Topology and Geometry of the Quantum Hall Effect

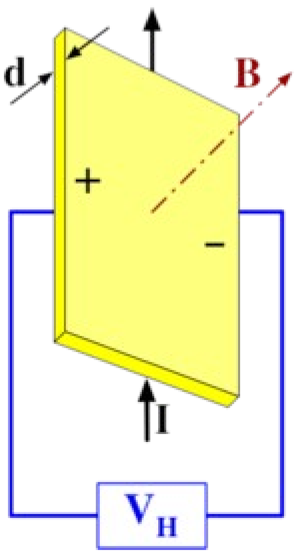
Xin Wan

Zhejiang Institute of Modern Physics, Zhejiang University

Outline

- Introduction on the topological aspects of quantum Hall effect
  - Topological quantum numbers
  - Ground state degeneracy
  - Chiral edge excitations
  - Bulk-edge correspondence
  - Quasiparticles and fractional (and non-Abelian) statistics
- Recent experiments on the 5/2 FQH state
- Geometrical aspects of quantum Hall effect
  - Example: Ultracold fermions with dipole-dipole interaction
  - Model anisotropic quantum Hall wavefunctions
  - Anisotropic Coulomb interaction: Laughlin liquid to Hall smectic

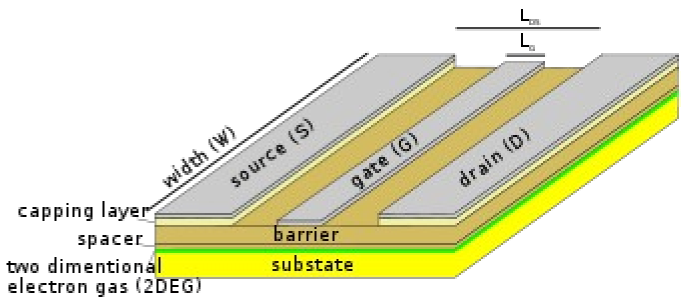
Hall Effect and 2DEG

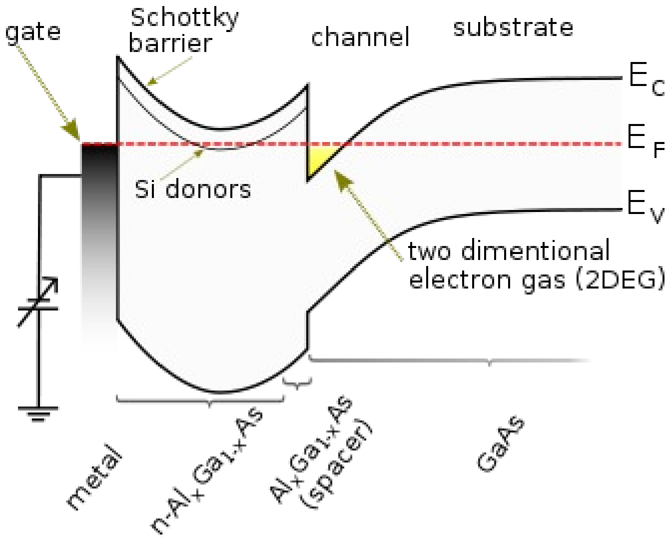


Edwin Hall
(1855-1938)

$$R_H = \frac{V_H}{I} = \frac{B}{nec}$$

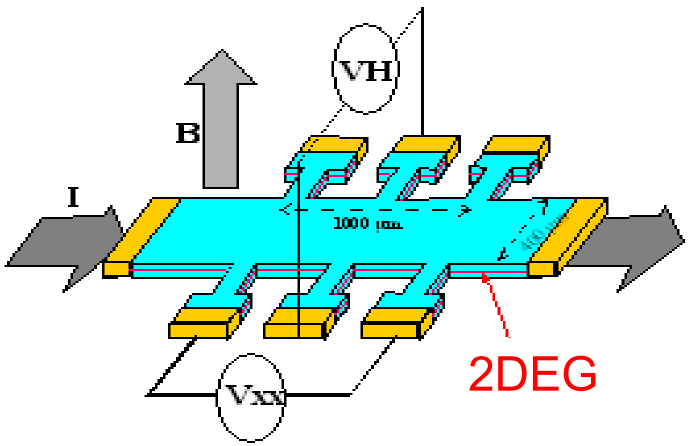
Useful to measure either the carrier density (and type) or the magnetic field

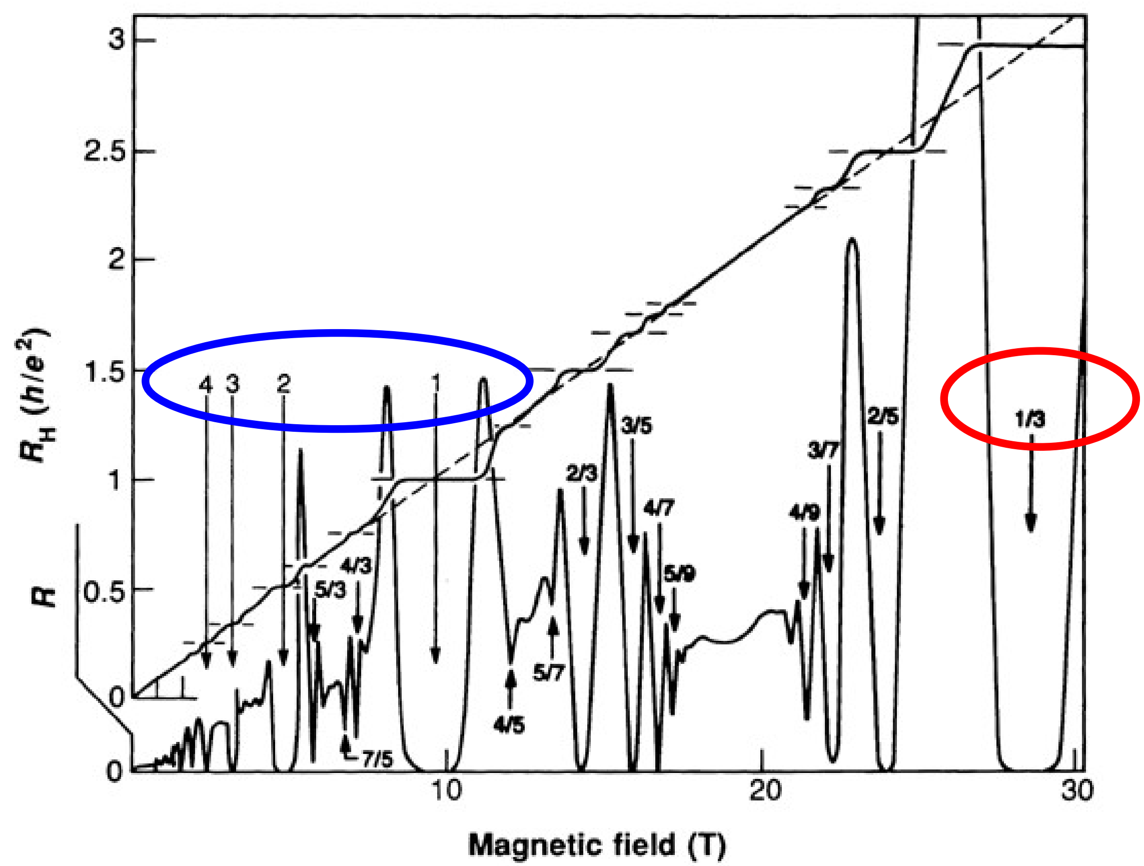




GaAs/AlGaAs heterostructure

Quantum Hall Effect





$$R_H = \frac{V_H}{I} = \frac{h}{\nu e^2}$$

$$R = \frac{V_{xx}}{I}$$

Nobel Prize (1985): K. von Klitzing

Nobel Prize (1998): D. C. Tsui, H. Stormer, R. B. Laughlin

Each plateau corresponds to a distinct topological phase.

Landau Levels (Symmetric Gauge)

- Single electron in a strong magnetic field: cyclotron motion

$$H_0=\frac{\Pi^2}{2m}, \quad \Pi=p-eA \qquad [\Pi_a,\Pi_b]=i\epsilon_{ab}(\hbar/l_B)^2 \longrightarrow [a,a^+]=1$$

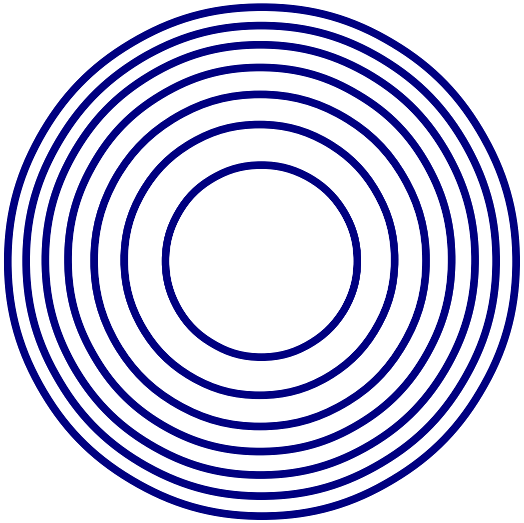
- Separate cyclotron motion from guiding-center motion

$$r=(l_B^2/\hbar)z\times\Pi+R \qquad [R_a,R_b]=-i\epsilon_{ab}l_B^2 \longrightarrow [b,b^+]=1$$

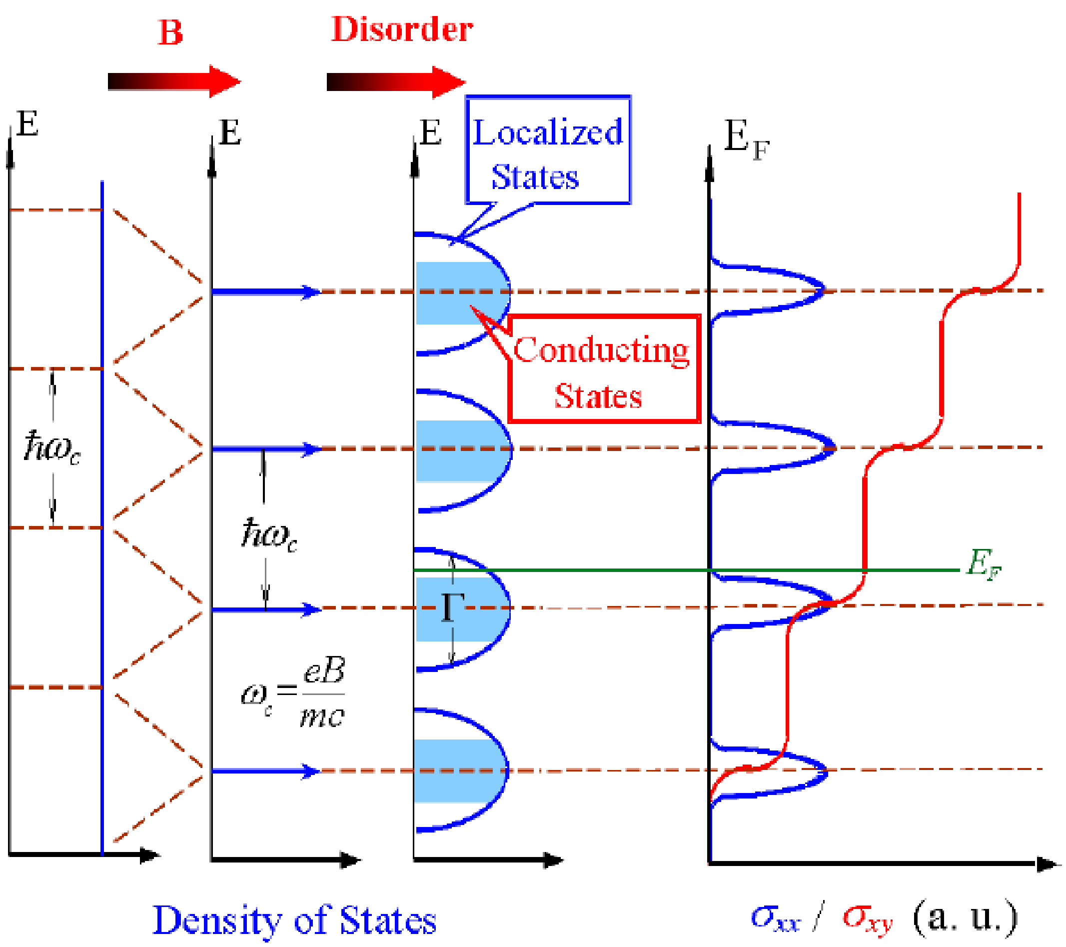
- Two sets of ladder operators – (a) inter- and (b) intra-Landau levels

nLL: $|nm\rangle=\frac{(a^+)^n(b^+)^m}{\sqrt{n!m!}}|00\rangle$

0LL/LLL: $|0m\rangle=\frac{1}{\sqrt{2\pi 2^m m!}}z^m e^{-|z|^2/4}$ $\qquad z=x+iy$



Integer Quantum Hall Effect



$$\rho_{xy} = \frac{h}{i\,e^2}$$

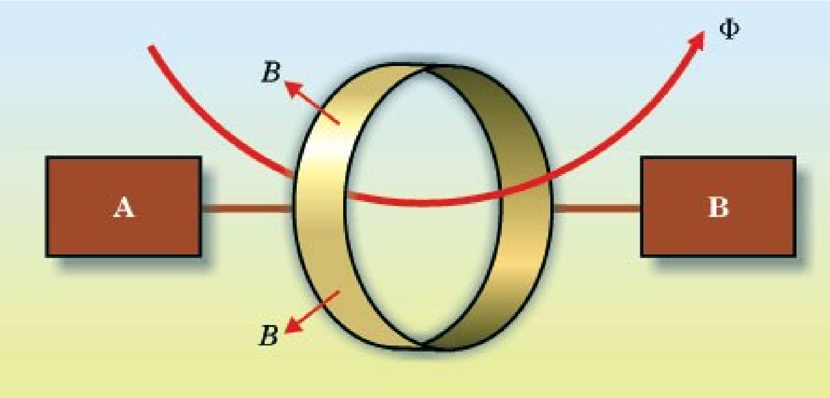
$$\sigma_{xy} = i\,\frac{e^2}{h}$$

LL spacing: different dancing patterns of electrons

LL degeneracy (broadened by disorder): guiding center motions

Topology of the IQHE

- "By gauge invariance, adding Φ_0 maps the system back to itself, … [which results in] the transfer of n electrons." – R. B. Laughlin



- Geometry links Hall conductance with topological invariants (TKNN)

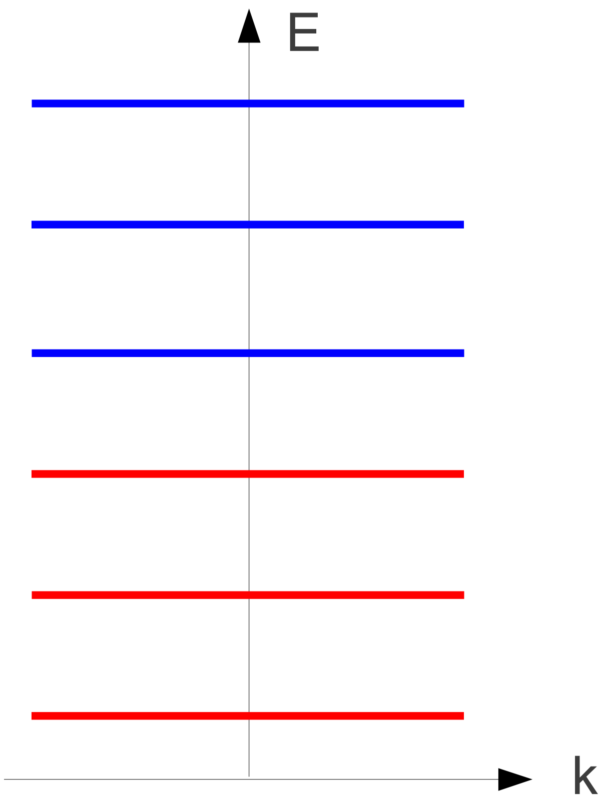
Gauge-Bonnet-Chern $\frac{1}{2\pi}\int_S K\,dA = 2(1-g)$

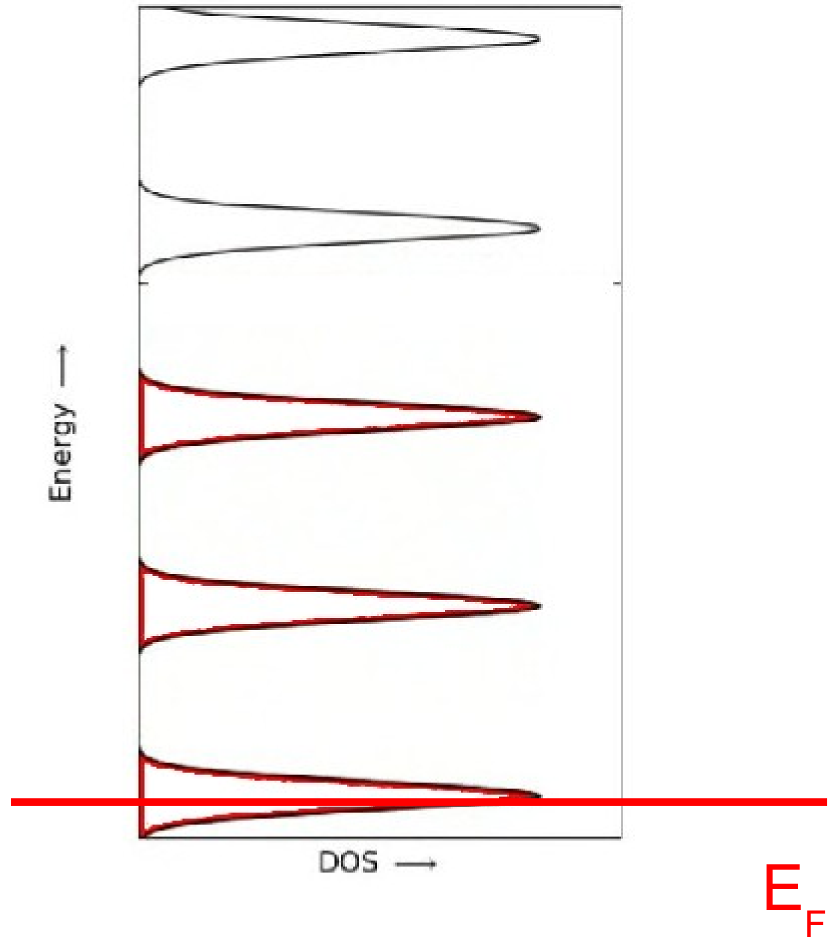
$$\sigma_{xy}(m) = \frac{e^2}{2\pi h}\int d\theta_x \int d\theta_y\, 2\Im\left\langle \frac{\partial\psi_m}{\partial\theta_x}\middle|\frac{\partial\psi_m}{\partial\theta_y}\right\rangle$$

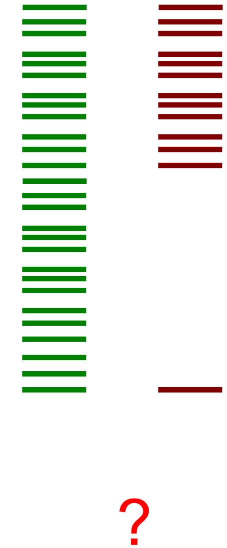
$$= C_1(m)\frac{e^2}{h}$$

Interaction Switched on

Classification of TIs







$$\nu = \frac{number\ of\ electrons}{Landau\ level\ degeneracy}$$

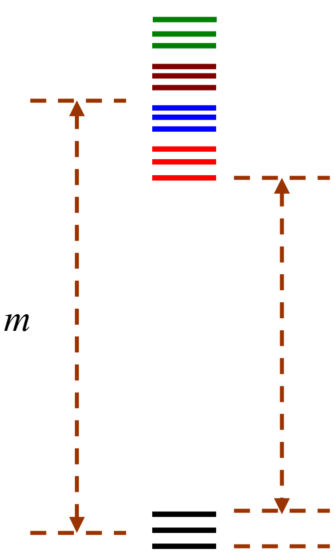
We consider the case interaction >> disorder.

