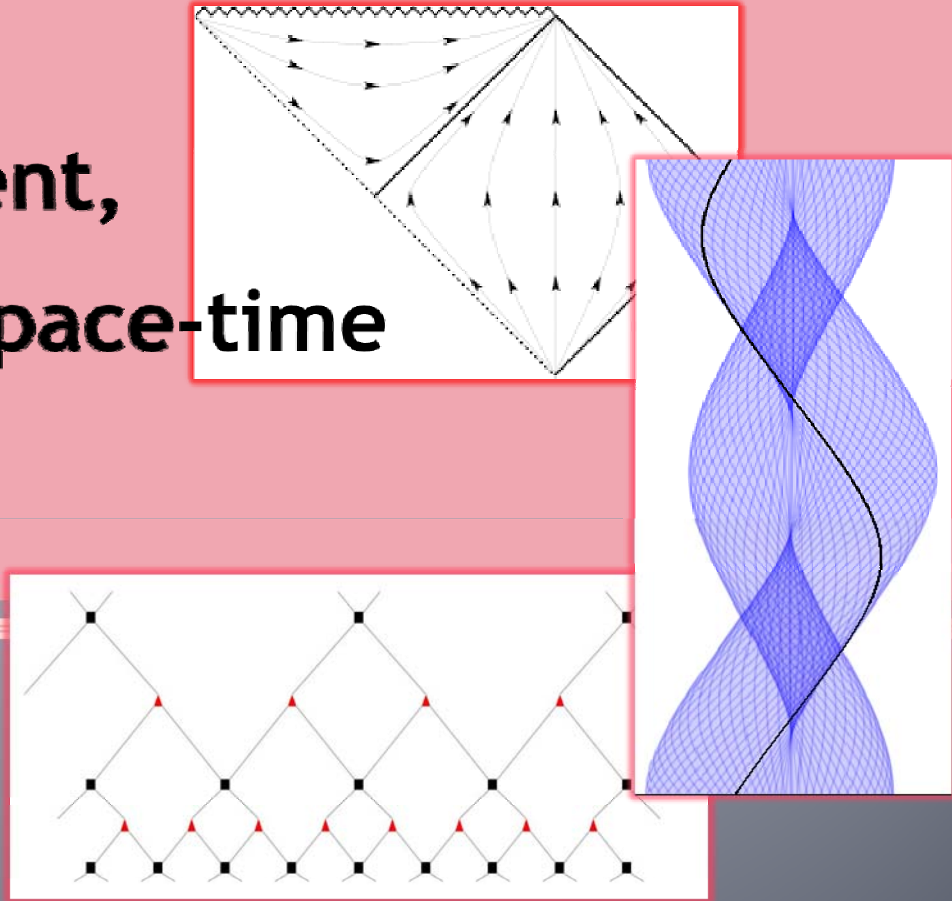
Quantum Entanglement, the Architecture of Space-time and Tensor Networks

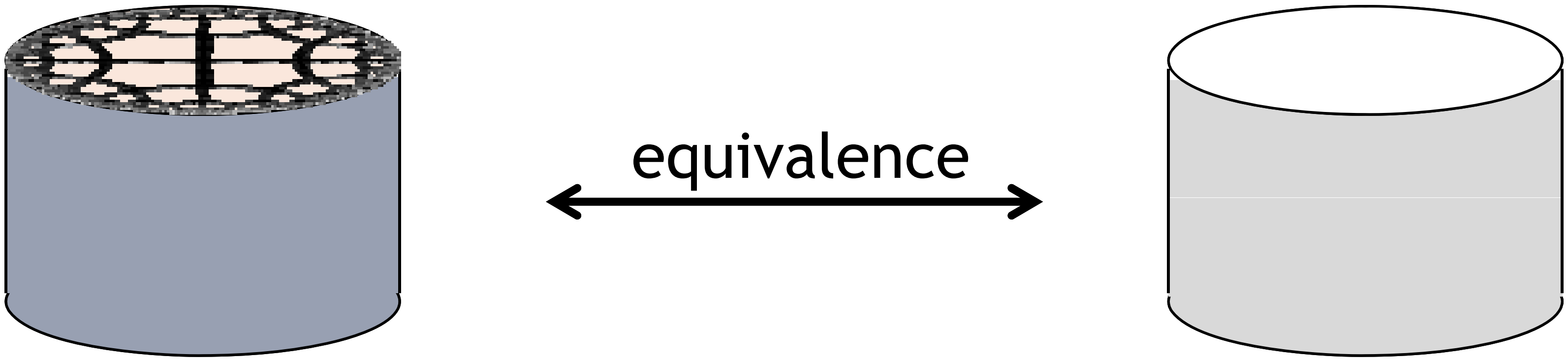


Bartłomiej Czech
Stanford University

Tsinghua University, 9 December 2015

Quantitative framework: AdS/CFT correspondence

Maldacena, 1997



Gravity in
anti-de Sitter space
(solid cylinder)

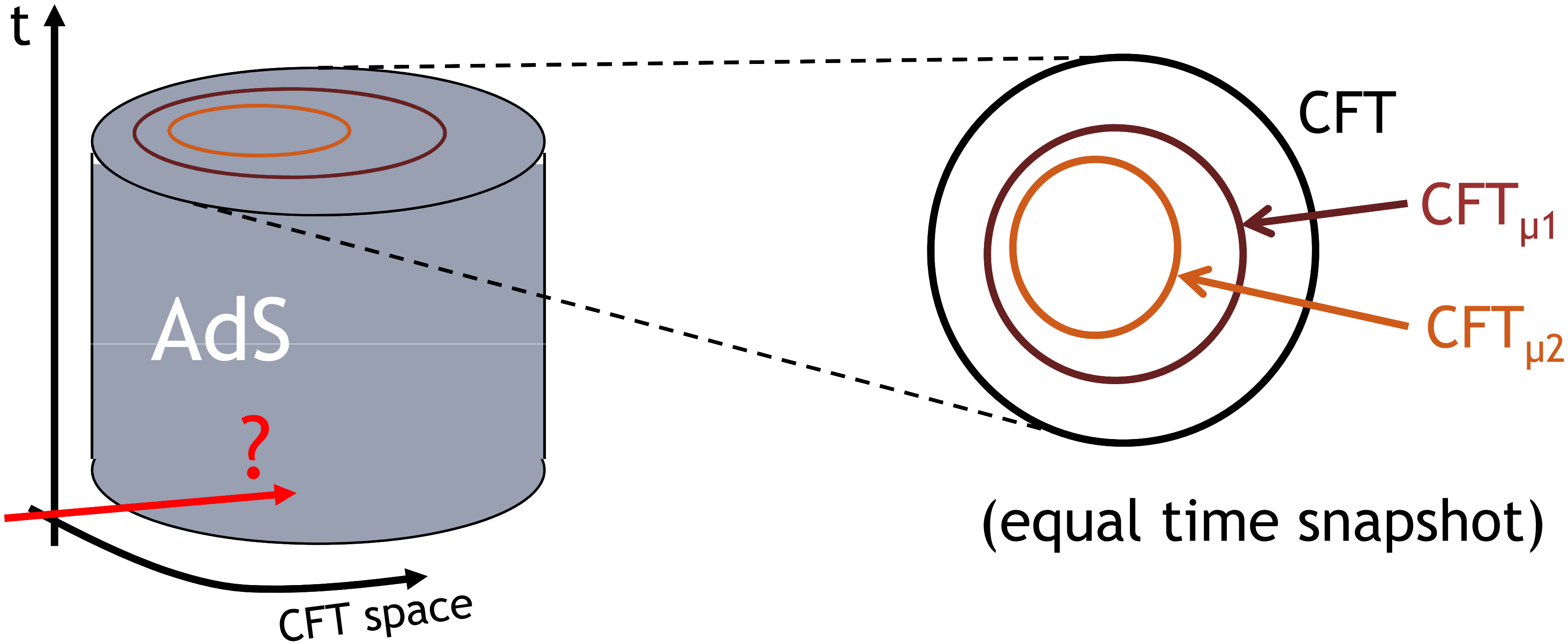
Conformal Field Theory (CFT)
on δAdS (hollow cylinder)

- States are asymptotically AdS geometries
- Homogeneous space-time with negative curvature

- Degrees of freedom organized into N x N matrices

L_{AdS} (curvature radius) ~ $N^{\#}$ (matrix size)

Extra dimension in AdS is RG scale in CFT



- radial slices - define CFTs at different cutoffs
- asymptotic boundary – CFT without a cutoff

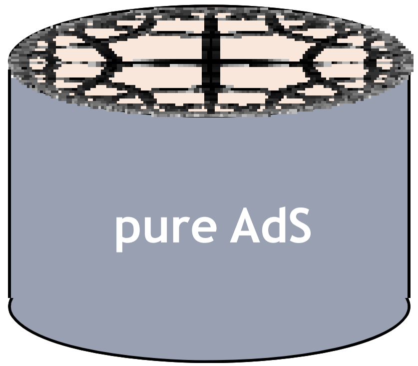
"CFT lives on the asymptotic boundary of AdS"

Let us see examples…

CFT states are AdS geometries

- CFT vacuum: |0>

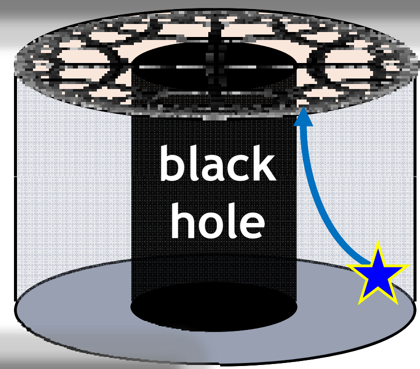
nothing to break
the symmetry



- CFT thermal state
  $\mathcal{Z}^{-1} e^{-\beta H}$

place an object
at thermal scale

BH is also characterized
by the Hawking temperature
of black hole radiation



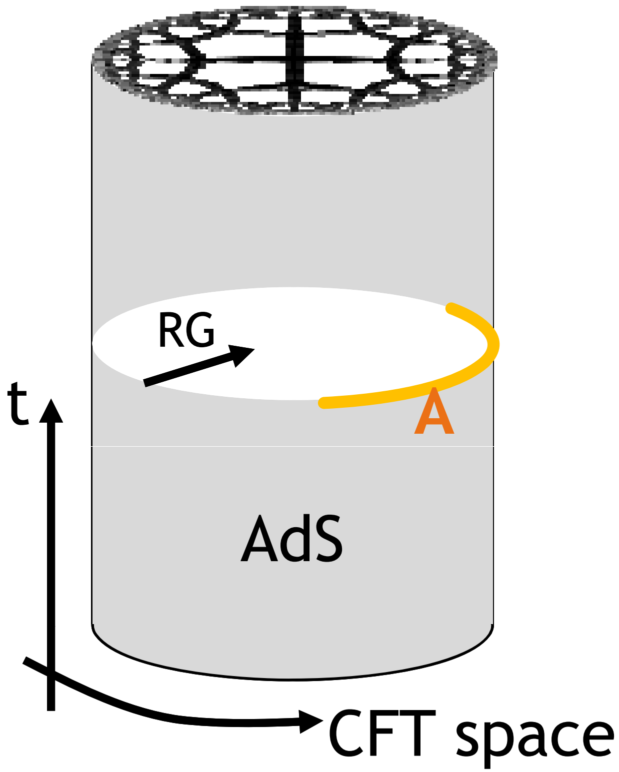
- CFT state
  from a given
  canonical ensemble

eigenstate
thermalization
hypothesis

things fall
into a black hole

Entanglement entropy in AdS/CFT

$S_{\rm ent}(A) =$



- Suppose $\mathcal{H}_{CFT} = \mathcal{H}_A \times \mathcal{H}_{env}$
- Given $|\Psi\rangle$ in $\mathcal{H}_{CFT}$, form $\rho_A = \mathrm{Tr}_{env} |\Psi\rangle\langle\Psi|$
- For every observable $O_A \times 1_{env}$ localized in A:

