Manipulating Quantum Defect-states of Topological States

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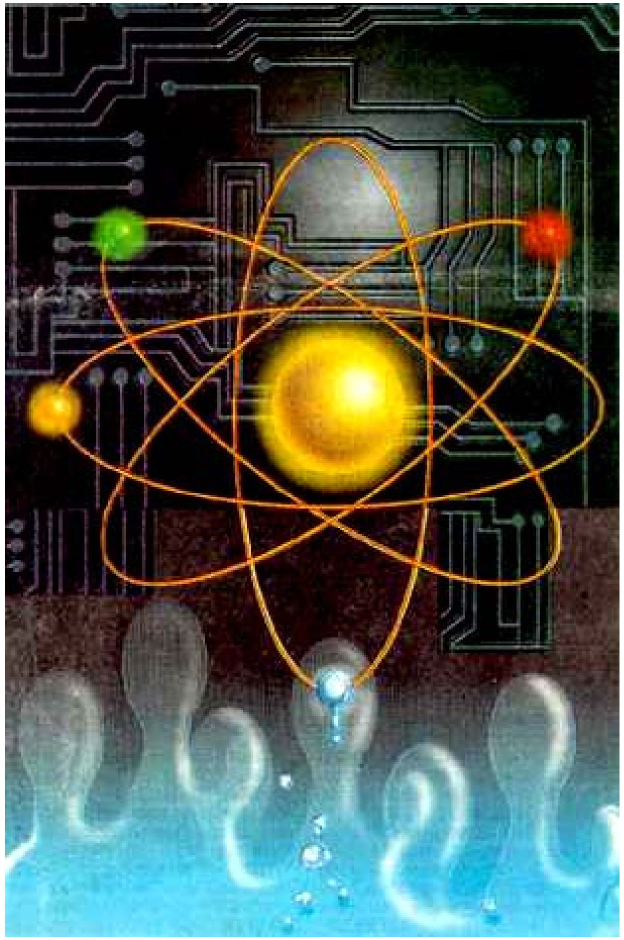
Outline

1. **Introduction to quantum computation**
2. **Quantum computation by manipulating topological qubit**
3. **Zero modes of lattice-vacancies in the topological insulators and topological superconductors**
4. **Conclusion**

- **Kou SP,** Quantum Computation via Quantum Tunneling Effect, PHYS. REV. LETT. **102**, 120402 (2009).
- Yu J and **Kou SP,** Macroscopic Quantum Tunneling Effect of *Z*2 Topological Order**,** PHYS. REV. **B 80**, 075107 (2009).
- **Kou SP**, Realization of Topological Quantum Computation with planar codes, PHYS. REV. **A 80**, 052317 (2009).
- Jing He, Ying-Xue Zhu, Ya-Jie Wu, Lan-Feng Liu, Ying Liang, and **Kou SP**, Protected Zero Modes on Vacancies in the Topological Insulators and Topological Superconductors on the Honeycomb Lattice, PHYS. REV. **B 87**, 075126 (2013).

I. Introduction to Quantum Computation

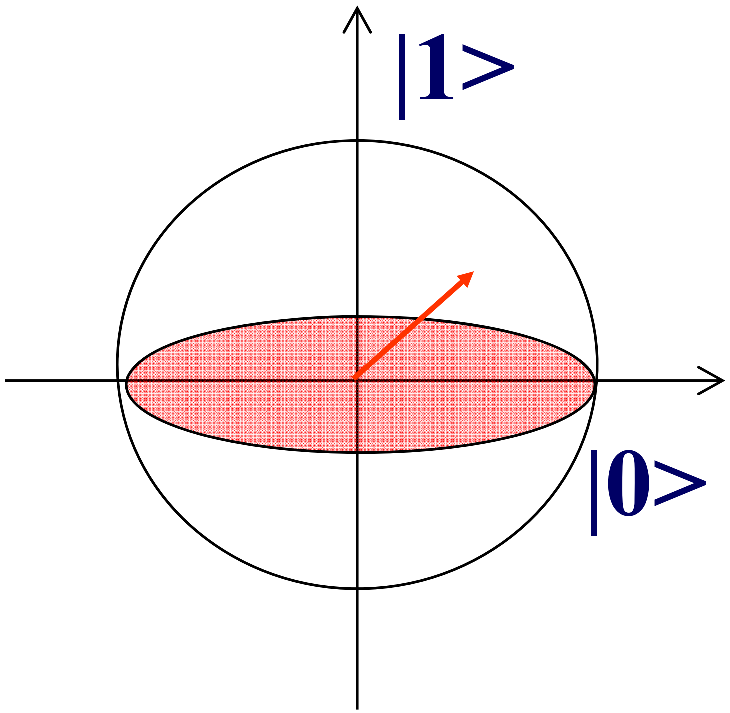
- **Quantum computers are predicted to use quantum states to perform memory and to process tasks.**



Five criteria of quantum computer
- D. P. DiVincenzo

- **Well defined extendible qubits - stable memory**
- **Preparable in the “000…” state**
- **Universal set of gate operations**
- **Single-quantum measurements**
- **Long decoherence time (>10^4 operation time)**

Quantum bit - Qubit

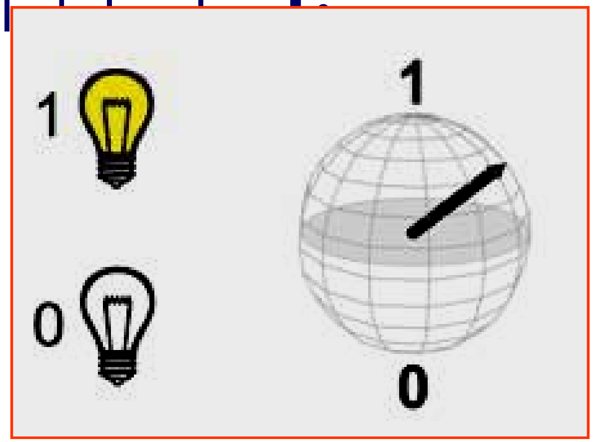


- Basis states |0>, |1>
- Arbitrary state:

|0>+ |1>,

,   complex,

| |²+| |²=1.



Physical qubits

- **Nuclear spin = orientation of atom's nucleus in magnetic field: $\uparrow = |0\rangle$, $\downarrow = |1\rangle$.**

- **Photons in a cavity:
No photon = $|0\rangle$, one photon = $|1\rangle$**

- **...**

Quantum Logic Gates

An arbitrary unitary operator may be

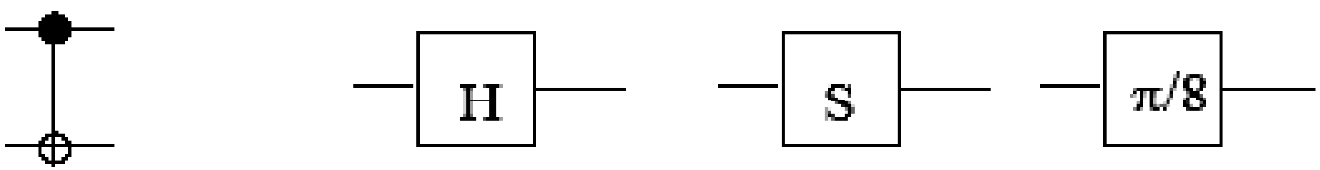
$$e^{i\alpha}\begin{bmatrix} e^{-i\beta/2} & 0 \\ 0 & e^{i\beta/2} \end{bmatrix}\begin{bmatrix} \cos\frac{\nu}{2} & -\sin\frac{\nu}{2} \\ \sin\frac{\gamma}{2} & \cos\frac{\nu}{2} \end{bmatrix}\begin{bmatrix} e^{-i\delta/2} & 0 \\ 0 & e^{i\delta/2} \end{bmatrix}$$

where α,β,ν, and δ are real-valued.

Four Universal Gate Sets

Hadamard + CNOT + phase +π/8

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix} \qquad \frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix} \qquad \begin{pmatrix}1&0\\0&i\end{pmatrix} \qquad \begin{pmatrix}1&0\\0&e^{i\pi/4}\end{pmatrix}$$

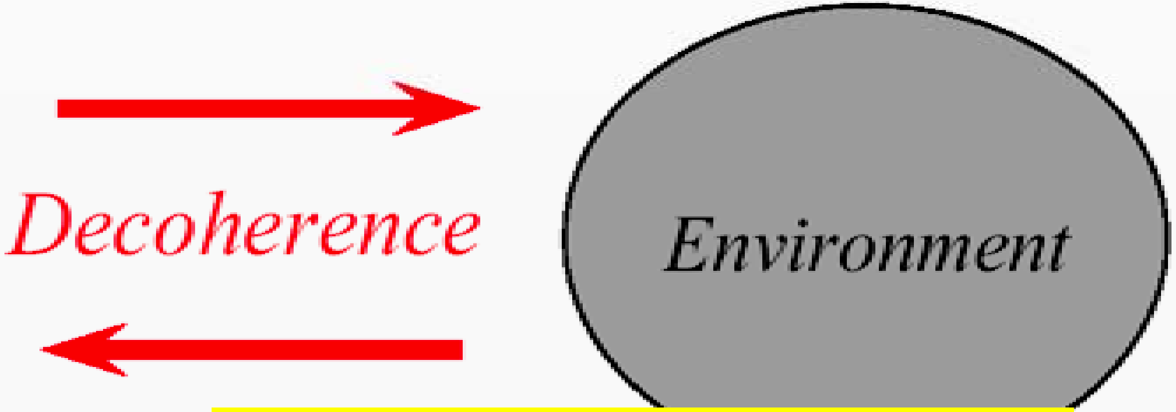


CNOT — Hadamard — Phase — π/8 (T) gate

Alternative set: Hadamard + CNOT + Phase + Toffoli

Physical systems actively considered for quantum computer implementation

- Liquid-state NMR
- NMR spin lattices
- Linear ion-trap spectroscopy
- Neutral-atom optical lattices
- Cavity QED + atoms
- Linear optics with single photons
- Nitrogen vacancies in diamond
- Electrons on liquid He
- Josephson junctions arrays
- Spin spectroscopies, impurities in semiconductors
- Coupled quantum dots
- …



Fault-Tolerance quantum computation

If quantum information is cleverly encoded, it *can* be protected from decoherence and other potential sources of error. Intricate quantum systems *can* be accurately controlled.

Courtesy to "Topological Quantum Computing for Beginners" by John Preskill

Topology : solution to decoherence

- Since the topological properties is not changed by small perturbations from the environment.

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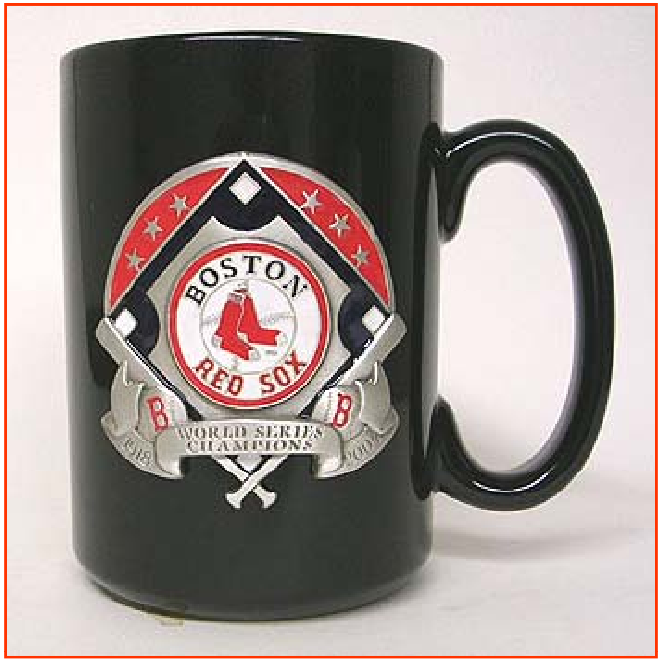
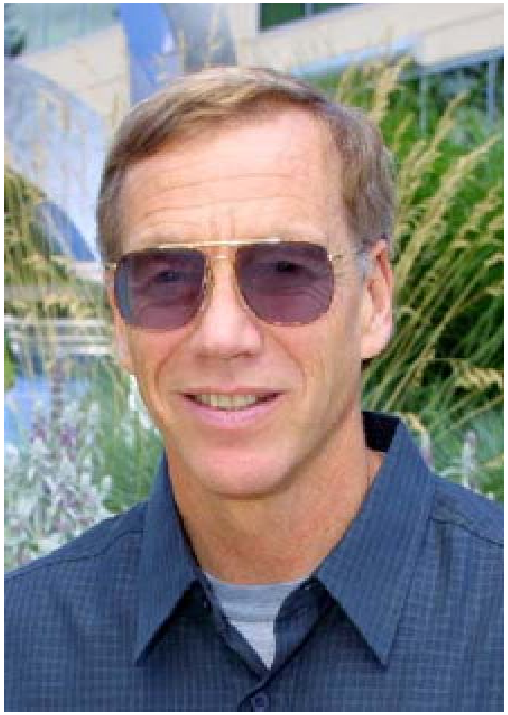


Milestone for topological quantum computation

- 1997, Kitaev proposed the idea of topological quantum bit and fault torrent quantum computation in an Abelian state.

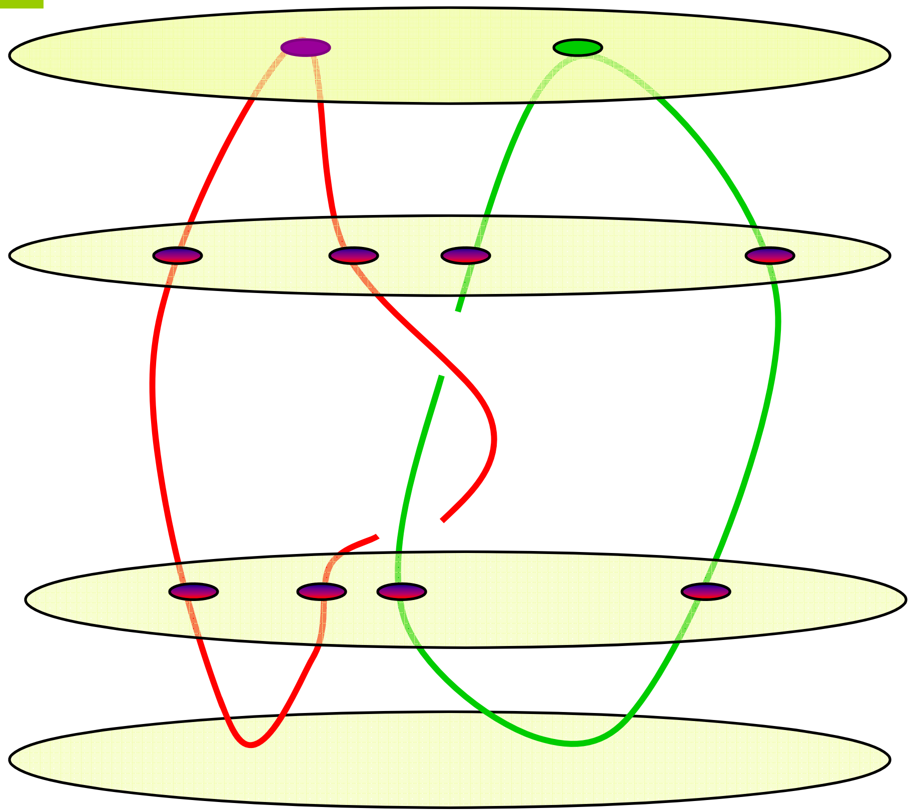
- 2001, Kitaev proposed the topological quantum compuation in a non-Abelian state

- 2001, Preskill, Freedman and others proposed a universal topological quantum computation



Topological Quantum Computation

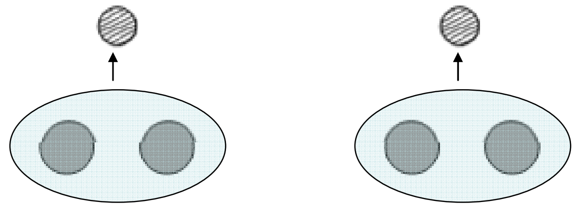
Computation	Physics
output	measure
operation	braid
initialize	create particles



Eric Rowell

Fermionic quantum computation

- **Quantum memory: isolated Majorana fermions**
- **Qubit: fermion parities of two Majorana fermions**



- **Majorana universal gates: coupling gate, interaction gate**

$$\left\{ \exp\left(\frac{\pi}{8} c_0 c_1\right), \ \exp\left(i \frac{\pi}{4} c_0 c_1 c_2 c_3\right) \right\}$$

$$\left\{ \begin{array}{l} \exp\left(i\frac{\pi}{4} a_0^\dagger a_0\right), \\ \exp\left(i\frac{\pi}{4}(a_0^\dagger a_1 + a_1^\dagger a_0)\right), \\ \exp\left(i\frac{\pi}{4}(a_1 a_0 + a_0^\dagger a_1^\dagger)\right), \\ \exp(i\pi a_0^\dagger a_0 a_1^\dagger a_1) \end{array} \right\}$$

- **Errors**

II. Quantum computation by manipulating topological qubits

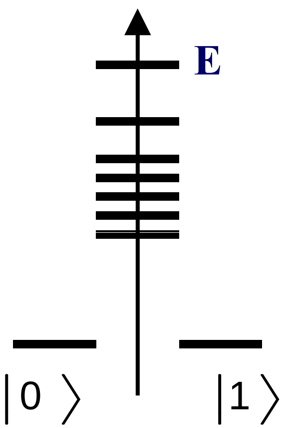
Topological qubit

A. Yu. Kitaev, Annals Phys. 303, 2 (2003) [quant-ph/9707021]

|0> and |1> are the ground-states of a topological order

which are degenerate because of the (non-trivial) topology.

$$\left|\Psi\right\rangle = \alpha\left|0\right\rangle + \beta\left|1\right\rangle$$



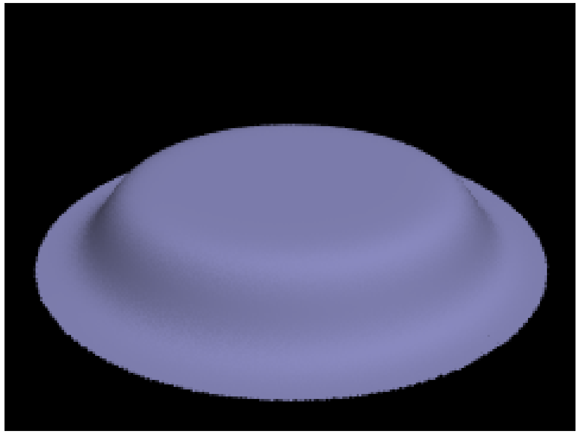
Advantage

The two states are *locally* indistinguishable

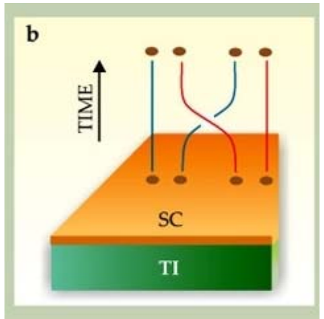
⇒ no local perturbation can introduce decoherence.

Ioffe, &, Nature 415, 503 (2002).