FQH Systems with Disorder

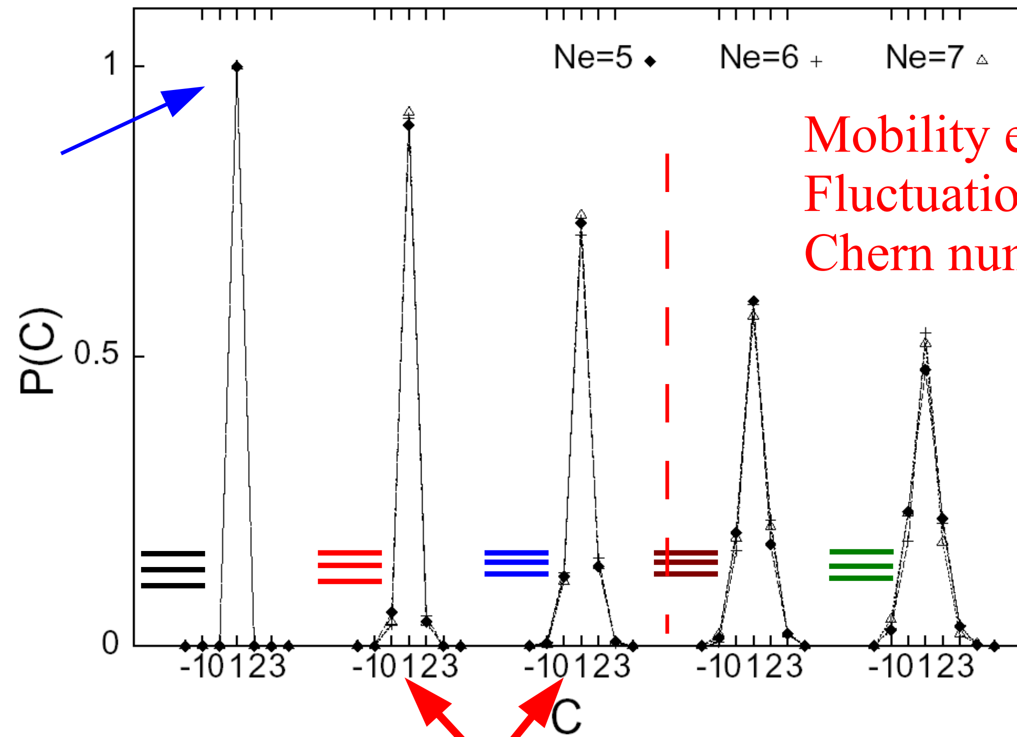
e.g., ν = 1/3

Shared by 3 states,
Hall conductance
1/3. FQHE!

$$\sum_{m_0}^{3} C_1(m_0) = 1$$



$$E_b \propto e^{-L(m^* \Delta)^{1/2}}$$



Mobility edge:
Fluctuation of
Chern numbers

Only contains localized
quasiparticle excitations

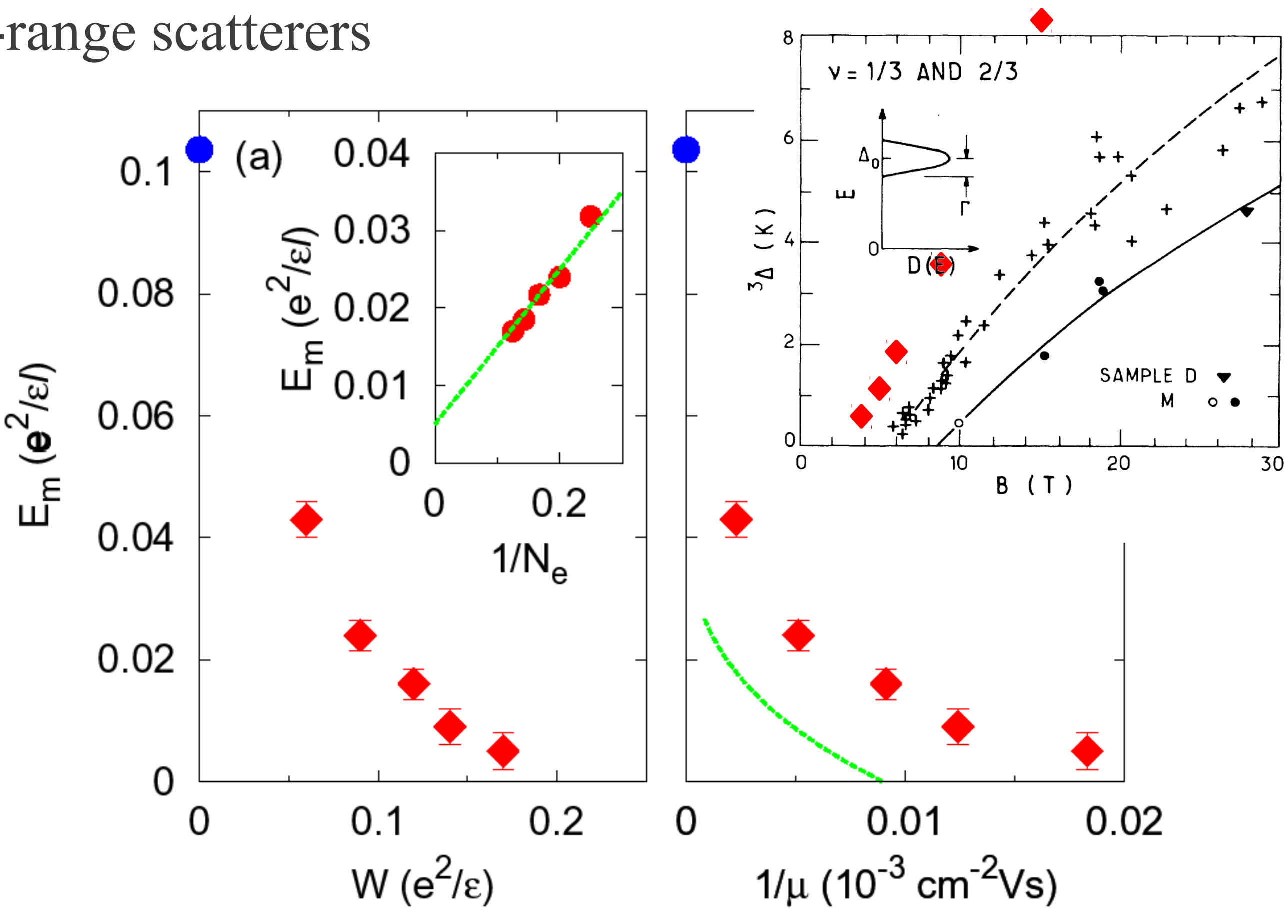
Sheng, XW, Rezayi, Yang, Bhatt & Haldane
PRL 90, 256802 (2003)

Degeneracy recovered in the thermodynamic limit (Wen & Niu, 90)
Robust against local perturbation, or local probe cannot distinguish degenerate ground states

Disorder Dependence of Mobility Gap

- To compare with experiment: mobility μ vs. disorder
  - Dominated by short-range scatterers
  - Born approximation

$$\mu = \frac{e h^3}{m^{*2} W^2}$$



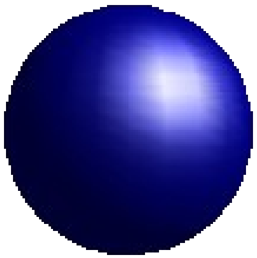
Blue dot: creation energy for quasiparticle-quasihole pair at large separation

Green curve in (b): converted from experimental data

Sheng, XW, Rezayi, Yang, Bhatt & Haldane, PRL 90, 256802 (2003)

XW, Sheng, Rezayi, Yang, Bhatt & Haldane, PRB 72, 075325 (2005)

Topology in the Laughlin State ($\nu = 1/m$)



sphere

degeneracy = 1

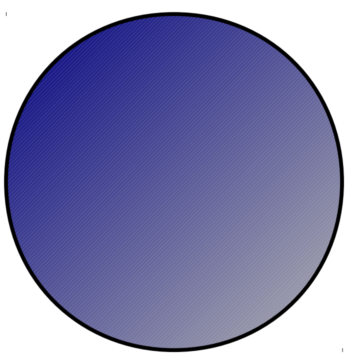
torus

degeneracy = m

two-holed torus

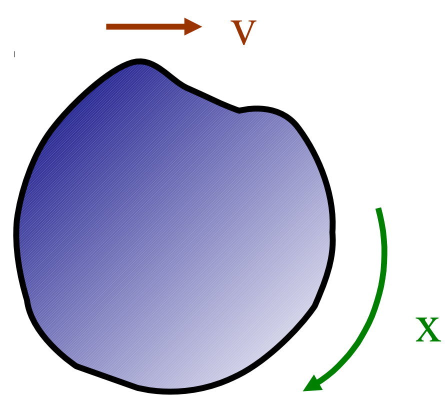
degeneracy = m^2

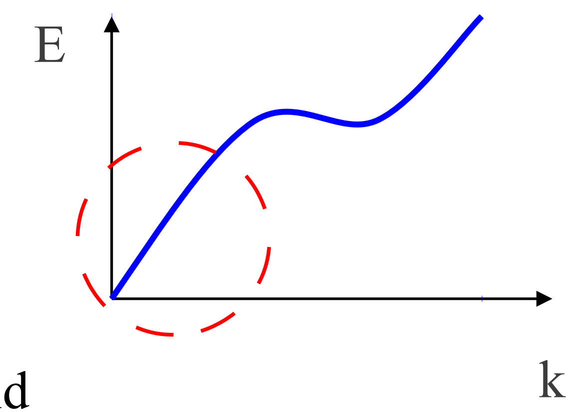
The ground state degeneracy of a fractional quantum Hall liquid is insensitive to disorder, but depends on the topology of the system (Wen & Niu, 1990).



disc

degeneracy = 1
gapless chiral edge mode

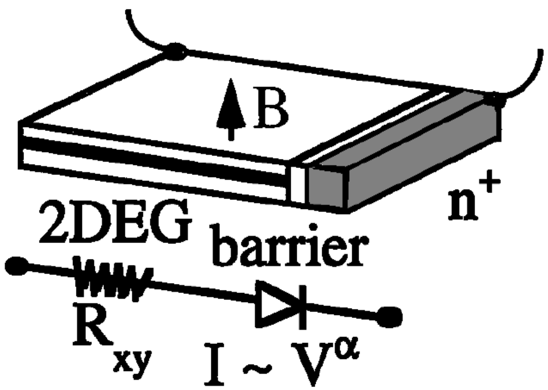




Chiral Luttinger liquid

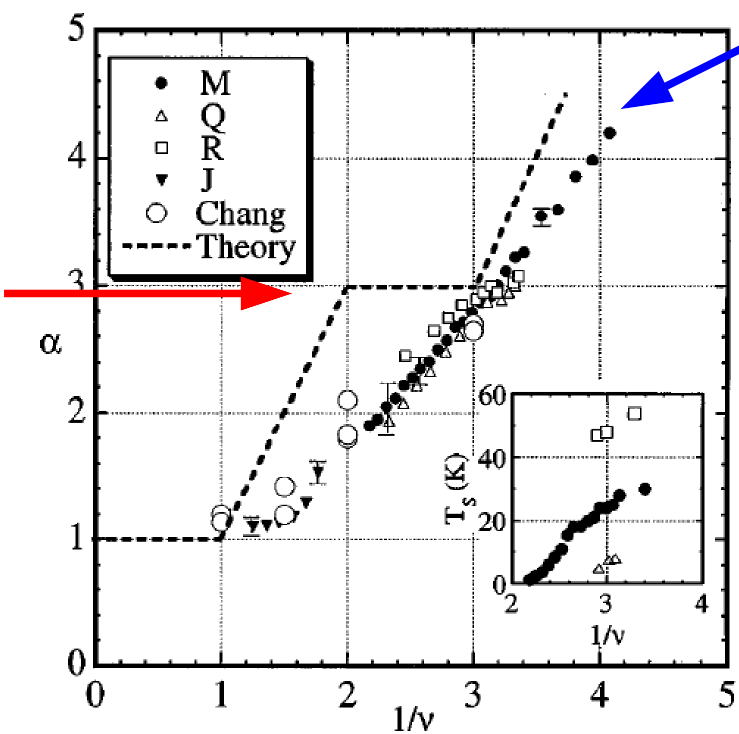
$$G(x,t) \sim \frac{1}{(x-vt)^m}$$

Edge Tunneling Experiment



Electrons tunnel across a barrier into the edge of an FQH liquid

Grayson, Tsui, Pfeiffer, West, Chang (98)



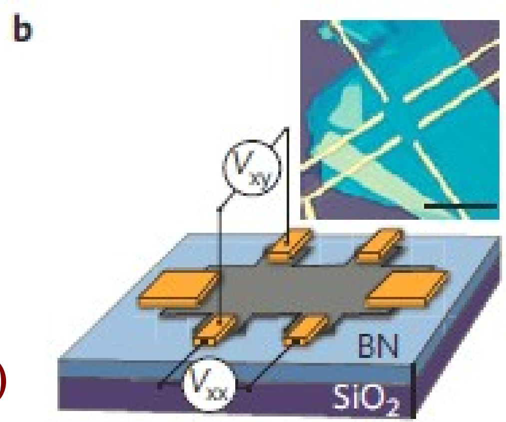
Experiment found

$$\alpha \approx 1/\nu$$

Not a universal law!

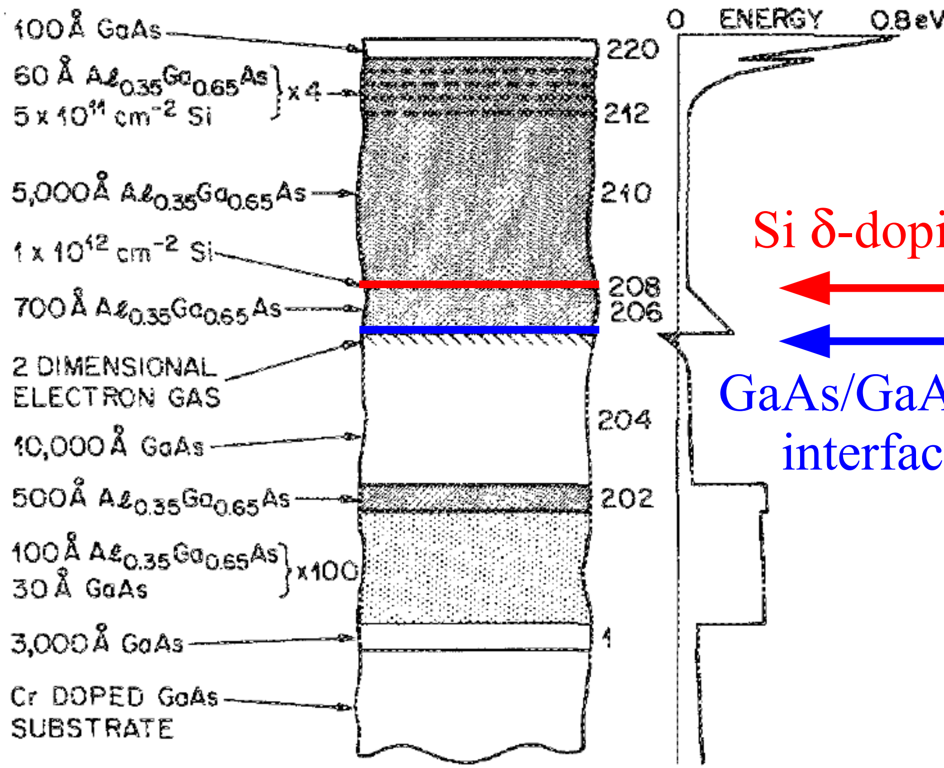
The chiral Luttinger liquid theory predicted a plateau between $\nu = 1/3$ and $1/2$ (Wen).

no counterpropagating edge modes!?



