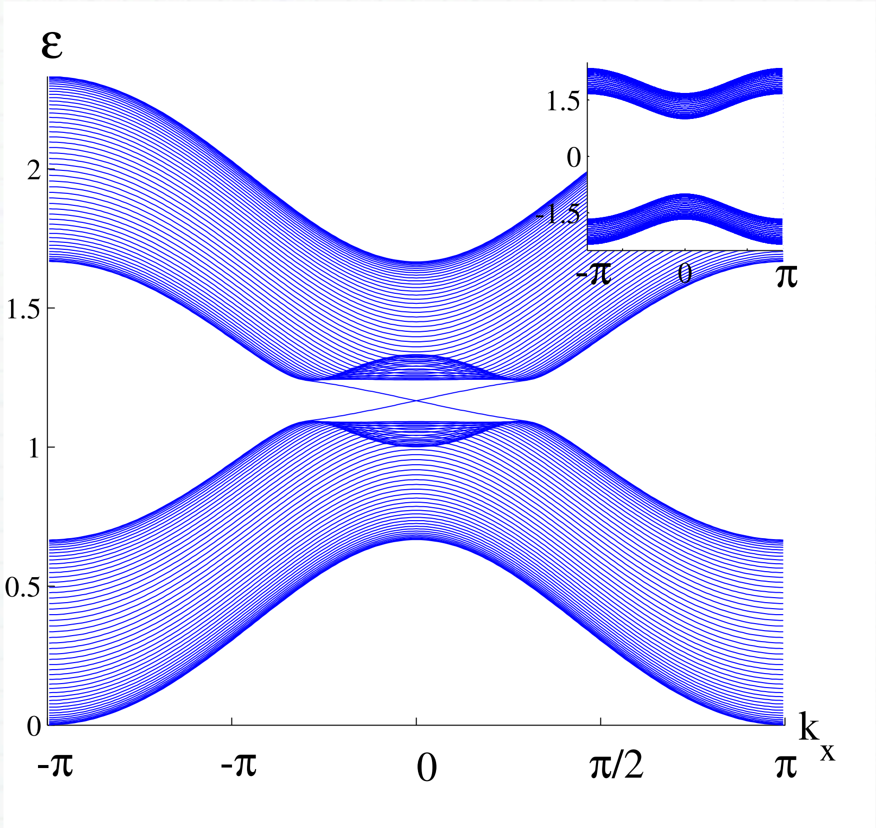
* Zincblende (4x4): $\mathcal{H} = \begin{pmatrix} H_k & 0 \\ 0 & H^*_{-k} \end{pmatrix}$

$$H_k = \vec{d}_k \cdot \vec{\sigma}$$

$$\vec{d}_k = \begin{pmatrix} A \sin k_x \\ A \sin k_y \\ M + 2B(2 - \cos k_x - \cos_k y) \end{pmatrix}$$

* Add time-dependent perturbation

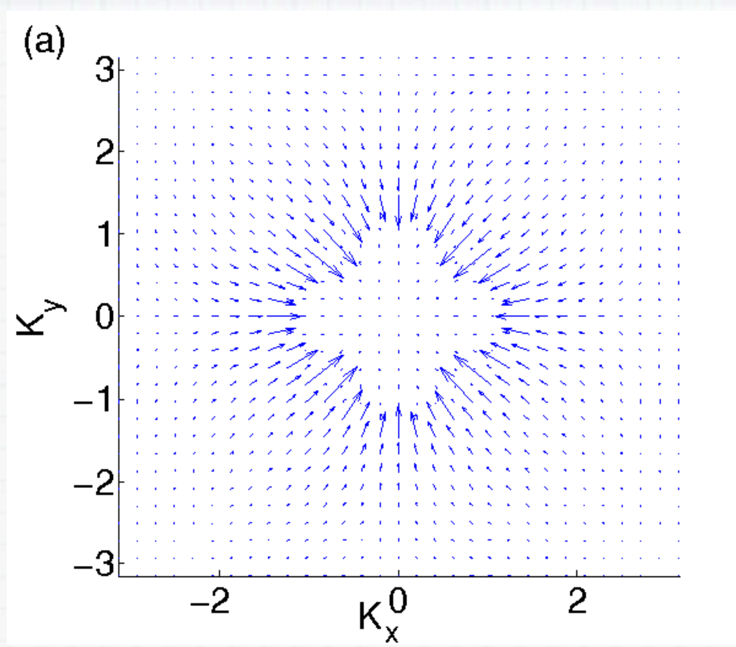
$$V(t) = V_0 \cos(\Omega t) \sigma_z$$



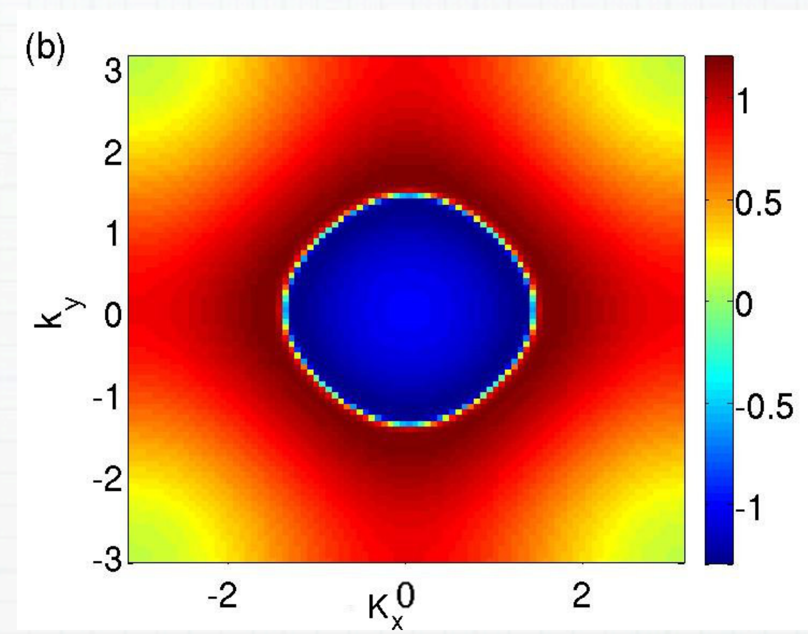
Lindner, Refael, Galitski, Nat. Phys. 2010