Topological order – an emergent world in a many-body system



- All excitations have mass gaps
- Topological excitations – anyons with fractional statistics
- Effective theory - topological field theory
- No local order parameters – string net condensation



String net condensation for the ground states

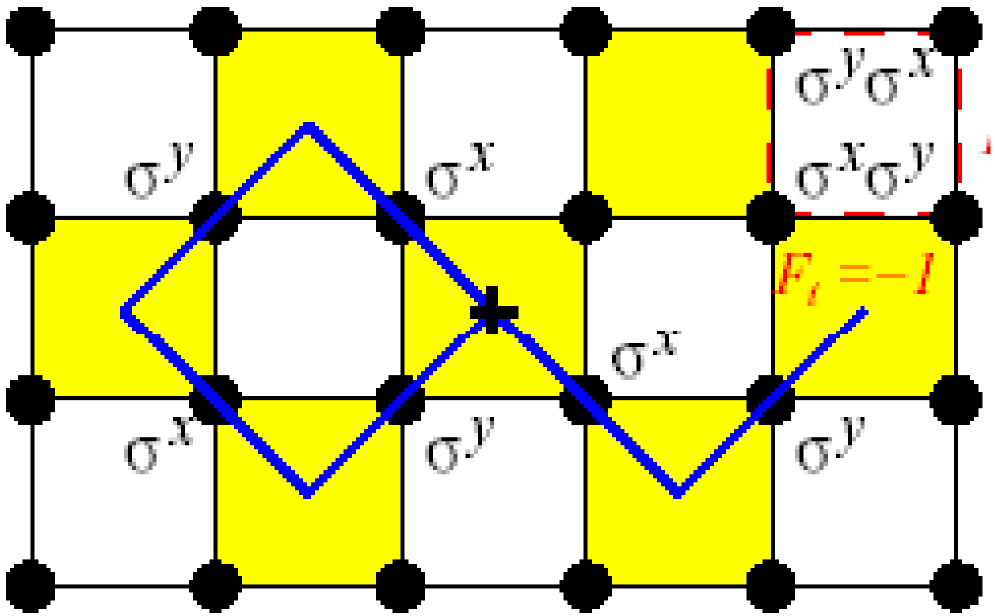
The string operators:

$$W(C) = \prod_m \sigma_{i_m}^{l_m}$$

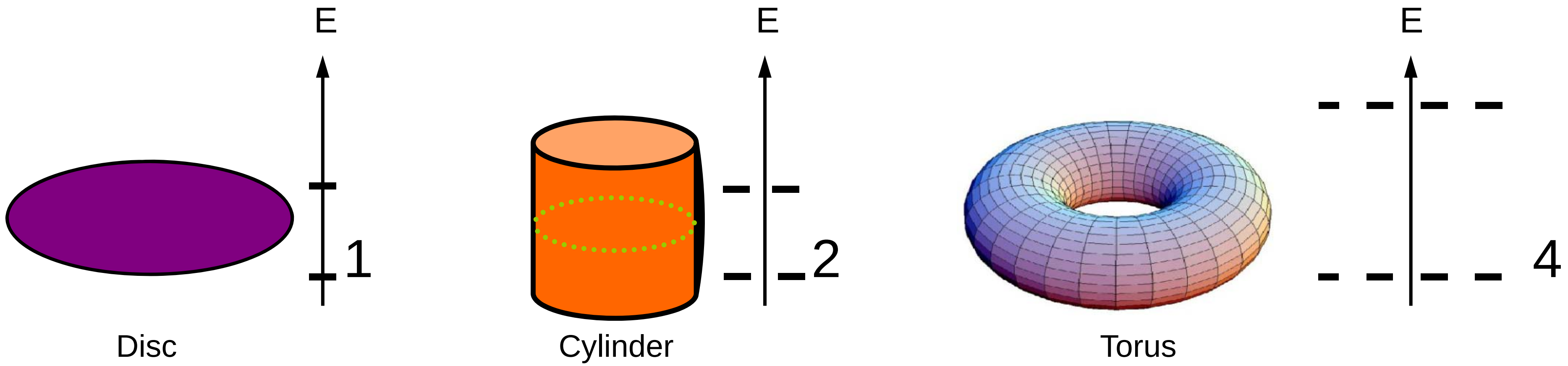
$W_c(C)$, $W_v(C)$和 $W_f(C)$,

For the ground state, the closed-strings are condensed

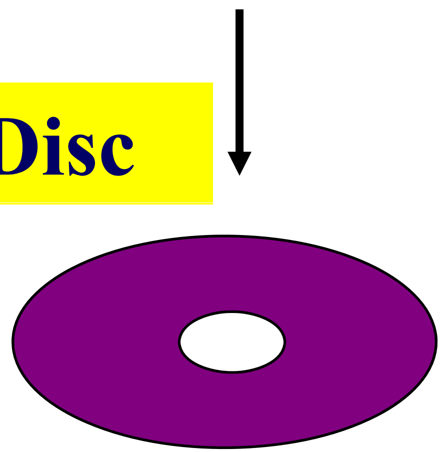
$$\langle W(C) \rangle \neq 0$$

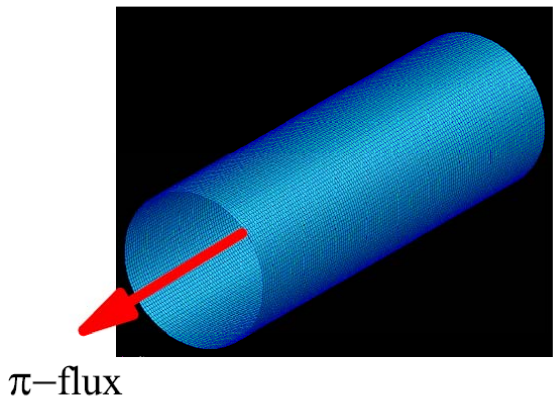


Topology of Z2 topological order

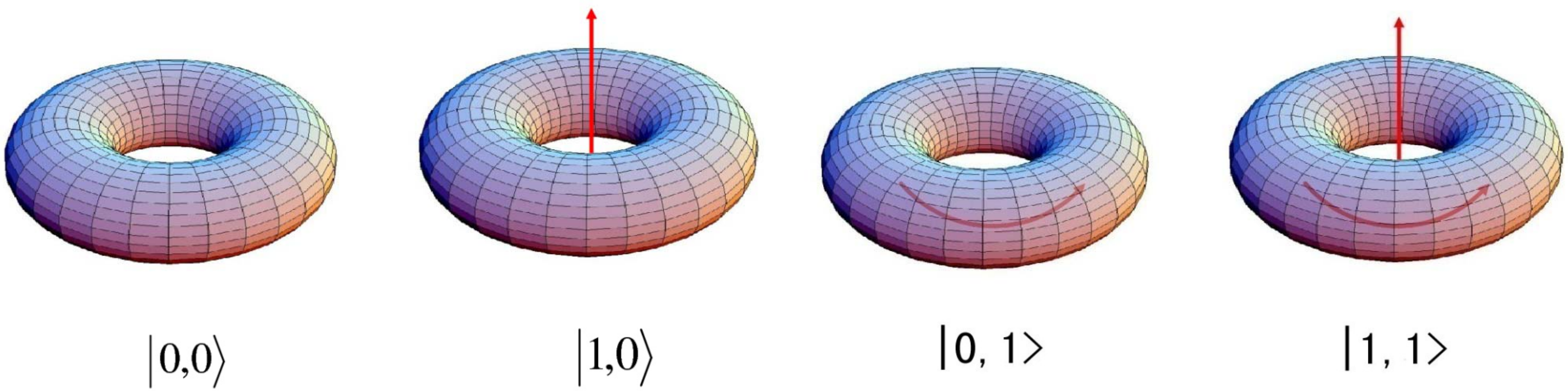


Hole on a Disc





Ground states with 4-fold degeneracy on a torus



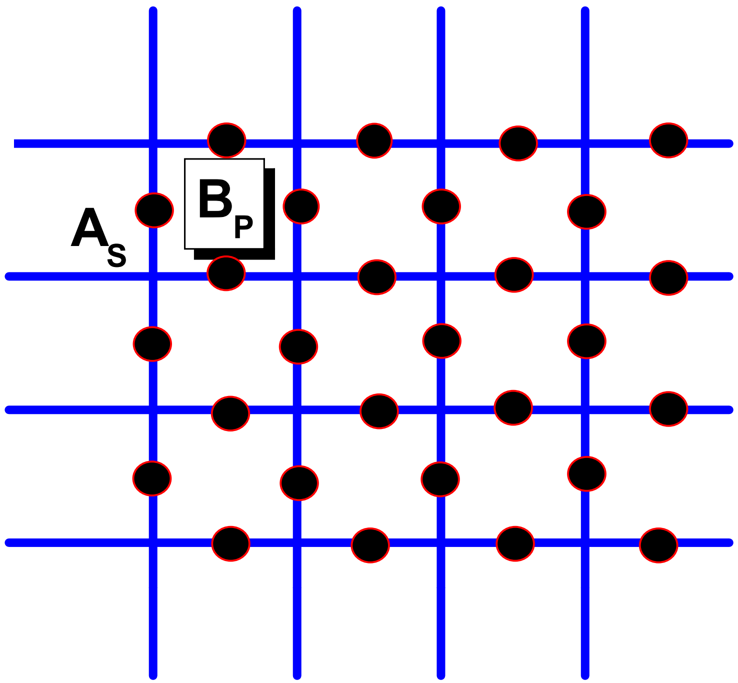
The topological degeneracy 4 means that the four ground states with same energy. Here m, n = 0, 1 labels the flux inside the holes of the torus。

X. G. Wen and Q. Niu, Phys. Rev. B 41, 9377 (1990).

Toric-code model

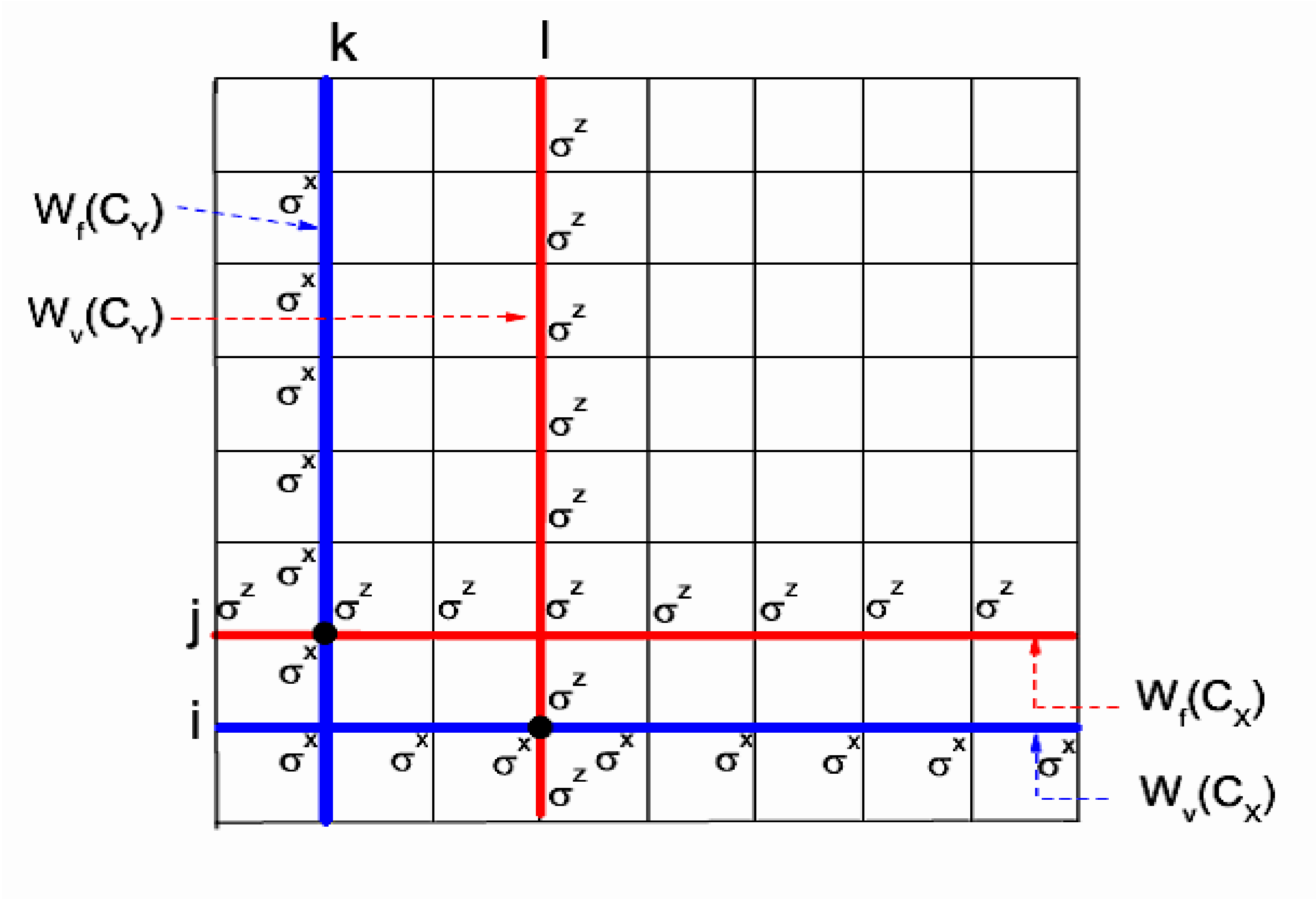
$$Z_i = s_i^z s_{i+\hat{e}_x}^z s_{i+\hat{e}_x+\hat{e}_y}^z s_{i+\hat{e}_y}^z, \quad X_i = s_i^x s_{i+\hat{e}_x}^x s_{i+\hat{e}_x+\hat{e}_y}^x s_{i+\hat{e}_y}^x$$

$$H_{\rm tc} = -A \sum_{i \in \rm even} Z_i - B \sum_{i \in \rm odd} X_i,$$

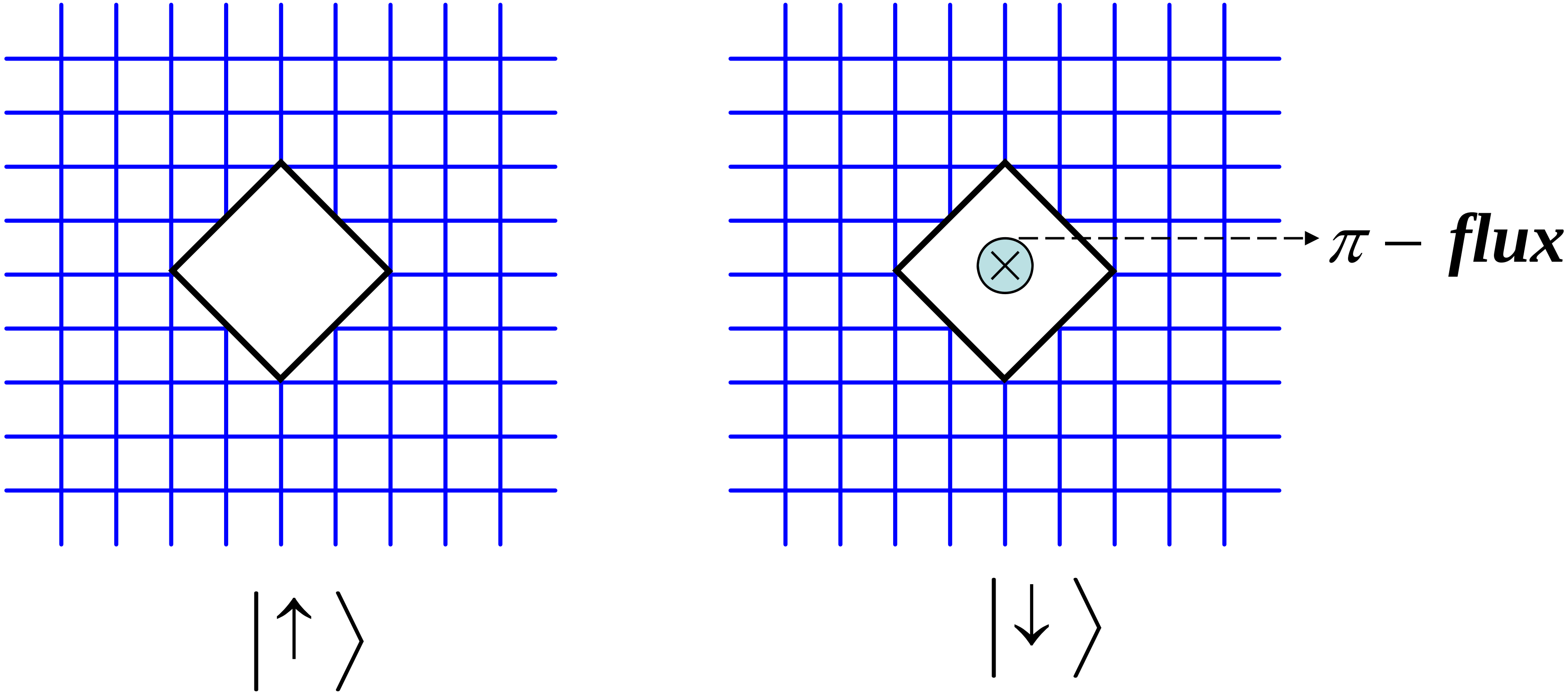


A. Y. Kitaev, Annals
Phys. 303, 2 (2003)

Topological closed string operators on torus – topological qubits



Topological qubits (planar code) of Z2 topological order



L. B. Ioffe, et al., Nature 415, 503 (2002).

How to control the topological qubits in Abelian states?

A．Y．Kitaev，Annals Phys. 303, 2 (2003)

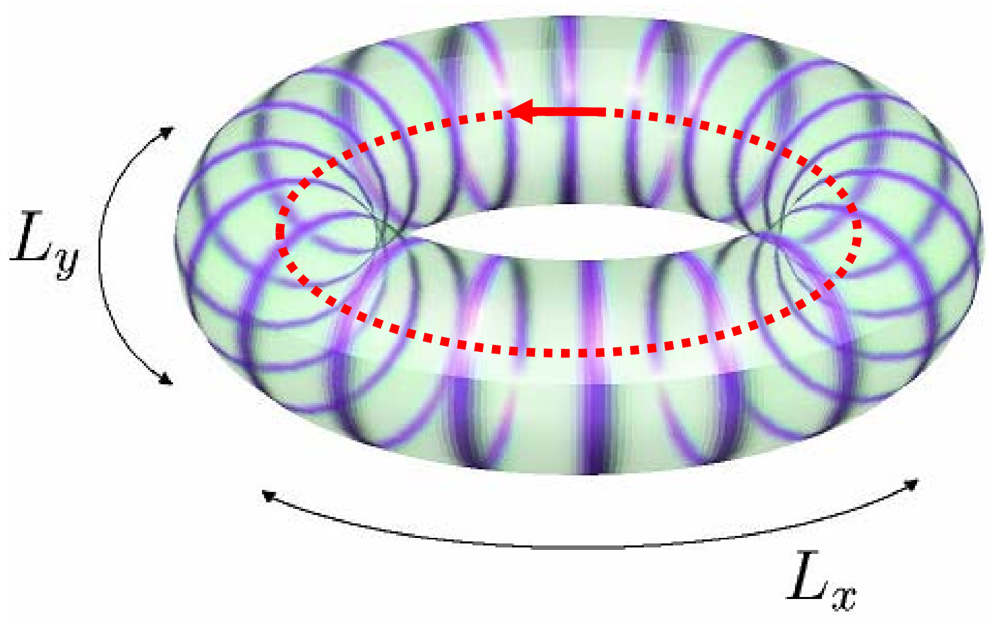
A. Y. Kitaev :

"Unfortunately, I do not know any way this quantum information can get in or out. Too few things can be done by moving abelian anyons. All other imaginable ways of accessing the ground state are uncontrollable."