Universal edge tunneling exponent in graphene?
Zi-Xiang Hu, R. N. Bhatt, XW, and Kun Yang, PRL (2011)

Theoretical Modeling Based on Sample Design



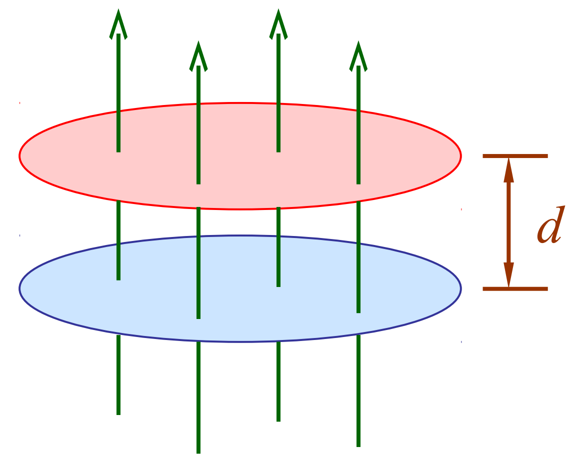
Si δ-doping

GaAs/GaAlAs interface

Background charge (+Ne)

Electron layer (-Ne)

$\Phi = N\Phi_0 / \nu$



$$H = \frac{1}{2}\sum_{mnl} V_{mn}^{l} c_{m+l}^{+} c_{n}^{+} c_{n+l} c_{m} + \sum_{m} U_{m} c_{m}^{+} c_{m}$$

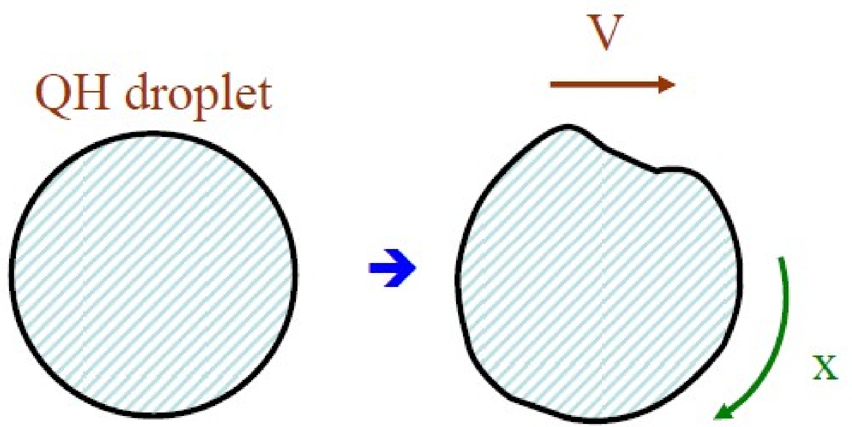
Pfeiffer et al., Appl. Phys. Lett. **55**, 18 (1989)

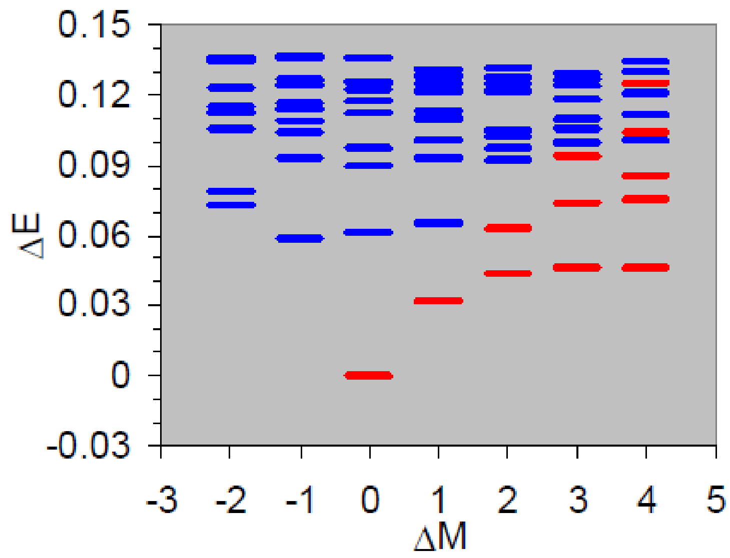
Coulomb interaction

Confining potential

Edge Excitations

- ν = 1/3 Abelian edge: chiral Luttinger liquid

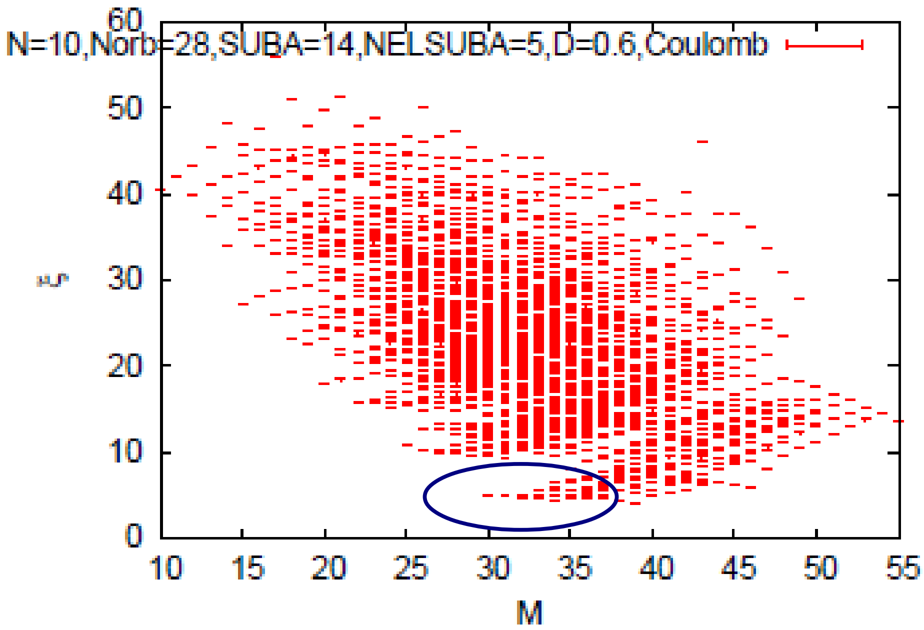




Number of edge states: 1 1 2 3 5 7 11 …

- Entanglement spectrum

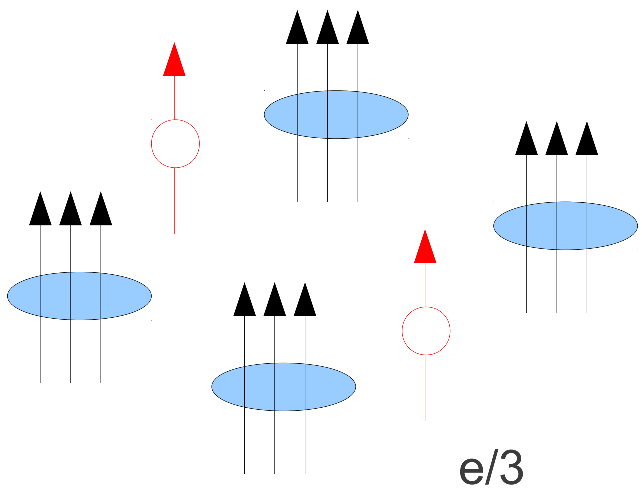
$$|\Psi\rangle = \sum_i e^{-\xi_i/2} |\Psi_i^A\rangle \otimes |\Psi_i^B\rangle$$

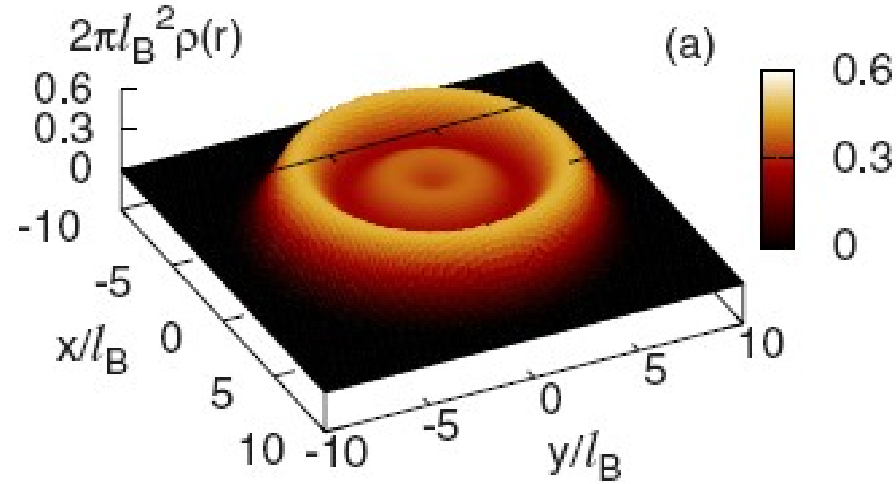


Gap and weight depends on interaction (i.e., geometry), counting does not.

Abelian Laughlin Quasiholes

- FQHE for electrons (ν = 1/3, 1/5, …)
  - Condensate of composite bosons

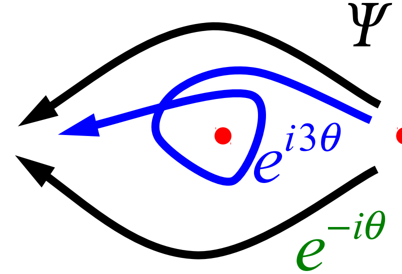




$$\Psi_L = \prod_{i<j} (z_i - z_j)^m$$

$$\Psi_\xi^{qh} = \prod_j (z_j - \xi_1)(z_j - \xi_2) \prod_{i<j} (z_i - z_j)^m$$

$$\Psi \to \Psi e^{i\theta}$$

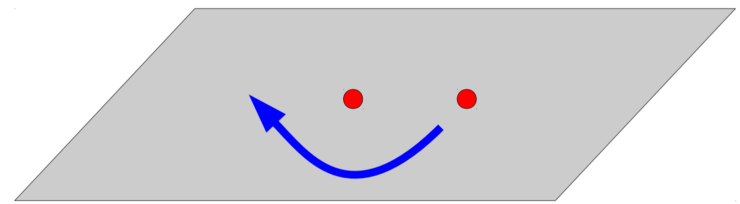


Path equiv. in 3D; NOT equiv. in 2D:

Abelian anyons (i.e., different by a phase)

Statistics in 2D

- Fermions $\Psi \to -\Psi$
- Bosons $\Psi \to \Psi$



point-like particles in 2D

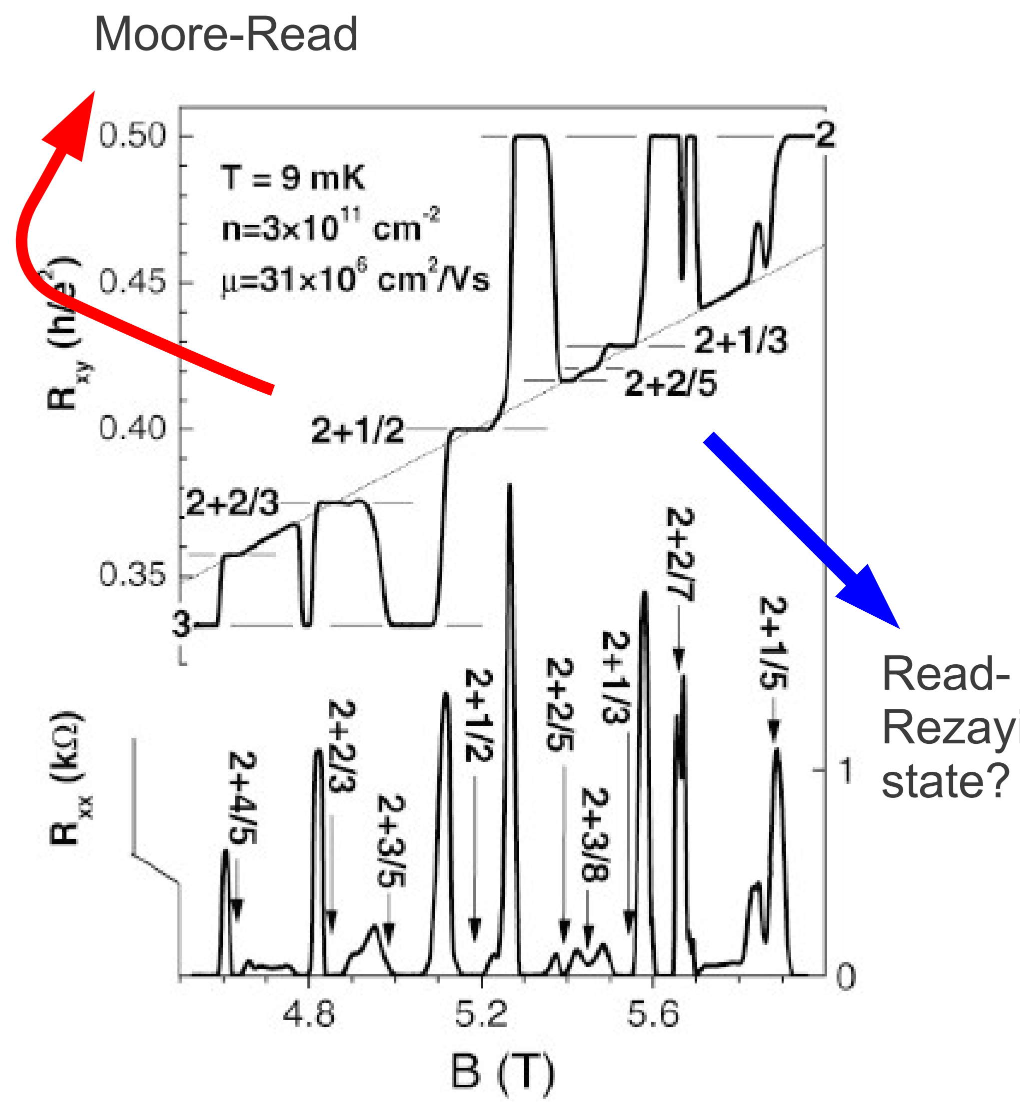
- Anyons
  - Abelian $\Psi \to e^{i\theta}\Psi$
  - non-Abelian $\Psi_a \to M_{ab}\Psi_b$

GS degeneracy robust against local perturbation

topological quantum computation

Milestones for the 5/2 State

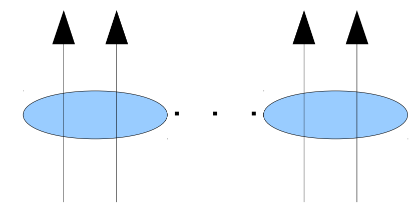
- Discovered by R. Willett, 1987
- Spin-polarized wavefunction based on Ising conformal field theory, Moore & Read, 1991
- Numerical verification, Morf, 1998; Rezayi & Haldane, 2000
- Proposal of topologically protected qubits, Das Sarma, Freedman & Nayak, 2005
- And many more ......



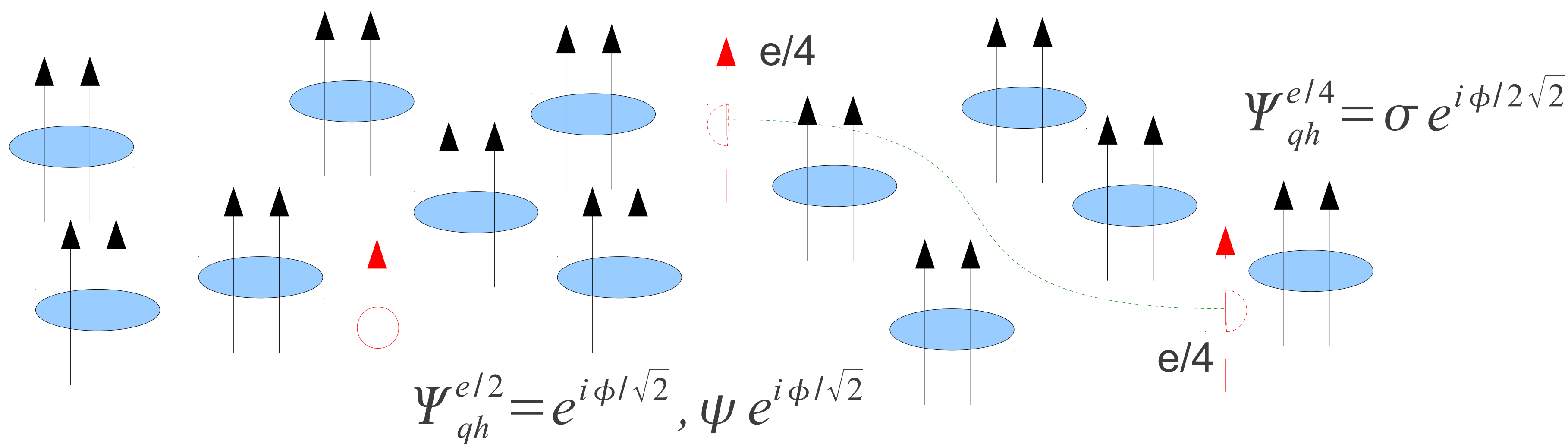
Xia et al., PRL (04)

A Cartoon of the Moore-Read State

- Half-filling ν = 1/2: CF at zero effective field (B* = 0)
  - 0LL (or LLL): Fermi sea of composite fermions
  - 1LL: Superfluid of Cooper pairs of composite fermions
  - 2+LL: Charge density wave



- Condensate of composite fermions (ν = 5/2 = 2 + 1/2)



e/4 quasihole = charge-e/4 boson + neutral Majorana fermion mode

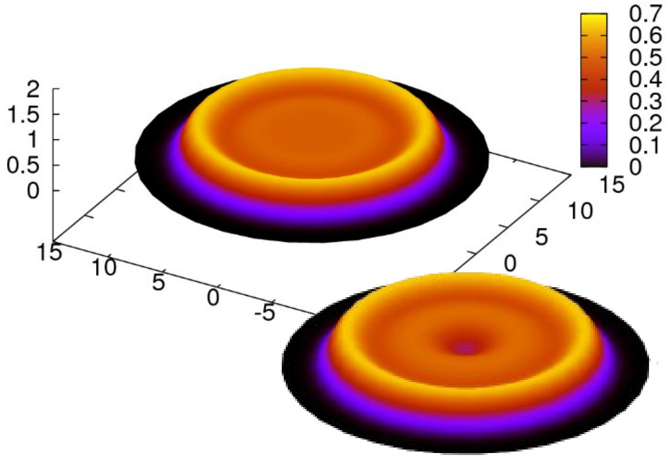
Moore-Read State

- Moore-Read state

$$\psi_e(z)=\psi(z)e^{i\alpha\phi(z)}, \alpha=\sqrt{m} \qquad \Psi_{Pf}=\langle\psi_e(z_1)\cdots\psi_e(z_N)\rangle=Pf\left(\frac{1}{z_i-z_j}\right)\prod_{i<j}(z_i-z_j)^m$$

- Quasiholes in Moore-Read condensate
  - Charge e/2, Abelian (Laughlin type)

$$\prod_i(z_i-\xi_1)(z_i-\xi_2)\,Pf\left(\frac{1}{z_i-z_j}\right)\prod_{i<j}(z_i-z_j)^2$$



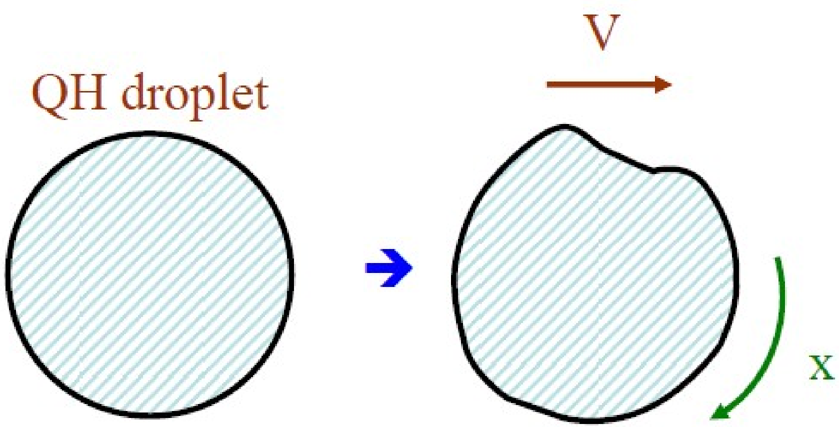
  - Charge e/4, non-Abelian

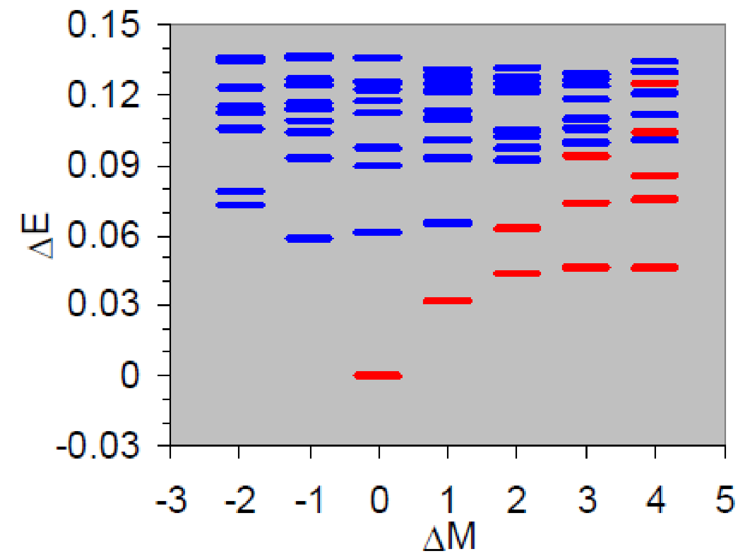
$$\Psi_{(12)(34)}=Pf\left(\frac{(z_i-\xi_1)(z_i-\xi_2)(z_j-\xi_3)(z_j-\xi_4)+i\Leftrightarrow j}{z_i-z_j}\right)\prod_{i<j}(z_i-z_j)^2$$