  $\langle\Psi|O_A|\Psi\rangle = \mathrm{tr}\, O_A\, \rho_A$
- This is a mixed state on A, which mimics all the properties of $|\Psi\rangle$ as far as A-observables are concerned.
- If we do not look at the environment, the pure state $|\Psi\rangle$ appears mixed.
- **Entanglement entropy** quantifies this:

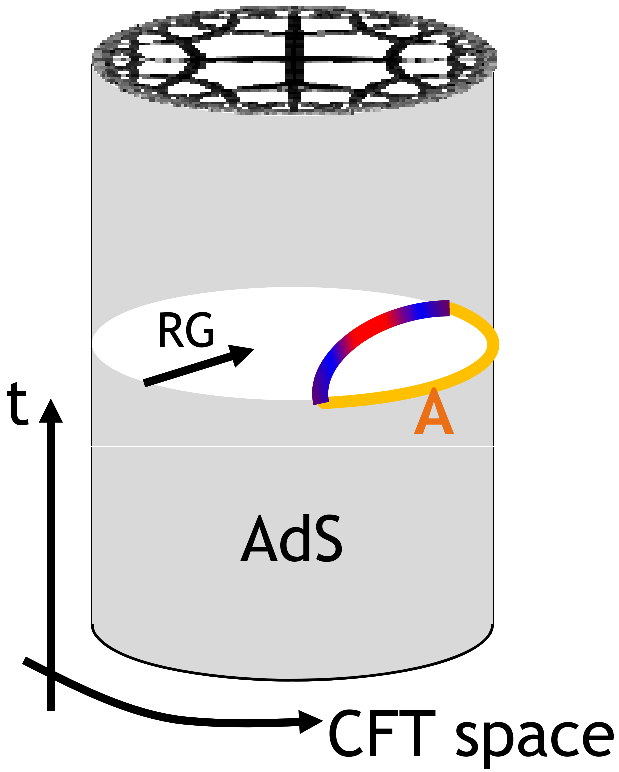
  $S_{ent}(A) = -\mathrm{Tr}\, \rho_A \log \rho_A$
- Entanglement entropy measures how much effect the environment has on A.

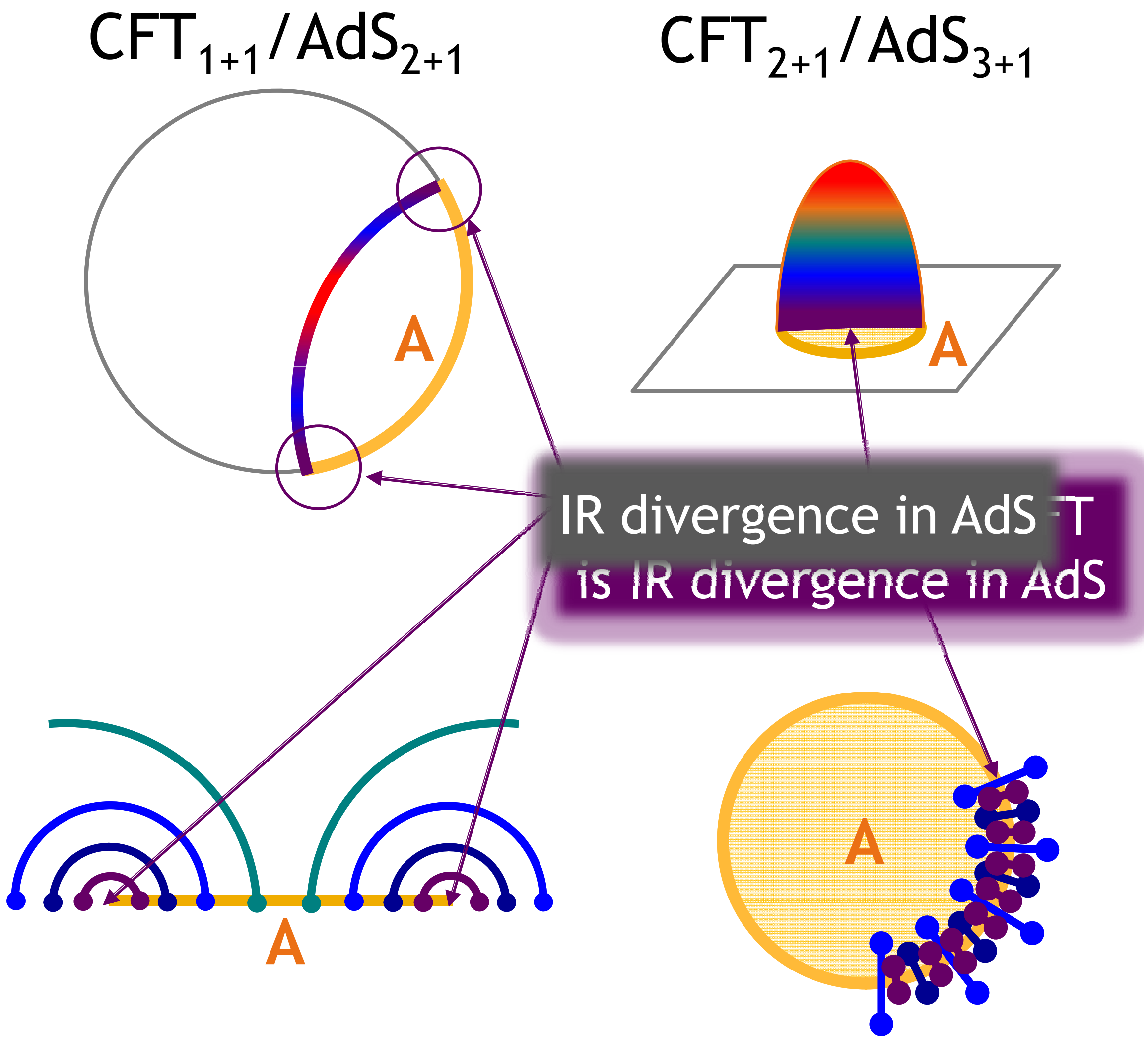
Entanglement entropy quantifies correlations

Ryu-Takayanagi proposal

Ryu-Takayanagi, 2006

$S_{\mathrm{ent}}(A) =$

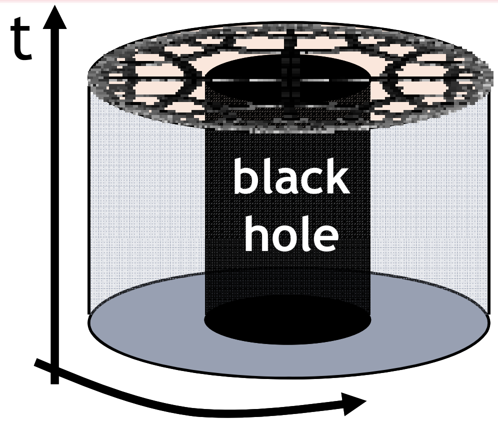




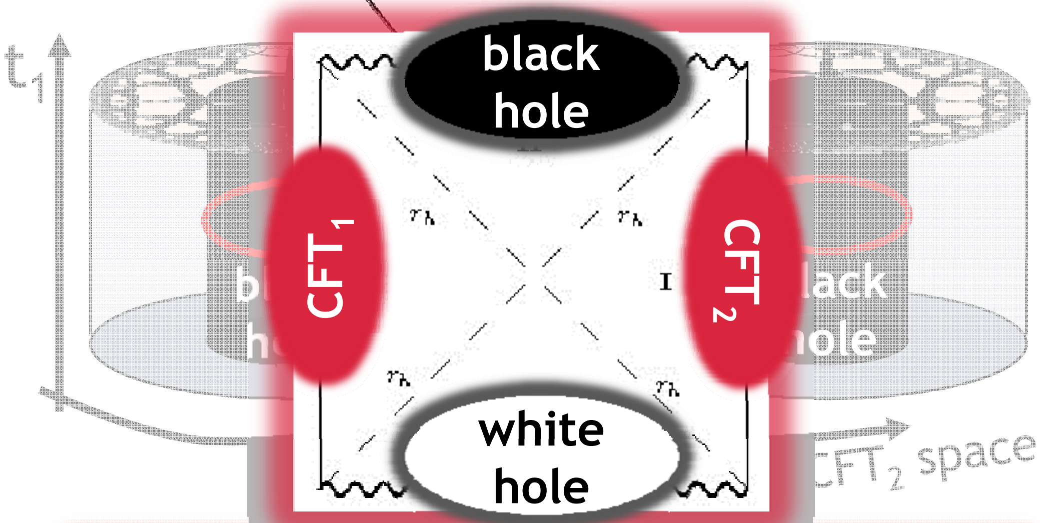
Application: two-sided black hole

Maldacena, 2001

- CFT thermal state
  $\mathcal{Z}^{-1} e^{-\beta H}$
- canonically purify thermal state:
  $\mathcal{Z}^{-1/2} \sum_i e^{-\beta E_i/2} |i\rangle_1 \otimes |i\rangle_2$
- *"THERMOFIELD DOUBLE STATE"* is a pure state in $\mathrm{CFT}_1 \otimes \mathrm{CFT}_2$ with identical one-sided properties as the thermal state



identifies black hole entropy (area) with entanglement entropy



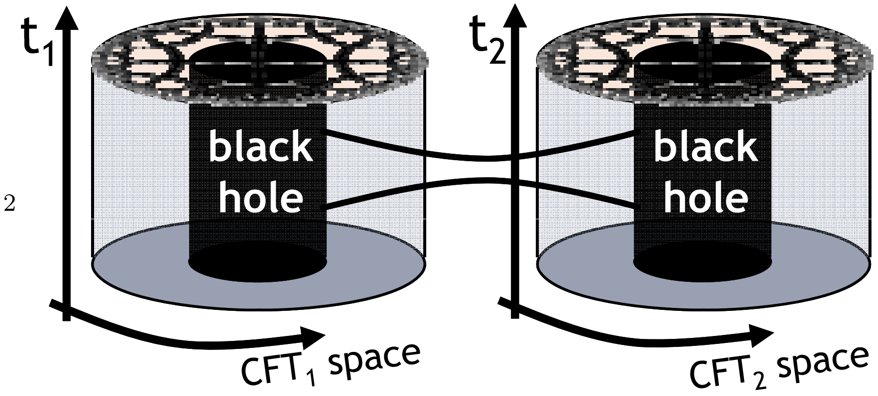
two-sided black hole in Kruskal coordinates

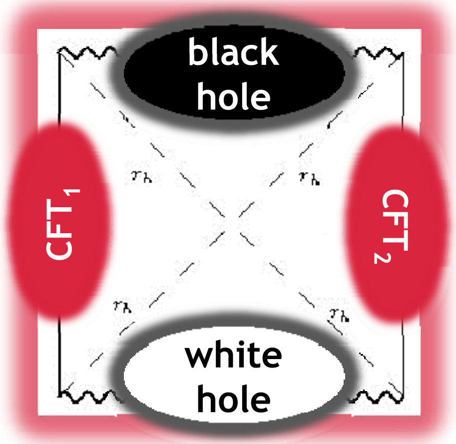
Connectedness is entanglement

van Raamsdonk, 2009; Czech et al., 2012

- thermofield double state:

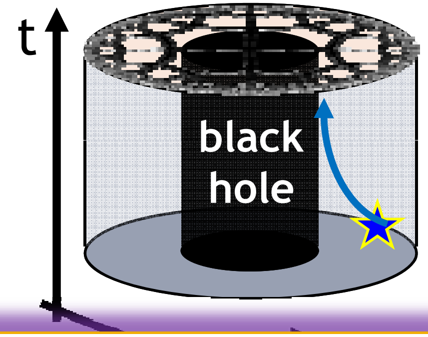
$$\mathcal{Z}^{-1/2}\sum_i e^{-\beta E_i/2}\,|i\rangle_1 \otimes |i\rangle_2$$

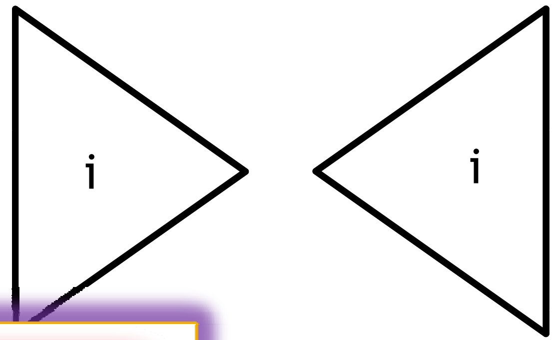




- CFT energy eigenstates:

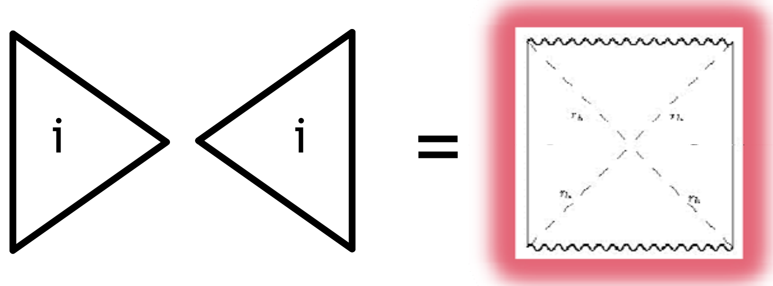
β $|i\rangle$





- substitute:

$\Sigma_i(\ldots)$



Entangling disjoint space-times connects them!

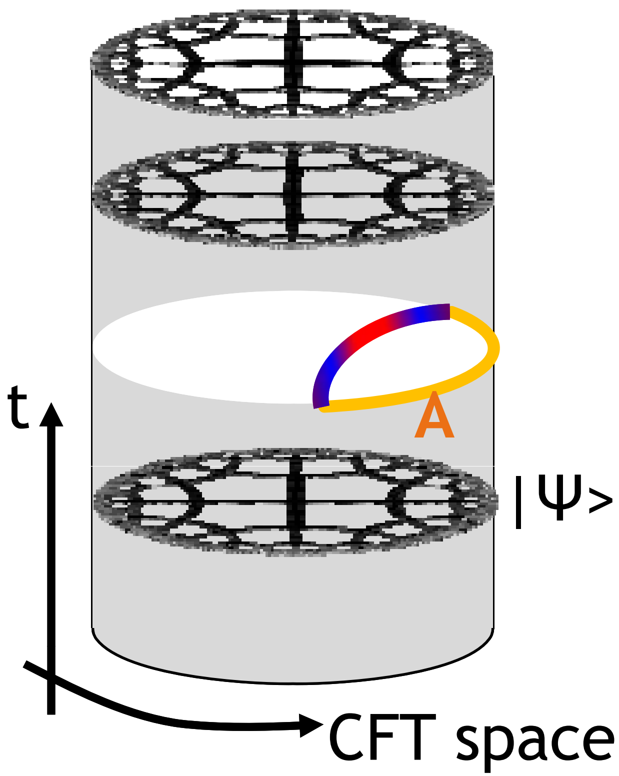
If entanglement is connectedness, then...

- we are learning about
  the **architecture of space** and maybe **space-time**

- space is a **map of the entanglement**
  in the quantum state living
  at asymptotic boundary

- what are **maps of entanglement**
  and how to use them?

Tensor Networks



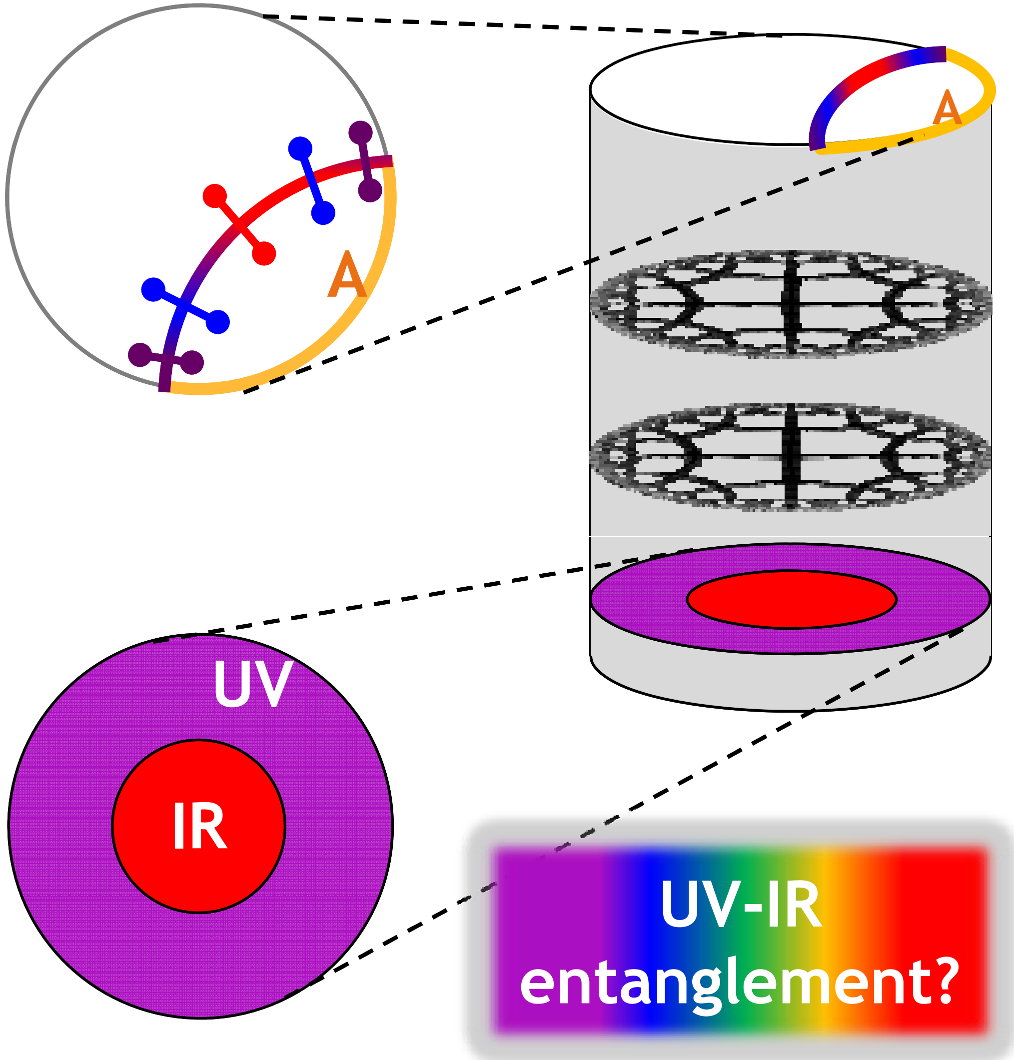
How to read the AdS map of entanglement?

What we already know:

- Minimal surfaces are entanglement entropies
- Connectedness across a minimal surface comes from the entanglement between A and complement

Next, we want to know:

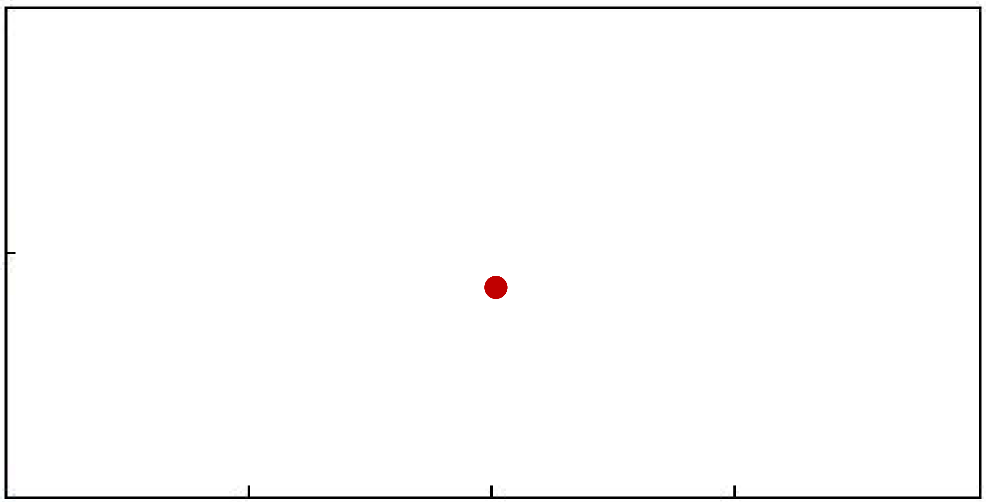
- What is responsible for connectedness between center and periphery?

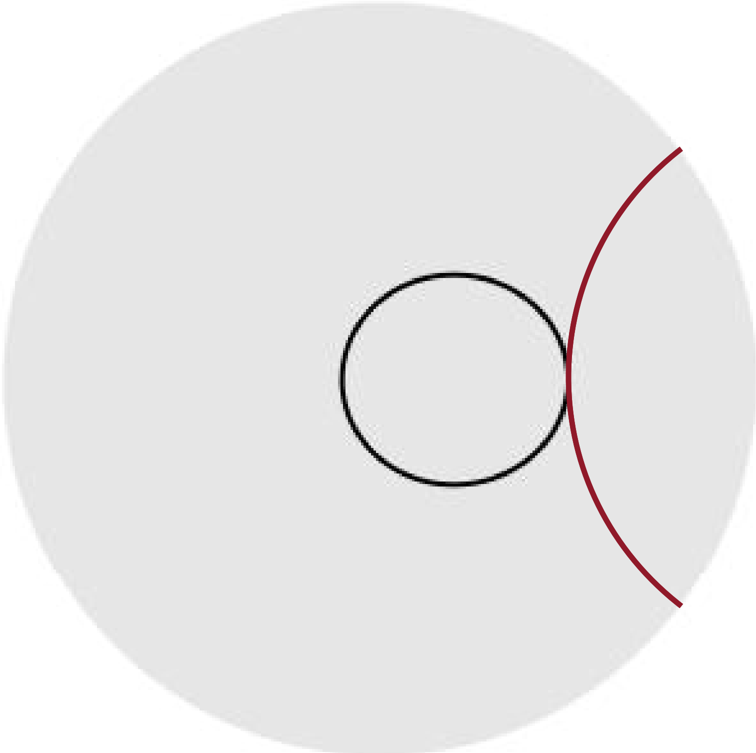


How to describe a center?