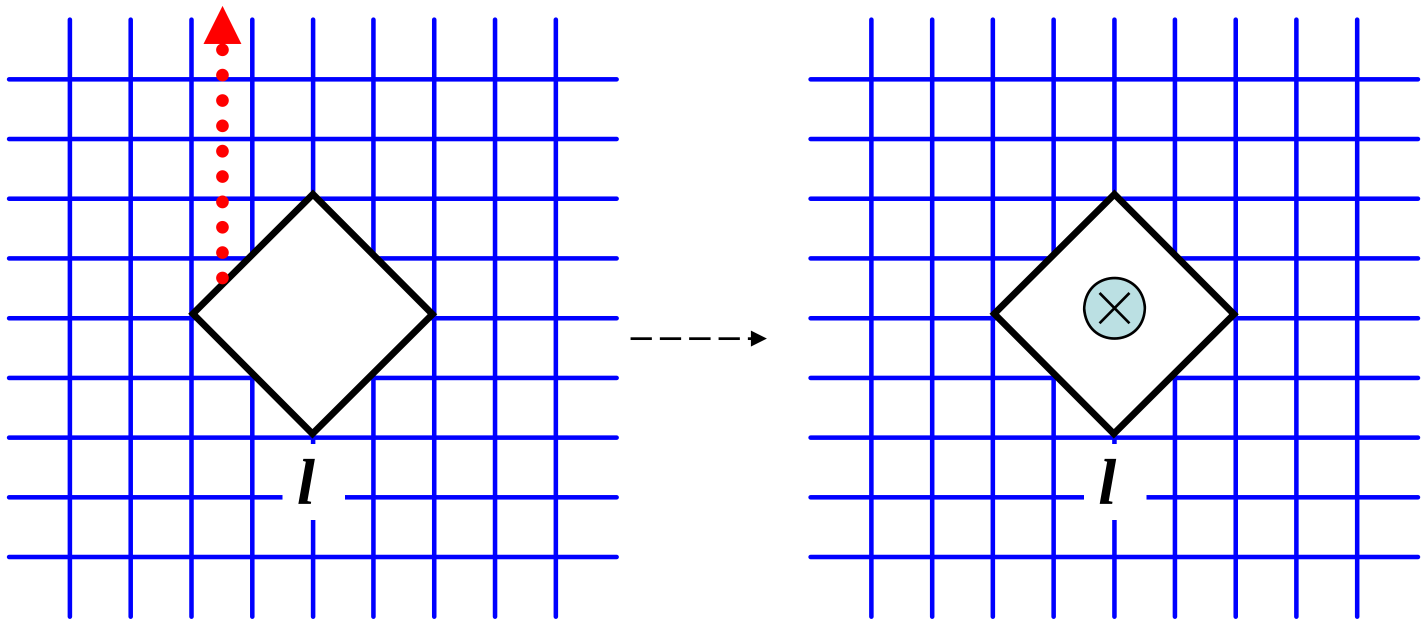
- **Kou SP,** PHYS. REV. LETT. **102**, 120402 (2009).
- Yu J and **Kou SP,** PHYS. REV. **B 80**, 075107 (2009).
- **Kou SP**, PHYS. REV. **A 80**, 052317 (2009).

(1) Quantum tunneling effects in Z2 topological order

Tunneling processes : a virtual quasi-particle moves to changing the topological class of the ground states:

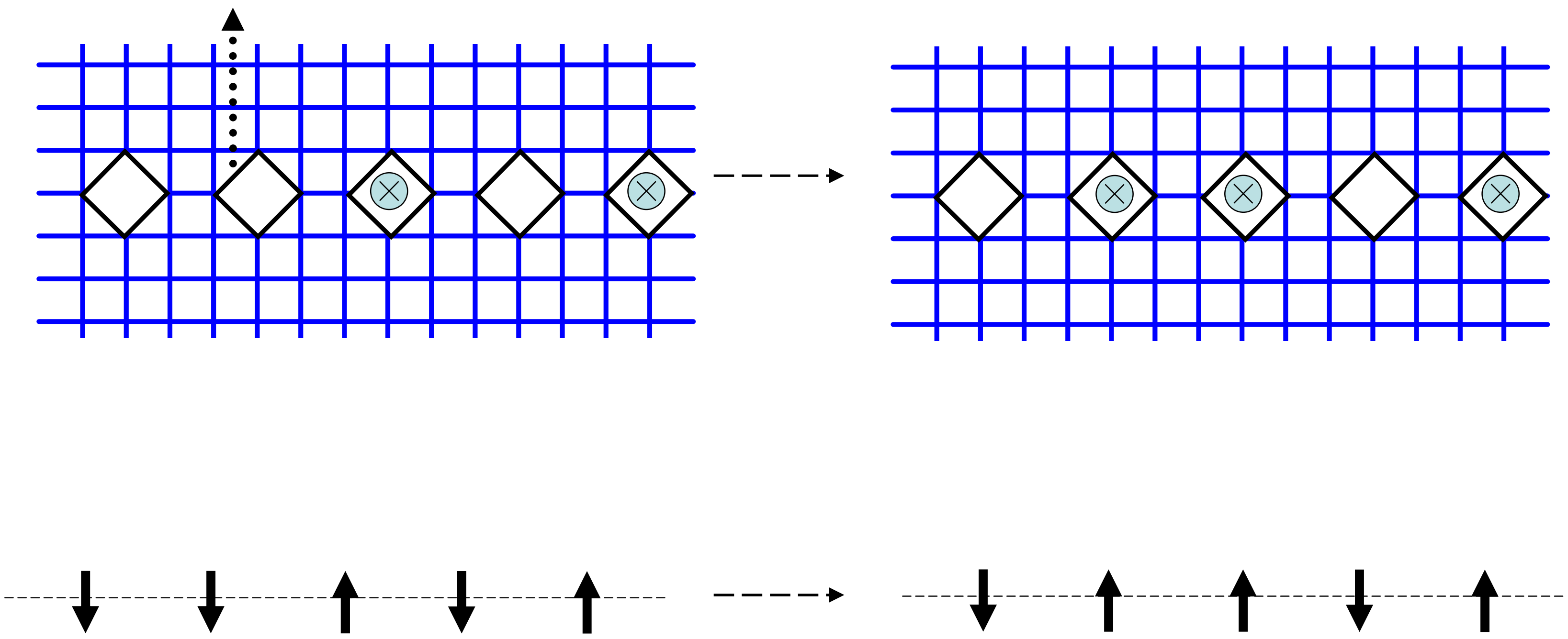


Tunneling process of Z2 vortex

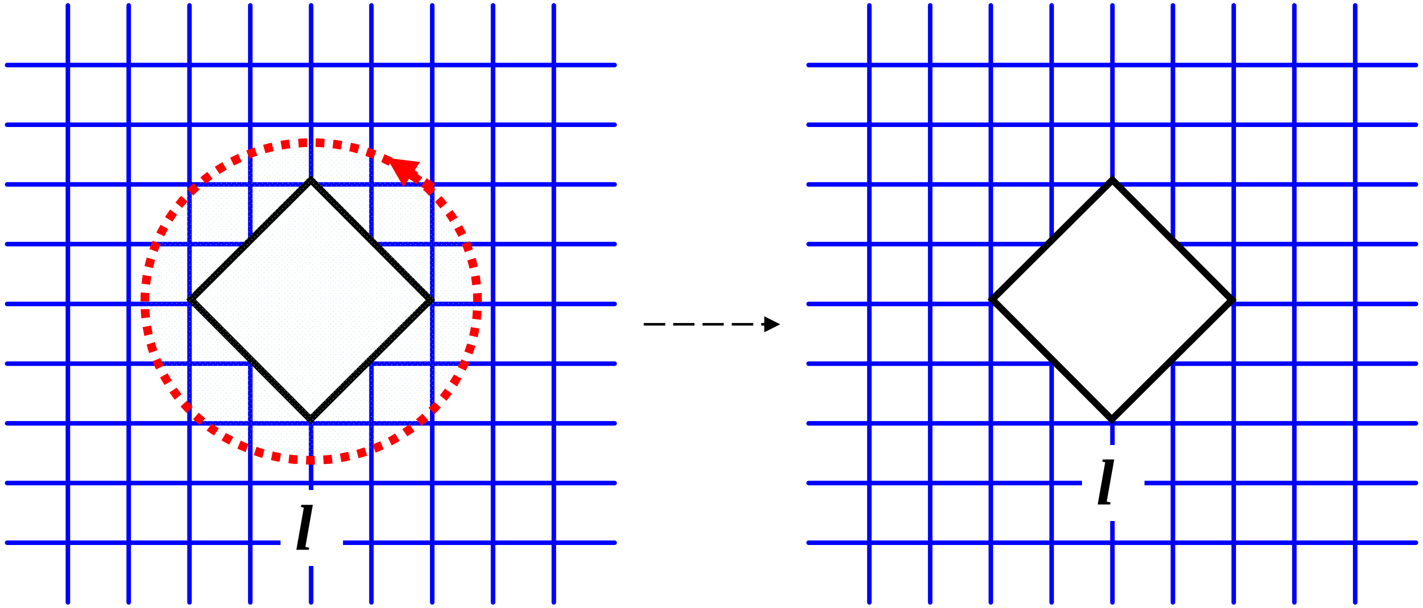


$$|\uparrow\rangle \dashrightarrow |\downarrow\rangle$$

Tunneling process of Z2 vortex on one-hole



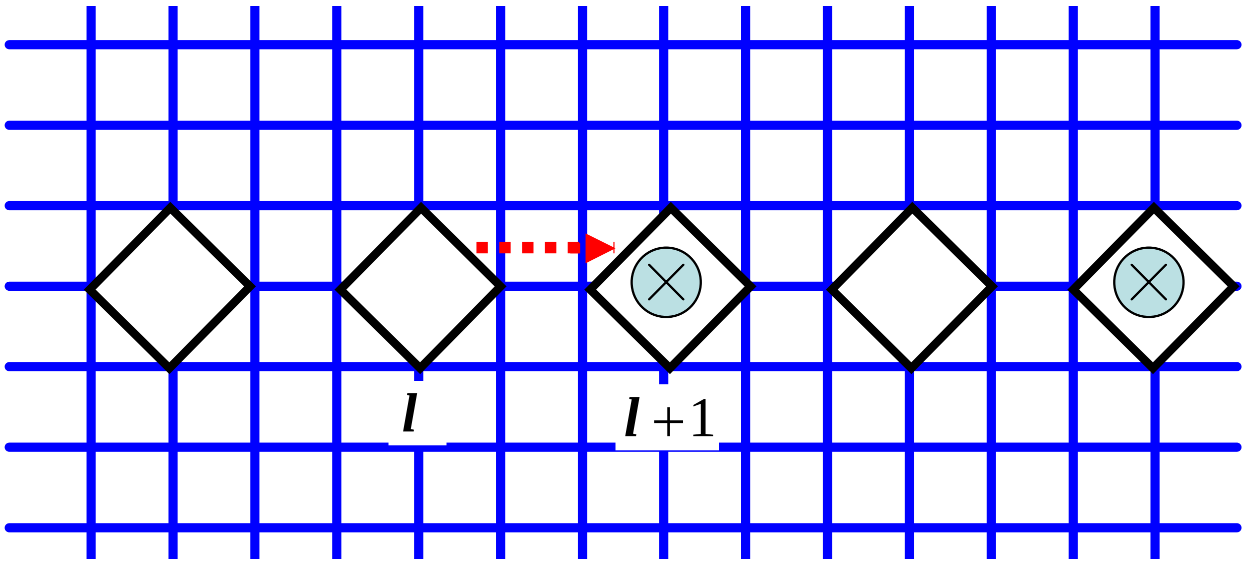
Tunneling process of Fermion



$$|\uparrow\rangle \dashrightarrow +|\uparrow\rangle$$

$$|\downarrow\rangle \dashrightarrow -|\downarrow\rangle$$

Tunneling process of Z2 vortex on 2-hole



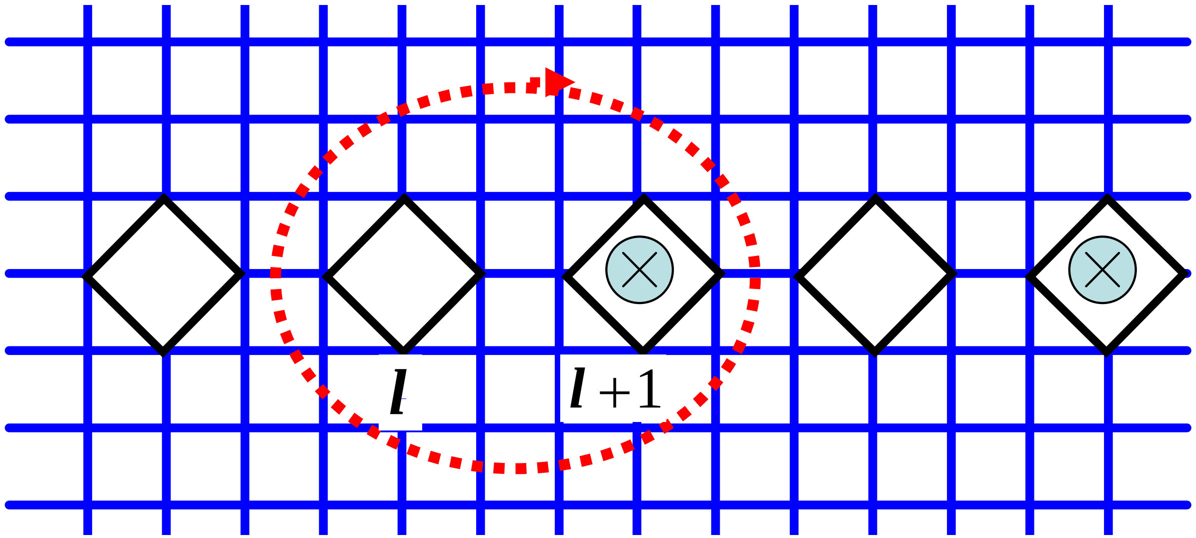
$Pseudo-spin$

$operator$

$\tau_1^x \otimes \tau_2^x$

$$|\uparrow,\uparrow\rangle \rightarrow |\downarrow,\downarrow\rangle$$
$$|\downarrow,\uparrow\rangle \rightarrow |\uparrow,\downarrow\rangle$$
$$|\uparrow,\downarrow\rangle \rightarrow |\downarrow,\uparrow\rangle$$
$$|\downarrow,\downarrow\rangle \rightarrow |\uparrow,\uparrow\rangle$$

Tunneling process of Fermion on 2-hole



Pseudo − spin operator

$\tau_1^z \otimes \tau_2^z$

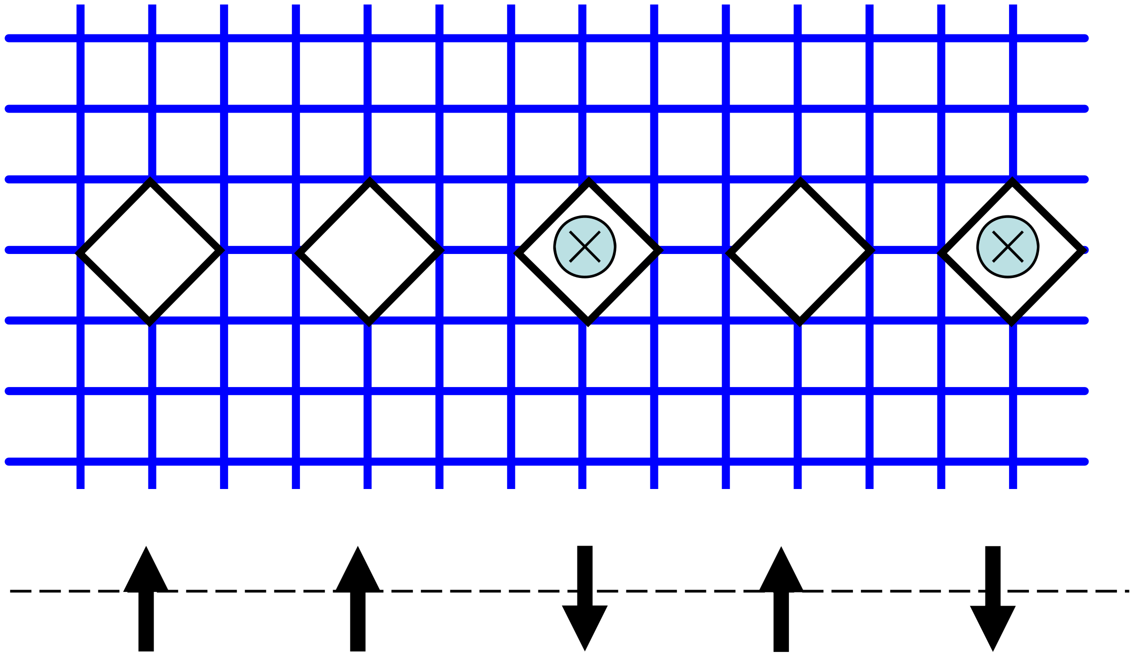
$|\uparrow,\uparrow\rangle \rightarrow +|\uparrow,\uparrow\rangle$

$|\downarrow,\uparrow\rangle \rightarrow -|\downarrow,\uparrow\rangle$

$|\uparrow,\downarrow\rangle \rightarrow -|\uparrow,\downarrow\rangle$

$|\downarrow,\downarrow\rangle \rightarrow +|\downarrow,\downarrow\rangle$

Effective model of the degenerate ground states of multi-hole



$$H_{eff} = \sum_{ij} J^z_{ij}\tau^z_i\tau^z_j + \sum_{ij} J^x_{ij}\tau^x_i\tau^x_j + \sum_i h^z_i\tau^z_i + \sum_i h^x_i\tau^x_i$$

The four parameters Jz, Jx, hx, hz are determined by the quantum effects of different quasi-particles.