Unerasable excitations: Excitations of the topological phase that cannot be created or destroyed by local operators and that are not degenerate in energy with the ground state. – N. Read in Physics Today (July 2012)

Paired FQH State at $\nu = 5/2$

- $\nu = 1/3$ Abelian edge: chiral Luttinger liquid



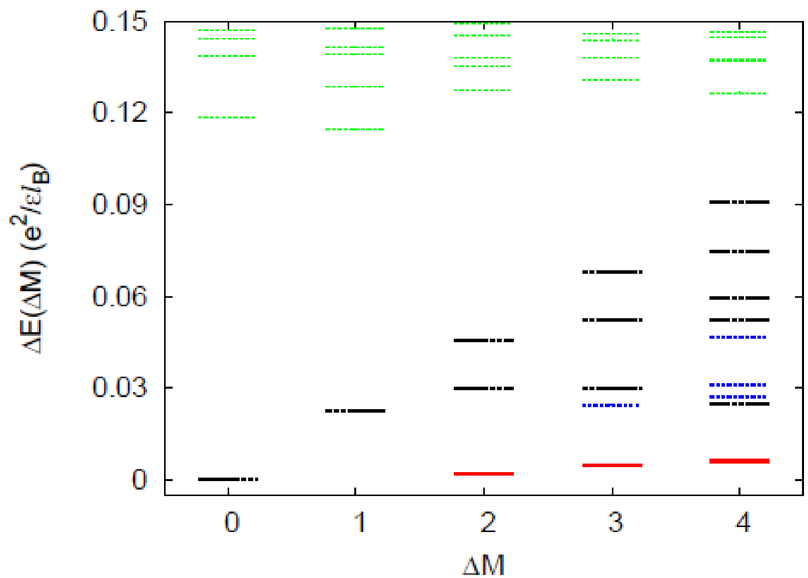


Number of edge states: 1 1 2 3 5 7 11 …

- $\nu = 5/2$ non-Abelian edge:

  - Charged mode (density deformation)
  - Neutral mode (Majorana fermion)

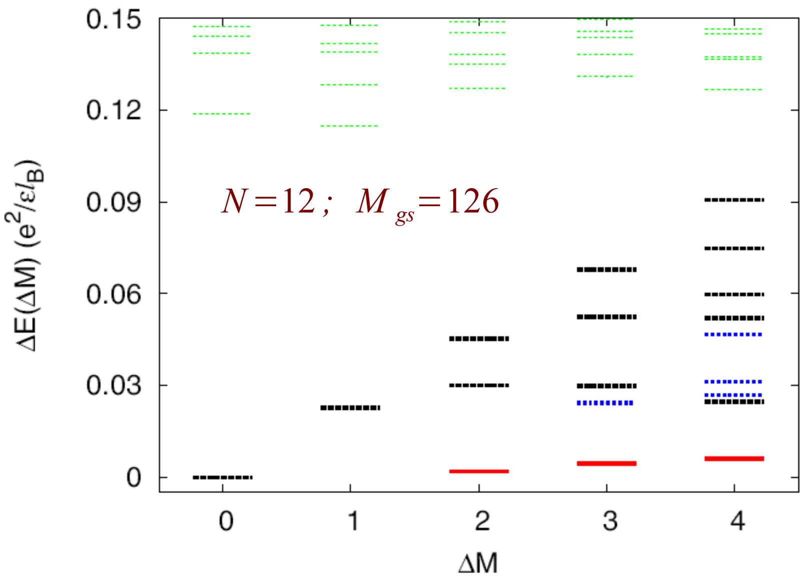
  Total number of edge states: 1 1 3 5 10 ...



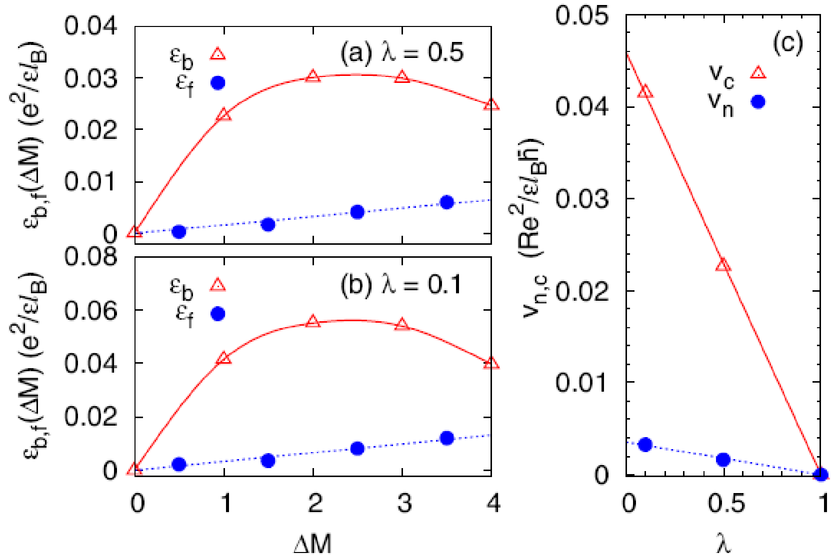
Read & Milovanovic, Phys. Rev. B (1996)

XW, Yang & Rezayi, Phys. Rev. Lett. (2006)

Edge-mode Velocities



$$H_\lambda = (1-\lambda) H_C + \lambda H_{3B}$$



<u>Bose-Fermi separation</u>:
Fermionic edge-mode velocity is much lower than the bosonic edge-mode velocity.

$v_c = 5\times10^6 cm/s$ $v_n = 4\times10^5 cm/s$

XW, Yang & Rezayi, PRL (2006)

XW, Hu, Rezayi & Yang, PRB (2008)
[PRB Editors' Suggestion]

Experimentally,

$v_c = 8 \sim 15\times10^6 cm/s$

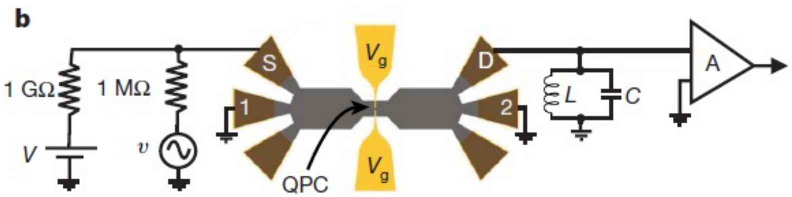
ν = 1
Marcus group
arXiv:0903.5097

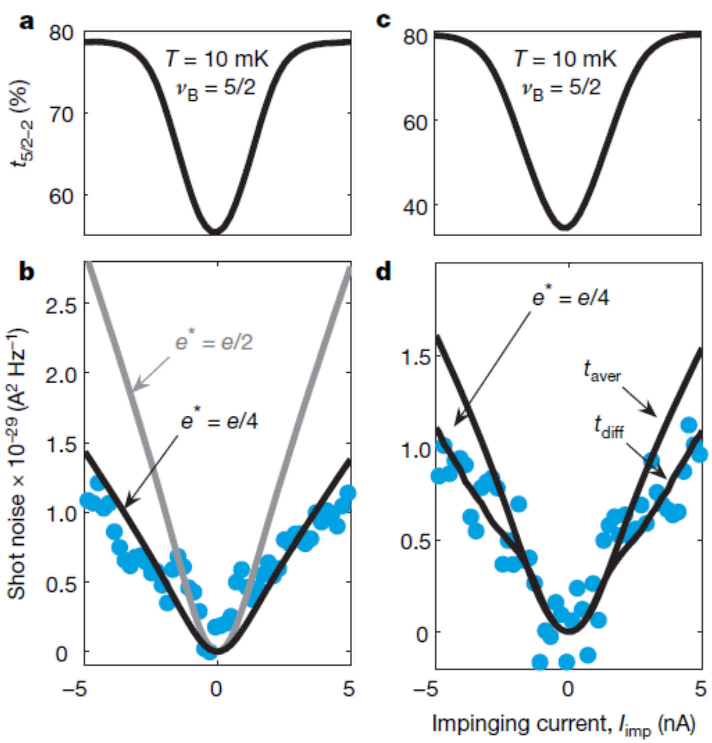
$v_c = 4\times10^6 cm/s$

ν = 1/3
Goldman group
PRB (2006)

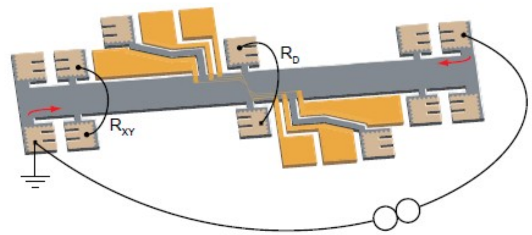
Anyon There?

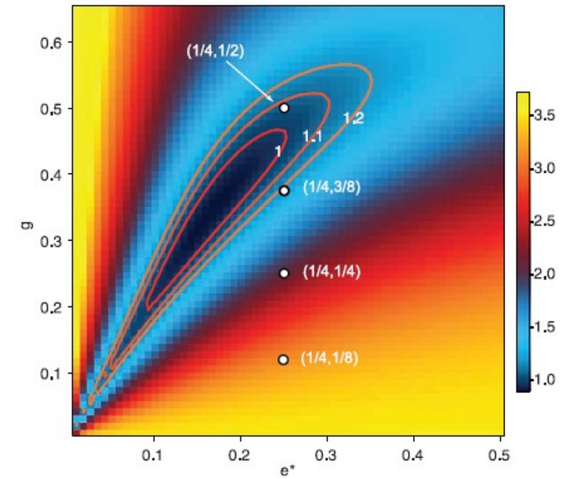
Dolev et al., Nature 452, 829 (2008)



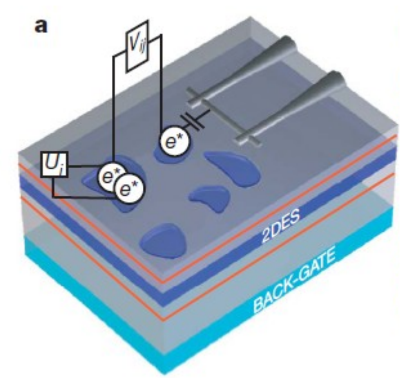


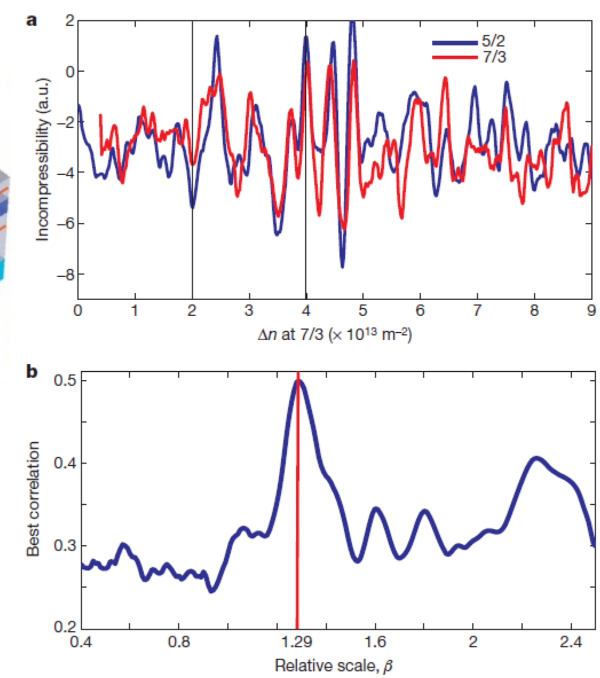
Radu et al., Science 320, 899 (2008)





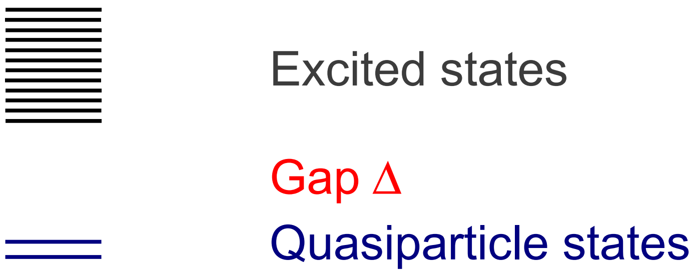
Venkatachalam et al., Nature 469, 285 (2011)

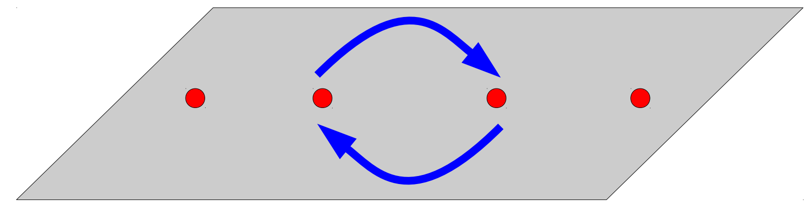




Noise, tunneling conductance, and local compressibility support the existence of e/4 anyons. But what about their statistics?

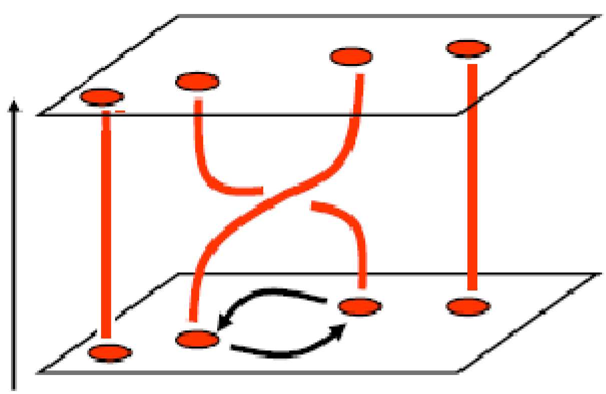
Non-Abelian Statistics





$$\Psi_a \rightarrow M_{ab} \Psi_b$$

Quasiparticle state degeneracy robust against local perturbation!



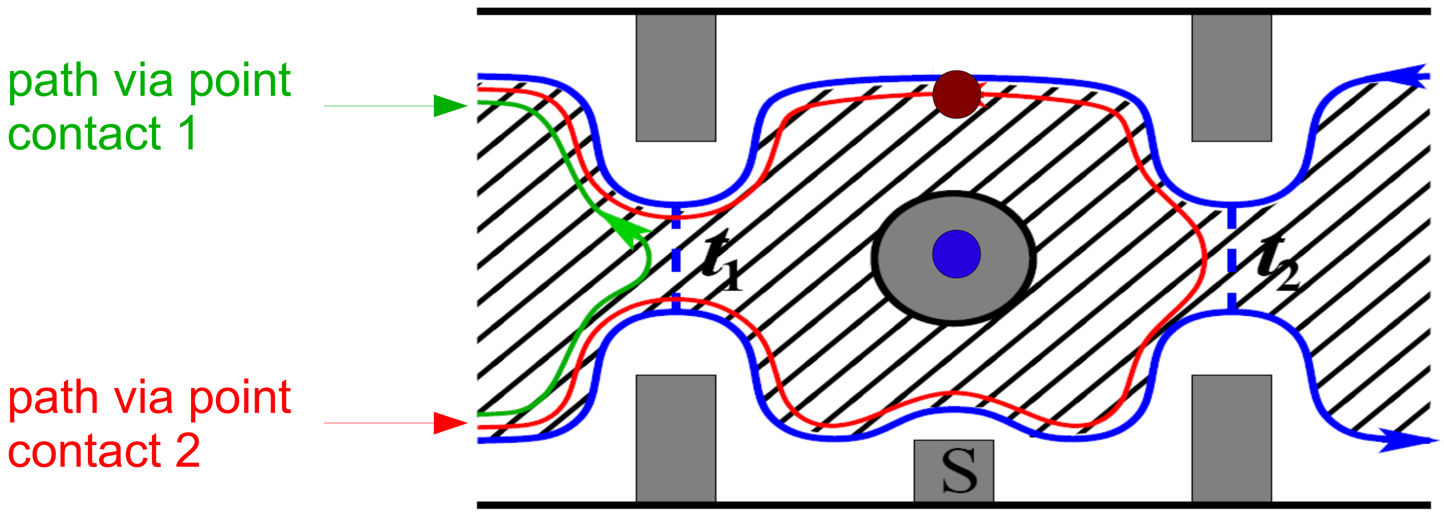
$$|\psi_f\rangle = \tilde{\alpha}|\psi_0\rangle + \tilde{\beta}|\psi_1\rangle$$

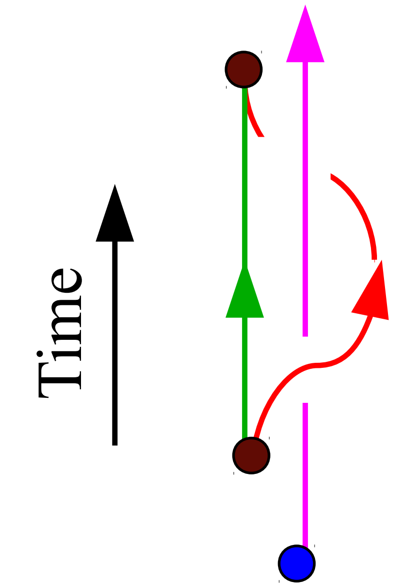
$$|\psi_i\rangle = \alpha|\psi_0\rangle + \beta|\psi_1\rangle$$

degenerate states

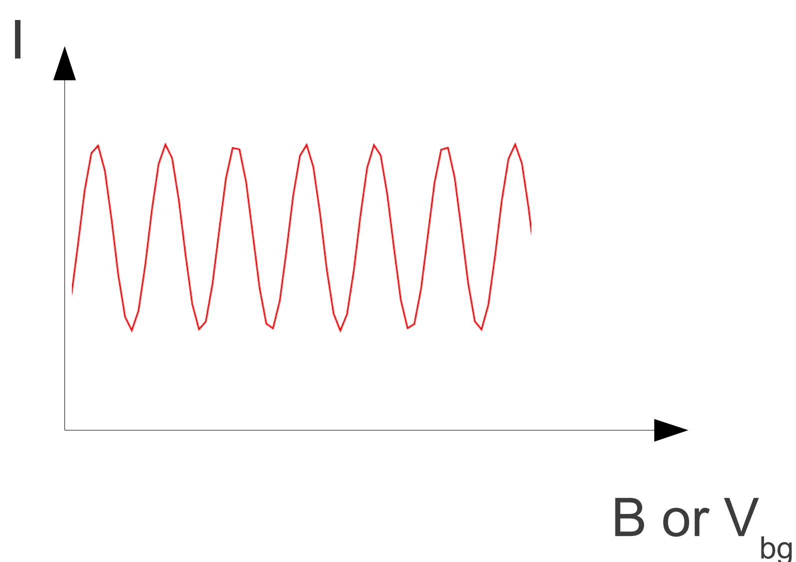
Braiding via Interference

Stern & Halperin (06); Bonderson, Shtengel & Kitaev (06)





edges of the filled Landau levels not included



Competition in Interference

- Competition between non-Abelian e/4 and Abelian e/2 quasiparticles

$$\Psi_{qh}^{e/4} = \sigma e^{i\phi/2\sqrt{2}} \qquad \Psi_{qh}^{e/2} = e^{i\phi/\sqrt{2}}, \psi e^{i\phi/\sqrt{2}} \qquad (\sigma \times \sigma = 1 + \psi)$$

odd-even effect:

(q = e/4)

$$s^{e/4} = \begin{cases} \pm 1/\sqrt{2} & n \ \ even \\ 0 & n \ \ odd \end{cases}$$

Aharonov-Bohm effect

$$I_{12}^{q} \propto s^{q} |\Gamma_1^q \Gamma_2^{q\,*}| e^{-|x_1 - x_2|/L_\phi} \cos\left(2\pi \frac{q}{e}\frac{\Phi}{\Phi_0} + \phi_q + arg(\Gamma_1^q \Gamma_2^{q\,*})\right)$$

tunneling amplitude

coherence length due to thermal smearing

favors e/4 qps?

$$L_\phi = \frac{1}{2\pi k_B T}\left(\frac{g_c}{v_c} + \frac{g_n}{v_n}\right)^{-1}$$

favors e/2 qps

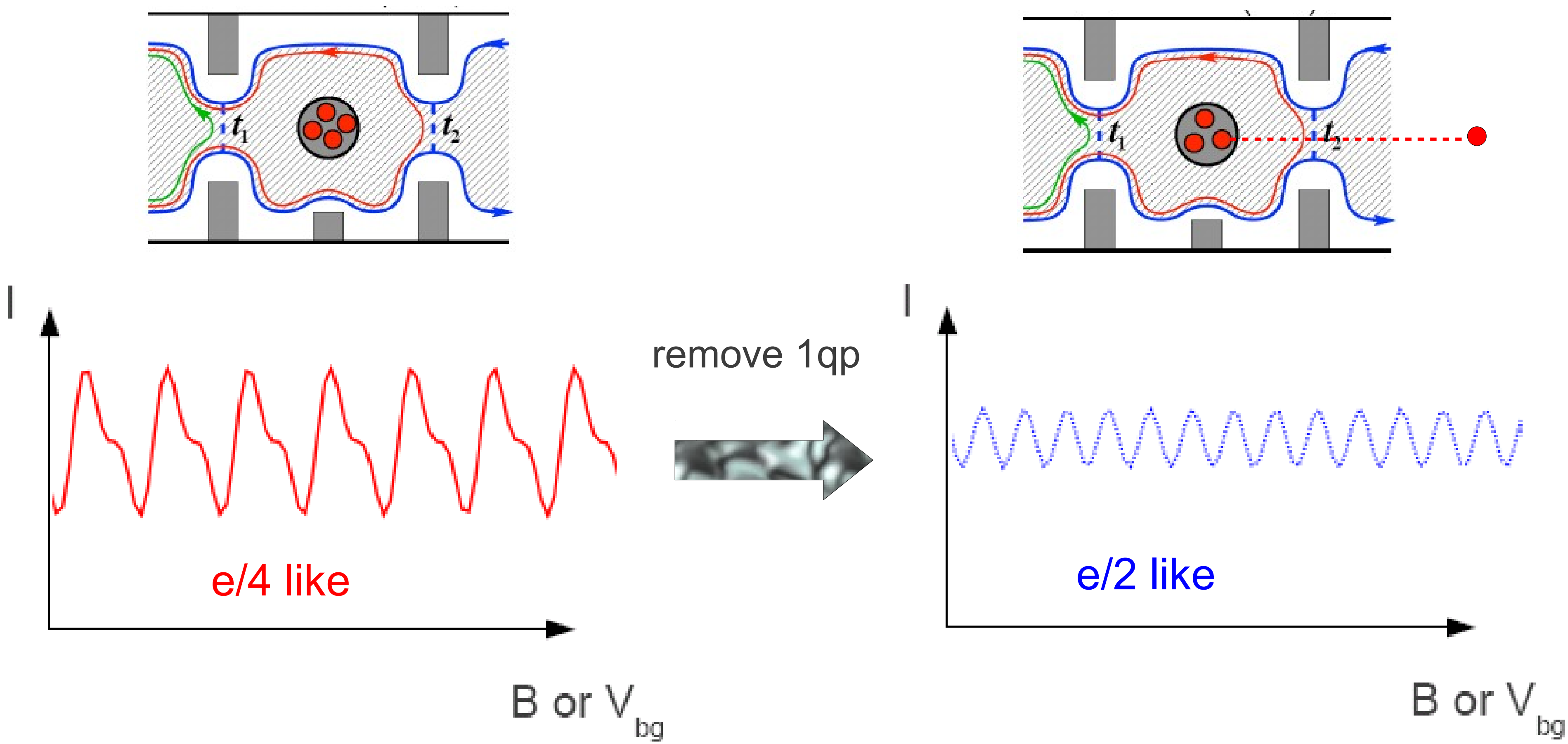
Bishara & Nayak, PRB (2008)

$L_\phi T \sim 40$ μm· mK

XW, Hu, Rezayi & Yang, PRB (2008)

Signature of Non-Abelian Statistics

XW, Hu, Rezayi & Yang, PRB (2008)

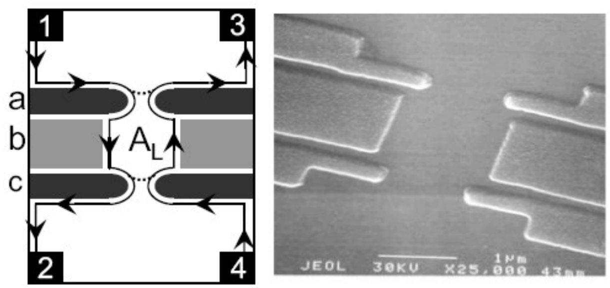


Even number of non-Abelian quasiparticles inside the interference loop

Odd number of non-Abelian quasiparticles inside the interference loop

Experiment in Agreement with Theory

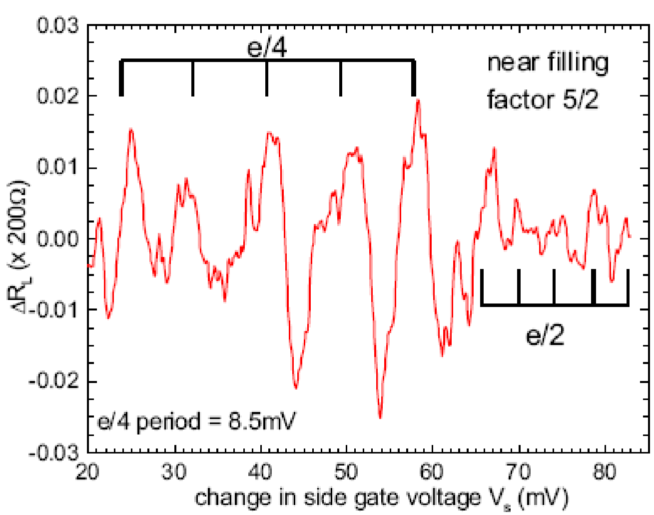
- 25 mK, size ~ 1 μm



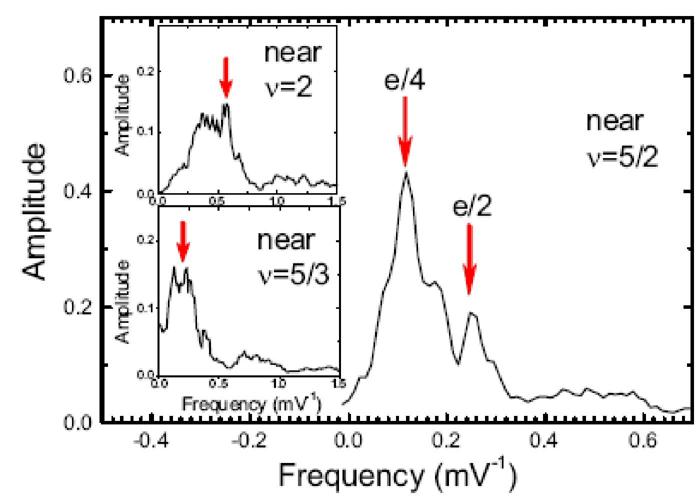
Willett, Pfeiffer & West, PNAS (2009)

- At 10 mK, e/4 pattern observable only when device size < 4 μm
  - At 25 mK, < 1.6 μm
- Both e/4 and e/2 interference patterns observable

Wan, Hu, Rezayi & Yang, PRB (2008)

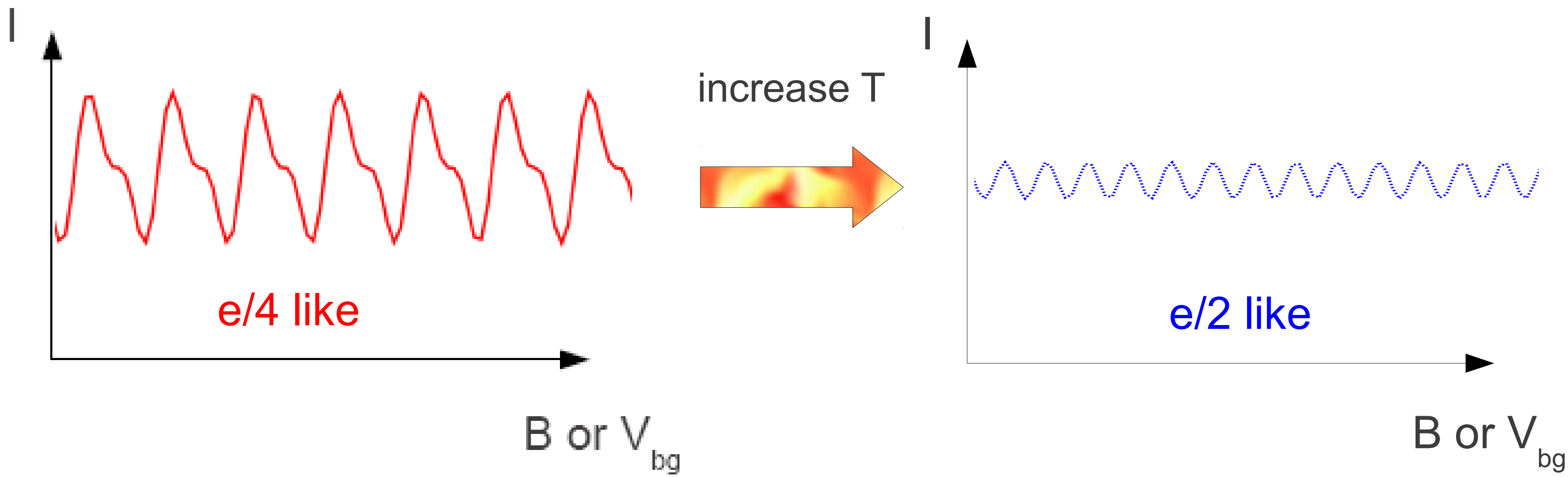


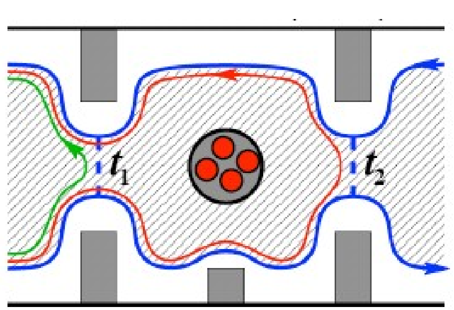
Fourier transform



e/4 Pattern Suppressed at Higher T

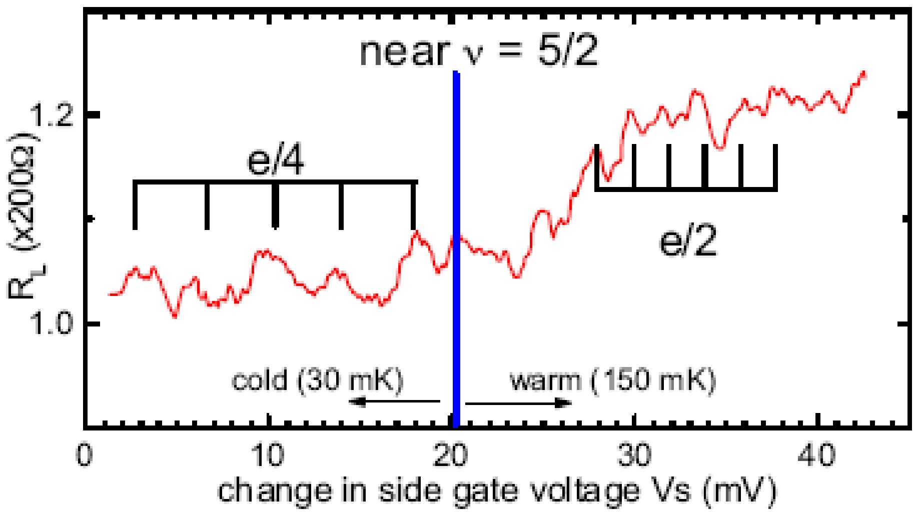
XW, Hu, Rezayi & Yang, PRB (2008)

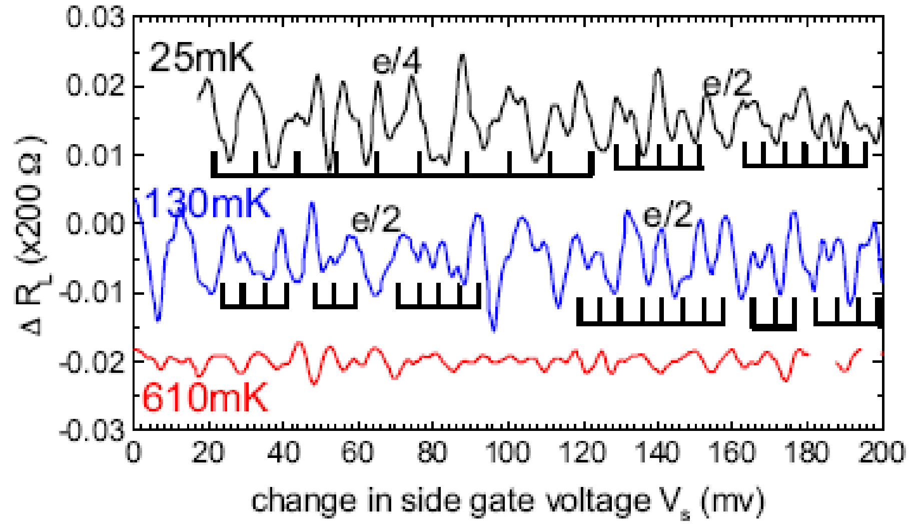




Even number of non-Abelian quasiparticles inside the interference loop

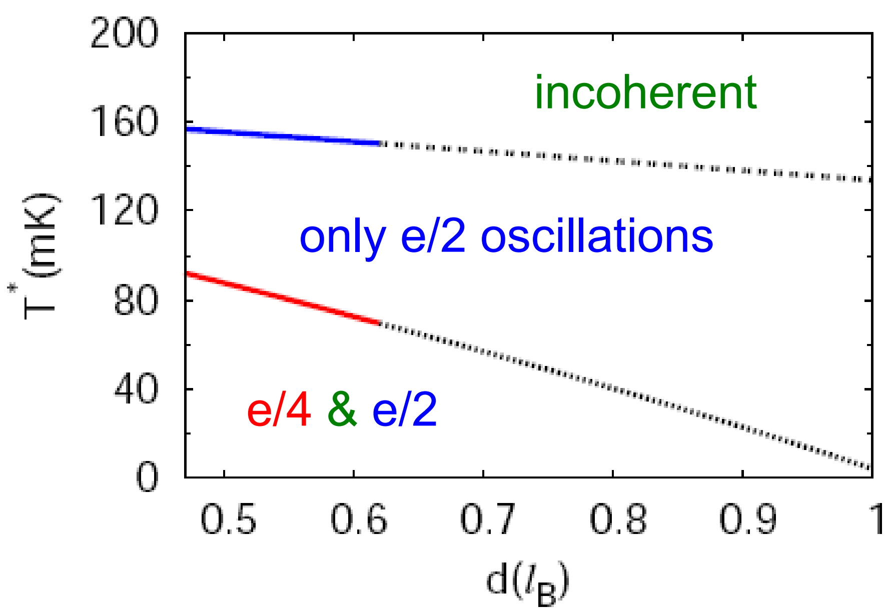
Temperature Dependence





period lines in the swept side-gate data. (*C*) Data indicate temperature dependence of e/4 and e/2 oscillations: e/2 oscillations may be made more prevalent with an increase in temperature. The temperature of the sample was taken from

Willett et al., PNAS (2009)



← Hu, Rezayi, XW & Yang, PRB (2009)

e/2: less sensitive

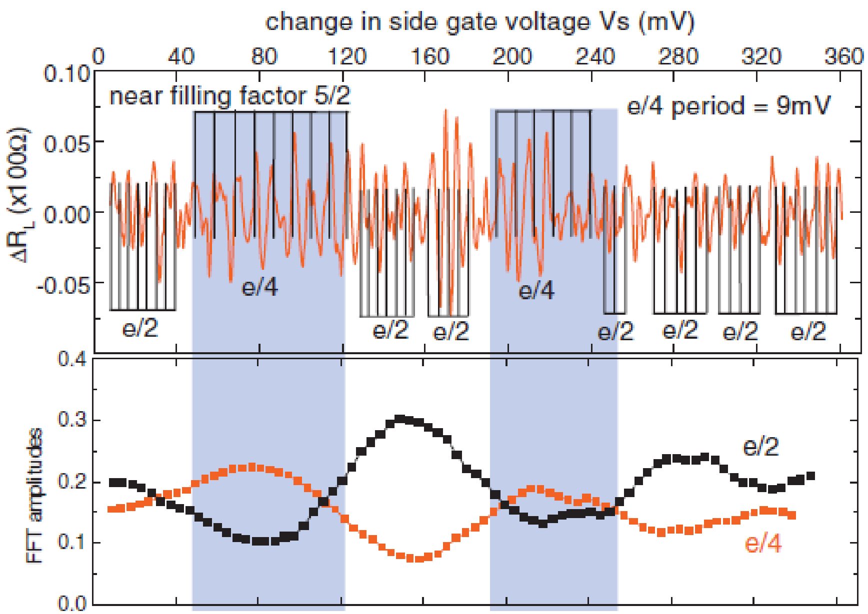
Parameters:
~~B = 6 T~~
ε = 13.1
$|x_1 - x_2| = 1\ \mu m$