Example: HgTe

- Floquet Hamiltonian: $H_k^F = \vec{n}_k \cdot \vec{\sigma}$



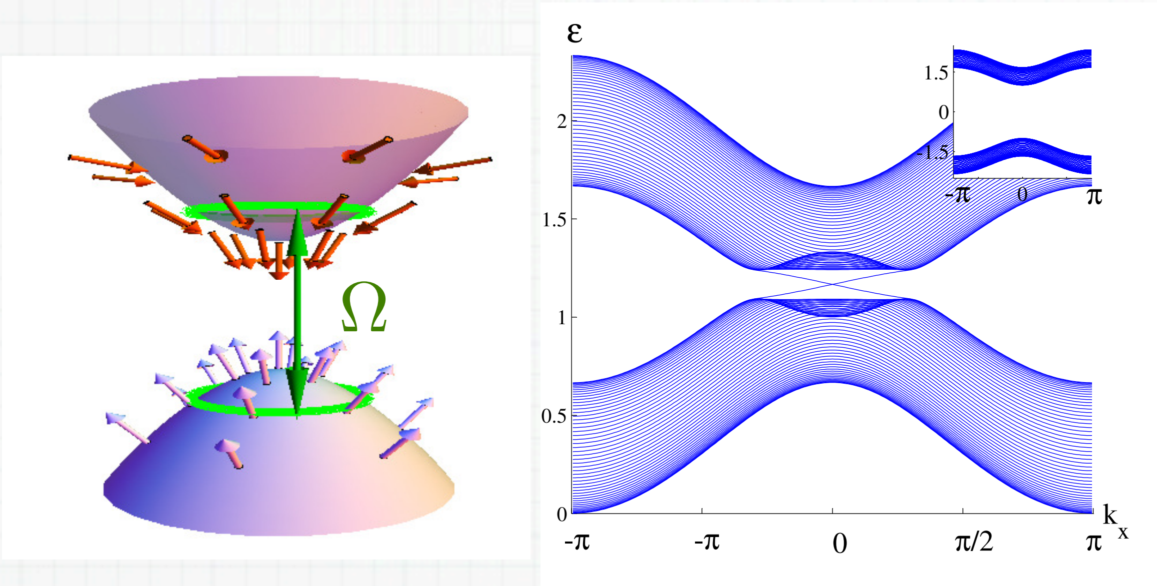
$n_k^x,\, n_k^y$



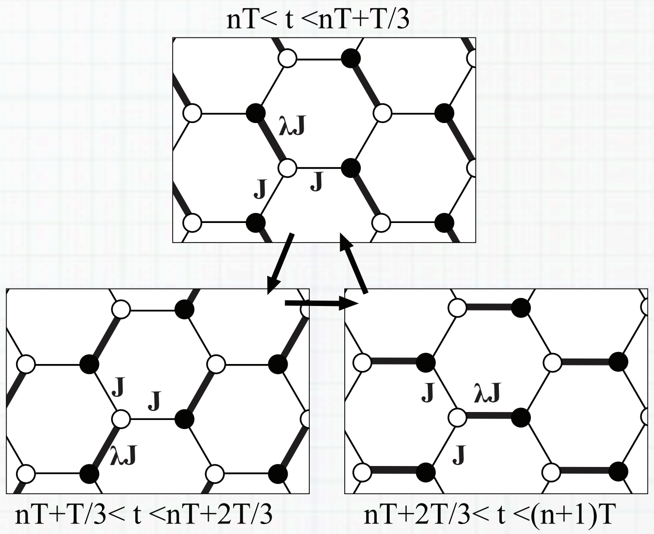
n_k^z

$$C_F = \frac{1}{4\pi} \int_{BZ} d^2k \left(\partial_{k_x} \hat{n}_k \times \partial_{k_y} \hat{n}_k\right) \cdot \hat{n}_k$$

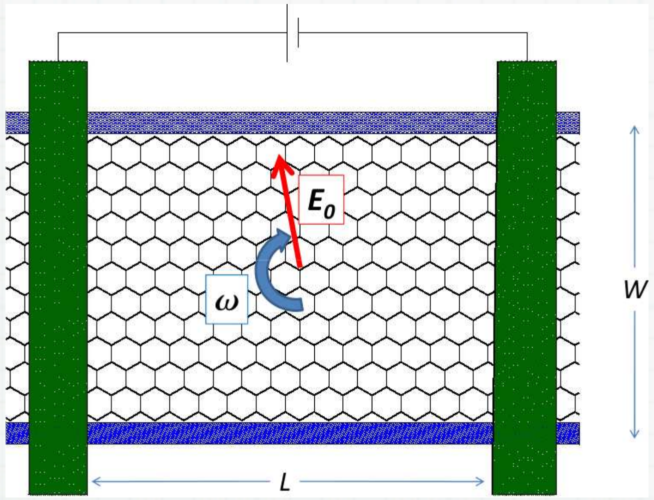
Proposals And Realizations



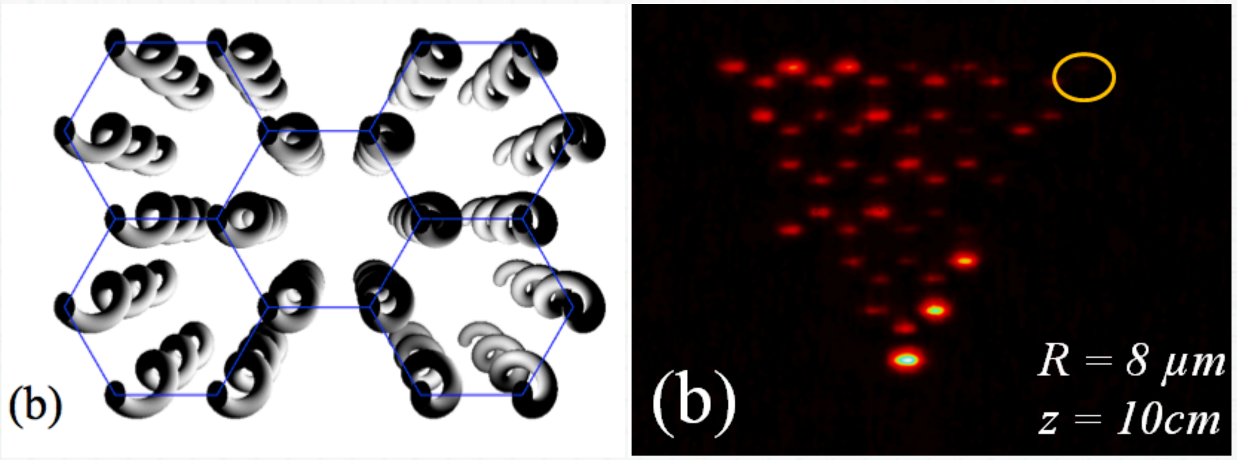
Lindner, Refael, Galitski, Nat. Phys. 2010