The energy splitting from higher order (degenerate) perturbation approach

$$\delta \mathrm{E}_{\mathrm{ij}}^{(\mathrm{s})} = \langle \varphi_i \,|\, \hat{H}'\left(\frac{\hat{H}'}{\hat{H}_0 - E_0}\right)^{s-1} |\, \varphi_j \rangle$$

$$\boldsymbol{\delta E} \rightarrow \delta\varepsilon \left(\frac{\boldsymbol{t_{eff}}}{\delta\varepsilon} \right)^{\boldsymbol{L}}$$

L : Hopping steps of quasi-particles

t_{eff} : Hopping integral

$\delta\varepsilon$: Excited energy of quasi-particles

The answer : control the quantum tunneling effect to control the topological qubits

- **How to control the quantum tunneling effect of the topological qubits?**

Keywords : controllable topological order

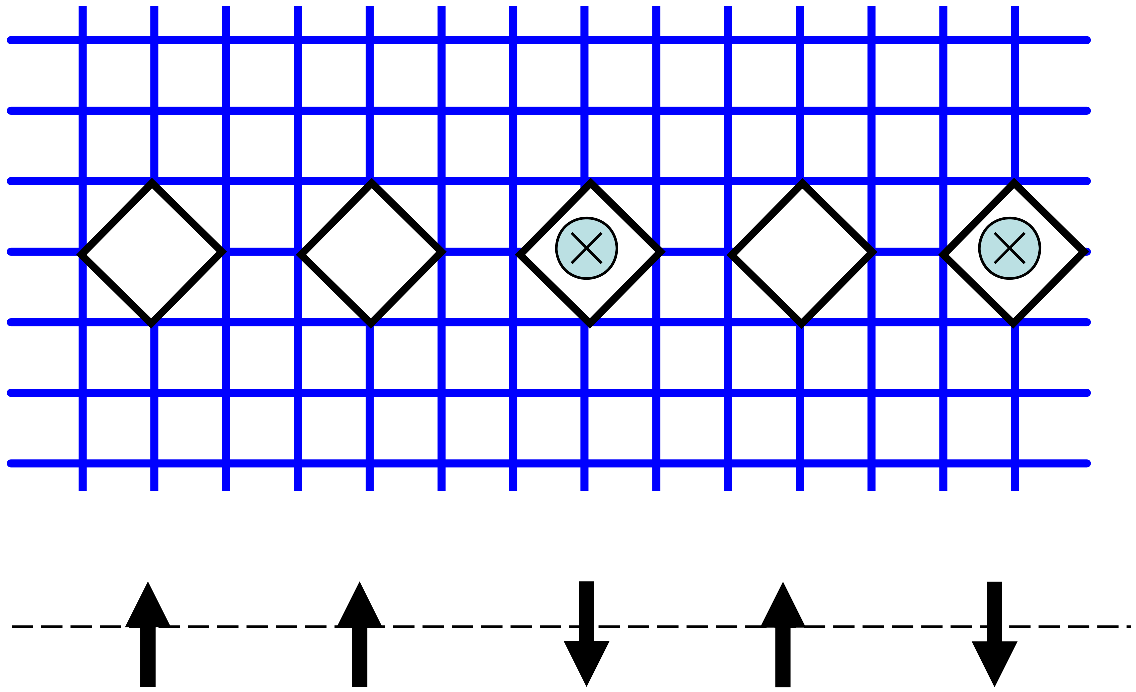
In a controllable topological order, quasi-particles' dispersions and the energy splitting of the degenerate ground states can be manipulated.

(2) Quantum computation by Z2 topological order

1. Quantum computer of topological qubits
2. Initialization
3. Unitary operations

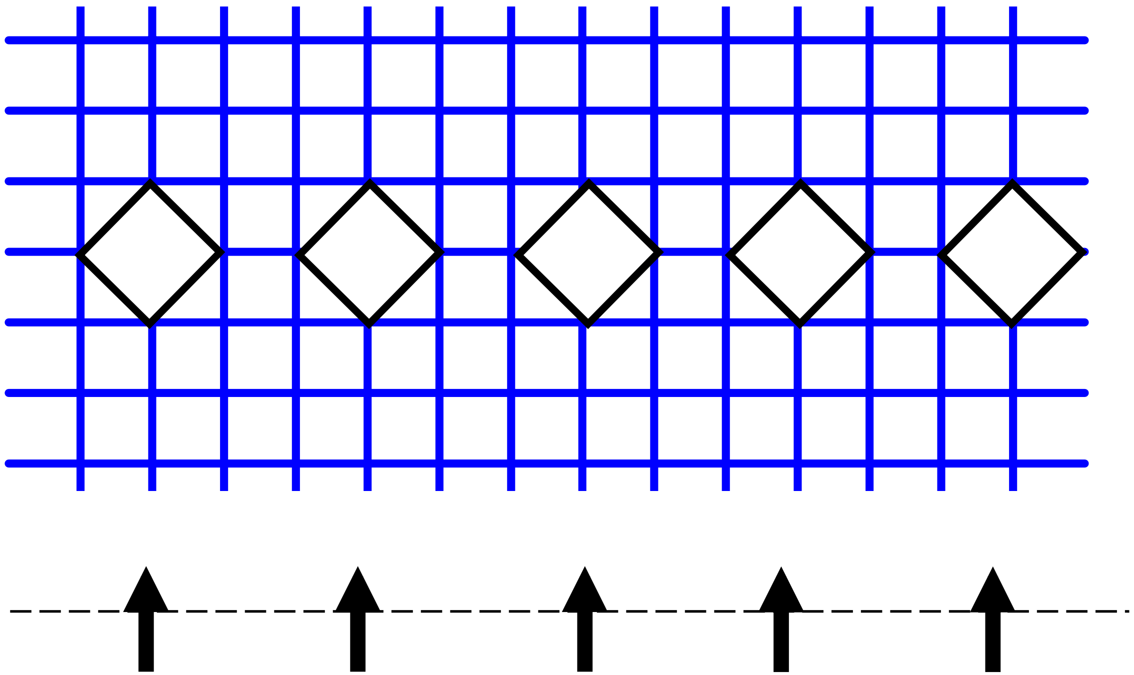
Quantum computer of topological qubits

A line of holes in a controllable topological order of the toric code model



Initialization

- **Applied a external fields along y-directions, only fermion can move, then the effective model becomes :** $H_{eff} = \sum_{ij} J^z \tau_i^z \tau_j^z + \sum_i h^z \tau_i^z$



Unitary operations

- **A general operator becomes :**

$$U = e^{-\frac{i}{\hbar}\gamma\tau_z} e^{-\frac{i}{\hbar}\varphi\tau_x} e^{-\frac{i}{\hbar}\theta\tau_z}$$

For example , Hadamard gate is

$$U_{\theta,\varphi}(\gamma = \frac{\pi}{4}, \theta = \frac{7\pi}{4}, \varphi = \frac{\pi}{4})$$

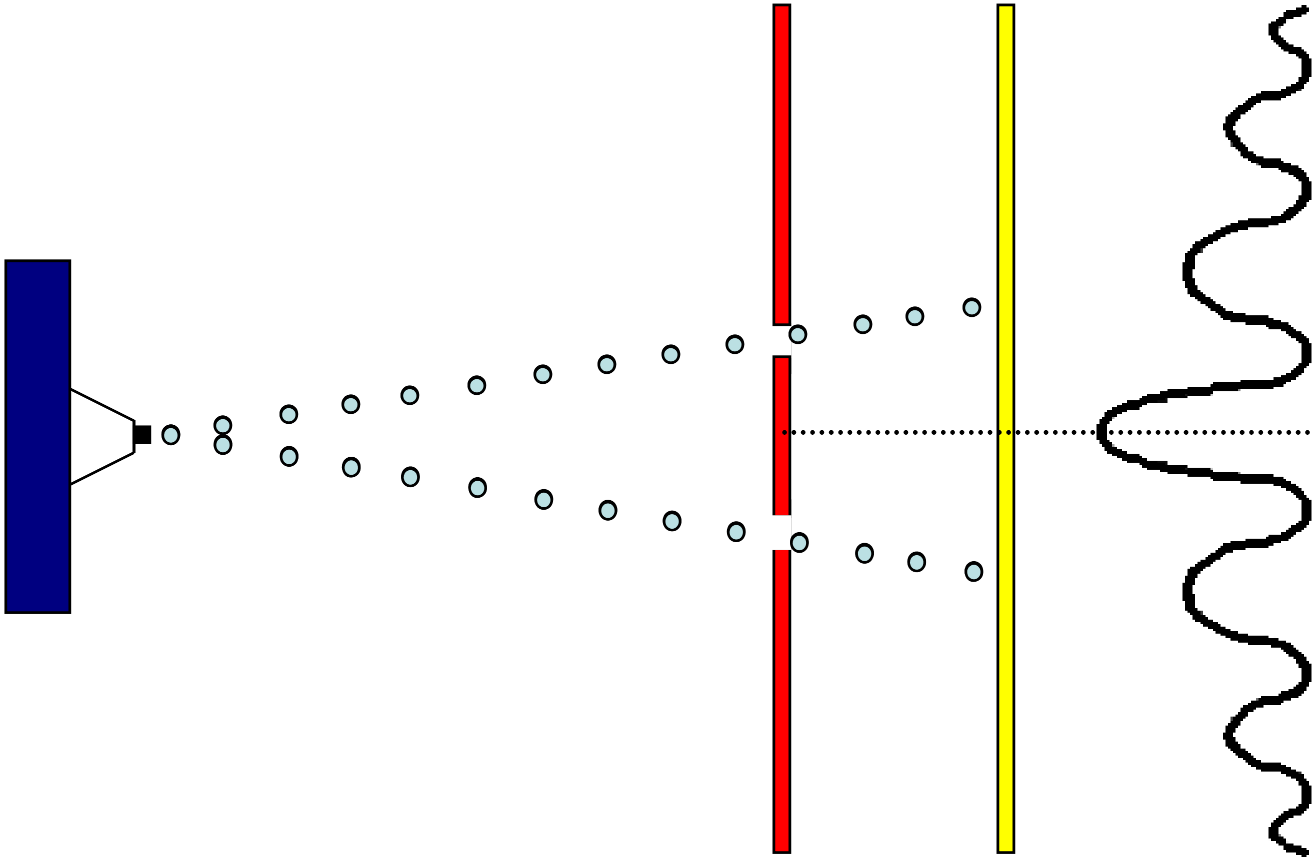
Measurement

- We want to determine the state

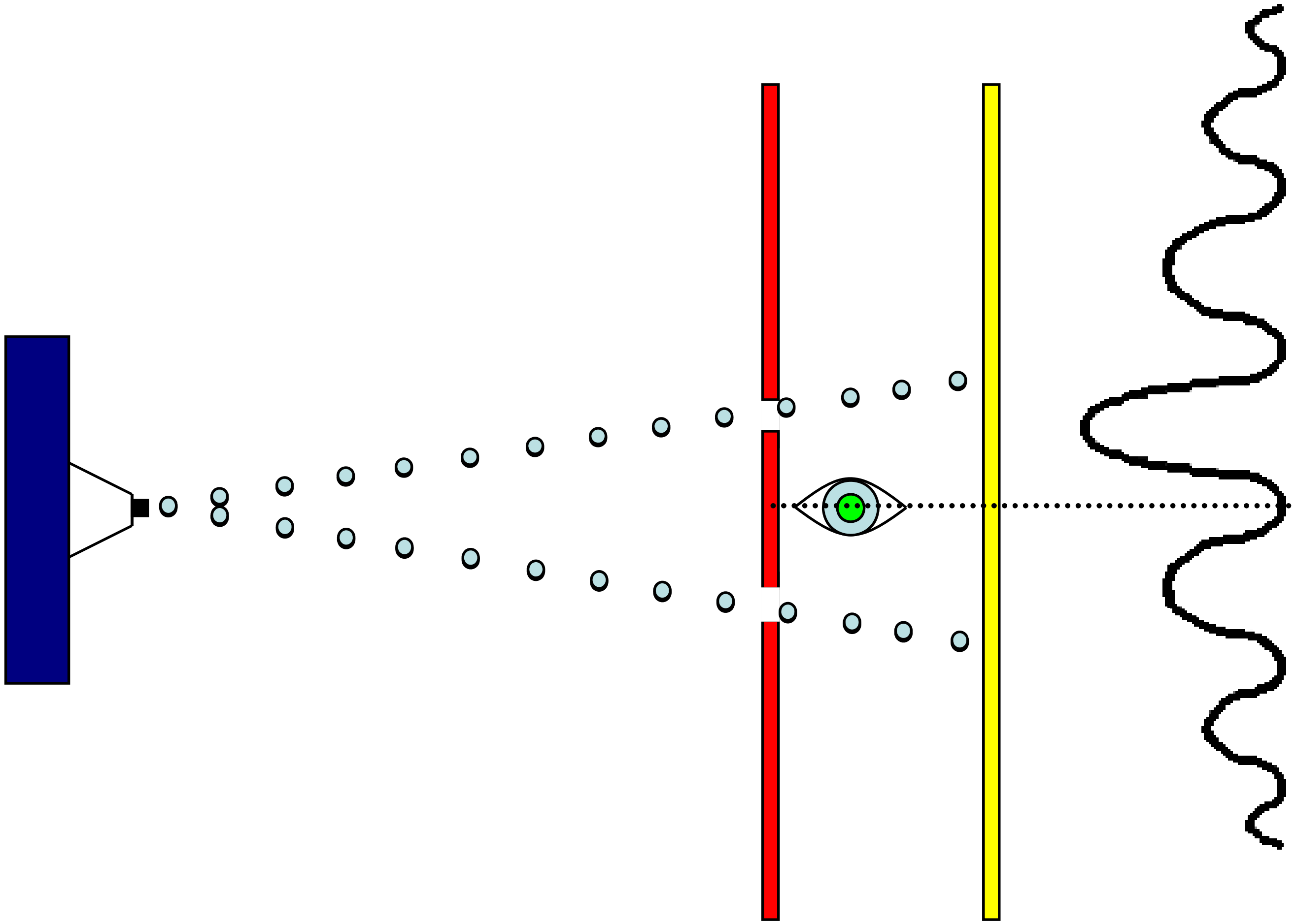
$$\left| vac \right\rangle = \alpha \left| \uparrow \right\rangle + \beta e^{i\phi} \left| \downarrow \right\rangle$$

- The interference from Aharonov-Bohm (AB) effect allows one to observe distinction between the processes with or without a flux inside the loop.