Czech et al., 2013-5

SPACE of MINIMAL SURFACES

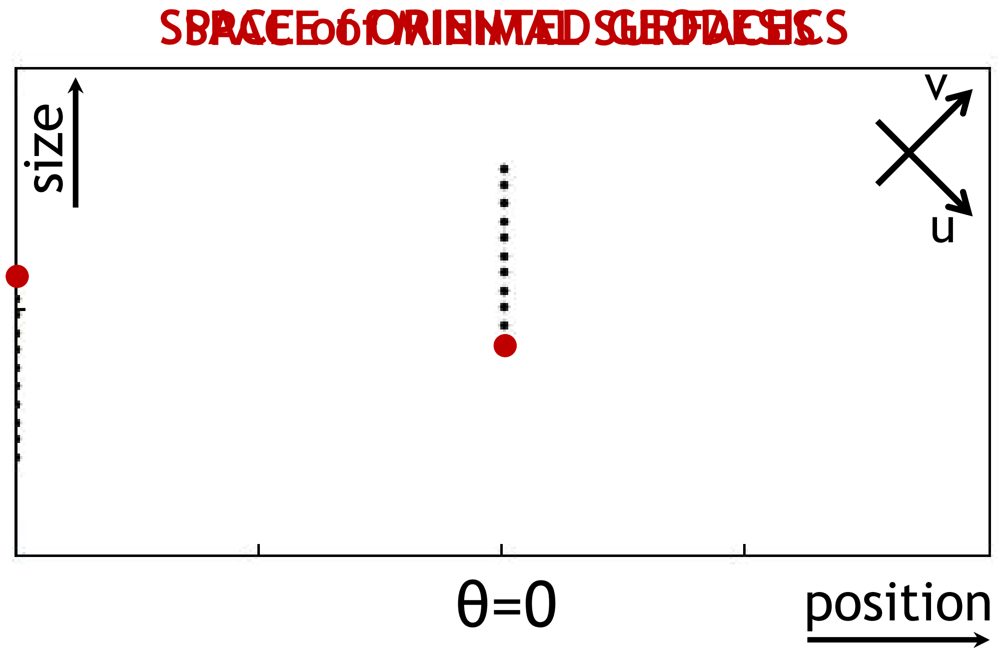


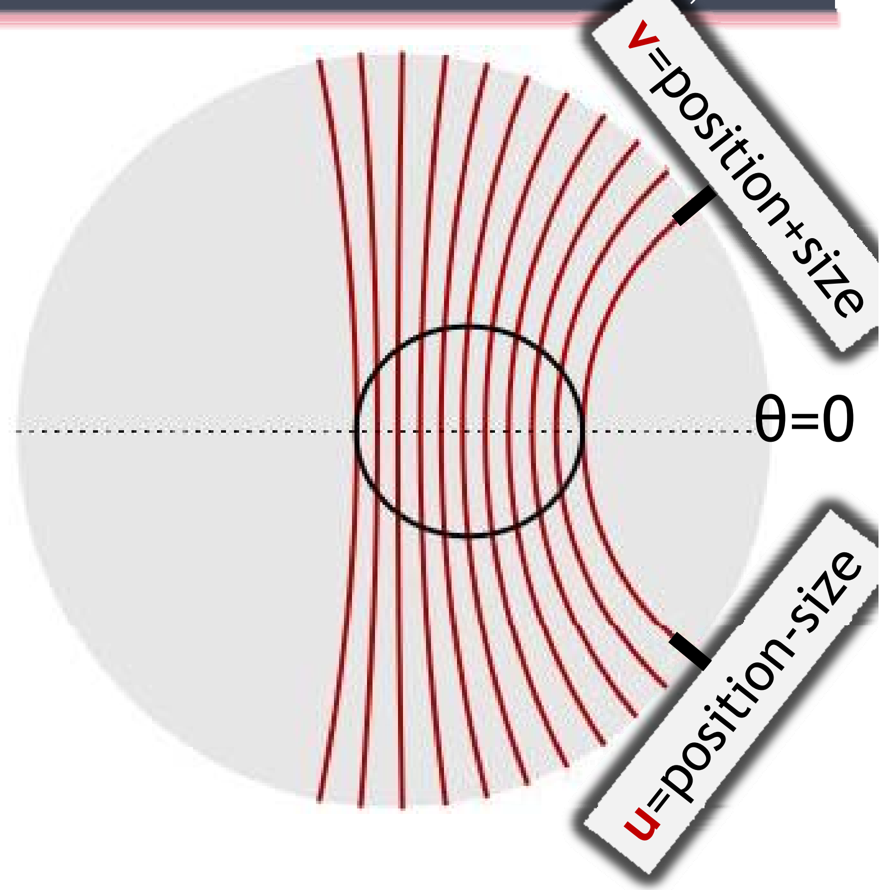


- So far, we only know minimal surfaces
- Let us use them!

How to describe a center of AdS_3?

Czech et al., 2013-5





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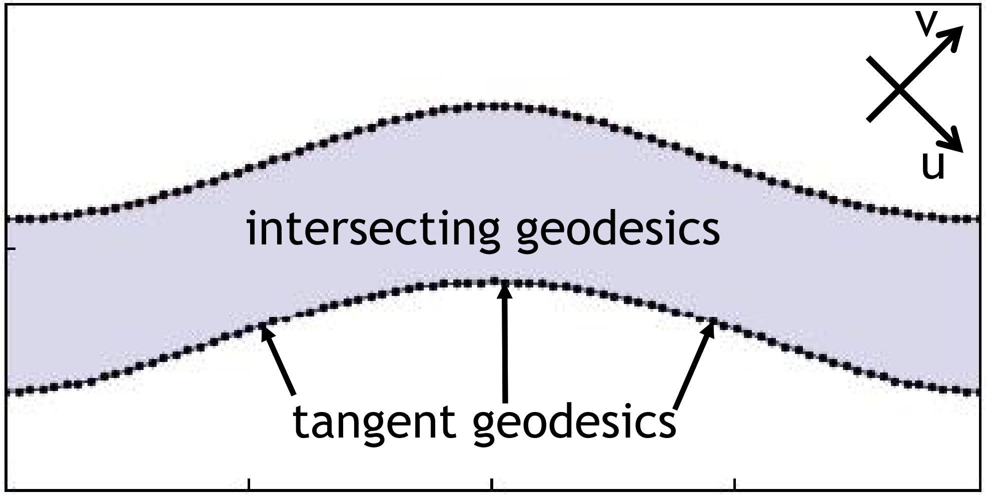
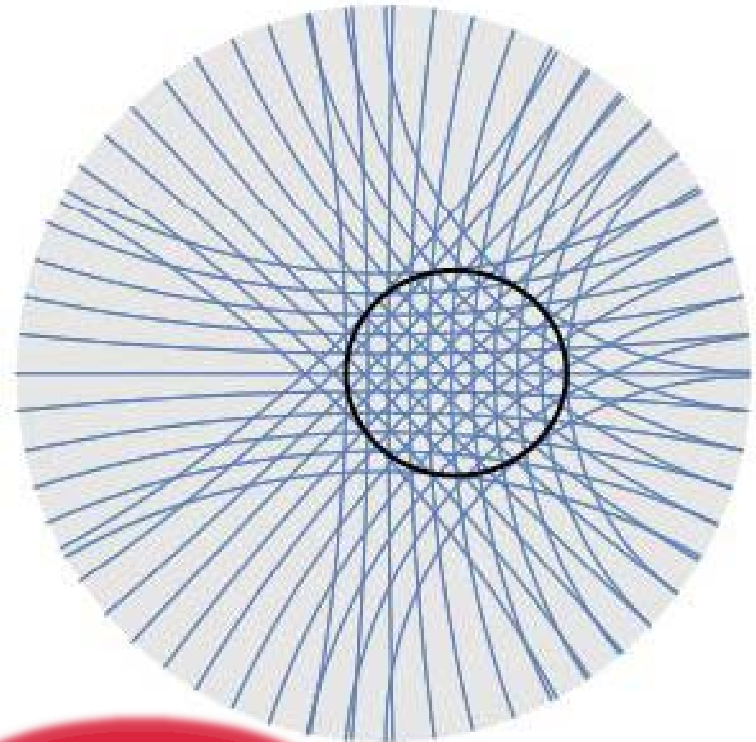
SPACE of ORIENTED GEODESICS





$$\frac{\text{circumference}}{4G} = \int_{\text{intersect}} \frac{\partial^2 S(u,v)}{\partial u\, \partial v}\, du\, dv$$

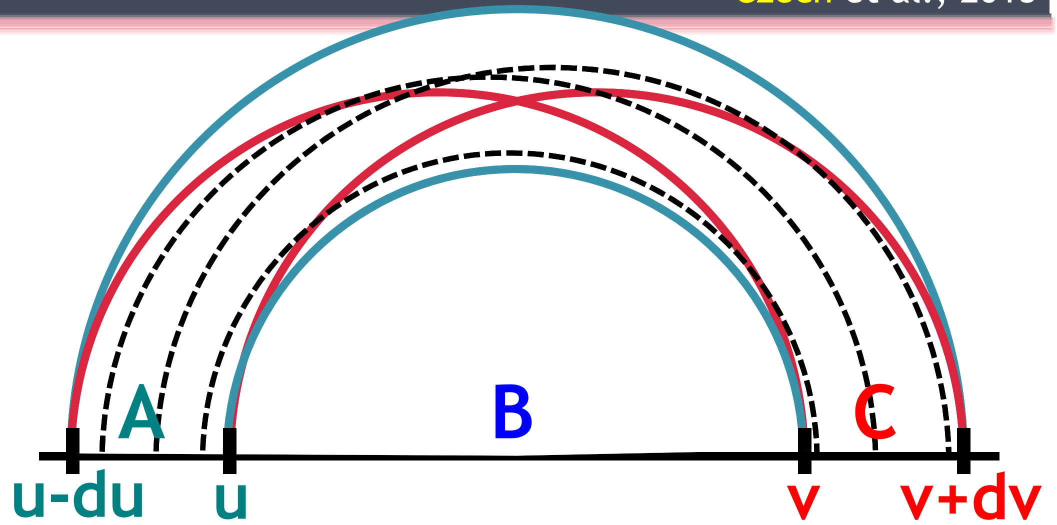
density of geodesics

- The **density of geodesics** only depends on entanglement entropy.
- I (re-)discovered, then generalized this formula.
- It was known in special cases (flat space – Crofton, 1869).

What is the density of geodesics?