Kitagawa, Berg, Rudner, Demler, PRB 2010



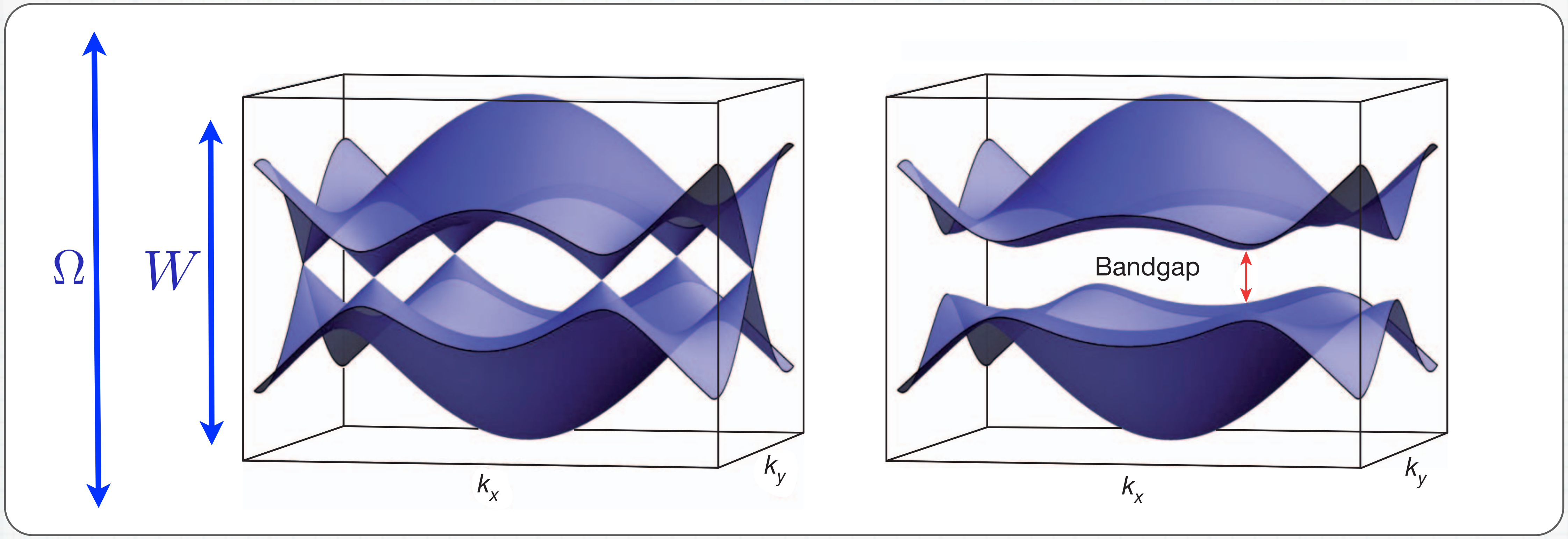
Gu, Fertig, Arovas, Auerbach, PRL 2011



Rechtsman, Zeuner, Plotnik, Lumer, Podolsky, Dreisow, Nolte, Segev, Szameit, *Nature* 496, 196 2013

Graphene + circularly polarized light

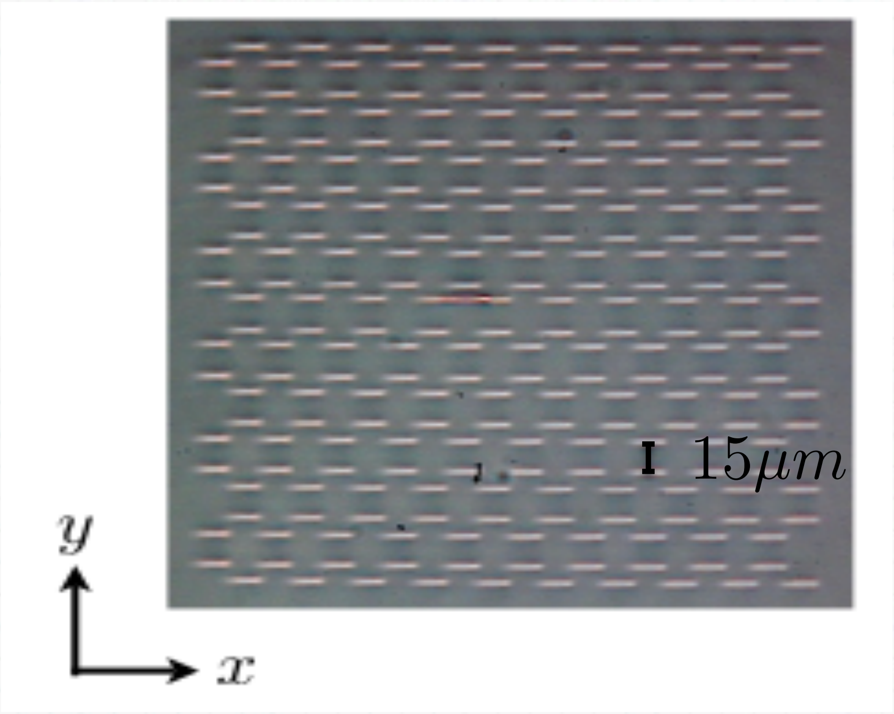
- For $\Omega > W$, non-resonant bandgap opens:

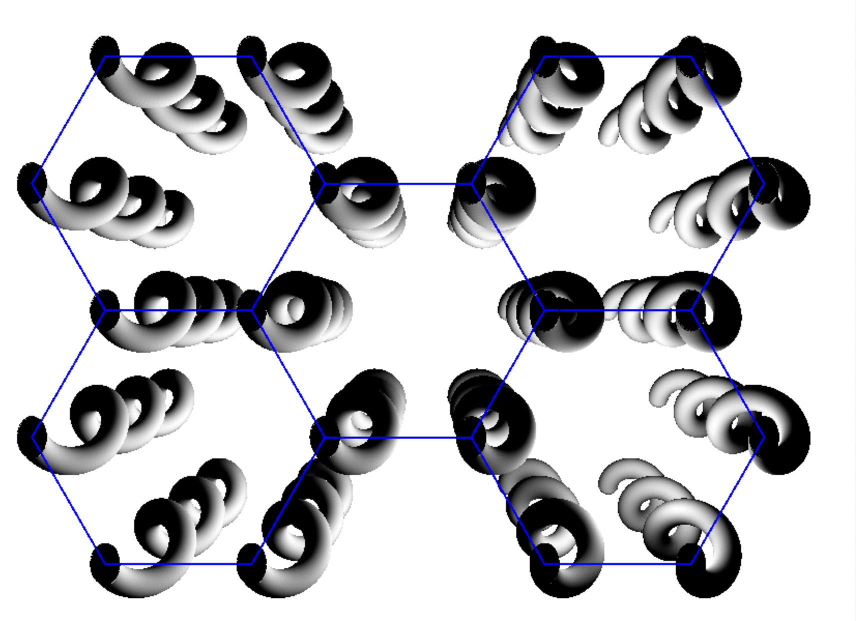


- Dirac cones receive opposite masses ==> (Chern #) = 1.
- Light polarization chooses the chirality of edge modes