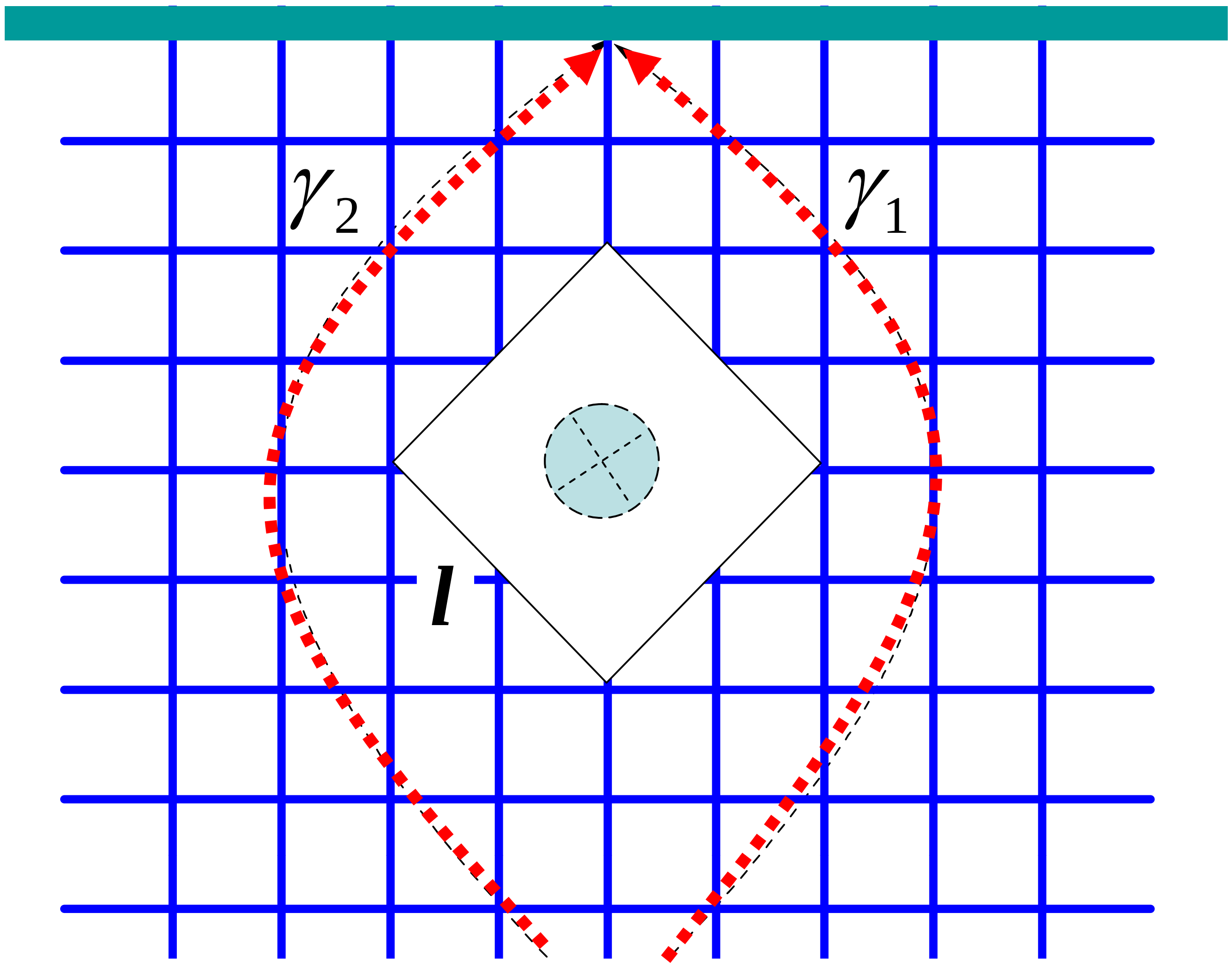
Interference in double slits



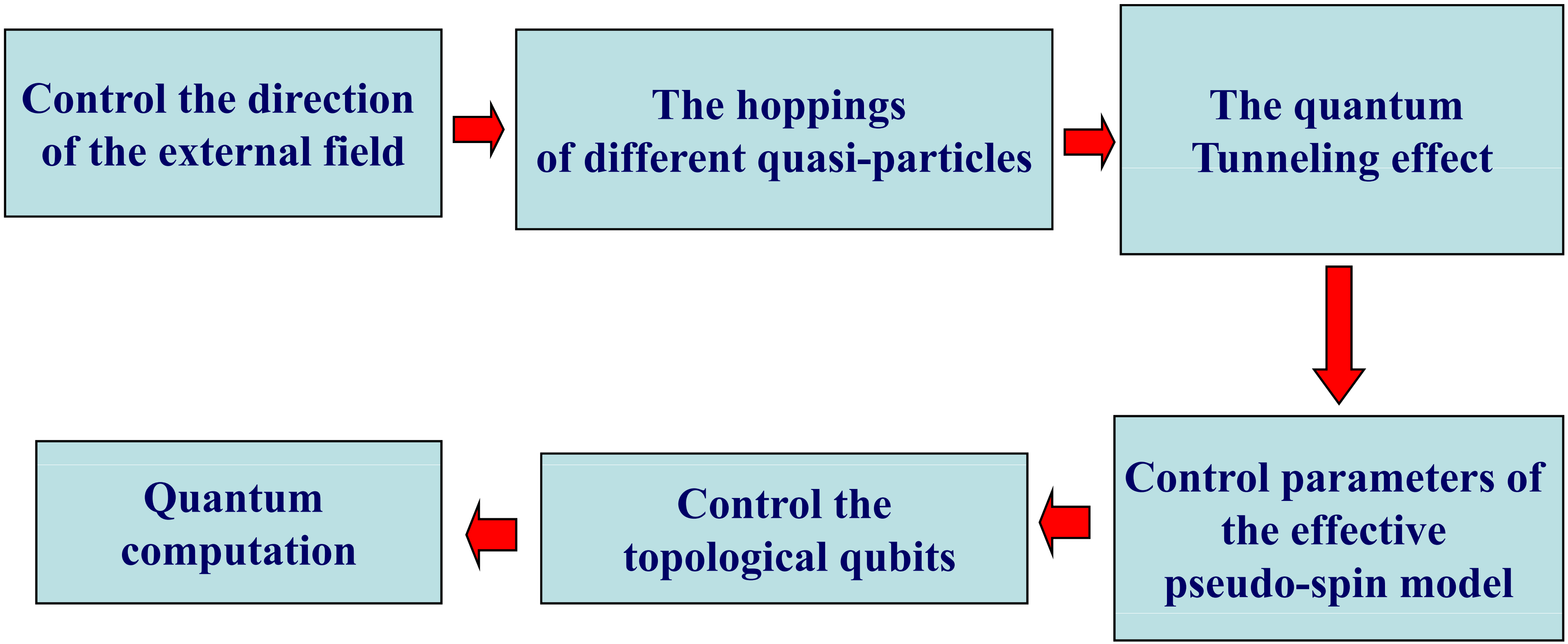
Observing AB effect in double slits



Measure topological qubit



Road map of Quantum Computation by Topological Qubits



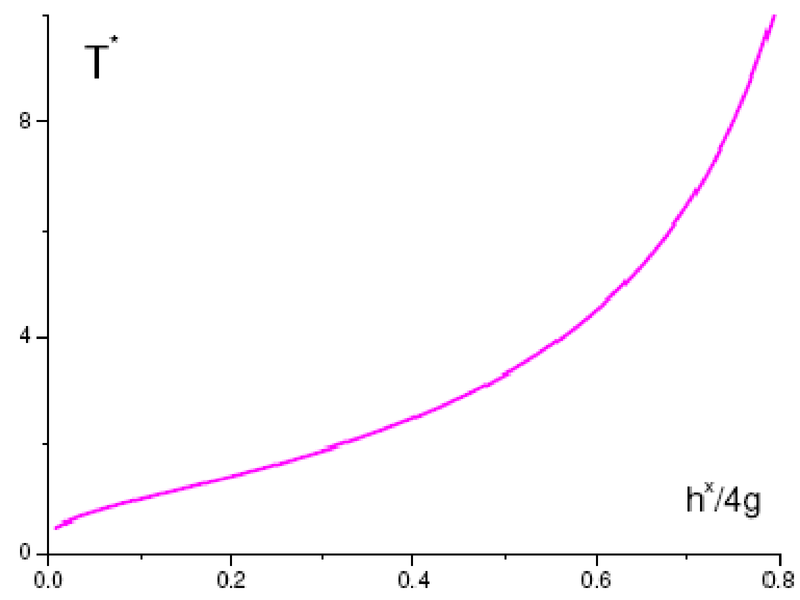
Control quantum tunneling effet in a controlled topological order

Errors

- **Thermal effect : at finite temperature, real quasi-particle exist, their moving leads to error. The probability is about $\exp(-\frac{\Delta}{T})$. Here △ is the energy gap of the quasi-particle.**
- **Real quasi-particles will also lead to errors on the storage and the measurement.**
- **Topological quantum computation ≠ quantum computation with topological qubits : whether the unitary transformation is topological?**

Classical - quantum crossover

- **T* is the crossover temperature divided quantum region and classical region,**
- **T>T*, the classical hopping processes dominate, one cannot do quantum computation;**
- **T<T*, the quantum tunneling processes dominate, the errors will be controlled.**



Threshold for Fault-Tolerance quantum computation

Theorem: ***There exists a threshold p_t such that, if the error rate per gate and time step is $p < p_t$, arbitrarily long quantum computations are possible.***

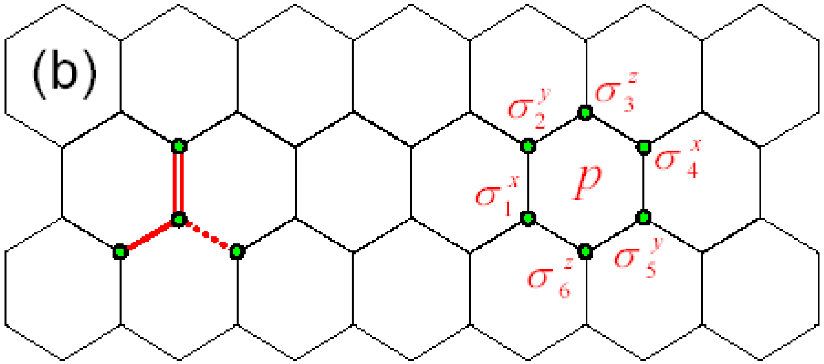
- **The concatenated 7-qubit Steane code has a threshold of $1.85 \times 10{-}5$.**
- **The concatenated Bacon-Shor code has a threshold of $2.02 \times 10{-}5$.**
- **2D *topological* codes has threshold of ~ $6 \times 10{-}3$ (Raussendorf, Harrington, quant-ph/0610082)**

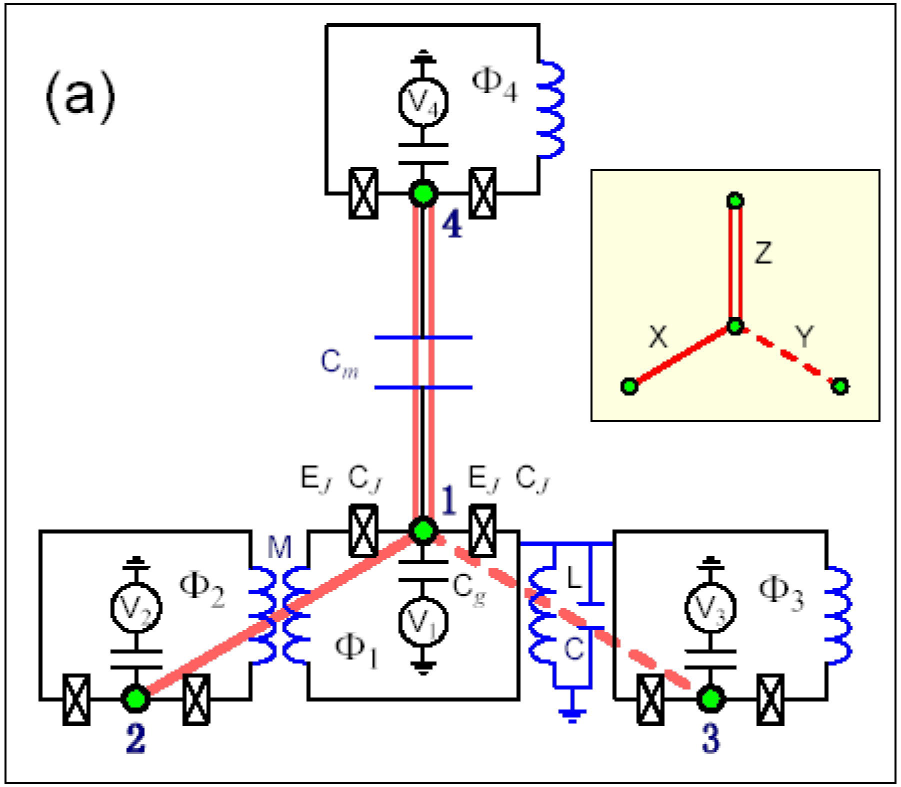
(3) Possibel experimental realization of controllable Z2 topological orders

1) Josephson junction array

2) Cold atoms

1. Possible realization in Josephson junction array

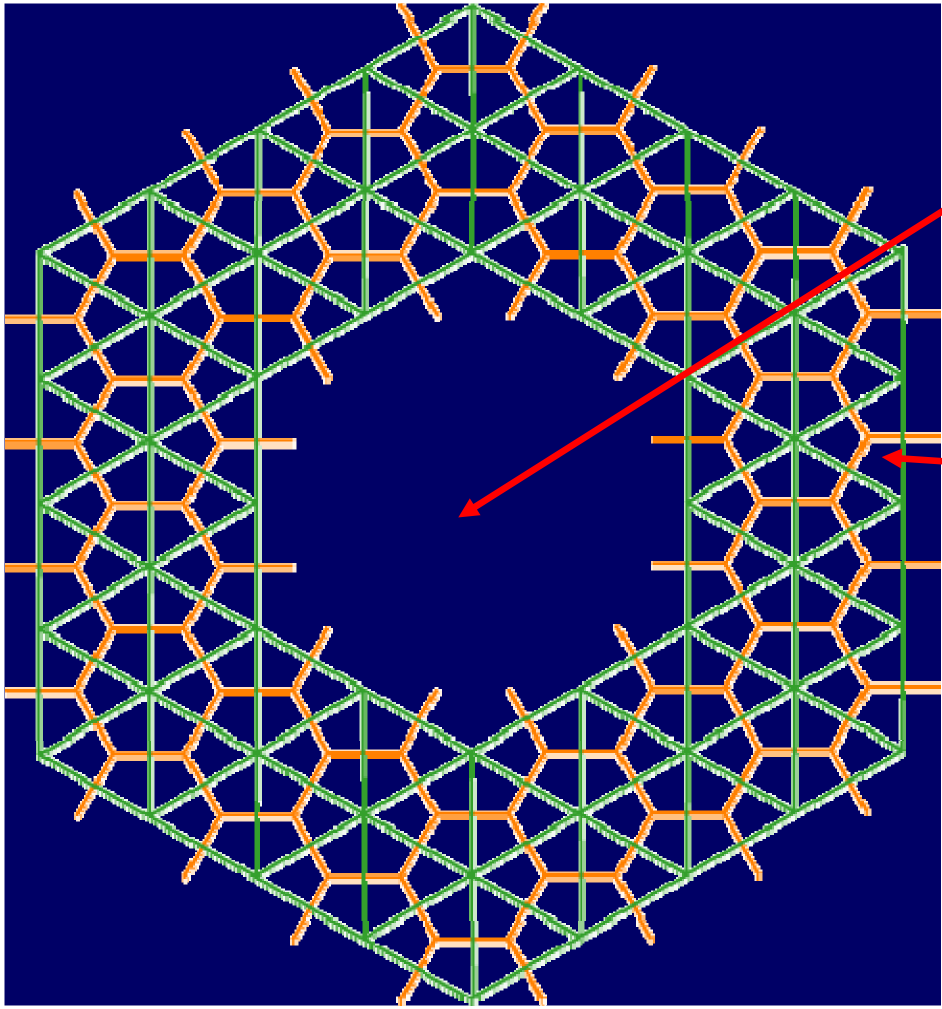




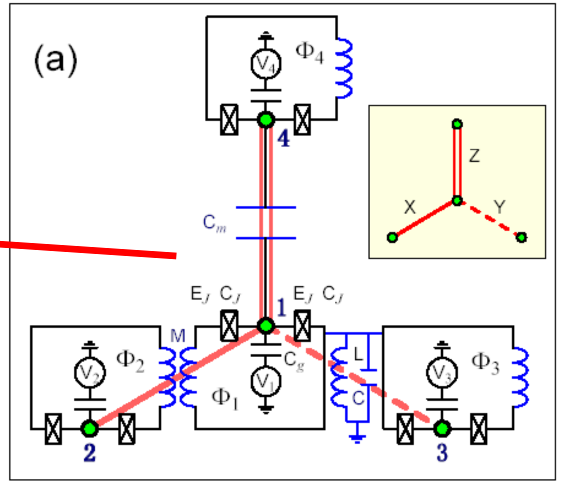
$$H_{eff} = J_x \sum_{x-link} \sigma_i^x \sigma_j^x + J_y \sum_{y-link} \sigma_i^y \sigma_j^y + J_z \sum_{x-link} \sigma_i^z \sigma_j^z + \sum_i h_x \sigma_i^x + \sum_i h_z \sigma_i^z$$

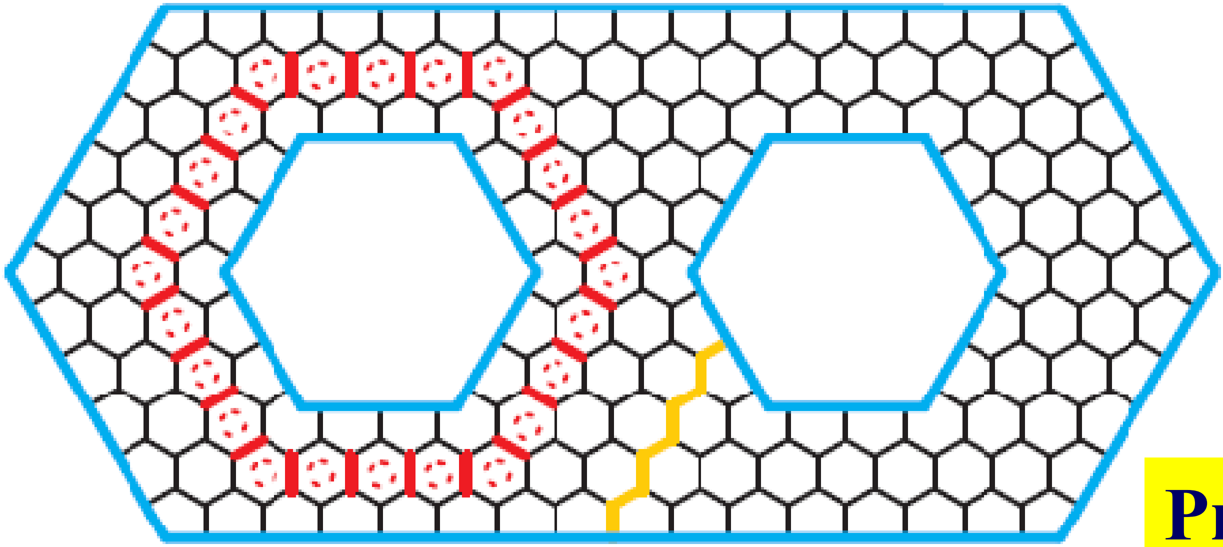
J. Q. You, X.-F. Shi, and F Nori, Phys. Rev. B 81, 014505 (2010)

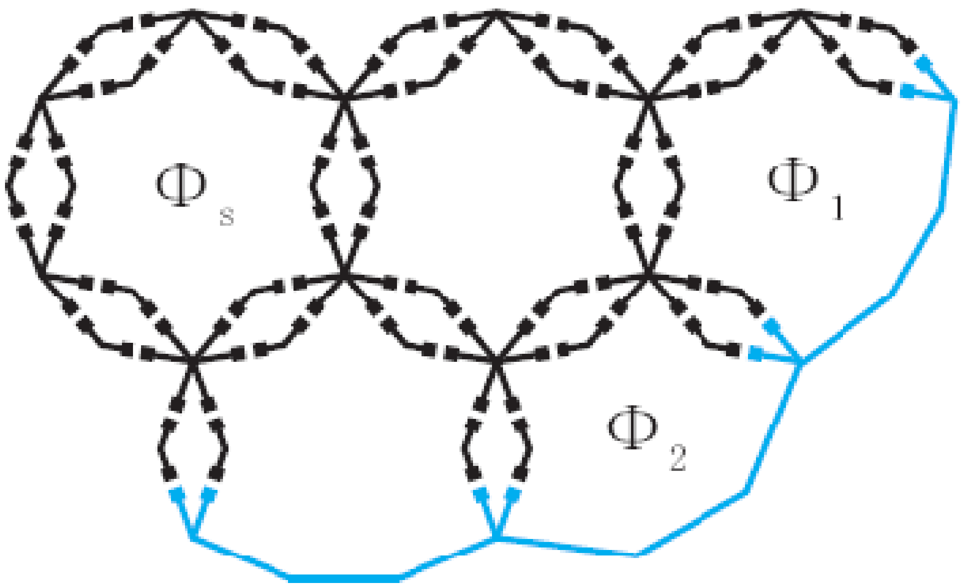
Possible realization of topological qubit in Josephson junction array : a hole in the designed model



Hole





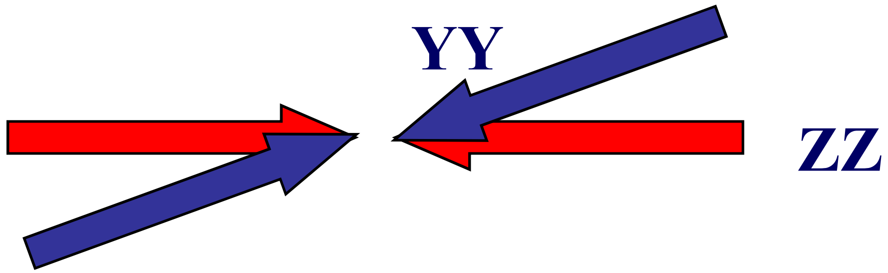


Predition of the topological qubits based on RK model

Zhi Yin, Sheng-Wen Li, and Yi-Xin Chen, Phys. Rev., A81(2010)012327

2. Possible realization in cold atoms

- **2D optical (honeycomb) lattice :**

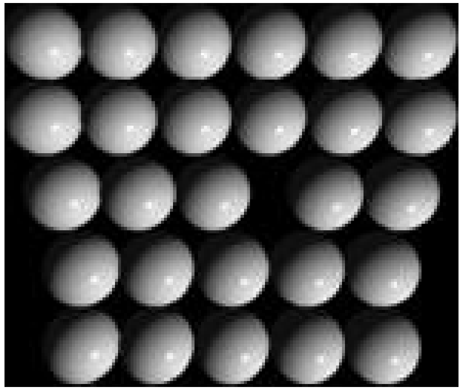


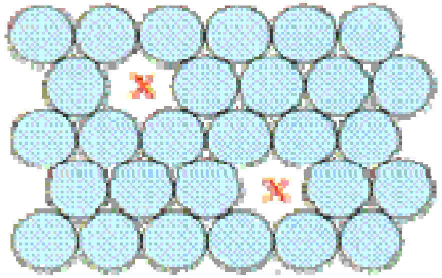
Kitaev model on honeycomb lattice can be created with 3 sets of light beams.

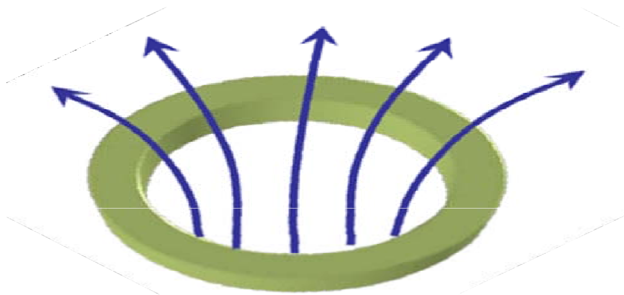
L.-M. Duan, E. Demler, and M. D. Lukin,
Phys. Rev. Lett. 91, 090402 (2003).