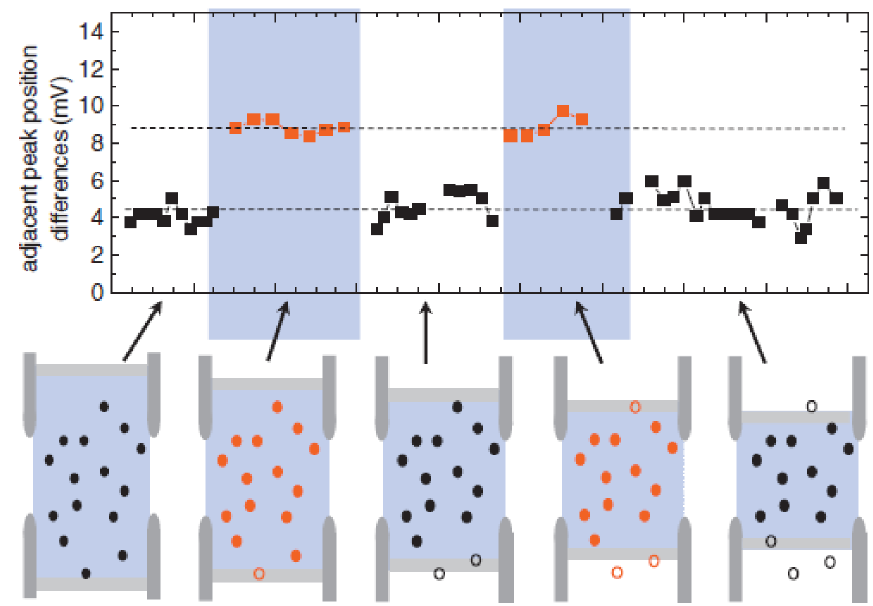
e/4: sensitive on interaction and confining potential

Opposite trend for anti-Pfaffian

Alternative e/4 and e/2 Patterns

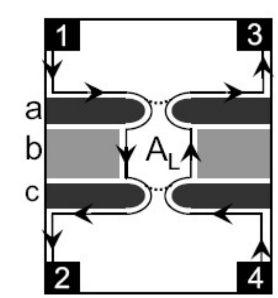
Willett et al., PRB (2010)

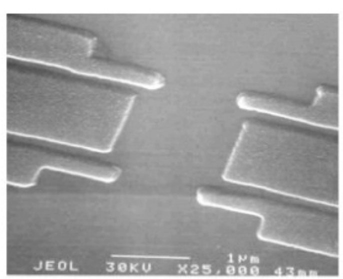




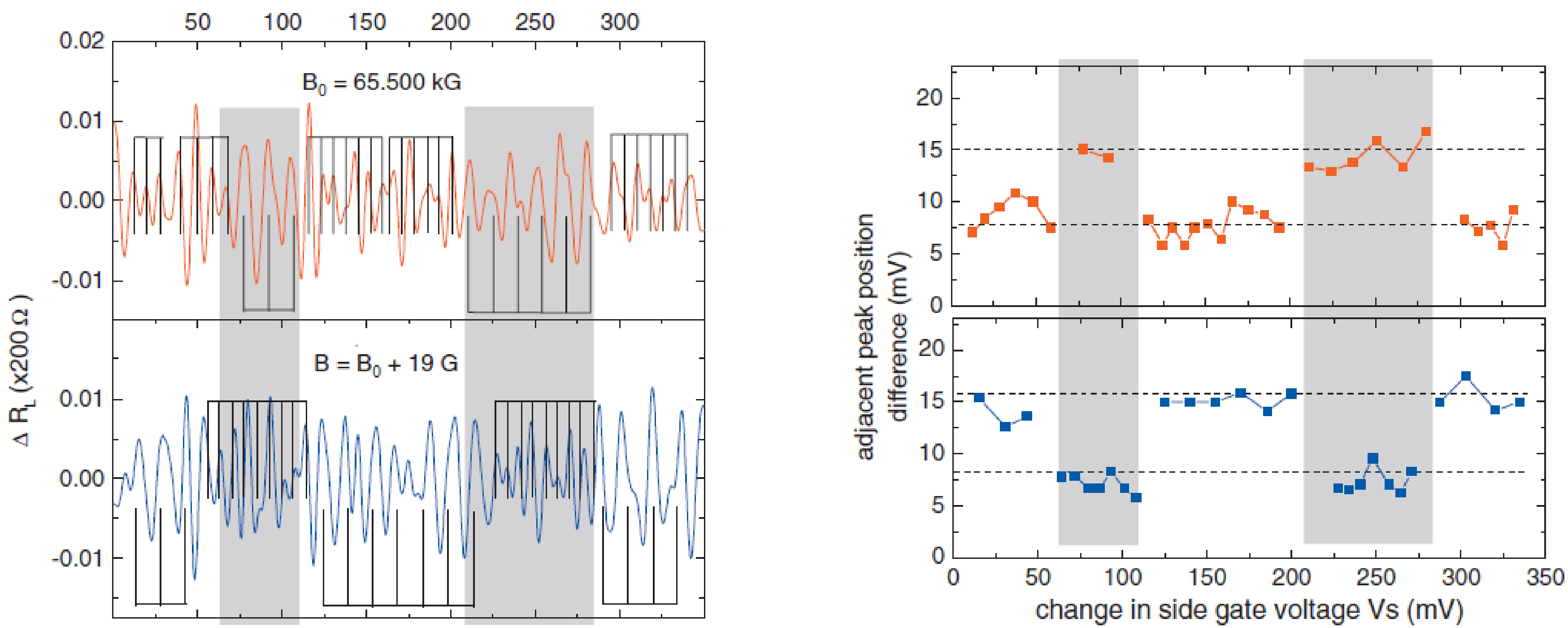
On thermal decoherence:
Bishara & Nayak, Phys. Rev. B 77, 165302 (2008)
XW, Hu, Rezayi & Yang, Phys. Rev. B 77, 165316 (2008)

On tunneling amplitude:
Bishara, Bonderson, Nayak, Shtengel & Slingerland,
Phys. Rev. B 80, 155303 (2009)
Chen, Hu, Yang, Rezayi & XW, Phys. Rev. B 80, 235305 (2009)





B-field Induced e/4 and e/2 Oscillation Swap



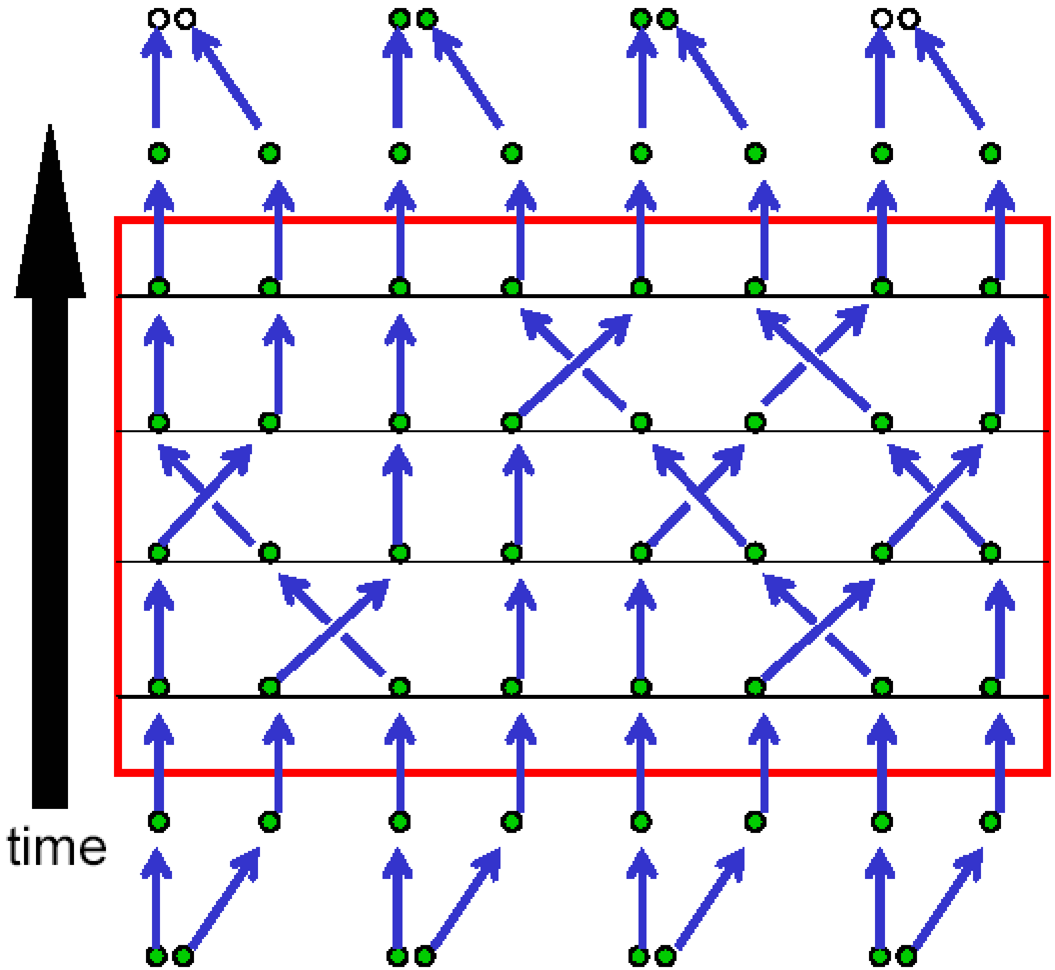
65 kG (upper panel) → 65 kG + 19 G (lower panel)

A suitable adjustment of the applied magnetic field is expected to change the parity in the encircled localized quasiparticle number, thus change the pattern of aperiodic e/4 and e/2 observed over the same side-gate sweep.

Willett et al., PRB (2010)

Topological Quantum Computation

Readout by, e.g., interference measurement.




Kitaev

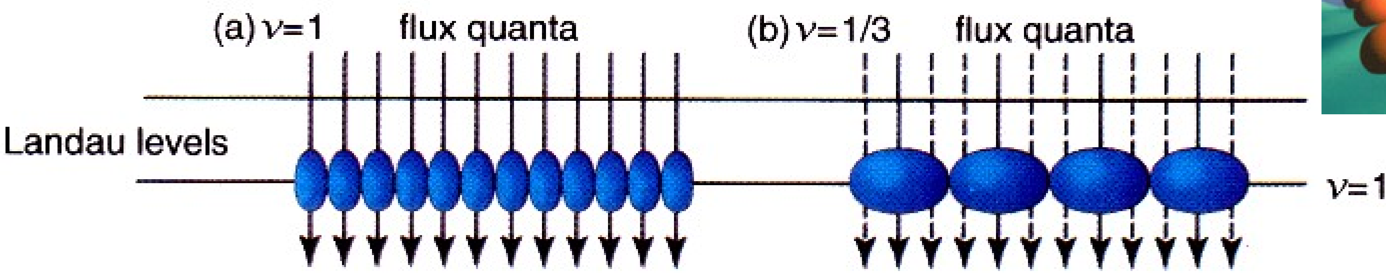

Freedman

Quantum gates constructed by a braid of anyons.

For example, 8 anyons encode 2 qubit of information.

Back to the Laughlin State

- FQHE for electrons (ν = 1/3, 1/5, …)
  - IQHE for composite fermions



$$\Psi_{IQH} = \prod_{i<j} (z_i - z_j) e^{-\sum_i z_i^2/4} \qquad \Psi_L = \prod_{i<j} (z_i - z_j)^2 \prod_{i<j} (z_i - z_j) e^{-\sum_i z_i^2/4}$$

Vandermonde determinant: manifestation of Pauli exclusion principle

Binding two additional vortices to the position of the other electrons.

Intrinsic Geometry: Mass vs Interaction Metric

Haldane, PRL (2011)

$$H=\sum_{i=1}^{N}\frac{1}{2m}g^{ab}\Pi_{ia}\Pi_{ib}+\frac{1}{A}\sum_{q}V(q)\sum_{i<j}e^{iq\cdot(r_i-r_j)}$$

$$m\,g_{ab}$$

$$\lim_{\lambda\to 0}\lambda V(\lambda q)\to\frac{e^2}{2\epsilon}\left(g_c^{ab}q_a q_b\right)^{-1/2}$$

effective mass metric

interaction metric

band structure

dielectric constant

ν = 1/3:

$$\Psi_L=\prod_{i<j}(z_i-z_j)^3 e^{-\sum_i |z_i|^2/4}$$

How can we generalize the Laughlin wavefunction to accommodate anisotropy?

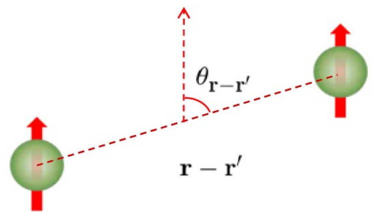
Difficulty: No obvious variational parameter to start with!

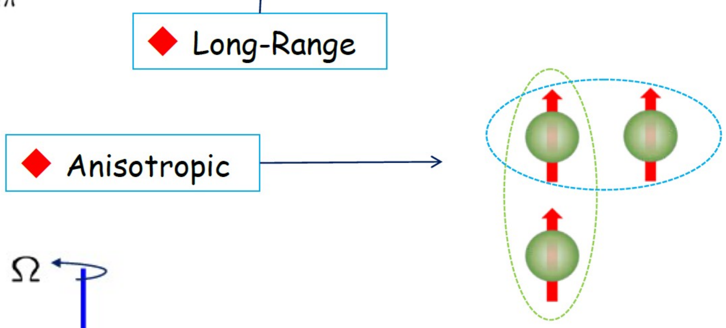
Ultracold Dipolar Systems

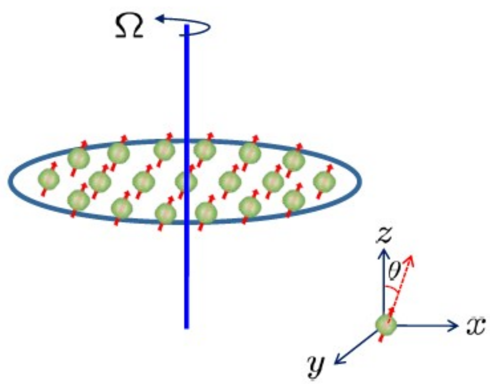
- FQH may be realized in particles with anisotropic dipolar-dipolar interaction
  - Fast rotation
  - Artificial gauge field

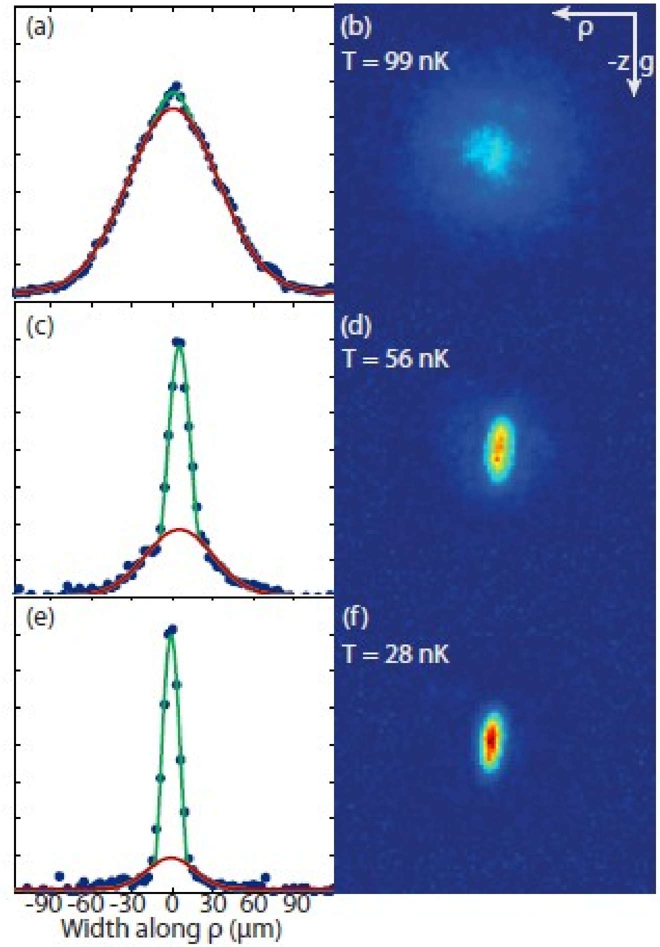
$$V_{\rm dd}(\mathbf{r}-\mathbf{r}') = c_d \frac{1-3\cos^2\theta_{\mathbf{r}-\mathbf{r}'}}{|\mathbf{r}-\mathbf{r}'|^3}$$

$$c_d = \frac{d^2}{4\pi\epsilon_0} \text{ or } \frac{\mu_0 d^2}{4\pi}$$







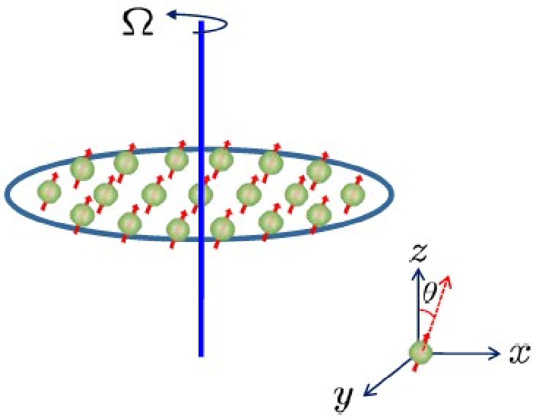


Strongly dipolar BEC of dysprosium realized recently
Mingwu Lu et al., PRL 107, 190401 (2011)

Broken Rotational Symmetry

$$H(\alpha,\theta) = H_{\text{kin}}(\alpha) + H_{\text{int}}(\theta)$$
$$= \alpha L^z + \frac{1}{2}\sum_{m_1 m_2 m_3 m_4} V_{1234}(\theta) f^\dagger_{m_1} f^\dagger_{m_2} f_{m_4} f_{m_3}$$

$\hbar(\omega-\Omega)\ell^3/c_d$ $\quad \sum_m m f^\dagger_m f_m$



$$\int d\boldsymbol{\rho}_1 d\boldsymbol{\rho}_2 \psi^*_{m_1}(\boldsymbol{\rho}_1)\psi^*_{m_2}(\boldsymbol{\rho}_2) V^{(2\text{D})}_\theta(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)\psi_{m_3}(\boldsymbol{\rho}_1)\psi_{m_4}(\boldsymbol{\rho}_2)$$

$$\theta = 0 : m_1 + m_2 = m_3 + m_4$$
$$\theta \neq 0 : m_1 + m_2 = m_3 + m_4 \pm 2,\ 0$$