Gu, Fertig, Arovas, Auerbach, PRL 2011

Helical rotation induces a gauge field





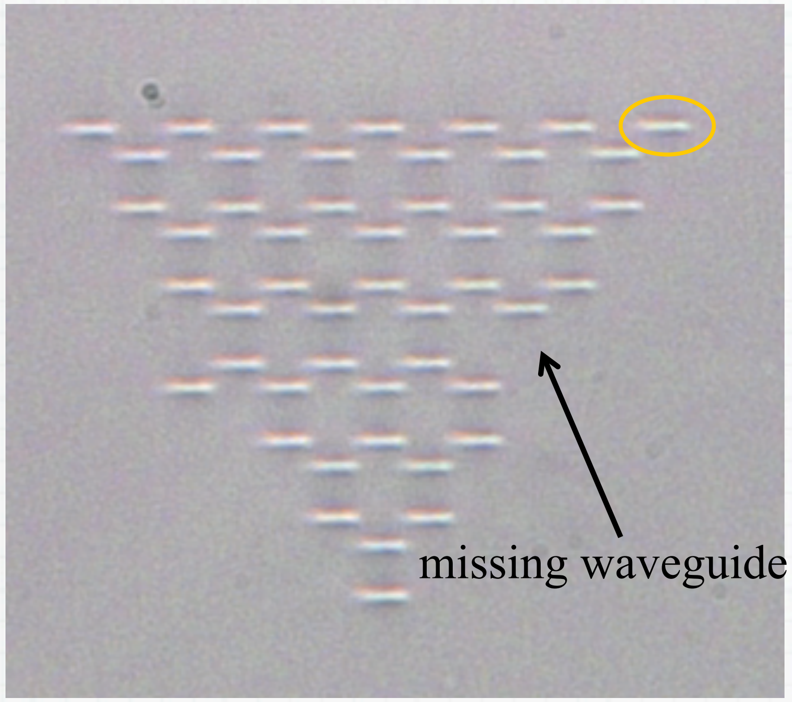
$$i\partial_z \psi = \frac{1}{2k_0}\left(i\nabla + \mathbf{A}(z)\right)^2 \psi - \frac{k_0 \Delta n(x,y)}{n_0}\psi - \frac{k_0 R^2 \Omega^2}{2}\psi$$

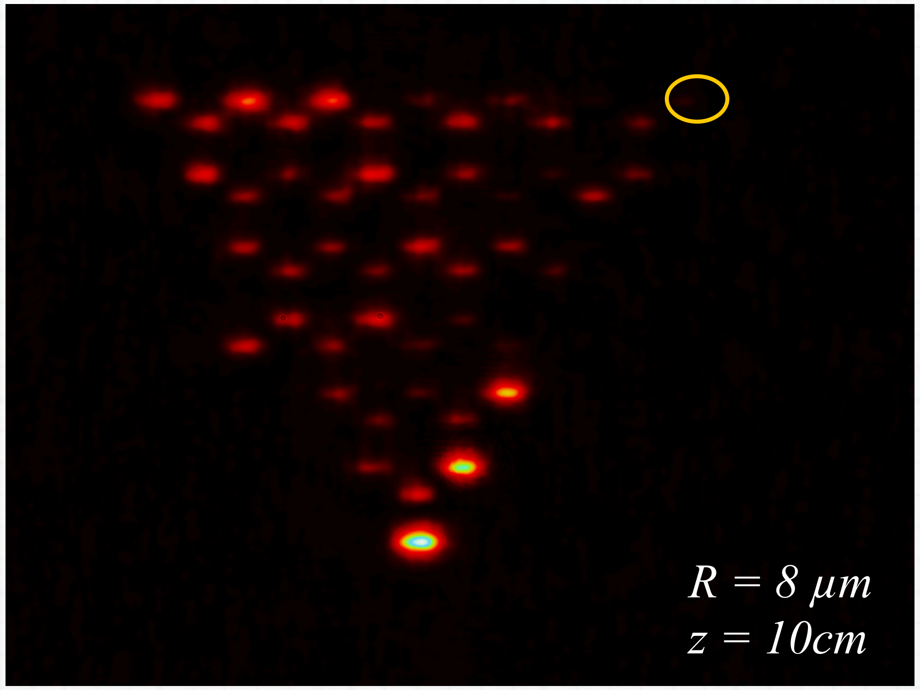
$$A(z) = k_0 R\Omega(\sin \Omega z, \cos \Omega z)$$

$$x' = x + R\cos \Omega z$$
$$y' = y + R\sin \Omega z$$
$$z' = z$$

$$\mathcal{H}(z) = \sum_{m,\langle n \rangle} e^{i\mathbf{A}(z)\cdot \mathbf{r}_{mn}} \psi_n^\dagger \psi_m$$