III. Zero modes of lattice-vacancies in the topological insulators and topological superconductors

Defects in topological states







Dislocation: topological defect

Vacancy: Non-topological defect

Vortex : topological defect

Classification of topological SCs : ten-fold way of random matrix

System	Cartan nomenclature	TRS	PHS	SLS	Hamiltonian	NLSM (ferm. replicas)
standard (Wigner-Dyson)	A (unitary)	0	0	0	$U(N)$	$U(2n)/U(n)\times U(n)$
	AI (orthogonal)	+1	0	0	$U(N)/O(N)$	$Sp(2n)/Sp(n)\times Sp(n)$
	AII (symplectic)	−1	0	0	$U(2N)/Sp(2N)$	$O(2n)/O(n)\times O(n)$
chiral (sublattice)	AIII (chiral unit.)	0	0	1	$U(N+M)/U(N)\times U(M)$	$U(n)$
	BDI (chiral orthog.)	+1	+1	1	$SO(N+M)/SO(N)\times SO(M)$	$U(2n)/Sp(n)$
	CII (chiral sympl.)	−1	−1	1	$Sp(2N+2M)/Sp(2N)\times Sp(2M)$	$U(2n)/O(2n)$
BdG	D	0	+1	0	$SO(2N)$	$O(2n)/U(n)$
	C	0	−1	0	$Sp(2N)$	$Sp(n)/U(n)$
	DIII	−1	+1	1	$SO(2N)/U(N)$	$O(2n)$
	CI	+1	−1	1	$Sp(2N)/U(N)$	$Sp(n)$

M. R. Zirnbauer, J. Math. Phys. 37, 4986 (1996). A. Altland and M. R. Zirnbauer, Phys. Rev. B 55, 1142 (1997).

Y. Kitaev, AIP Conf. Proc. 22, 1134 (2009).

S. Ryu, et al., New J. Phys. 12, 065010 (2010).

Classification of topological SCs : ten-fold way of random matrix

AZ\d	0	1	2	3	4	5	6	7	8	9
A	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	⋯
AIII	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	⋯
AI	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	⋯
BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	⋯
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	⋯
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	⋯
AII	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	⋯
CII	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	⋯
C	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	⋯
CI	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	⋯

IQHE
p+ip wave SC
polyacetylene
3He B
TMTSF
Z2 topological insulator
QSHE
d+id wave SC

M. R. Zirnbauer, J. Math. Phys. 37, 4986 (1996). A. Altland and M. R. Zirnbauer, Phys. Rev. B 55, 1142 (1997).
Y. Kitaev, AIP Conf. Proc. 22, 1134 (2009).
S. Ryu, et al., New J. Phys. 12, 065010 (2010).

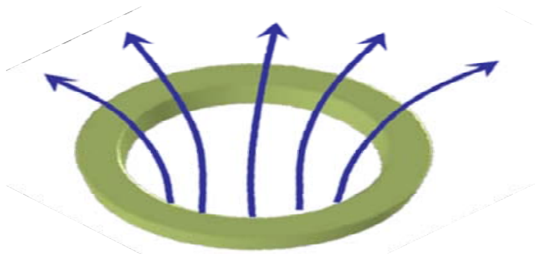
Zero modes of topological defect – vortex of topological insulator

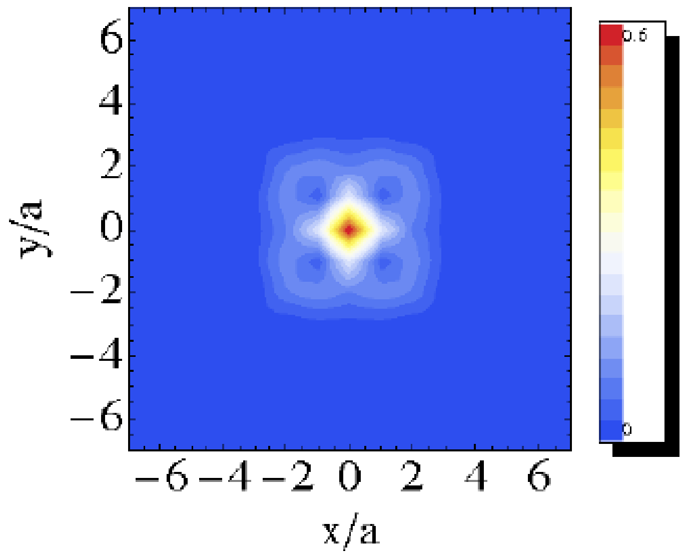
- **Zero modes of quantized vortex of topological insulator are protected by the topological index of the system**
- **The mathematic origin is the AS index theorem.**

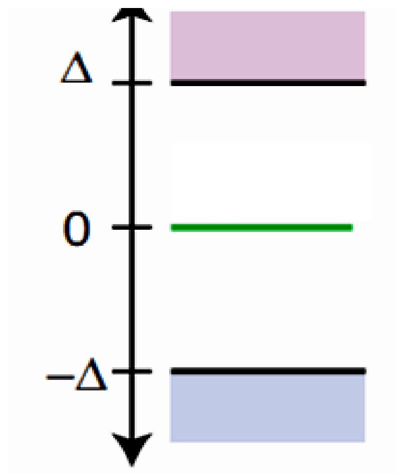
Topology

⇩

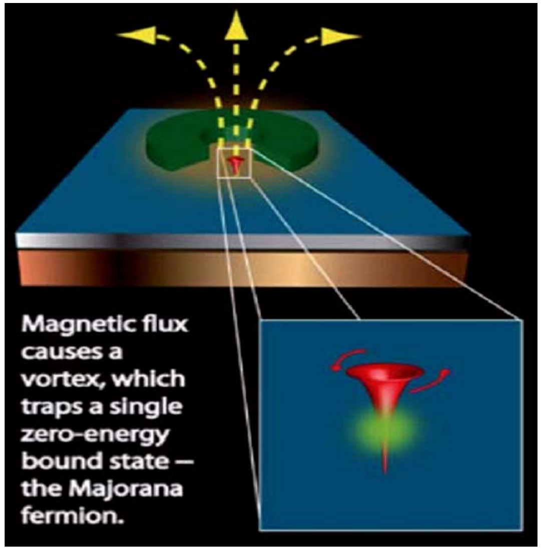
Zero modes

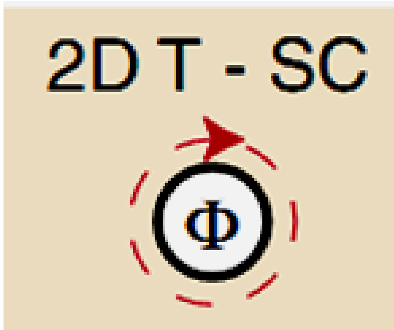


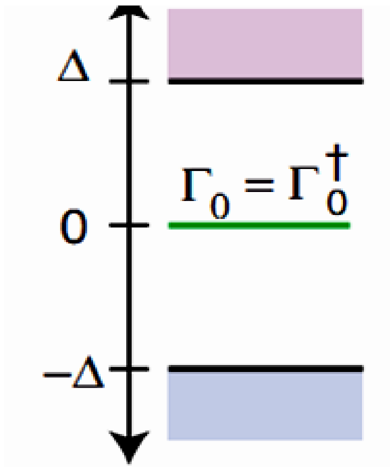




Majorana zero mode on topological defect in D-type TSC



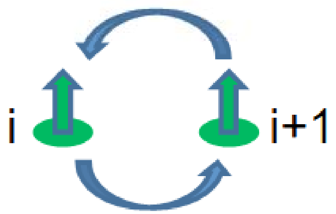




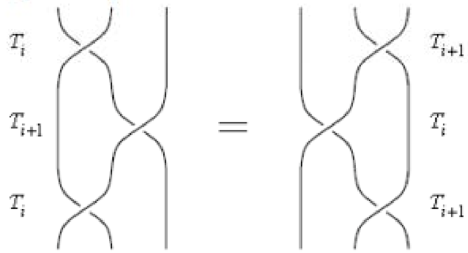
$$\varepsilon_n = n\omega_0, \quad \omega_0 \approx \Delta_0^2 / E_F$$

zero mode $\varepsilon_0 = 0$ $\Psi = \Psi^+ (= \gamma)$ Majorana (real) fermion!

interchanging vortices ⟹ braid groups, non-Abelian statistics



$$\gamma_i \to \gamma_{i+1}$$
$$\gamma_{i+1} \to -\gamma_i$$

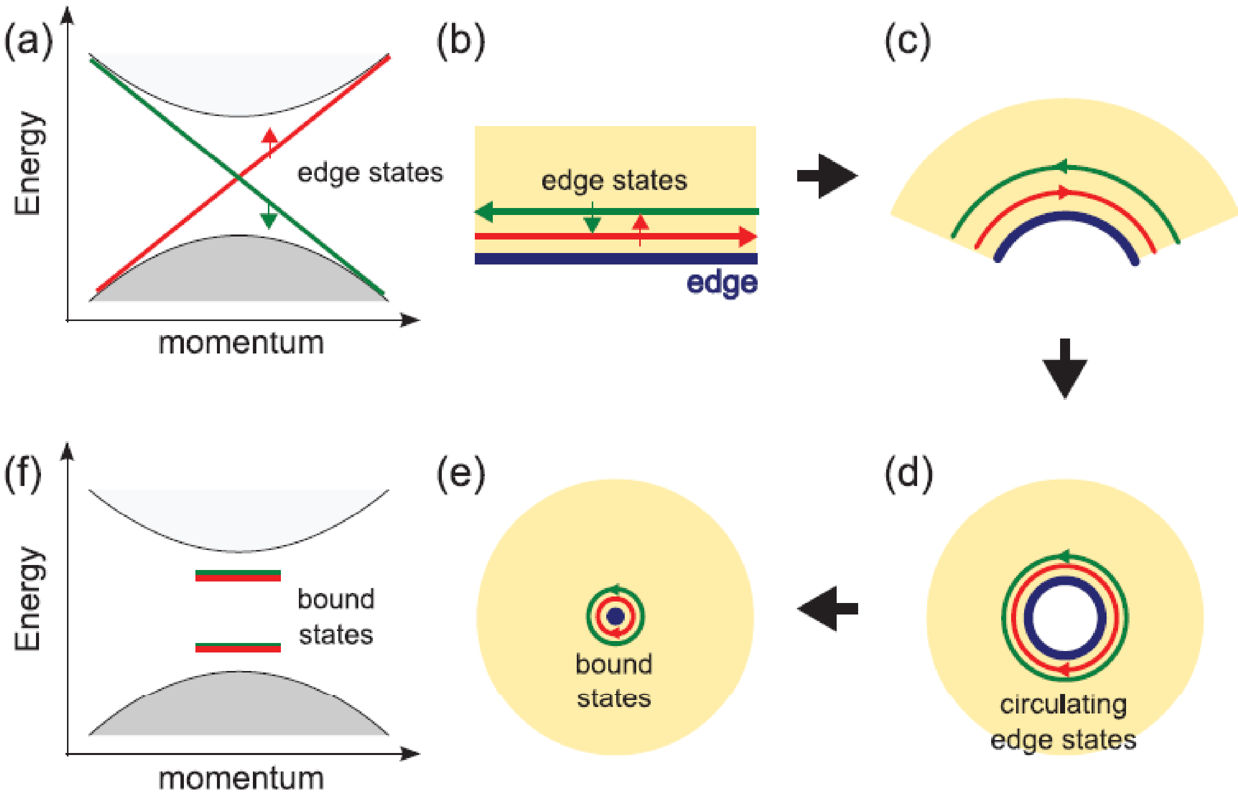


$$(\gamma_j^\dagger = \gamma_j)$$

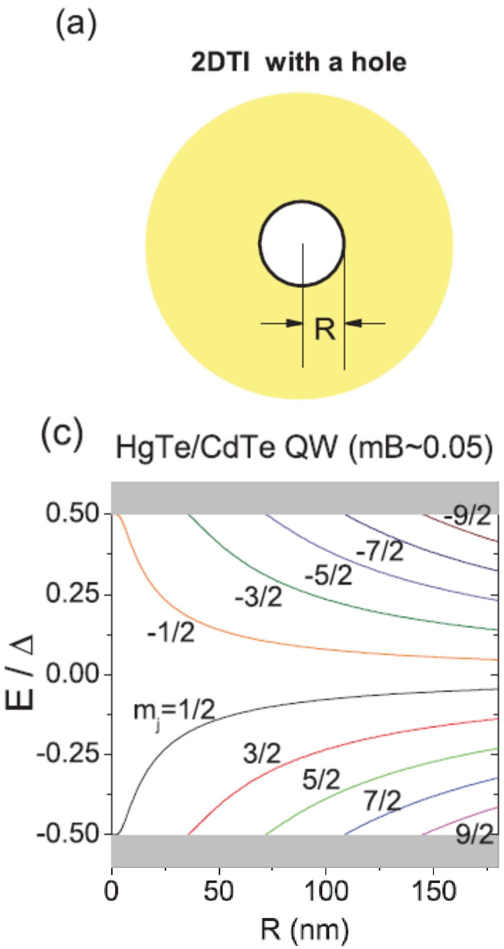
nonAbelian statistics

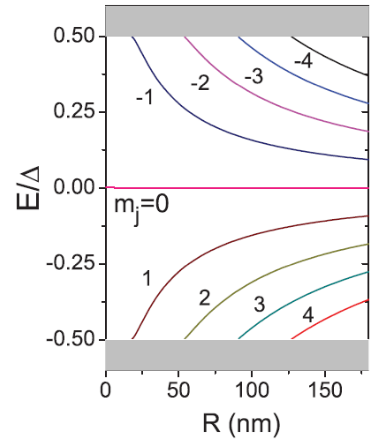
N. B. Kopnin and M. M. Salomaa, Phys. Rev. B 44, 9667 (1991).

二维拓扑绝缘体中的点缺陷：连续模型计算

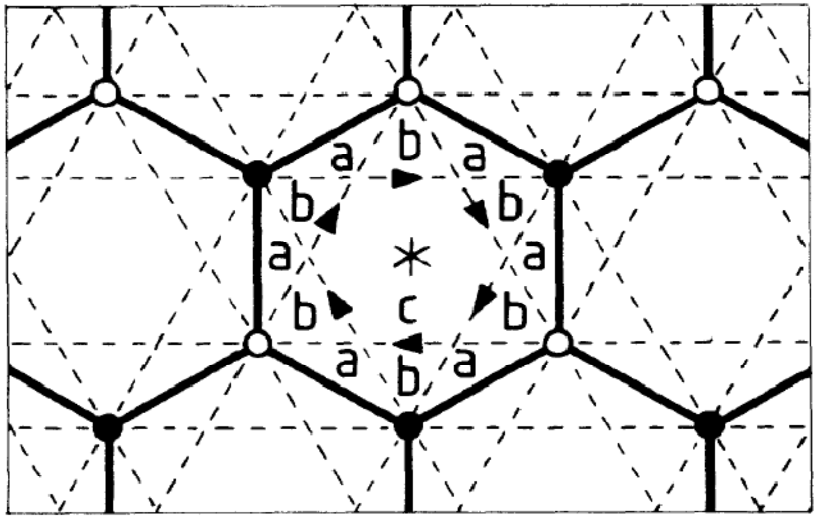


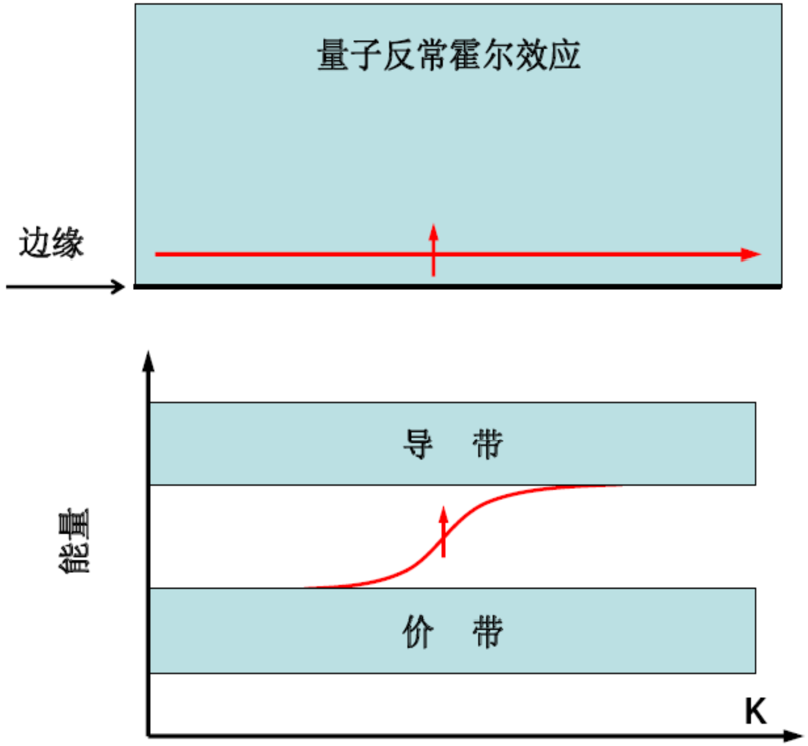
Wen-Yu Shan, et.al, PRB 84, 035307 (2011)

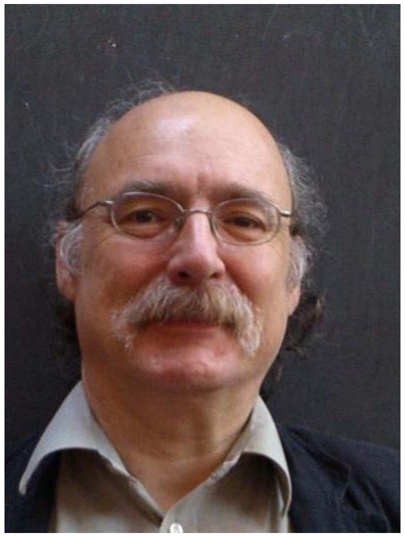




The Haldane model







$$H_{\mathrm{H}} = -t \sum_{\langle i,j\rangle,\sigma} \left(\hat{c}^{\dagger}_{i\sigma}\hat{c}_{j\sigma} + h.c.\right) - t' \sum_{\langle\langle i,j\rangle\rangle,\sigma} e^{i\varphi_{ij}}\,\hat{c}^{\dagger}_{i\sigma}\hat{c}_{j\sigma}$$

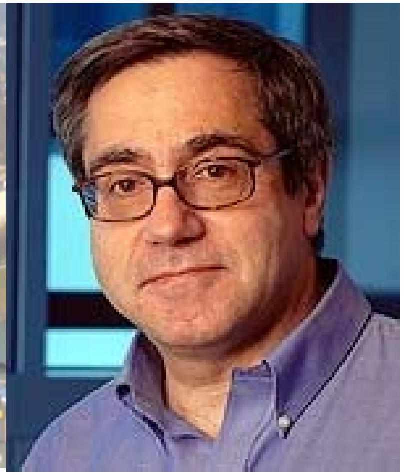
F. D. M. Haldane, Phys. Rev. Lett. 61, 2015 (1988).