Czech et al., 2015

- "How many" geodesics have endpoints in A = (u-du, u) and C = (v, v+dv)?



$$\frac{\partial^2 S(u,v)}{\partial u\, \partial v}\, du\, dv$$

= S(u-du, u) + S(u,v+dv) - S(u,v) - S(u-du, d+dv)

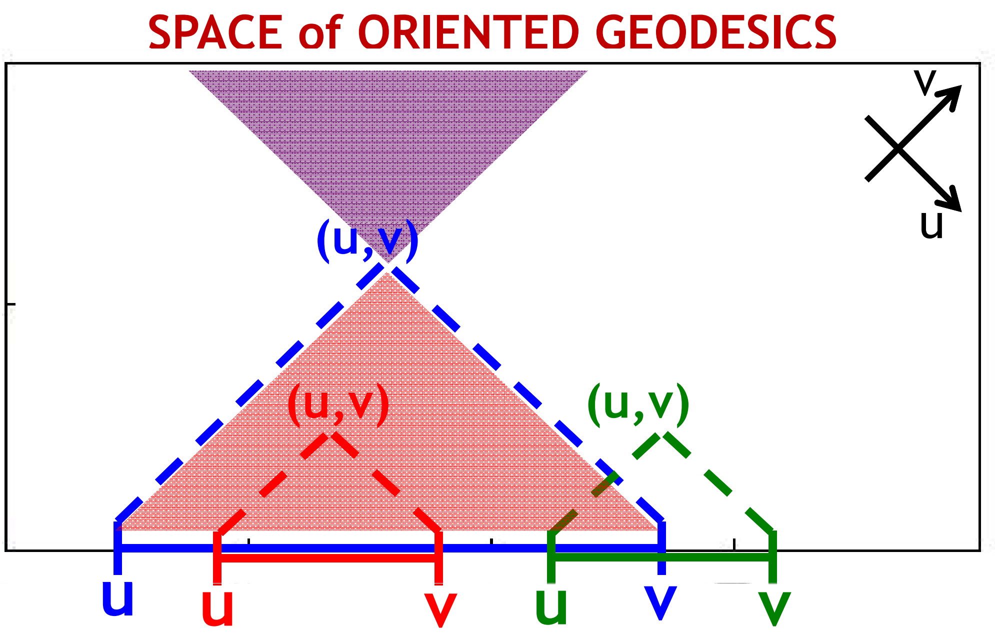
= S(AB) + S(BC) - S(B) - S(ABC)

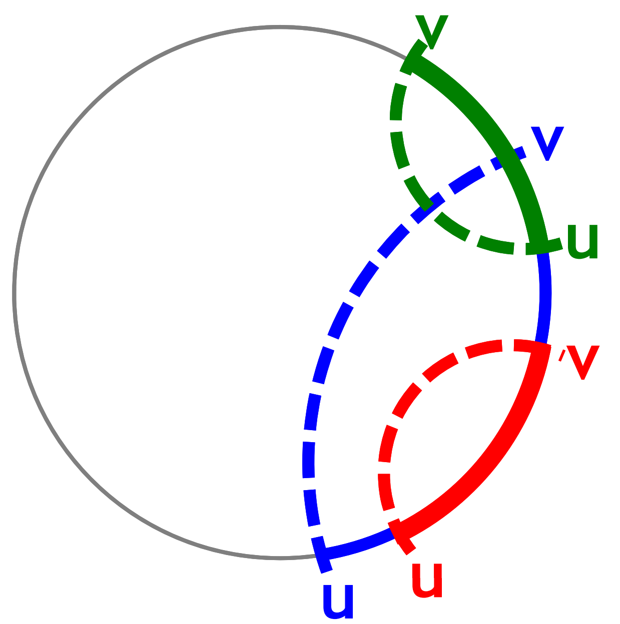
- this is **non-negative** by the **strong subadditivity of entanglement entropy**
- it is called the **conditional mutual information** I(A,C|B)
- quantifies the correlations between A and C not mediated by B

density of geodesics = density of correlations

Space of Geodesics has a "causal structure"

Czech et al., 2015





- **Timelike** separated (u,v): interval (u,v) contains (u,v)
- **Spacelike** separated (u,v): neither interval contains the other
- **Lightlike** separated: common endpoint left (**u** = u) or right (**v** = v)

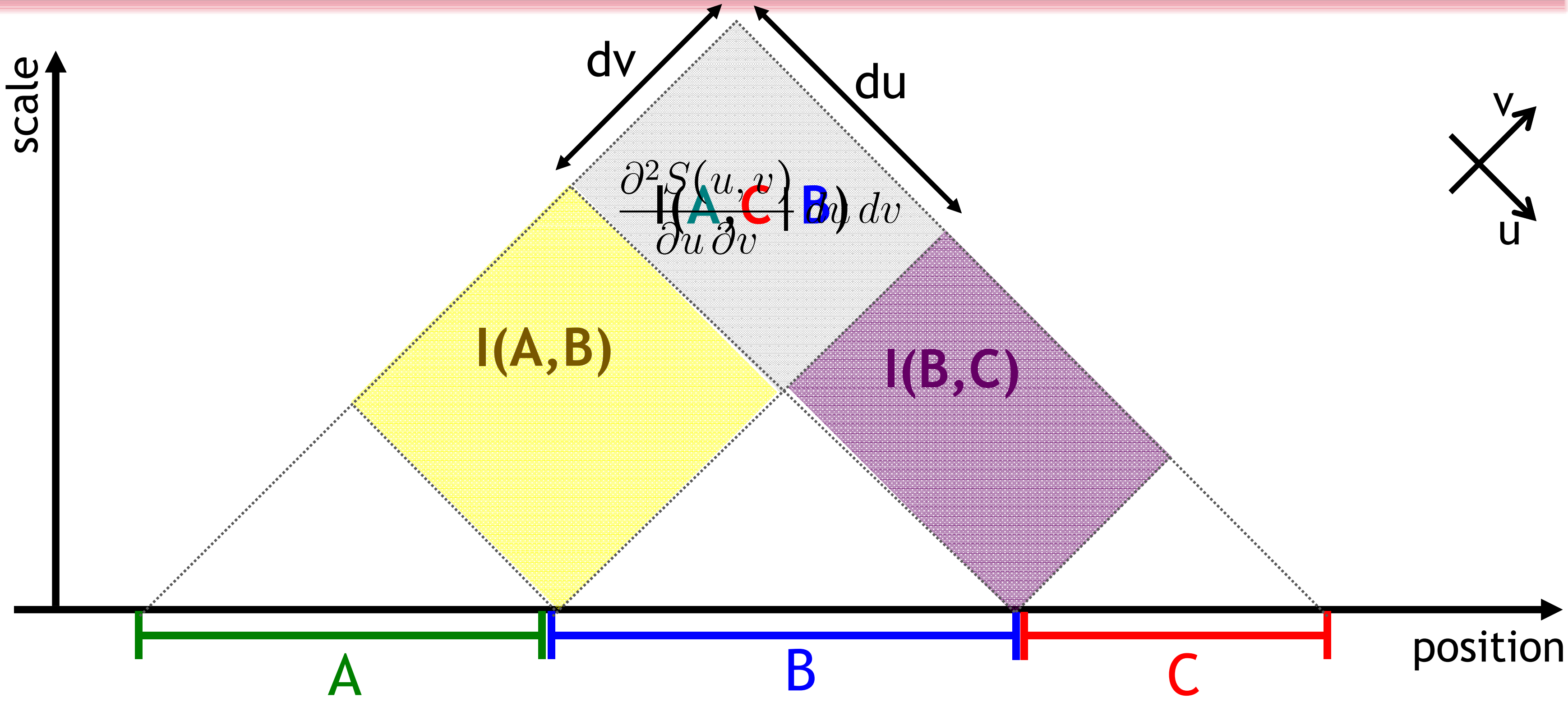
- **Past:** all intervals contained in (u,v)
- **Future:** all intervals containing (u,v)

Space of Geodesics is also the Space of Intervals

Endpoint coordinates u,v are lightlike

Structure of Kinematic Space

Czech et al., 2015

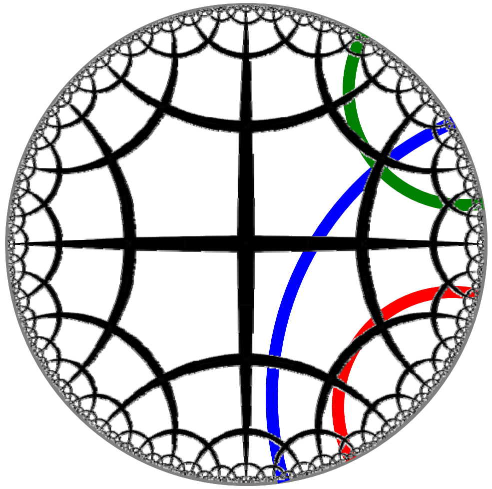


- Volumes of causal diamonds are conditional mutual informations
- Diamonds that extend all the way to the bottom are mutual informations

These volumes are “bouquets” of geodesics!

Summary so far

- Space is a fabric woven from geodesics.
- Geodesics are carriers of correlations.
- Density of geodesics
  = density of correlations
  = conditional mutual information **I(A,C|B)**
- Geodesics have a causal structure.
- All this is captured by
  the **Kinematic Space**.

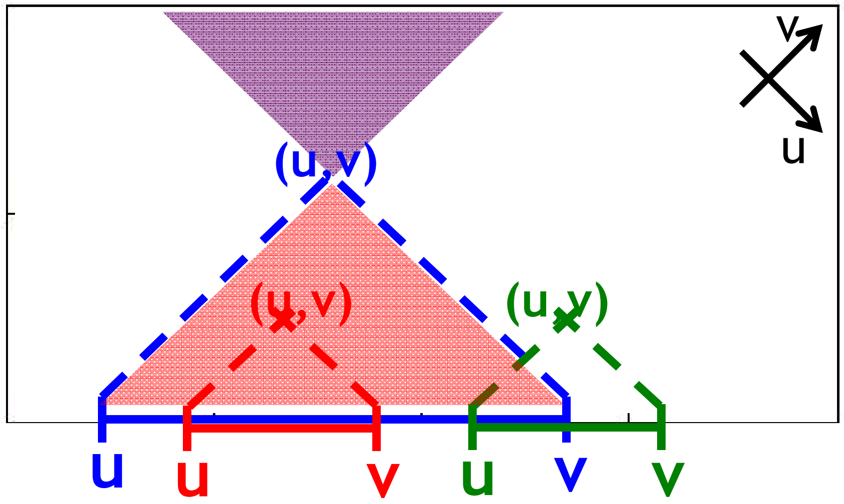


QUESTION:

- Have we seen a structure like this before?

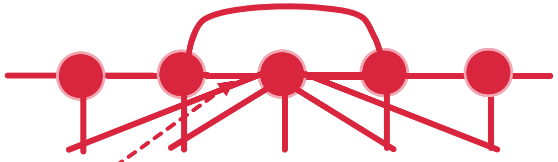
Tensor Networks

SPACE of ORIENTED GEODESICS / INTERVALS



What are Tensor Networks?

- A tool in condensed matter theory
- useful for efficiently representing many-body wavefunctions:



$$|\Psi\rangle = \sum_{i_1 i_2 \ldots i_N} \Psi_{i_1 i_2 \ldots i_N} |i_1 i_2 \ldots i_N\rangle$$

a wavefunction

a tensor

a vector

in some q-dimensional vector space

O(N) parameters

q - "bond dimension"

$O(\#^N)$ parameters

- This class of states does not cover the whole Hilbert space

efficient representation

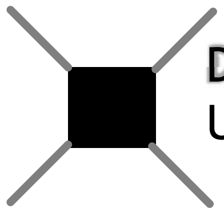
- The art is to define a class of tensor network states with desired physical properties
- For understanding the holographic architecture of AdS_3, use Multi-scale Entanglement Renormalization Ansatz: (Vidal, 2005)

MERA

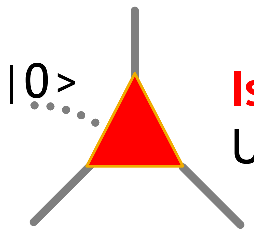
What is MERA?