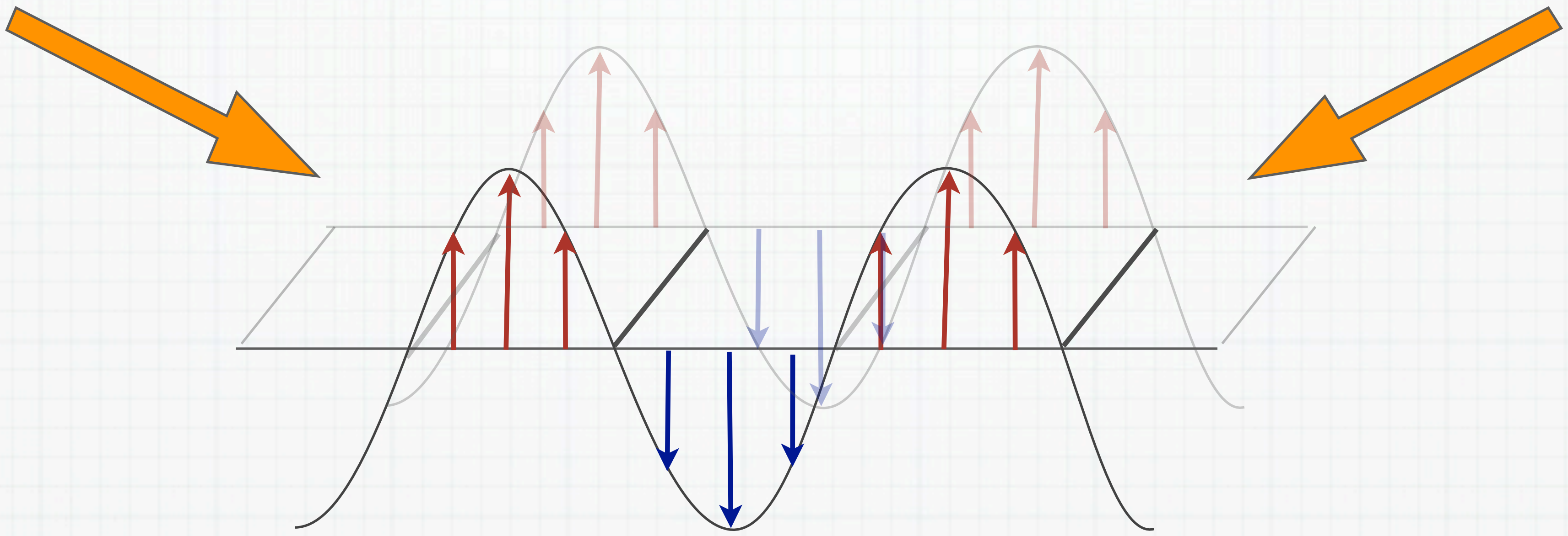
Triangular sample with defect





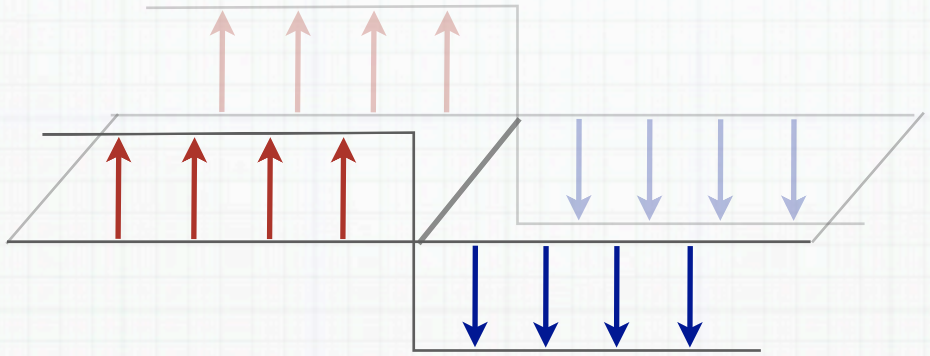
Rechtsman, Zeuner, Plotnik, Lumer, Podolsky, Dreisow, Nolte, Segev, Szameit, *Nature* 496, 196 (2013)

What happens if we add spatial modulation?



Domain wall in phase

* Interface between two regions with π phase shift:



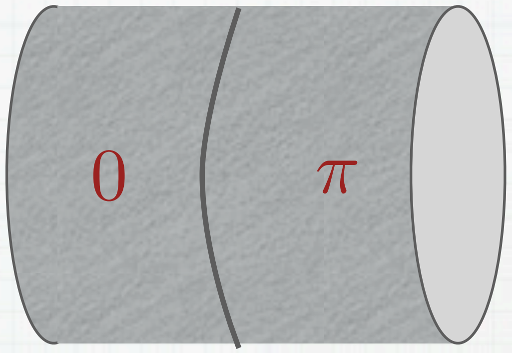
* Delay phase:

$$H = \sigma_z \Delta/2 + \sigma_x B \cos(\Omega t + \alpha)$$

* Effect of delay:

$$H_F \approx \sigma_z(\Delta - \Omega)/2 + B\left(\sigma_x \cos\alpha + \sigma_y \sin\alpha\right)/2$$

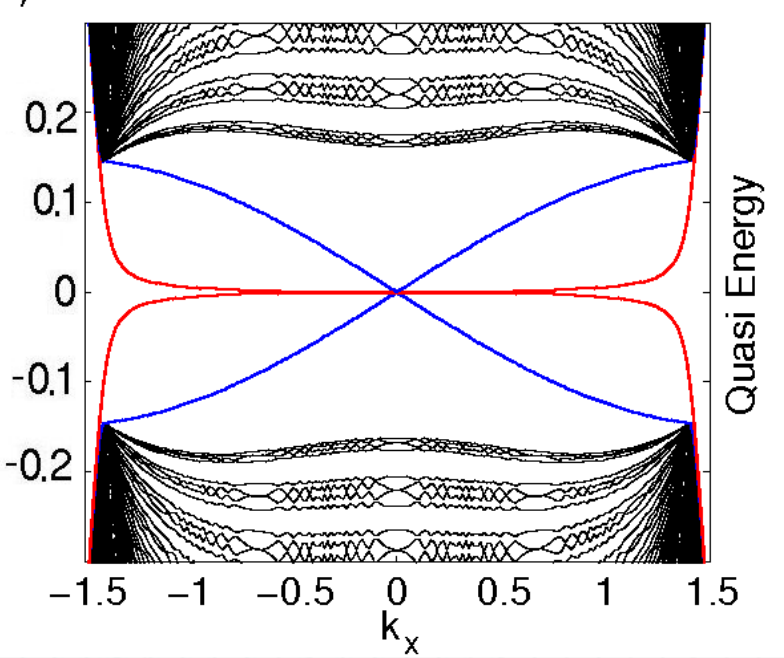
* Spectrum unaffected
==> Chern number independent of α



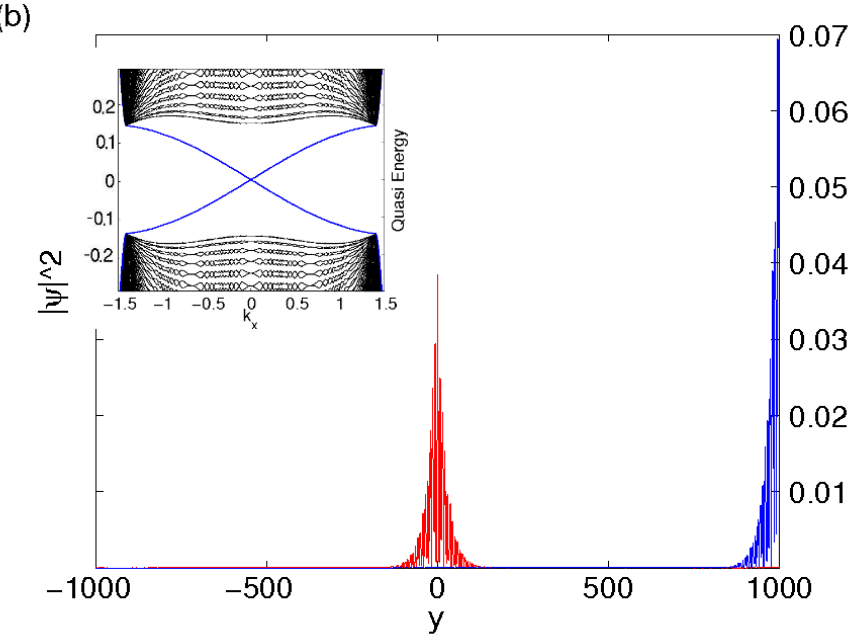
Domain wall

* Numerical simulation:

(a)



(b)



* Analytic understanding?