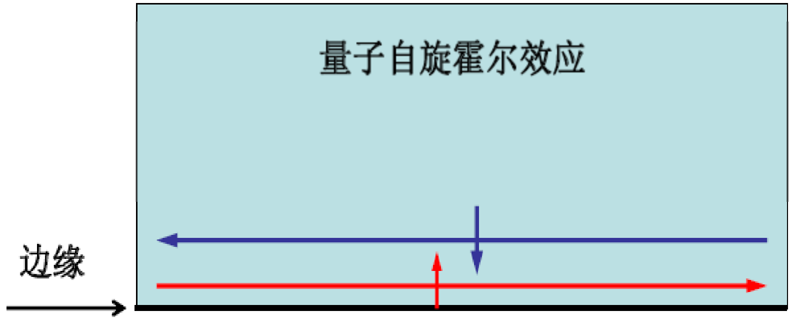
Kane-Mele model

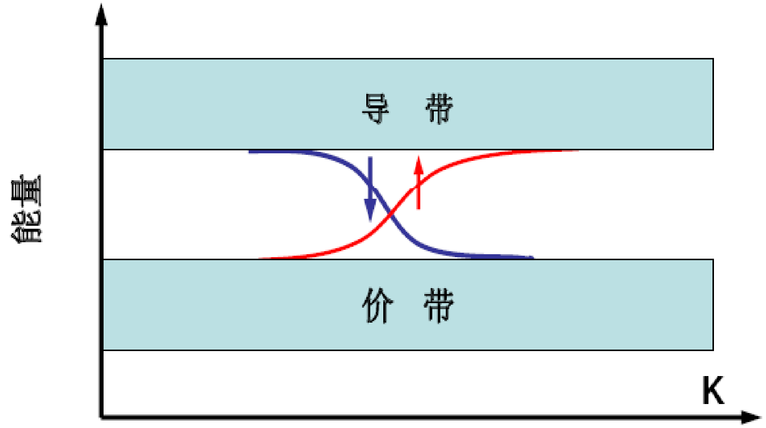
Kane and Mele, Phys. Rev. Lett. 95, 146802 (2005)

$$\mathcal{H} = -t \sum_{\langle ij \rangle} \sum_{\sigma} c_{i\sigma}^{\dagger} c_{j\sigma}$$

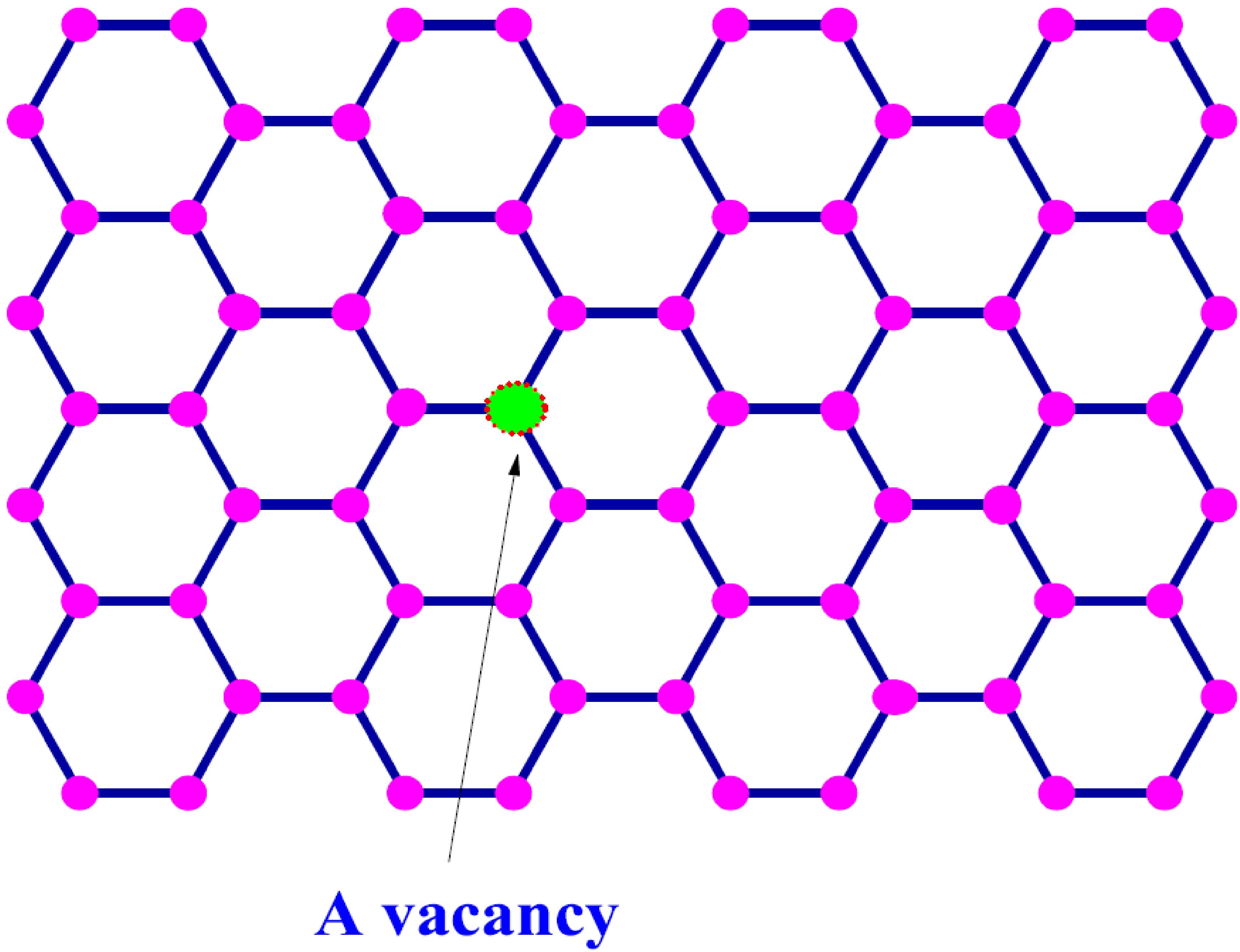
$$+ i\lambda \sum_{\langle\langle ij \rangle\rangle} \sum_{\sigma\sigma'} \nu_{ij}\, \sigma^{z}_{\sigma\sigma'}\, c_{i\sigma}^{\dagger} c_{j\sigma'}$$



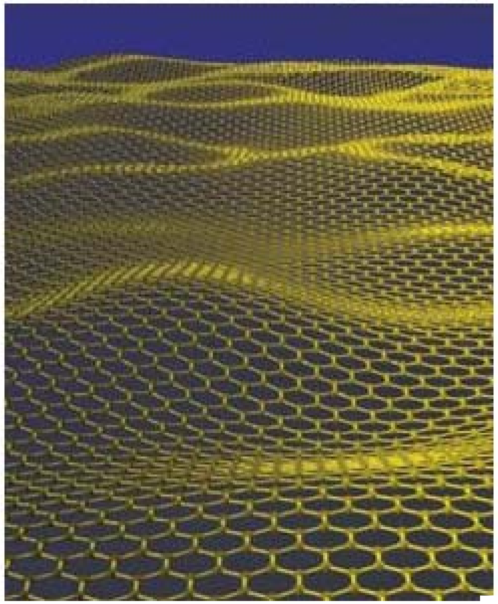


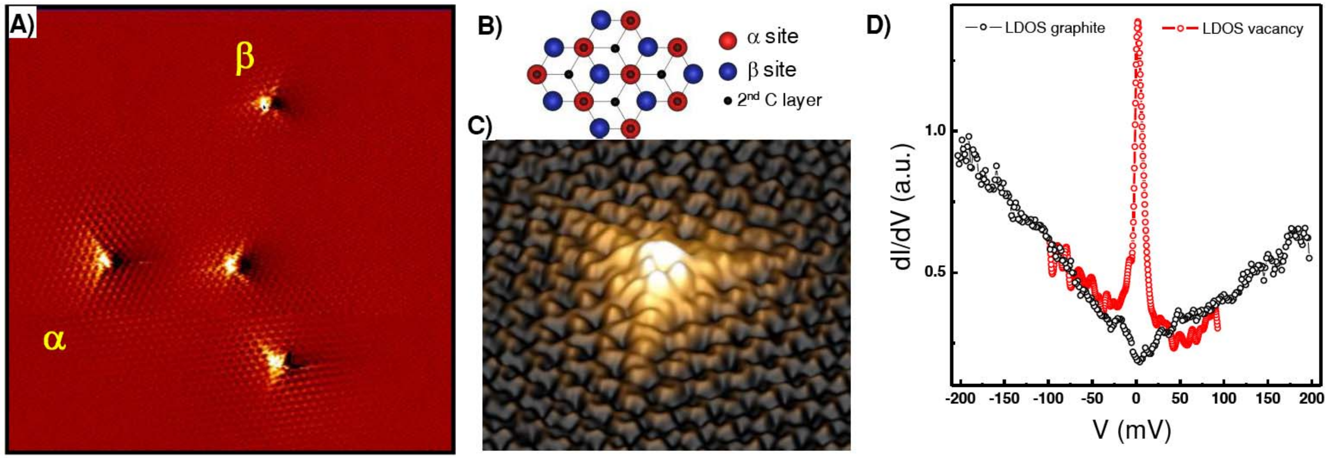


http://www.physics.upenn.edu/~kane/


A vacancy

石墨烯中的点缺陷: 零模束缚态和局域磁矩





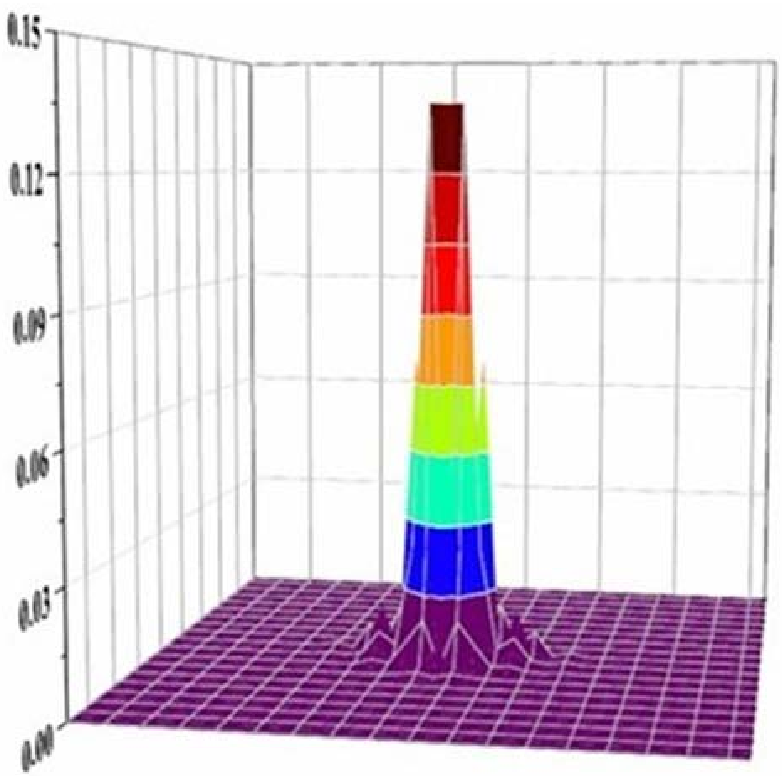
M. M. Ugeda, et.al, Phys. Rev. Lett. **104**, 096804 .

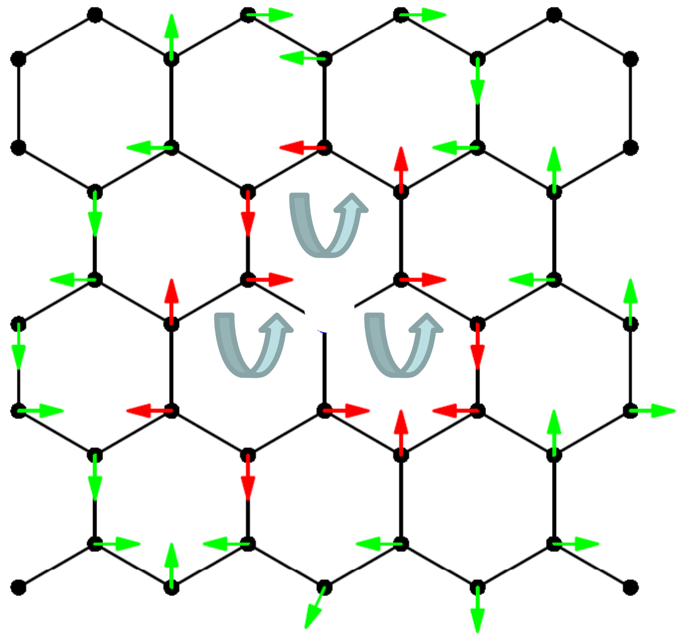
$$\Psi^{(L)}(x,y) \sim \int_{2\pi/3}^{4\pi/3} dk(-2\cos(k/2))^{2x/3} e^{iky/\sqrt{3}}$$

$$\approx \frac{e^{(4\pi i y)/(3\sqrt{3})}}{x+iy} + \frac{e^{2\pi i(x+y/\sqrt{3})/3}}{x-iy},$$

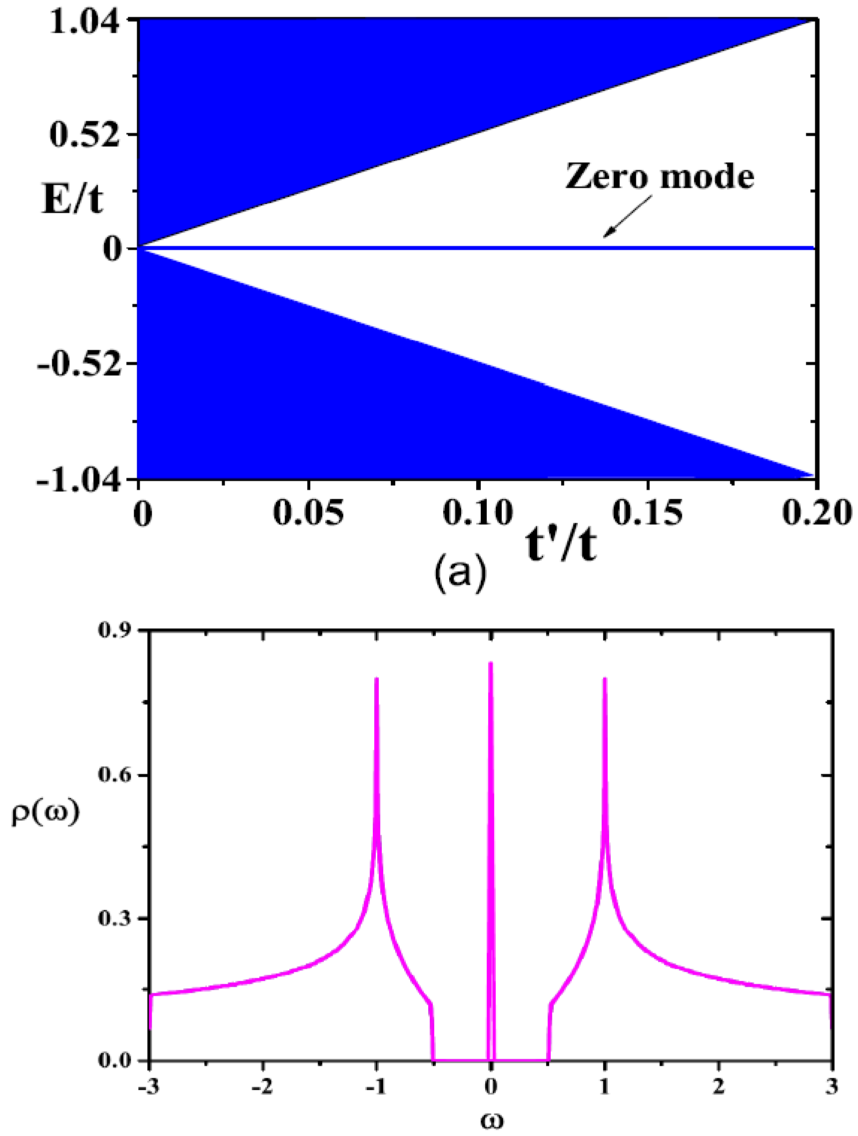
Pereira, V. M., Guinea, et.al, *Phys. Rev. Lett.* ***96, 036801 (2006)***

单个点缺陷的示意图
及零模态粒子的几率分布和波函数相位





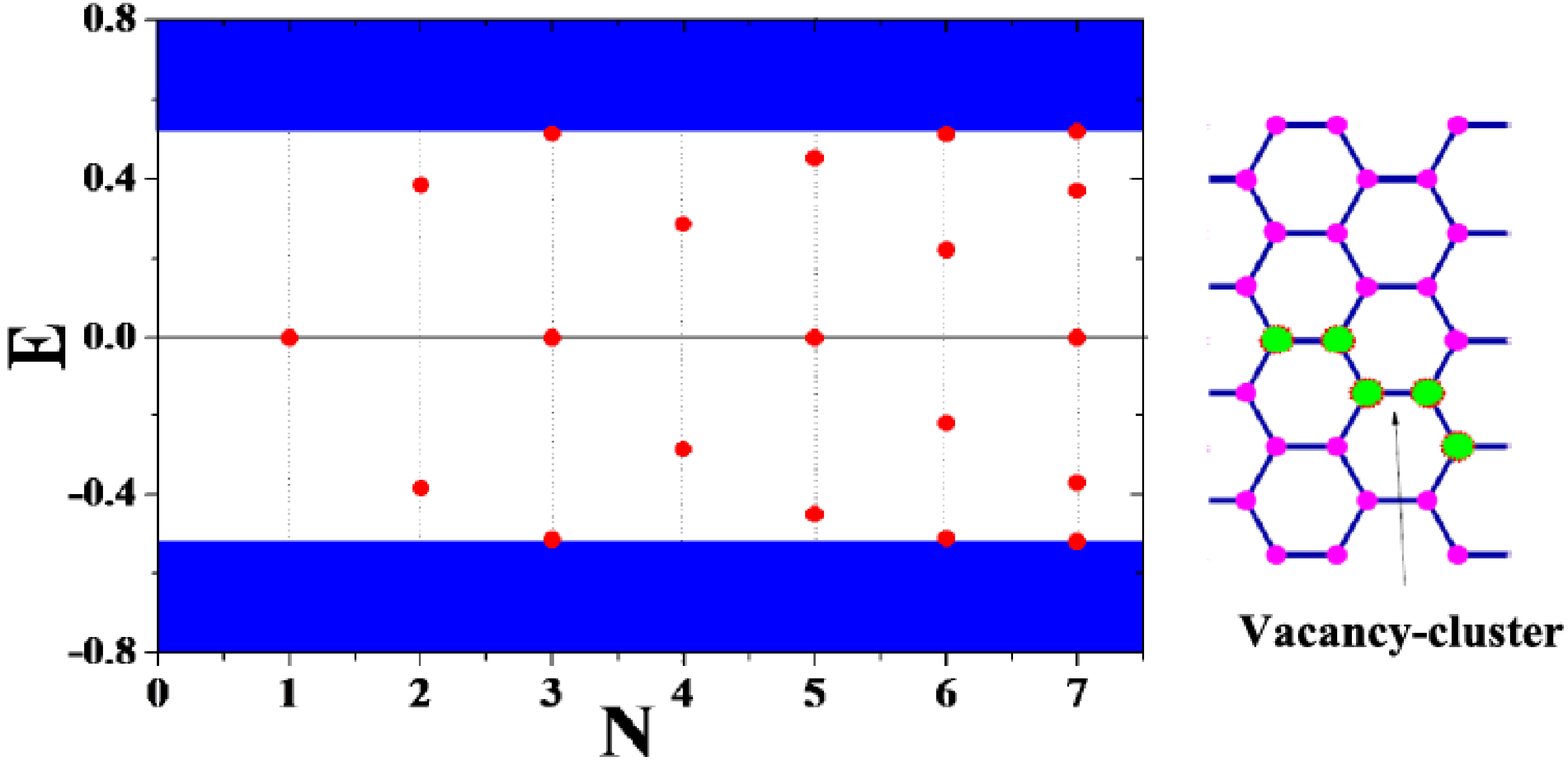
Energy spectrum and DOS



(a)