Vidal, 2005

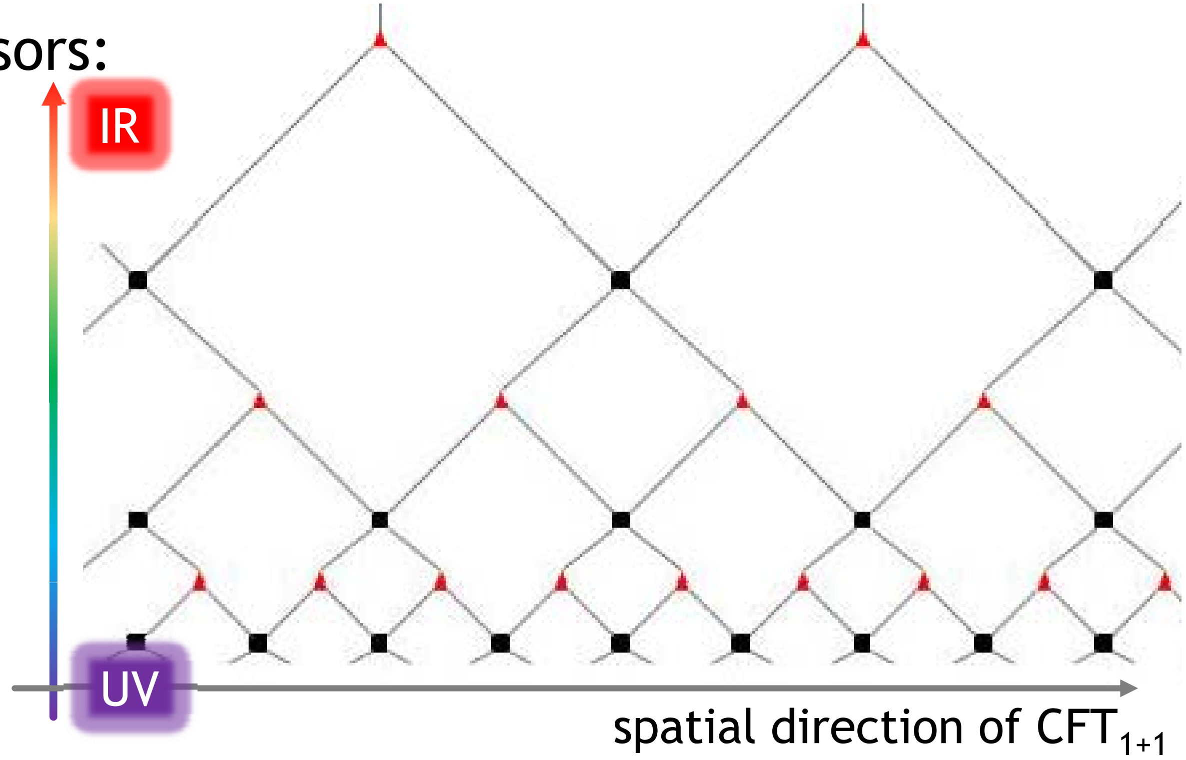
- Two types of unitary tensors:



Disentanglers remove UV entanglement



Isometries set aside UV degrees of freedom



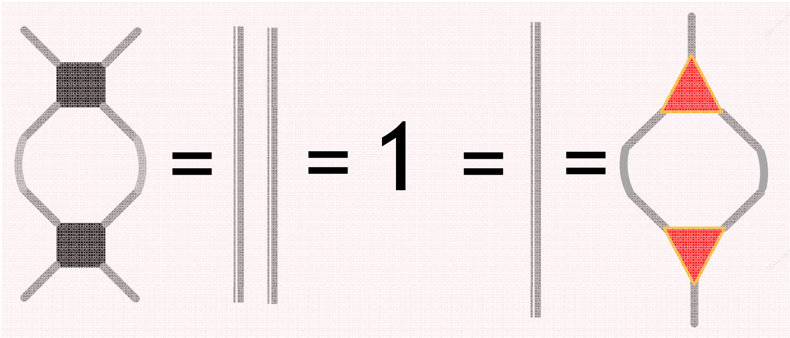
- Implements real space coarse-graining (renormalization group)
- A successful variational ansatz for finding ground states of 1+1-dimensional critical systems (e.g. Ising model)

a working model of CFT_{1+1}

Causal structure and locality in MERA

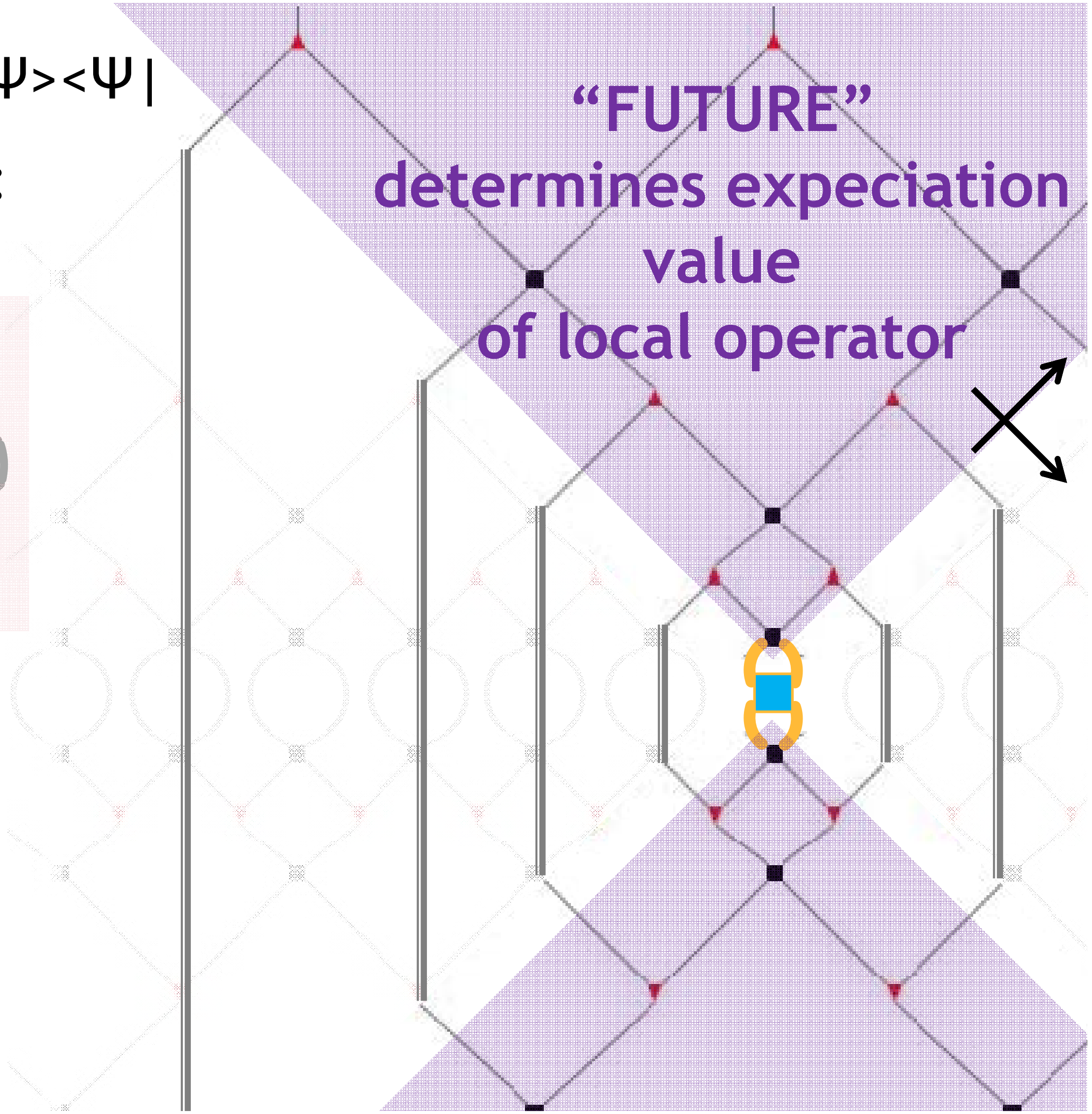
Vidal, 2005

- Compute <Ψ|O|Ψ> = Tr O|Ψ><Ψ|
- Unitarity of tensors implies:



Causal Structure

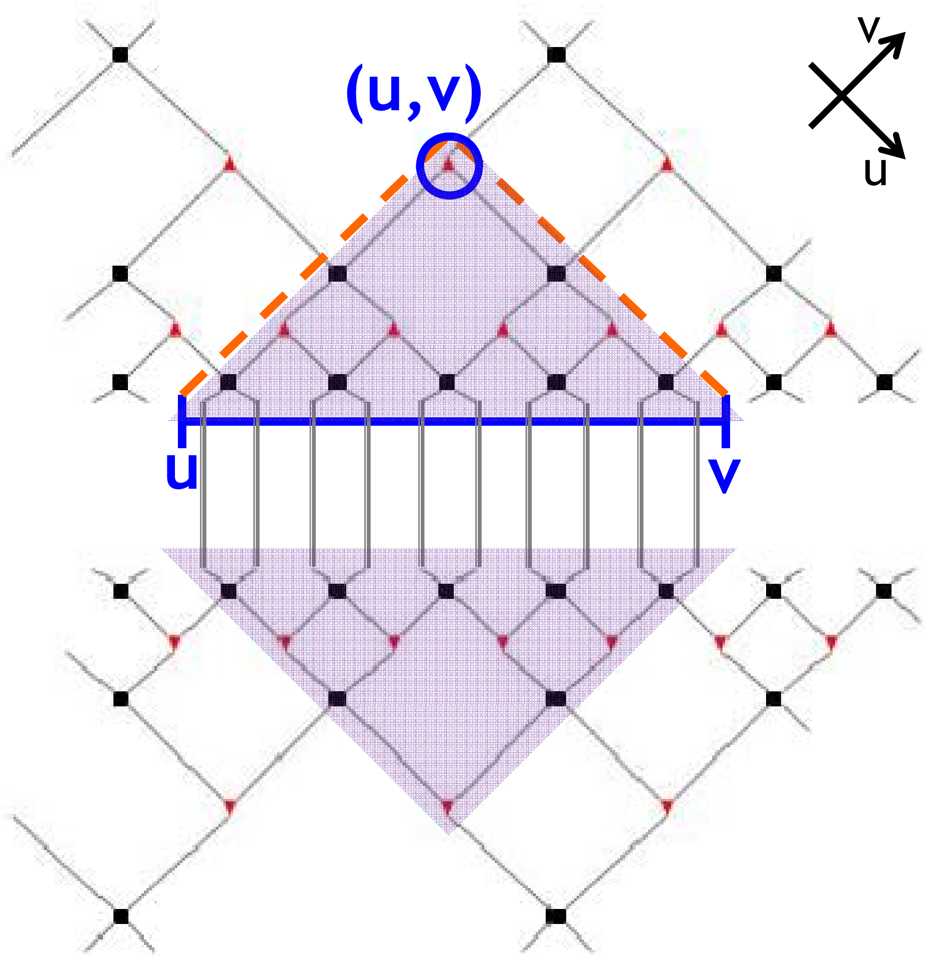
(in auxiliary time ↔ scale)



Null coordinates in MERA

Czech et al., 2015

- tensor at **(u,v)** is the last one which cancels out when interval **(u,v)** is traced out
- each field theory interval uniquely identifies a tensor
- the relation between the two is via causal cuts