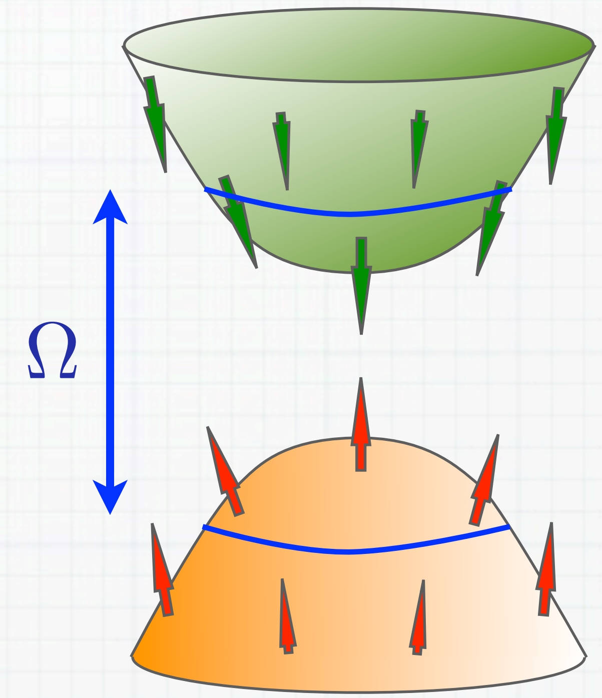
Analogy to p+ip superconductor

* Nambu Hamiltonian:

$$H_k^F \approx \begin{pmatrix} \xi_k & \Delta_k e^{-i\alpha} \\ \Delta_k^* e^{i\alpha} & -\xi_k \end{pmatrix}$$

$$\xi_k = |\vec{d}_k| - \Omega/2$$

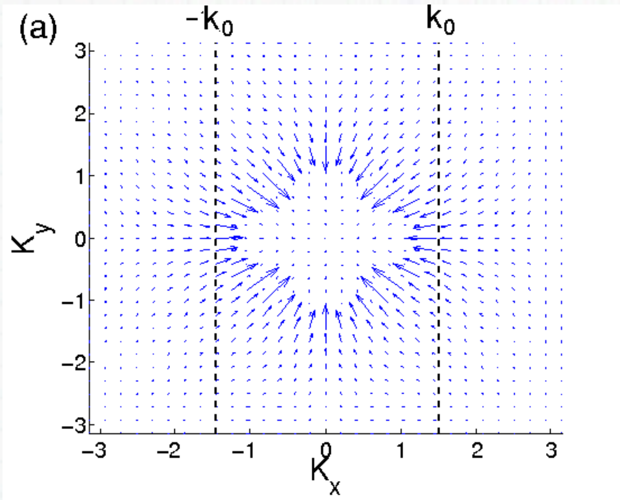
$$\Delta_k = (V_{\perp,k}/2k)\,(k_x + ik_y)$$

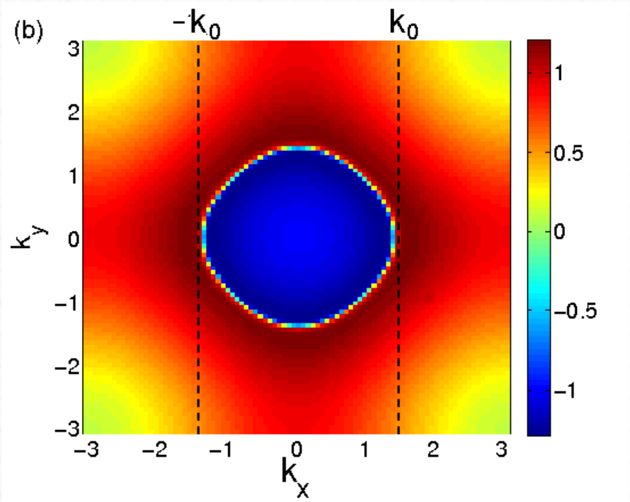


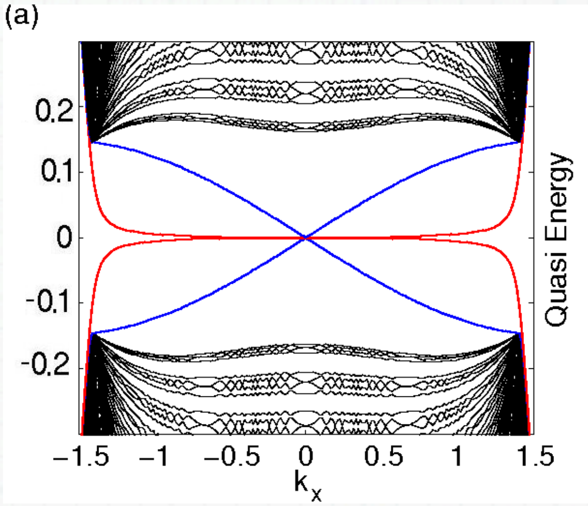
* When α depends on position, problem becomes BdG equation
* Domain wall ==> pi-junction in p+ip superconductor

Topological protection

- For fixed k_x, define $C'_{k_x} \equiv \text{winding in } (n_y, n_z)$



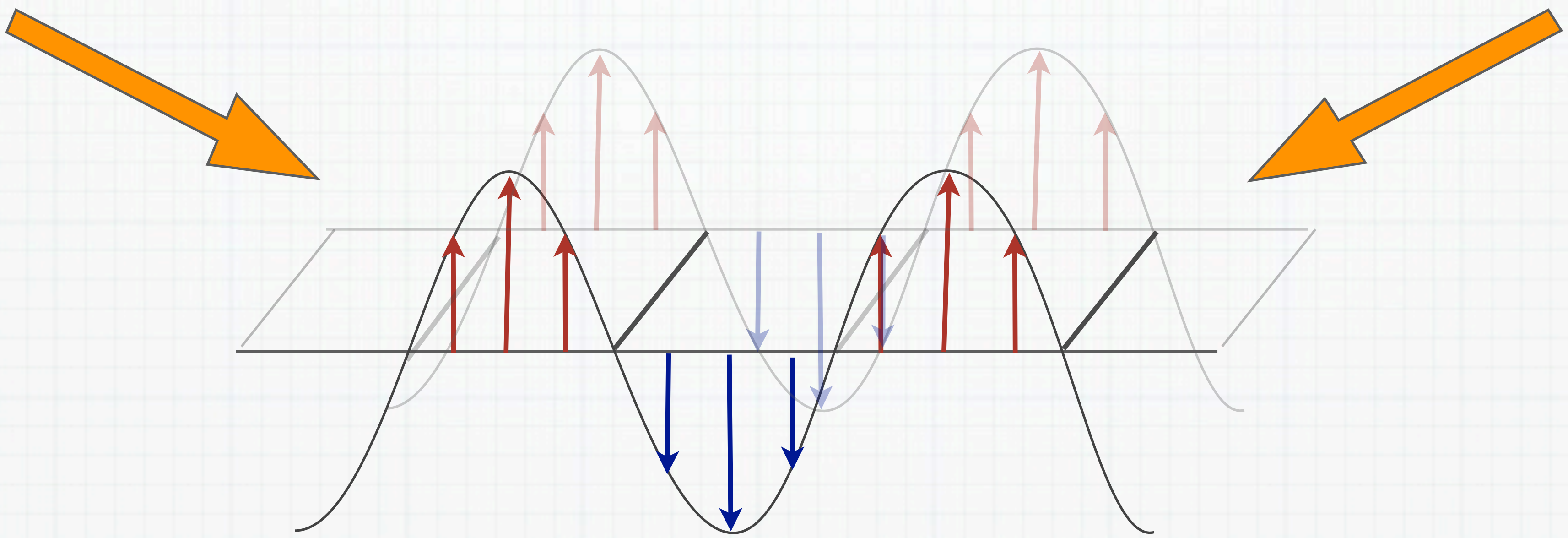




- For $\alpha = \pi$, C'_{k_x} changes sign
- Crossing of modes is protected by particle-hole symmetry
- Particle-hole symmetry breaking opens a small gap

Anisotropic transport?