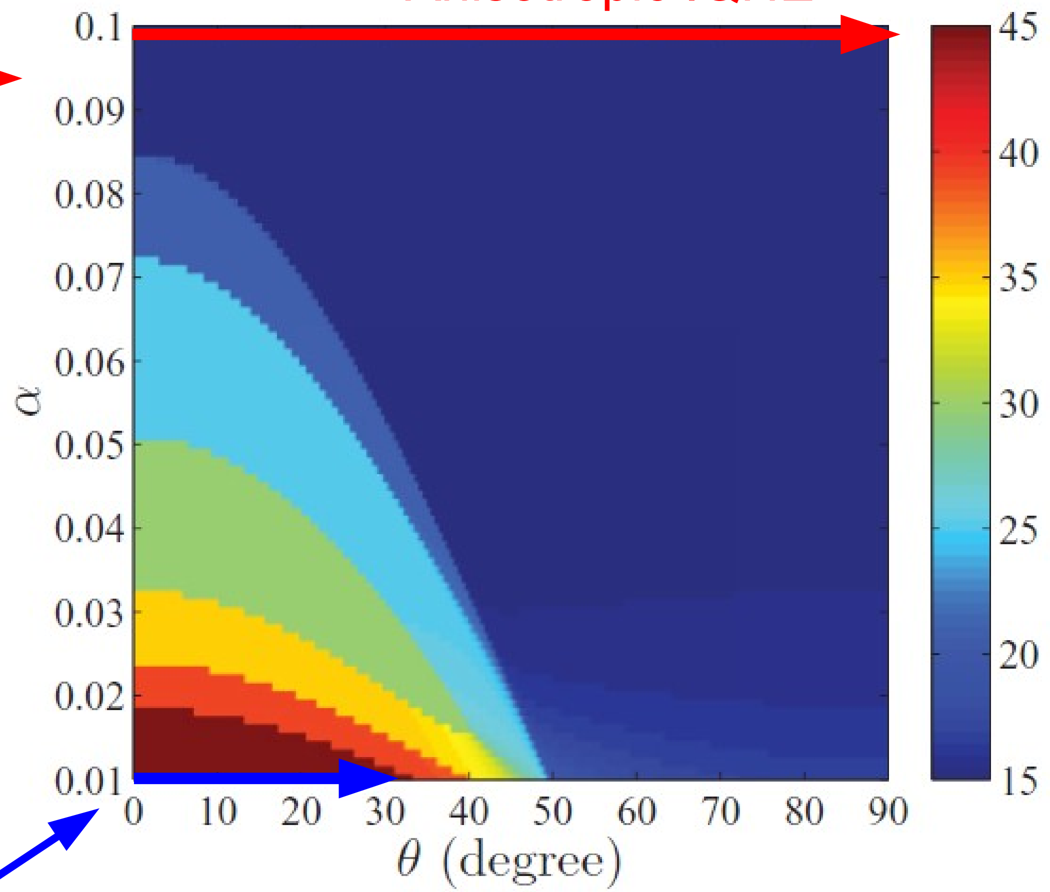
mean angular momentum $\quad \overline{M} = \left\langle \Psi^{(N)}(\alpha,\theta) \right| L^z \left| \Psi^{(N)}(\alpha,\theta) \right\rangle$

Anisotropic QH States

Qiu et al., Phys. Rev. A 83, 063633 (2011)

Anisotropic IQHE

Isotropic IQHE



Trapping potential strength

Isotropic Laughlin

A continuously-parametrized family of Laughlin states?

Anisotropy

Landau Levels Revisited

- Single electron in a strong magnetic field: cyclotron motion

$$H_0 = \frac{\Pi^2}{2m}, \quad \Pi = p - eA \qquad [\Pi_a, \Pi_b] = i\epsilon_{ab}(\hbar/l_B)^2 \longrightarrow [a, a^+] = 1$$

- Separate cyclotron motion from guiding-center motion

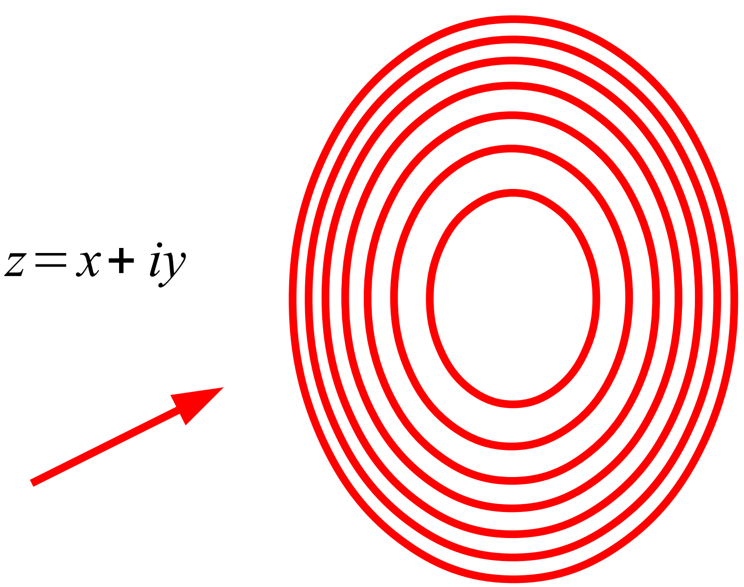
$$r = (l_B^2/\hbar) z \times \Pi + R \qquad [R_a, R_b] = -i\epsilon_{ab} l_B^2 \longrightarrow [b, b^+] = 1$$

- Two sets of ladder operators – (a) inter- and (b) intra-Landau levels

nLL: $$|nm\rangle = \frac{(a^+)^n (b^+)^m}{\sqrt{n!m!}} |00\rangle$$

0LL/LLL: $$|0\mathrm{m}\rangle = \frac{1}{\sqrt{2\pi 2^m m!}} z^m e^{-|z|^2/4} \qquad z = x + iy$$

Squeeze the guiding-center motion

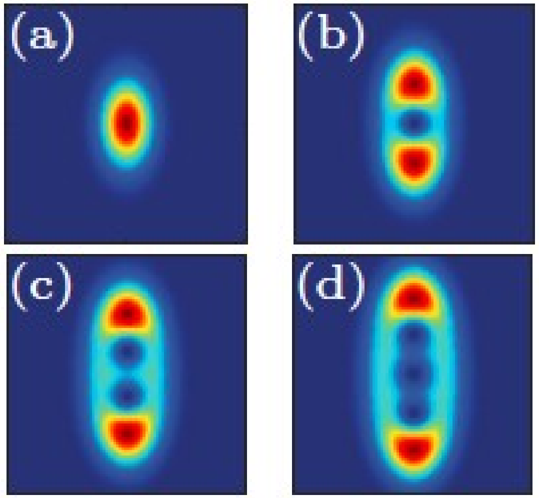


Squeezing the Guiding Center Motion

- Introduce a Bogoliubov transformation (preserving $[b_\gamma, b_\gamma^+]=1$) to the guiding-center motion

angular momentum not conserved

$$\begin{pmatrix} b_\gamma \\ b_\gamma^+ \end{pmatrix} = \frac{1}{\sqrt{1-\gamma\gamma^*}} \begin{pmatrix} 1 & \gamma \\ \gamma^* & 1 \end{pmatrix} \begin{pmatrix} b \\ b^+ \end{pmatrix}$$



- Anisotropic LLL wavefunctions

$$e^{-|z|^2/4} \Rightarrow \phi_{00}(z) \sim e^{-\gamma z^2/4} e^{-|z|^2/4}$$

$$z^m e^{-|z|^2/4} \Rightarrow \phi_{0m}(z) \sim e^{-\gamma z^2/4} e^{-|z|^2/4} (z+2 z_0^2 \partial_z)^m \cdot 1$$

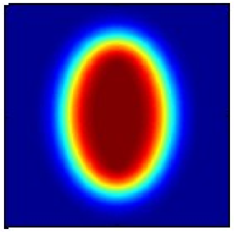
$$z_0^2 = \gamma^* / \sqrt{1-\gamma\gamma^*}$$

- For IQHE

Suppressed by edge confinement

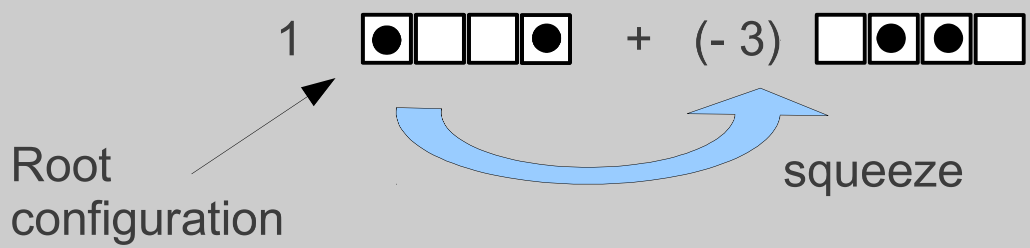
$$\Psi_{ani} = e^{-\sum_i \gamma z_i^2/4} \prod_{i<j} (z_i - z_j) e^{-\sum_i |z_i|^2/4}$$

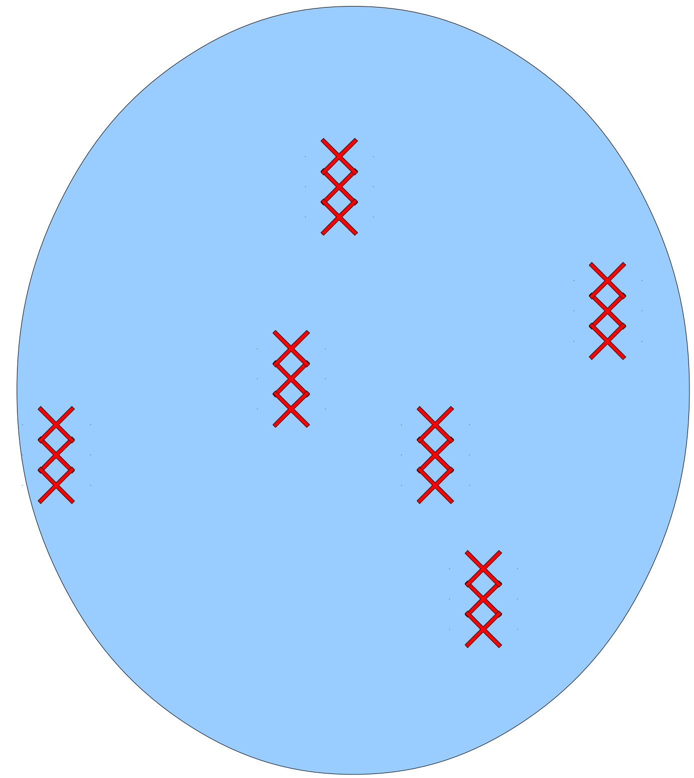
$$\sum_i z_i^2 = \frac{1}{N}\left[\left(\sum_i z_i\right)^2 + \sum_{i<j} (z_i - z_j)^2\right]$$



Splitting the Zeros in the Wavefunction

$$(z_1-z_2)^3 = 1\cdot(z_1^3 z_2^0 - z_1^0 z_2^3) + (-3)\cdot(z_1^2 z_2 - z_1 z_2^2)$$





$z = x + i\,y$

$$e^{-|z|^2/4} \Rightarrow \phi_{00}(z) \sim e^{-\gamma z^2/4} e^{-|z|^2/4}$$

$$z^m \Rightarrow \phi_{0m}(z) \sim (z + 2 z_0^2 \partial_z)^m \cdot 1$$

$$(z_1-z_2)^3 \Rightarrow \left[(z_1-z_2)^2 + 12 z_0^2\right](z_1-z_2) = \left[z_1 - z_2 + 2 z_0^2(\partial_1 - \partial_2)\right]^3 \cdot 1$$

$$z_0^2 = \gamma^* / \sqrt{1-\gamma\gamma^*}$$

Model Anisotropic Wavefunctions

Rui-Zhi Qiu et al., Phys. Rev. B 85, 115308 (2012)

- Laughlin state

$$\Psi_L^q(\{z_i\};\gamma)=\prod_i \phi_{00}(z_i;\gamma)\prod_{i<j}\left[z_i-z_j+2z_0^2(\partial_i-\partial_j)\right]^q\cdot 1$$

$$\phi_{00}(z;\gamma) = e^{-\gamma z^2/4}e^{-|z|^2/4}$$

$$z_0^2 = \gamma^*/\sqrt{1-\gamma\gamma^*}$$

- Moore-Read state

$$\Psi_{MR}^q(\{z_i\};\gamma)=\prod_i \phi_{00}(z_i;\gamma)\, Pf\left[\frac{1}{z_i-z_j+2z_0^2(\partial_i-\partial_j)}\right]\prod_{i<j}\left[z_i-z_j+2z_0^2(\partial_i-\partial_j)\right]^q\cdot 1$$

$$\phi_{00}(z;0) = e^{-|z|^2/4} \longrightarrow \phi_{00}(z;\gamma) = e^{-\gamma z^2/4}e^{-|z|^2/4}$$

$$(z_i-z_j) \longrightarrow \left[z_i-z_j+2z_0^2(\partial_i-\partial_j)\right]\cdot$$

Coulomb Interaction Metric

If the mass metric ($m\,g^{ab}$) and the interaction metric are the same, we recover the Laughlin state (say, at $1/q$ filling).

What if the mass and the interaction metrics are different?

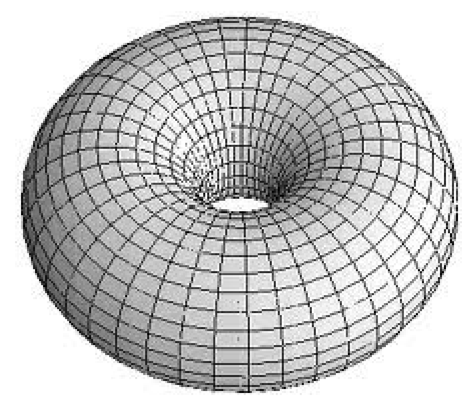
Hao Wang et al., PRB (2012)

$$H_c = \frac{1}{N_\phi}\sum_q V(q)\, e^{-q^2/2} \sum_{i<j} e^{i q\cdot(R_i - R_j)}$$

$$V_c(r) = \frac{e^2}{4\pi\epsilon |r|} \longrightarrow V_c(r) = \frac{e^2}{4\pi\epsilon\sqrt{A_c x^2 + y^2/A_c}}$$

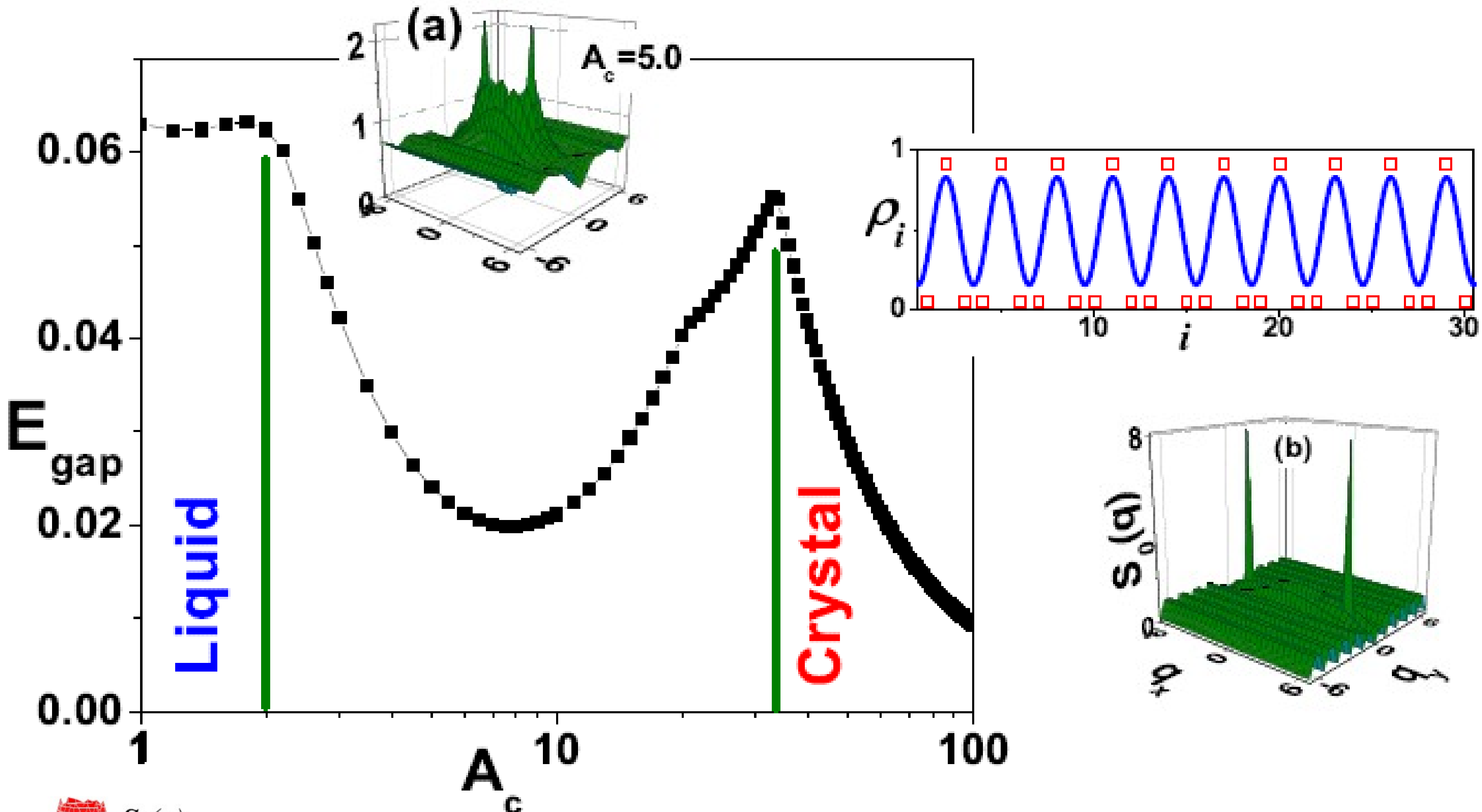
$$q^2 = q_x^2 + q_y^2 \longrightarrow q_g^2 = g_c^{ab} q_a q_b$$

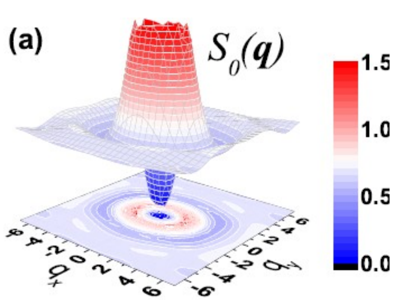
Inverse metric $\quad g_c = \begin{pmatrix} 1/A_c & 0 \\ 0 & A_c \end{pmatrix}$



torus geometry

Phase Diagram beyond α-Laughlin Liquid





$$S_0(q) = \frac{1}{N_e}\left\langle 0 \left| \sum_{i \neq j} e^{i q \cdot (r_i - r_j)} \right| 0 \right\rangle$$