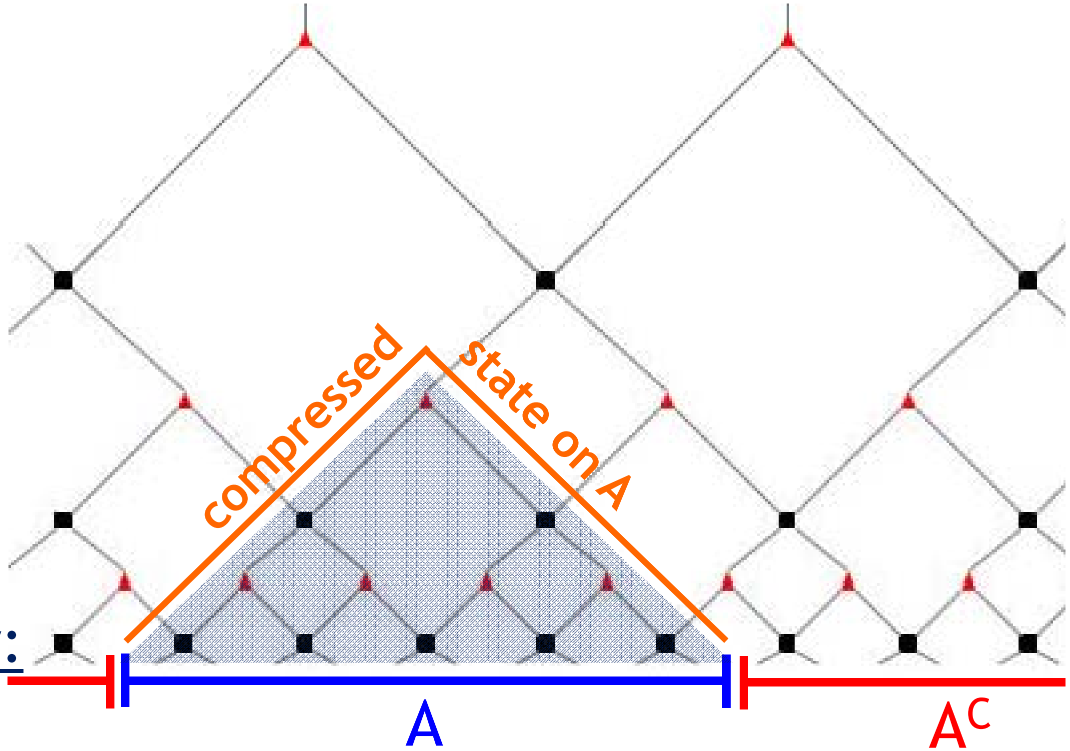
Causal cuts and entanglement entropy

Vidal, 2005

- The causal structure determines which tensors affect which expectation values
- The state on top of a **causal cut** is a compressed state on **A**
- This gives an upper bound for the entanglement entropy: S(A) ≤ #(**cuts**)
- It turns out that:

  S(A) ~ #(**cuts**)

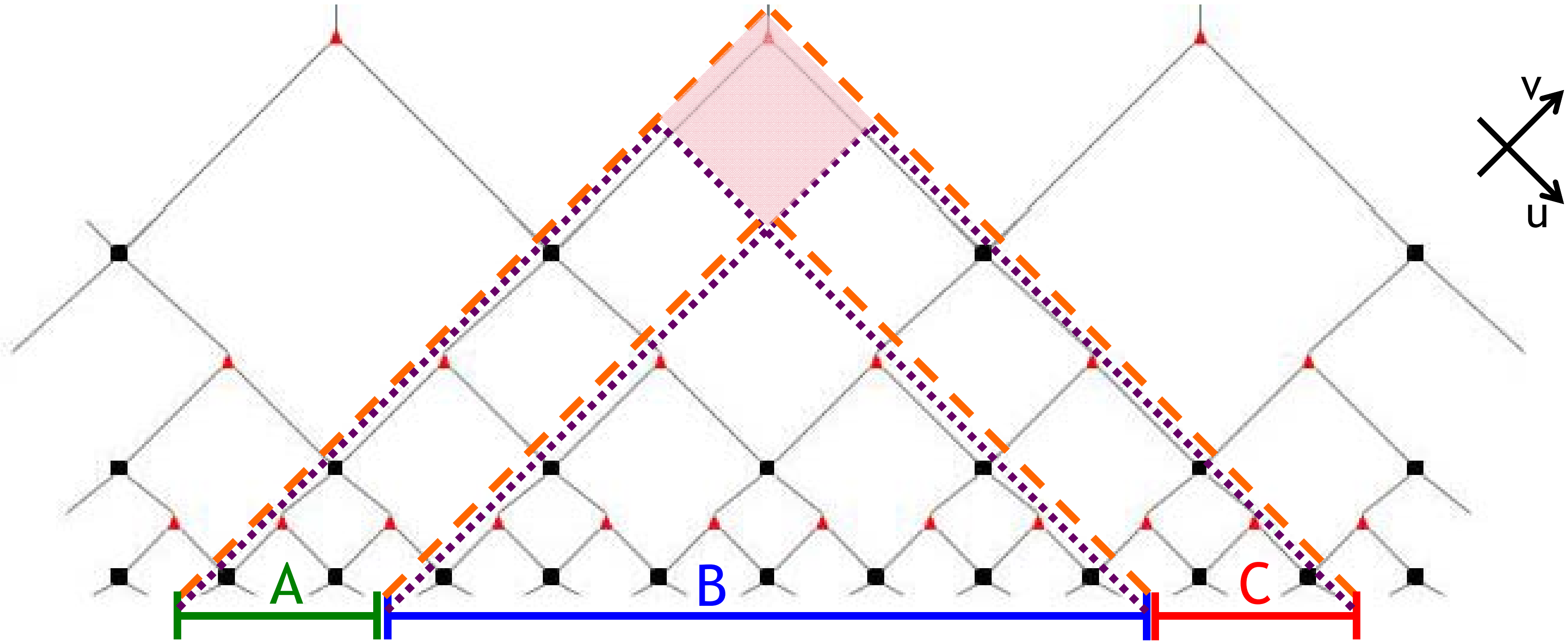
- This reproduces S(A) ~ log|A|



- these tensors do not affect expectation values of operators acting on A^C
- they form an isometry that acts within $\mathcal{H}_A$

Conditional mutual information in MERA

Czech et al., 2015

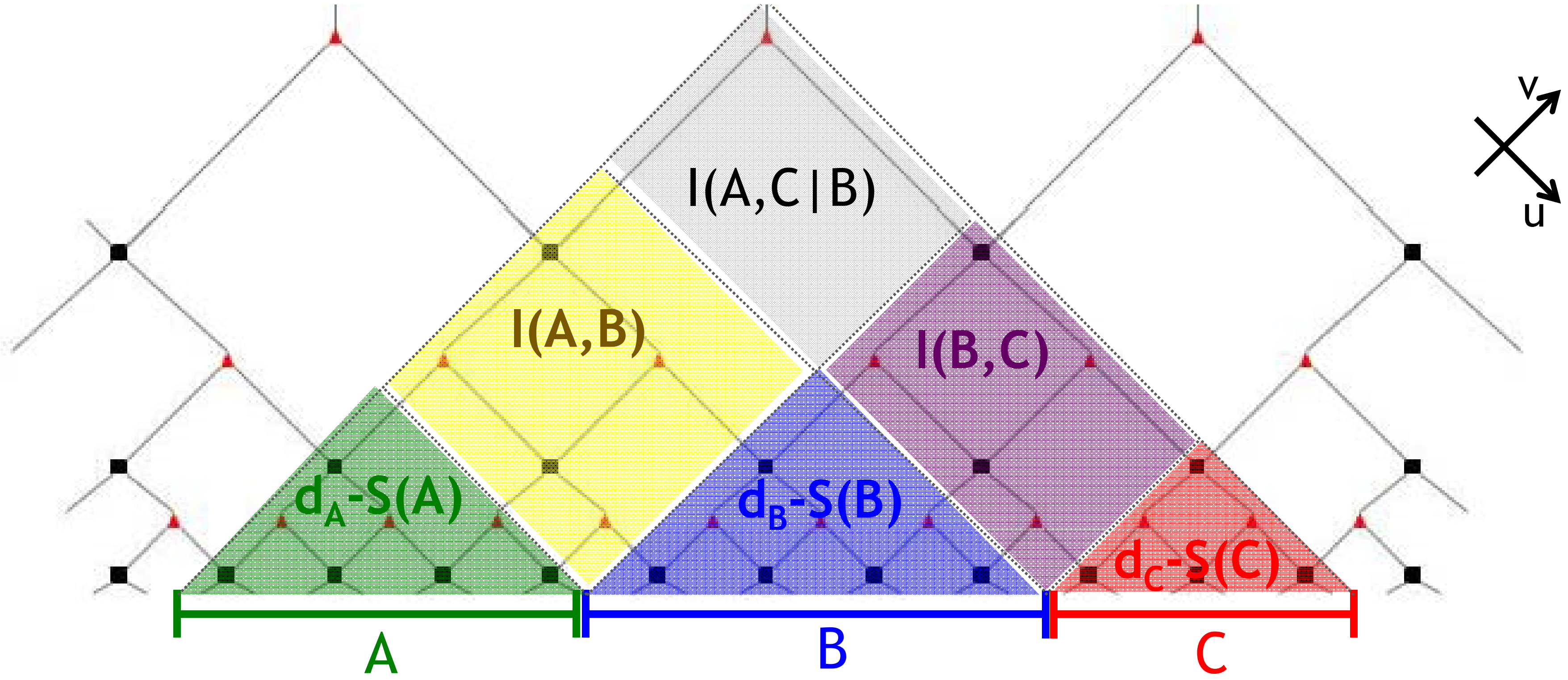


$$I(A,C|B) = S(AB) + S(BC) - S(ABC) - S(B)$$

- strong subadditivity of entropy: $I(A,C|B) \geq 0$ → $\#(\Delta) \geq 0$
- because of cancellations, this quantity localizes in the network
- it counts the number of isometries in a causal diamond