* Control density of nodes by angle of incident light

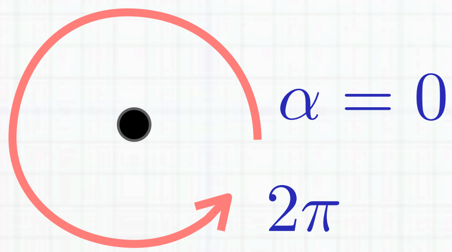


* Enhanced conductivity along the nodes

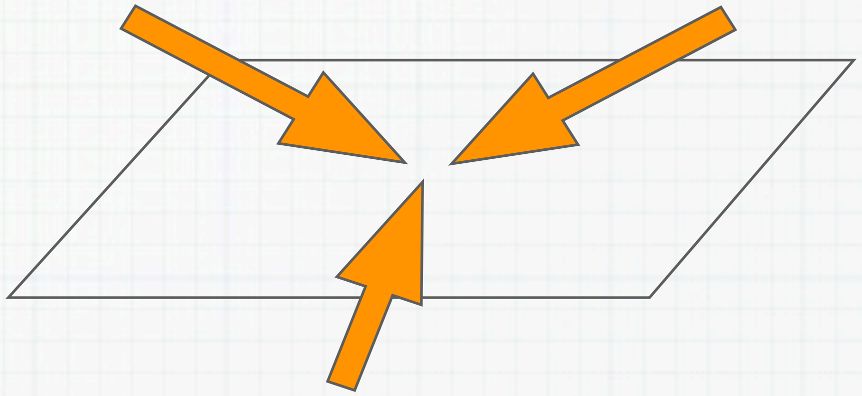
Gu, Fertig, Arovas, Auerbach, PRL 2011
Kundu and Seradjeh, arxiv:1301.4433

Vortex states

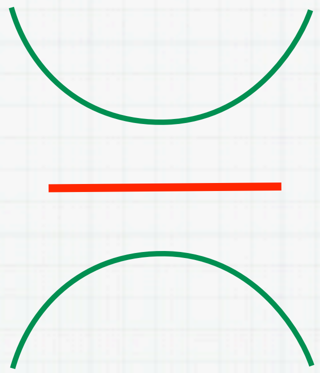
- Vortex in the phase



- Vortex lattice can be created using three lasers



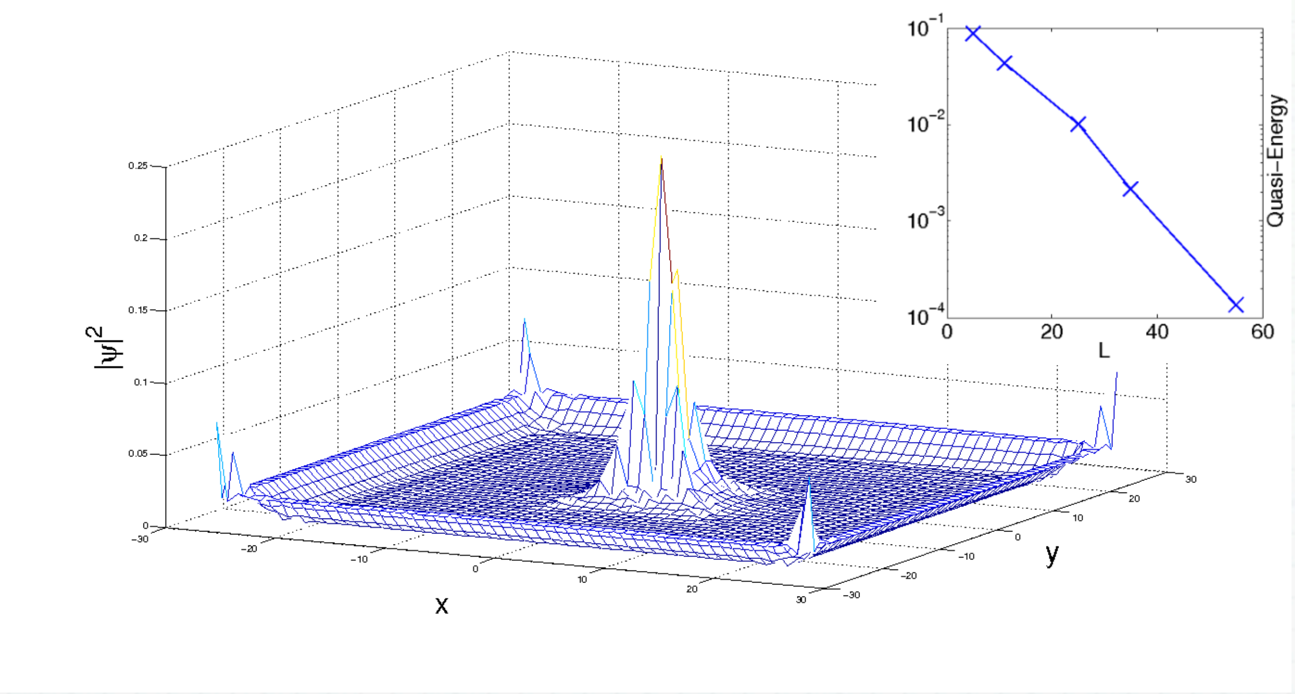
- Expect vortex core state with zero quasi-energy



Read & Green 2000; Ivanov 2001

Vortex states

* Numerical results:



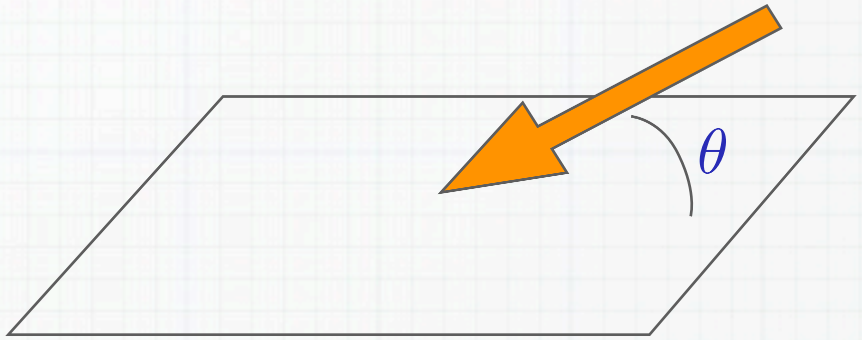
* Hybridization with edge mode exponentially small
* Analogous to Majorana, fractional excitation

Photogalvanic effect

* Analogue of supercurrents? $\vec{j} = \rho_s \vec{\nabla} \alpha$

* Noether's theorem:

$$\vec{j} = -\vec{\nabla}_k \left(\vec{d}_k \cdot \vec{\sigma} \right)$$



* Linear response

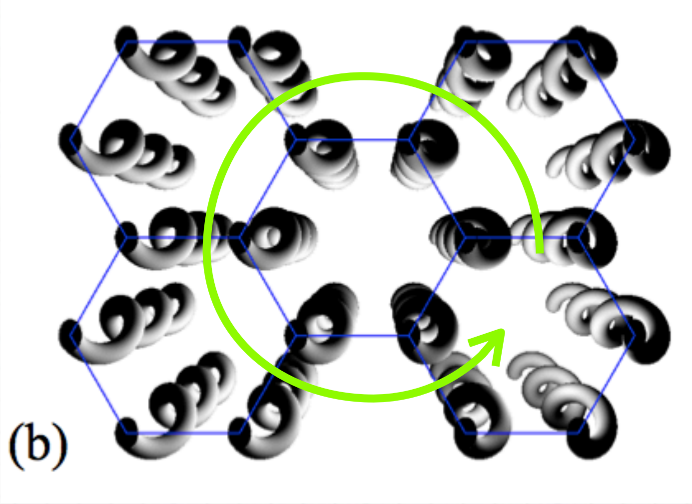
$$\frac{\langle j_x \rangle}{\partial_x \alpha} = V_0 \int \frac{d^2k}{8\pi^2} \frac{d_k}{n_k} \frac{\hat{n}_z}{\hat{n}_x^2 + \hat{n}_y^2} \left(\left(\hat{d}_k \times \partial_{k_x} \hat{n}_k \right) \cdot \hat{z} \right)^2$$

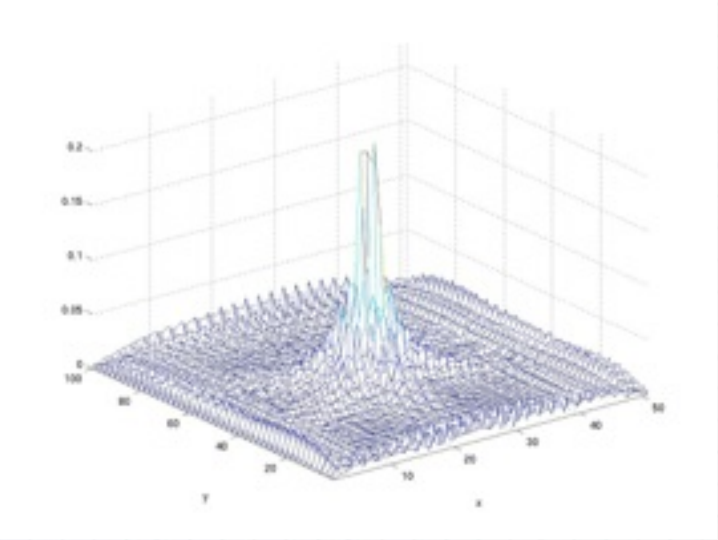
$$\frac{\langle j_y \rangle}{\partial_x \alpha} = 0$$

* Caveat: relies on occupation of "valence band"

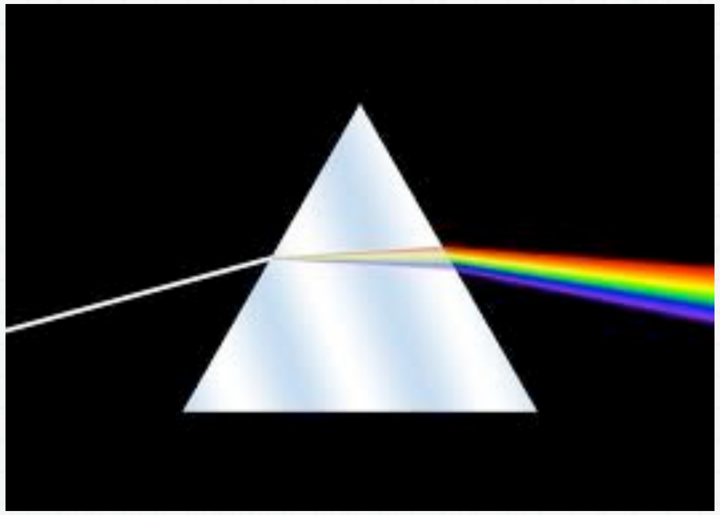
Some generalizations

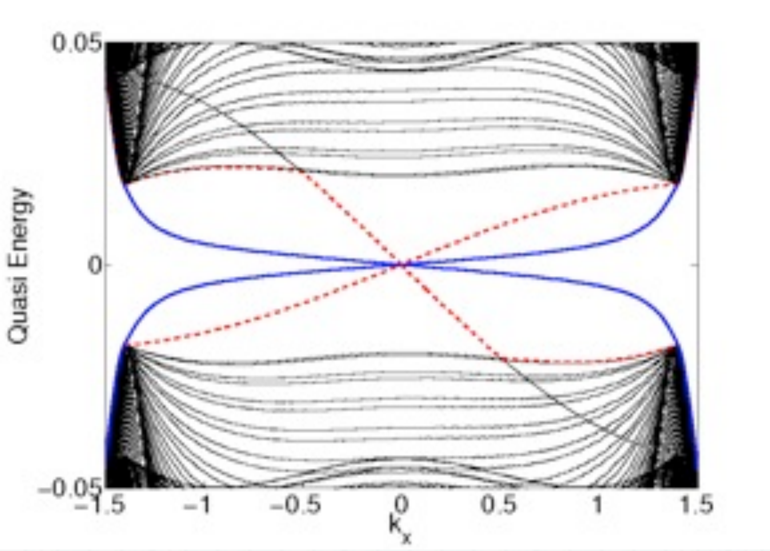
* Graphene with circularly polarized light





* Position-dependent frequency





Summary

* Modulation leads to interesting new effects

  - domain wall modes
  - vortex core states
  - photogalvanic effect

* Demonstration of the versatility of Floquet topological insulators

* Generalizations:

  - graphene
  - position-dependent frequency
  - 3D

Thank you!