(b)

Parity effect of vacancies



symmetry → zero modes

Particle-hole symmetry

$$\hat{c}^{\dagger}_{i\in A} \leftrightarrow -\hat{c}_{i\in A}$$

$$\hat{c}^{\dagger}_{i\in B} \leftrightarrow \hat{c}_{i\in B} \quad \Rightarrow \quad (E \Leftrightarrow -E)$$

+ 电荷共轭

Bipartite lattice : A, B sub-lattice

⇓

Odd number of electronic states with single vacancy

Particle-hole symmetry protected zero mode

- **General spinless fermion model on bipartite lattice with SC paring**

$$H = \sum_{ij} \left(\hat{c}_i^\dagger t_{ij} \hat{c}_j + \hat{c}_i^\dagger \Delta_{ij} \hat{c}_j^\dagger + h.c. \right)$$

Particle-Hole Symmetry:

$$\mathcal{P}\mathcal{H}\mathcal{P}^{-1} = -\mathcal{H} \text{ or } \{\mathcal{P}, \mathcal{H}\} = 0.$$

Particle-Hole Transformation:

$$\mathcal{P} = \mathcal{R}\mathcal{K}.$$

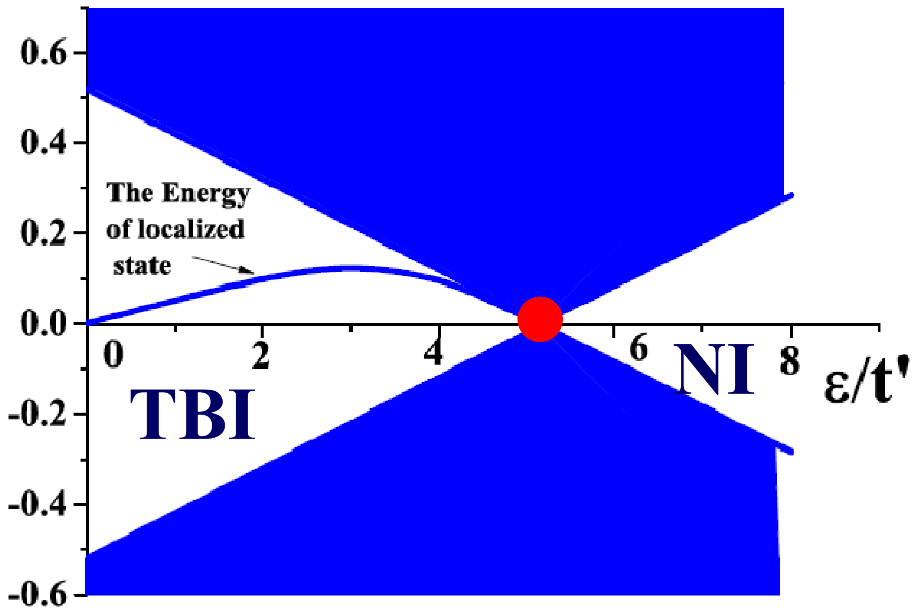
$\mathcal{R}$: $\hat{c}_{i\in A} \leftrightarrow -\hat{c}_{i\in A}$, $\hat{c}_{i\in B} \leftrightarrow \hat{c}_{i\in B}$,

$\mathcal{K}$: **Complex conjugate operator**

Energy levels are paired as: $(E_m, \ -E_m)$

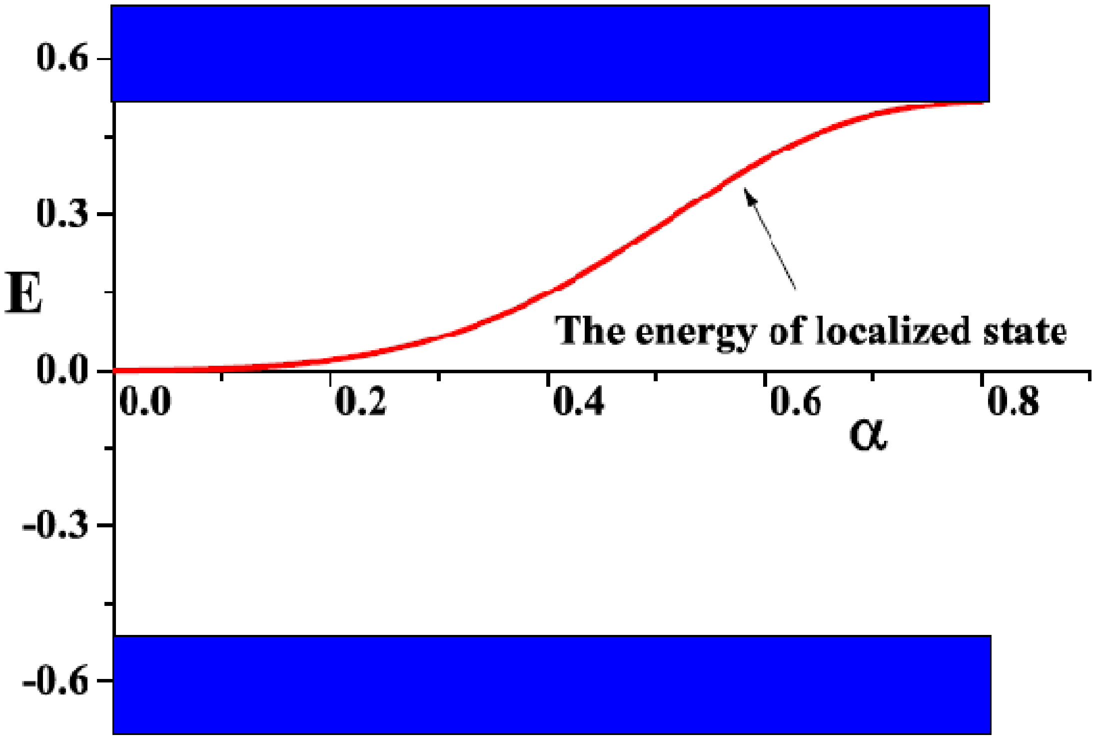
One vacancy → **One unpaired states left** → **One zero mode**

Particle-hole symmetry



$$H_{\mathrm{H}} = -t \sum_{\langle i,j \rangle} \left(\hat{c}_i^{\dagger} \hat{c}_j + h.c. \right) - t' \sum_{\langle\langle i,j \rangle\rangle} e^{i\phi_{ij}} \hat{c}_i^{\dagger} \hat{c}_j + \varepsilon \sum_{i \in A} \hat{c}_i^{\dagger} \hat{c}_i - \varepsilon \sum_{i \in B} \hat{c}_i^{\dagger} \hat{c}_i$$

$$
\begin{aligned}
H_{\mathrm{H-V}} = &-t \sum_{\langle i\neq i_0, j\neq i_0\rangle} \left(\hat{c}_i^\dagger \hat{c}_j + h.c.\right) - t' \sum_{\langle\langle i\neq i_0, j\neq i_0\rangle\rangle} e^{i\phi_{ij}} \hat{c}_i^\dagger \hat{c}_j \\
&- \alpha t \sum_{\langle i_0, j\rangle} \left(\hat{c}_i^\dagger \hat{c}_j + h.c.\right) - \alpha t' \sum_{\langle\langle i_0, j\rangle\rangle} e^{i\phi_{ij}} \hat{c}_i^\dagger \hat{c}_j \\
&+ V_0 \hat{c}_{i_0\in A}^\dagger \hat{c}_{i_0\in A}. \qquad (2)
\end{aligned}
$$



α为局域无序的强度:
当α接近于零时, 为点缺陷;
当α接近于1时, 点缺陷消失,体系恢复平移不变性.

半填满时点缺陷量子态:

$$|\uparrow_-\rangle\otimes|\downarrow_+\rangle \quad |\uparrow_+\rangle\otimes|\downarrow_-\rangle$$

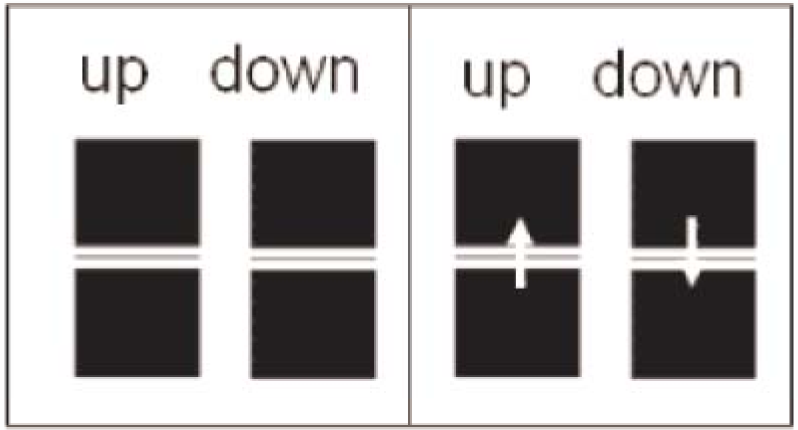
点缺陷量子态的磁矩:

$$\hat{S}^z\,|\uparrow_-\rangle\otimes|\downarrow_+\rangle = \frac{1}{2}\,|\uparrow_-\rangle\otimes|\downarrow_+\rangle,$$

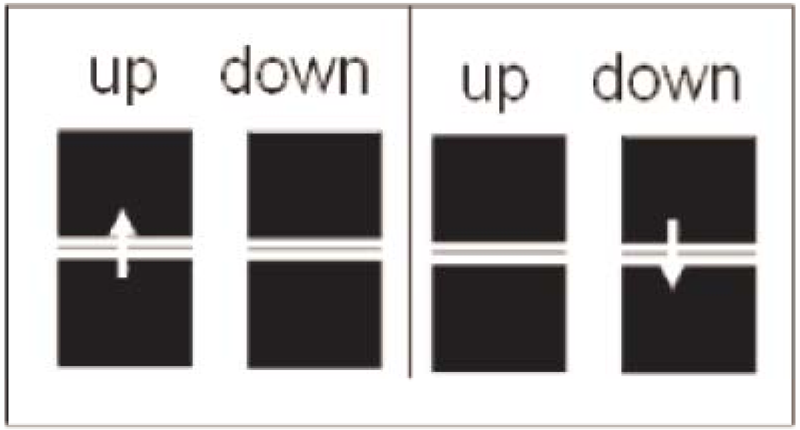
$$\hat{S}^z\,|\uparrow_+\rangle\otimes|\downarrow_-\rangle = -\frac{1}{2}\,|\uparrow_+\rangle\otimes|\downarrow_-\rangle.$$

点缺陷的磁矩磁化率:

$$\chi_s = \beta\left[\langle\hat{M}_z^2\rangle - \langle\hat{M}_z\rangle^2\right] \sim N_{\rm ls}(k_BT)^{-1}$$



$$|\uparrow_+\rangle\otimes|\downarrow_+\rangle, \quad |\uparrow_-\rangle\otimes|\downarrow_-\rangle,$$



$$|\uparrow_-\rangle\otimes|\downarrow_+\rangle \quad |\uparrow_+\rangle\otimes|\downarrow_-\rangle.$$

拓扑超导体中的晶格缺陷

Tow Non-topological Majorana modes around a vacancy in the TSC with particle-hole symmetry on honeycomb lattice

$$H_{\mathrm{TSC}} = H_{\mathrm{H}} + \Delta_{\mathrm{induce}} \sum_{\langle ij \rangle} \hat{c}_i \hat{c}_j + \mathrm{H.c.}$$

	Haldane model	Kane-Mele model	TSC on honeycomb lattice
π-flux	Abelian anyon with statistical angle $\pi/4$	spin moment	Non-Abelian anyon
vacancy	$e/2$ charge	spin moment	Majorana fermion mode

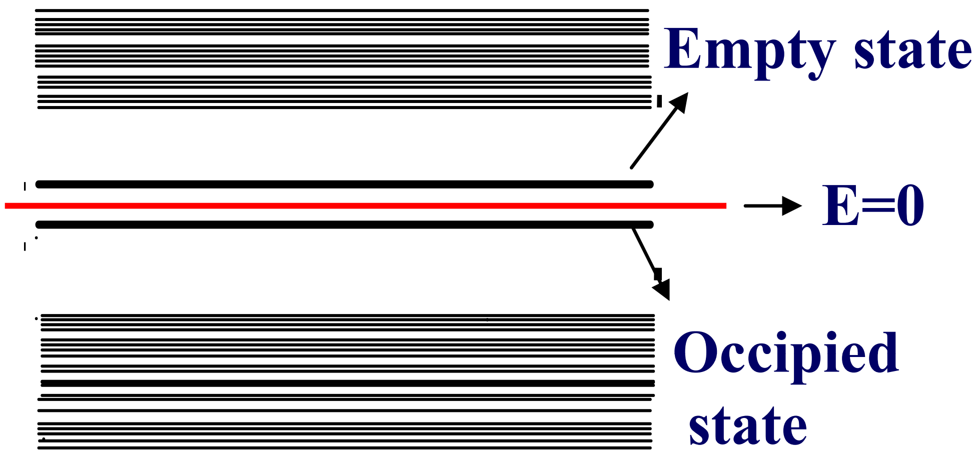
Two Majorana modes of a vacancy

A vacancy is a two-level system from two Majorana modes γ_1 and γ_2: fermion occupied state and fermion empty state

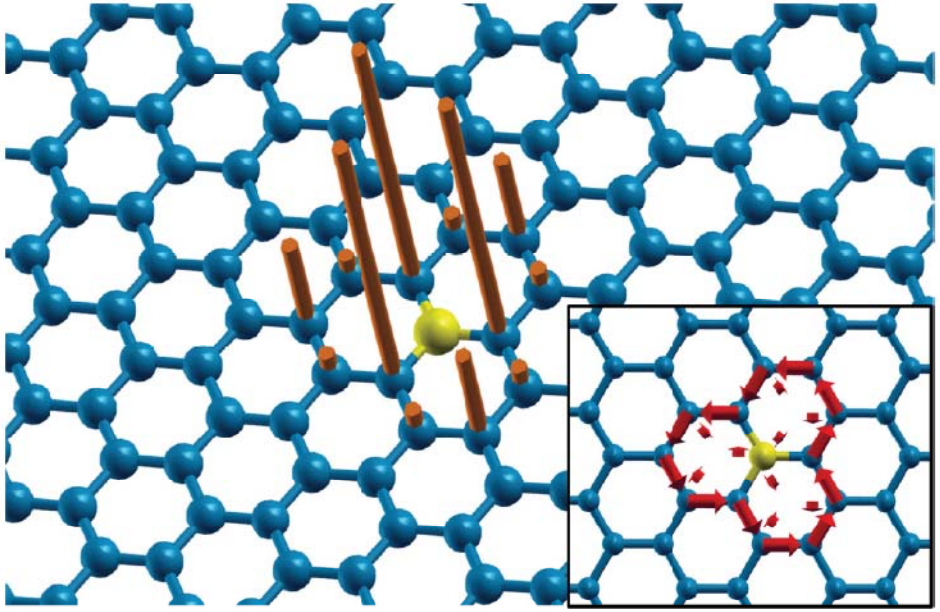
$$H = \Delta E \psi^{+} \psi = i \frac{\Delta E}{2} \gamma_1 \gamma_2$$

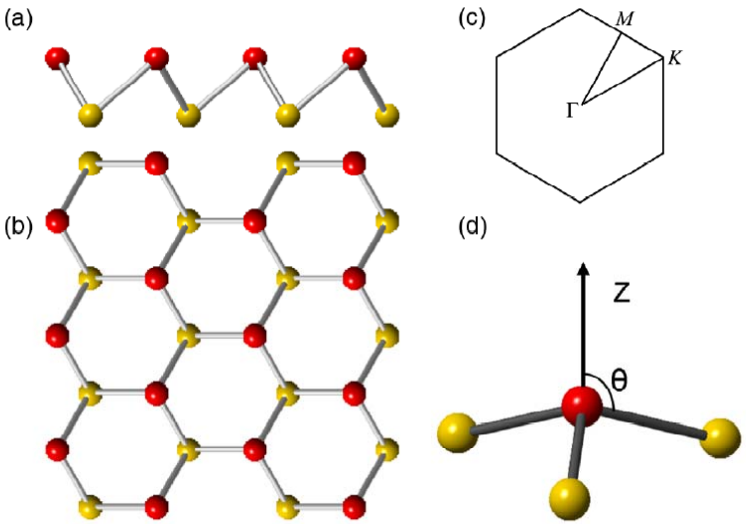
$$\gamma_1 = c + c^\dagger, \; \gamma_2 = (c - c^\dagger)/i$$

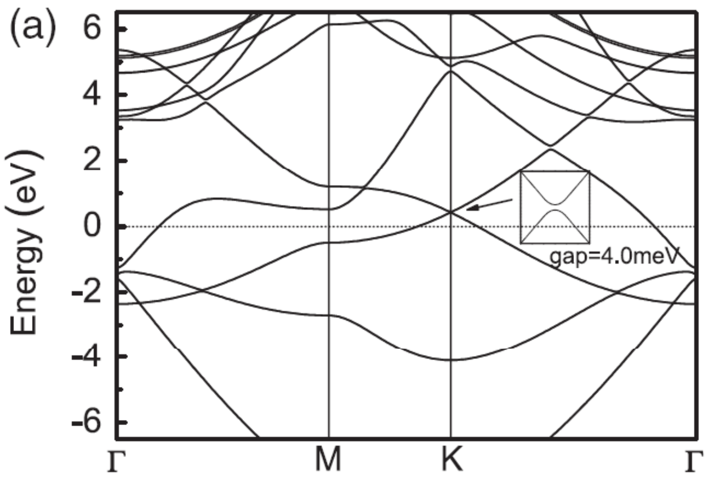
$$\gamma_1 + \gamma_2 = I + \psi$$



Possible observation on Silicene







C. C. Liu, W. Feng, and Y. Yao, PRL 107, 076802 (2011).

Conclusion: symmetry ⇒ zero modes

For topological band insulators and topological superconductors on honeycomb lattice with particle-hole symmetry, each lattice vacancy has one zero mode for the Haldane model and two zero modes for the Kane-Mele model.

In TSCs on honeycomb lattice with particle-hole symmetry, we found the existence of the non-topological Majorana zero modes around the vacancies.

These zero energy modes are protected by particle-hole symmetry of these topological sates.

Jing He, Ying-Xue Zhu, Ya-Jie Wu, Lan-Feng Liu, Ying Liang, and **Kou SP**, PHYS. REV. **B 87**, 075126 (2013).

V. Conclusion

Lattice defects always have trivial quantum properties in solid state physics. While in topological states, the lattice defects may have nontrivial quantum effects.

By manipulating these quantum defect-states, we found new ways towards fault-torrent quantum computation:

We used the degenerate ground states of *Z*2 topological order on a plane with holes (the planar codes) to do universal topological quantum computation.


谢谢！