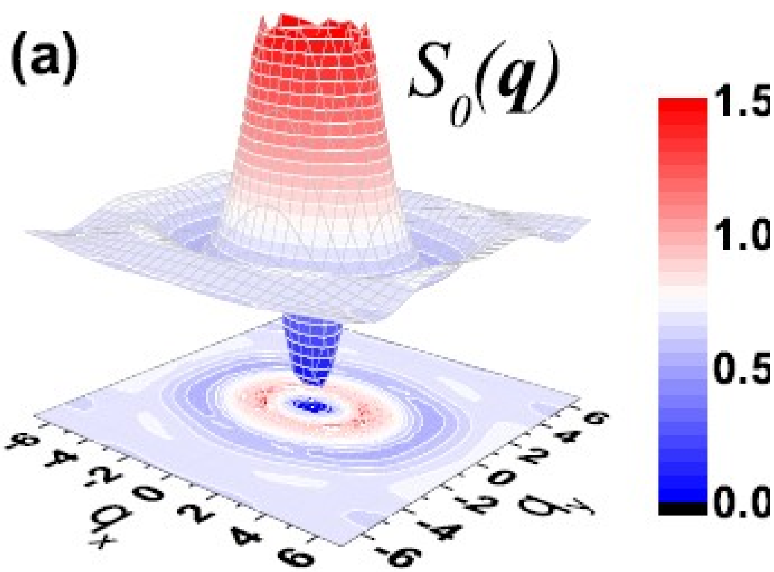
Small Anisotropy – α-Laughlin Liquid

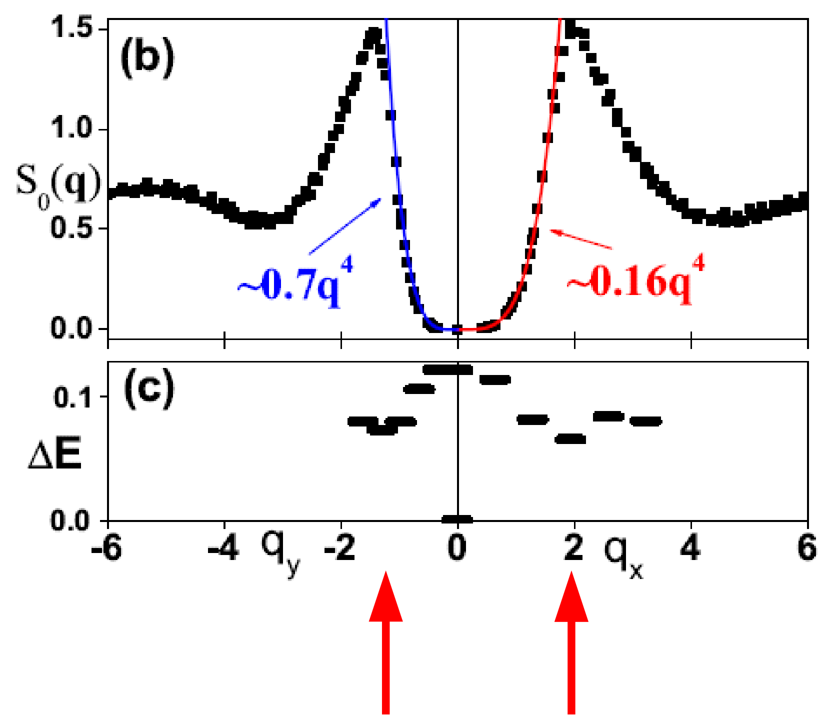
Projected static structure factor:

$$S_0(q) = \frac{1}{N_e} \left\langle 0 \left| \sum_{i \neq j} e^{i q \cdot (r_i - r_j)} \right| 0 \right\rangle \sim q^4$$

Coulomb metric $A_c = 1.8$

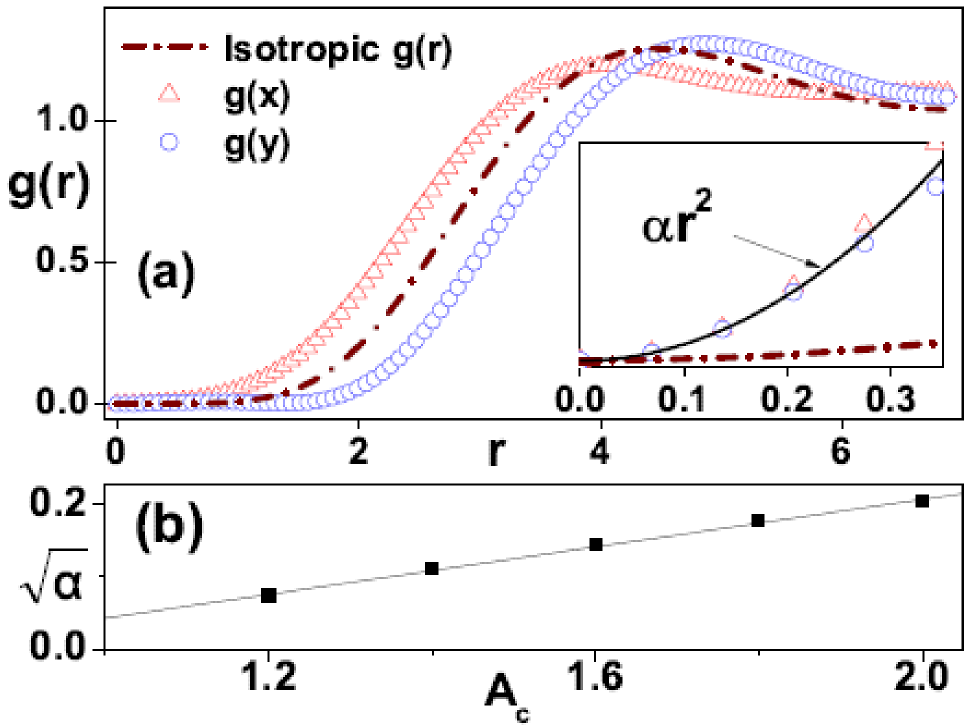
Guiding center metric: $A_L = \sqrt[4]{\frac{0.7}{0.16}} = 1.45$





Pair Correlation Function

$$g(r) = \frac{L_x L_y}{N_e(N_e - 1)} \left\langle 0 \left| \sum_{i \neq j} \delta(r - (r_i - r_j)) \right| 0 \right\rangle$$



Expect

$$g(r) \to \alpha r^2 \quad \text{as} \quad r \to 0$$

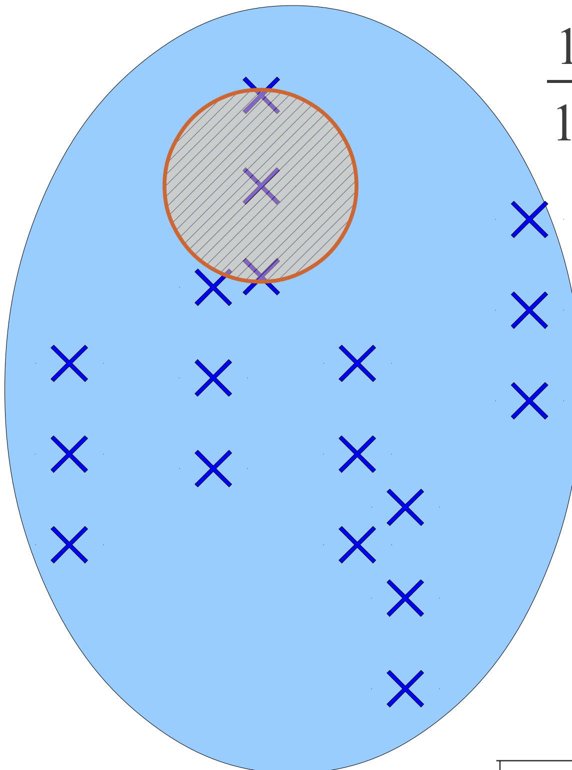
$$\alpha \sim |\gamma|^2 \sim |z_0|^4$$

Expect

$$\sqrt{\alpha} \sim (\sqrt{A_L} - 1) \sim (A_L - 1)$$

$$w(z_i) \sim \prod_{i<j} (z_i - z_j)\left[(z_i - z_j)^2 + 12 z_0^2\right]$$

Breakdown of the α-Laughlin State



$$\frac{12\pi|\gamma|}{1-\gamma\gamma^*}=\frac{2\pi}{\nu}$$

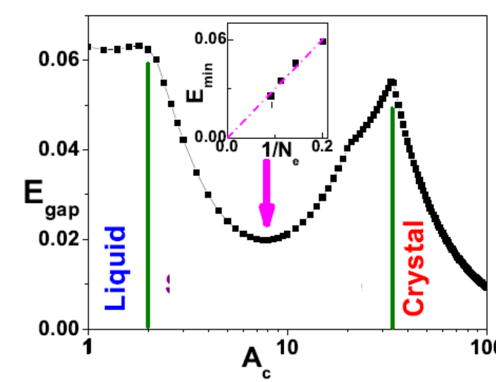
Isotropic Laughlin state

$$\Psi_L=\prod_{i<j}(z_i-z_j)^3 e^{-\sum_i |z_i|^2/4}$$

Characterized by the triple zeros at the locations of other electrons in the wavefunction

Anisotropic Laughlin state

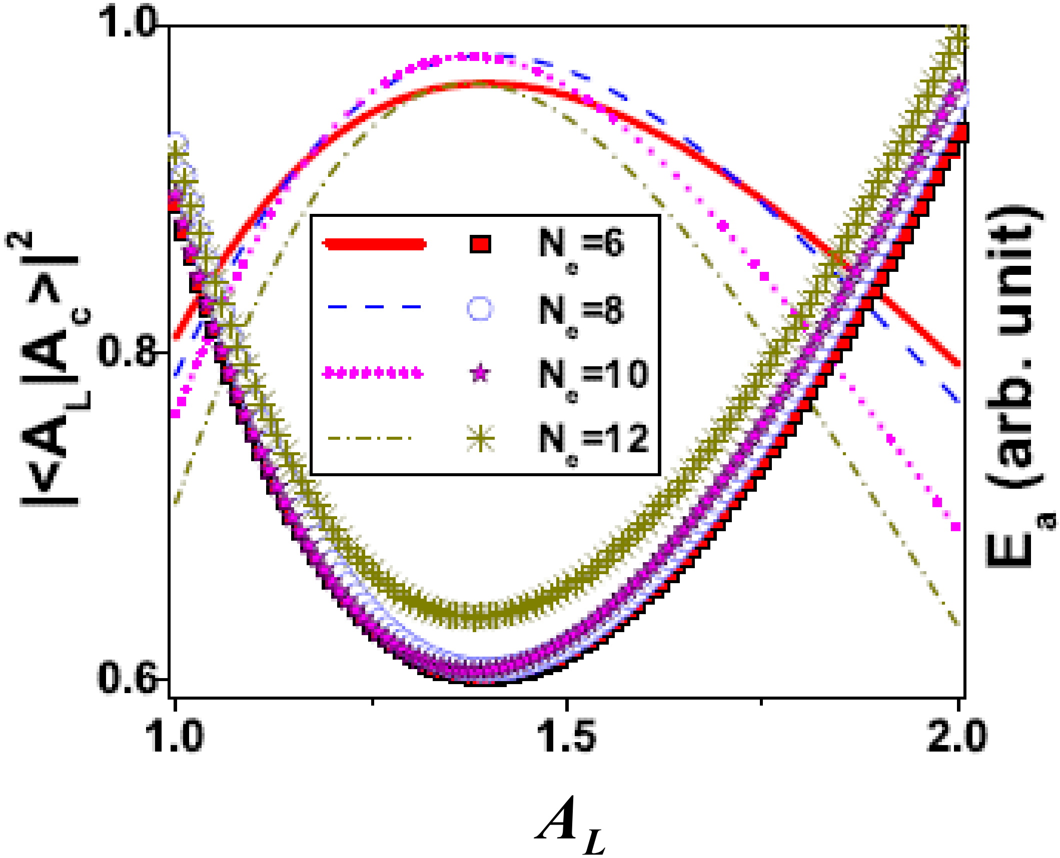
$$\Psi_{\alpha L}=\prod_{i<j}\left[(z_i-z_j)^2+12z_0^2\right](z_i-z_j)e^{-\gamma\sum_i z_i^2/4}e^{-\sum_i |z_i|^2/4}$$



The splitting of zeros proportional to anisotropy parameter z_0

Postulate: breakdown when $12\pi|z_0|^2$ ~ area per particle → $A_L \sim 2$ (or $A_c \sim 3$)

Calibrating Anisotropy



Guiding-center metric can be calibrated by finding either the maximum of the variational-wavefunction overlap or the minimum of the variational energy.

Geometrical Description

Haldane, PRL (2011)

$m\, g^{ab}$

$$\lim_{\lambda \to 0} \lambda V(\lambda q) \to \frac{e^2}{2\epsilon} \left(g_c^{ab} q_a q_b \right)^{-1/2}$$

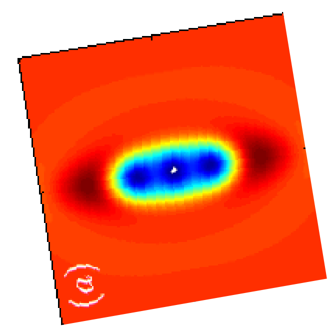
Effective mass metric

Interaction metric

$$\left[(z_1 - z_2)^2 + 12 z_0^2 \right] (z_1 - z_2)\, e^{-\gamma \left(z_1^2 + z_2^2 \right)/4}$$

$$z_0^2 = \gamma^* / \sqrt{1 - \gamma \gamma^*}$$

guiding center metric



Numerical demonstration:

- Bo Yang, Z. Papic, E. H. Rezayi, R. N. Bhatt, and F. D. M. Haldane, Phys. Rev. B 85, 165318 (2012).
- Hao Wang, Rajesh Narayanan, Xin Wan, and Fuchun Zhang, Phys. Rev. B 86, 035122 (2012).

Tilted Magnetic Field (i.e., Mass Anisotropy)

$\nu = 7/3$

with in-plane magnetic field

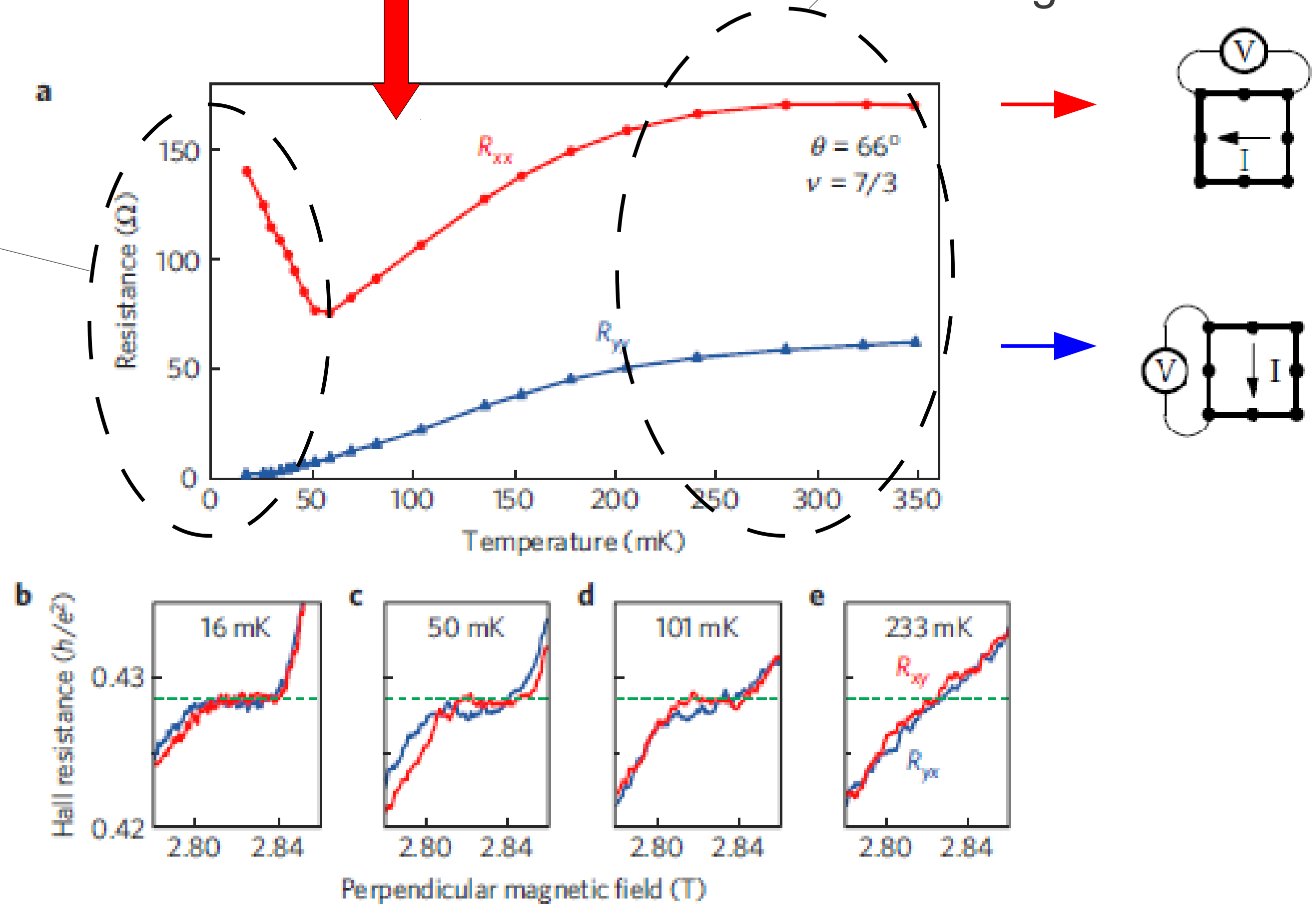
Anisotropic FQHE?

Anisotropic transport
no Hall plateau

Similar to half-filling at high Landau levels

$R_{yy} \rightarrow 0$, but still with Hall plateau

suggests that the system may develop FQHE bubbles/stripes



J. Xia et al., Nature Physics **7**, 845 (2011)

Summary

- The topological aspects of the quantum Hall effect include topological Chern numbers, gapless edge excitations, gapped bulk quasiparticles with fractional charge, fractional and possibly non-Abelian statistics, and nontrivial entanglement spectra.
- Recent experiments and theoretical understandings suggest that the filling factor 5/2 state supports non-Abelian quasiparticle excitations.
- For quantum Hall systems with anisotropic mass or interaction, one can introduce a family of wavefunctions with identical topological characteristics, but with different geometrical information, encoded in the so-called guiding-center metric. The guiding-center metric serves as a variational parameter for fractional quantum Hall states.

Supported by NSFC and MoST-China