Structure of MERA

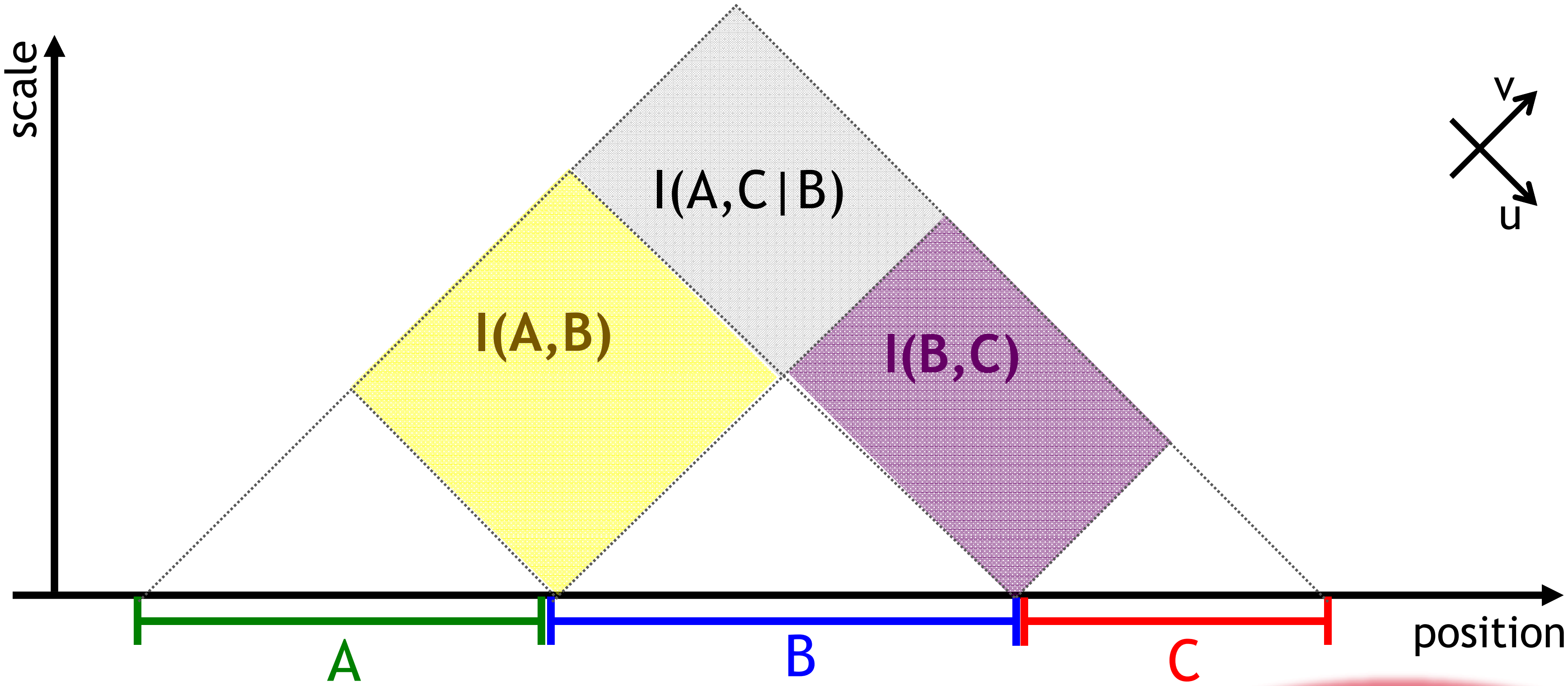
Czech et al., 2015



- Causal diamonds are conditional mutual informations
- Diamonds that extend all the way to the bottom are mutual informations
- Past causal diamonds of kinematic points characterize the isometric embedding of a compressed state in the Hilbert space

Structure of Kinematic Space

Czech et al., 2015



Kinematic Space and MERA share:

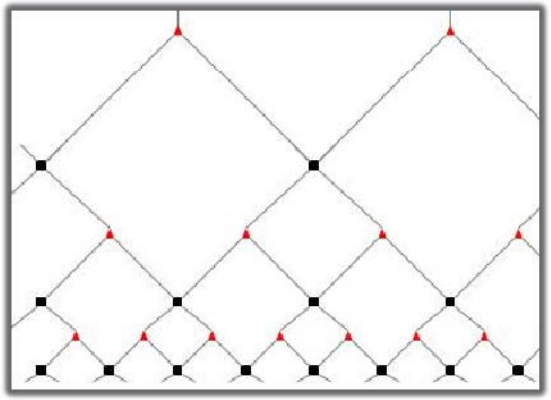
- the same **causal structure**
- the same **localization** of conditional mutual information

kinematic volume counts isometries in MERA

MERA is a discretization of Kinematic Space!

Application to many-body physics

- If MERA ≈ Kinematic Space then…
- Facts about Kinematic Space must carry over to MERA
- We will use one such fact to learn two new things about MERA in CFT_{1+1}:



CFT ground state

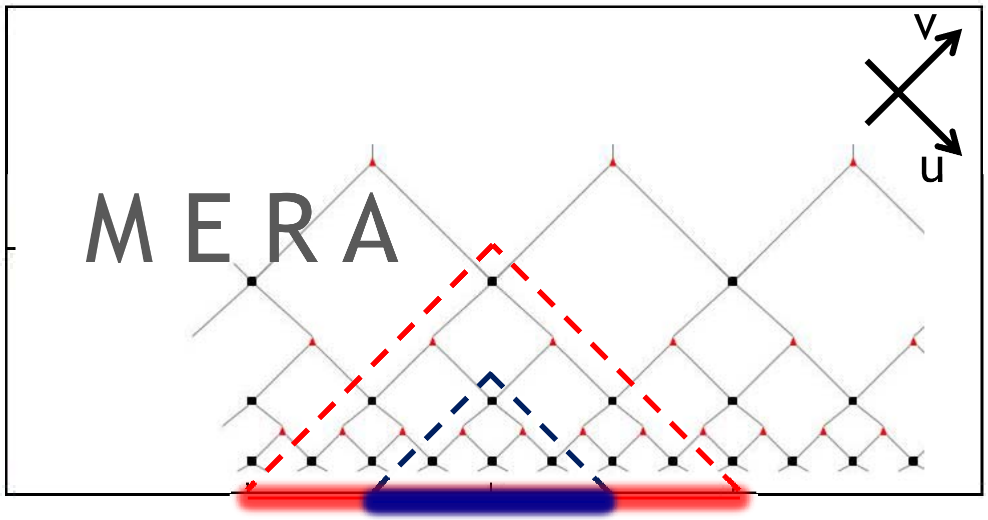
→ thermal density operator

?

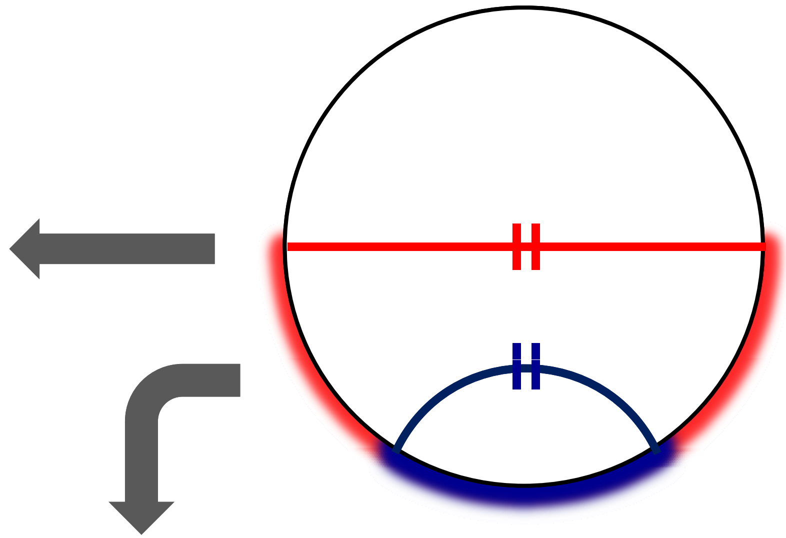
Black hole is a quotient of AdS_3

Banados et al., 1993

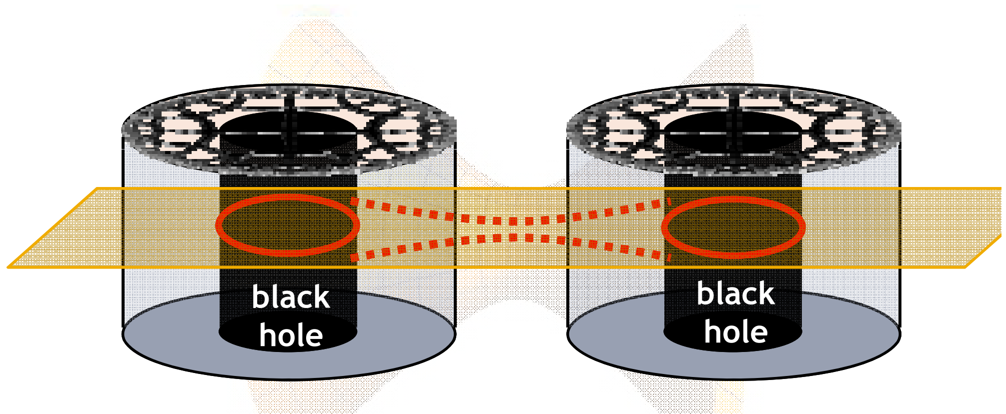
SPACESPACE ORIENTED INTERVALS GEODESICS



t=0 snapshot of AdS_3



- identify these geodesics
- this produces the dual of the thermofield double state
- perform the same identification in MERA!

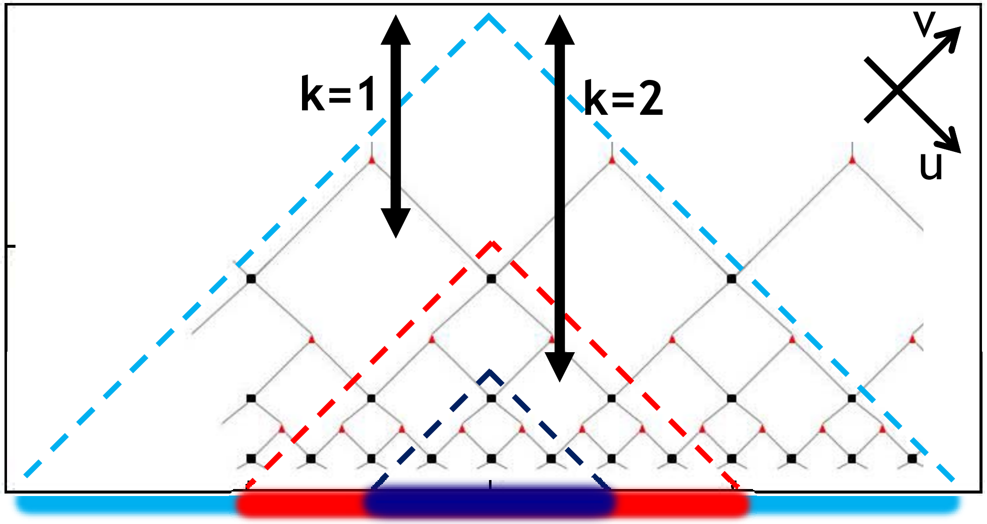


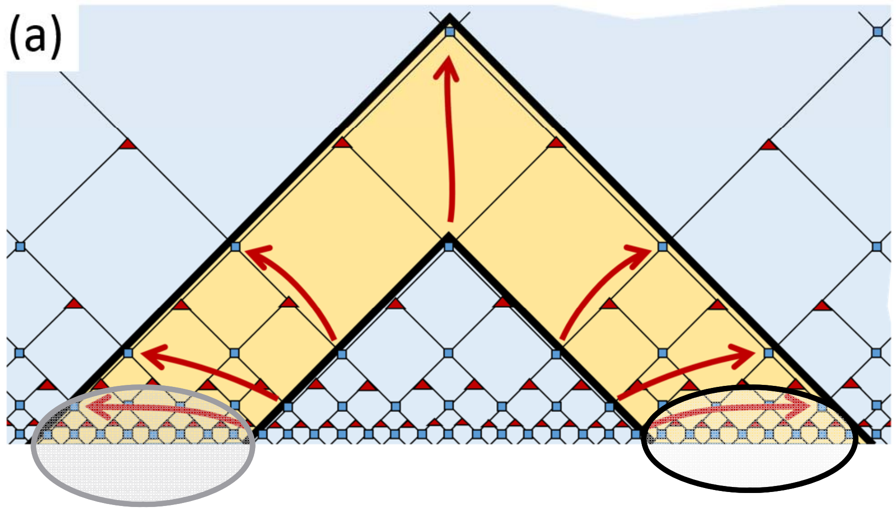
two-sided black hole

MERA quotient prepares thermal state

Czech et al., 2015

KINEMATIC SPACE / MERA





- this is a density operator with two sets of open indices
- the TFD spectrum should be $e^{-\beta\Delta/2}$
- β is given in terms of parameter k:

  $\beta = 4\pi^2/k(\log 2)$

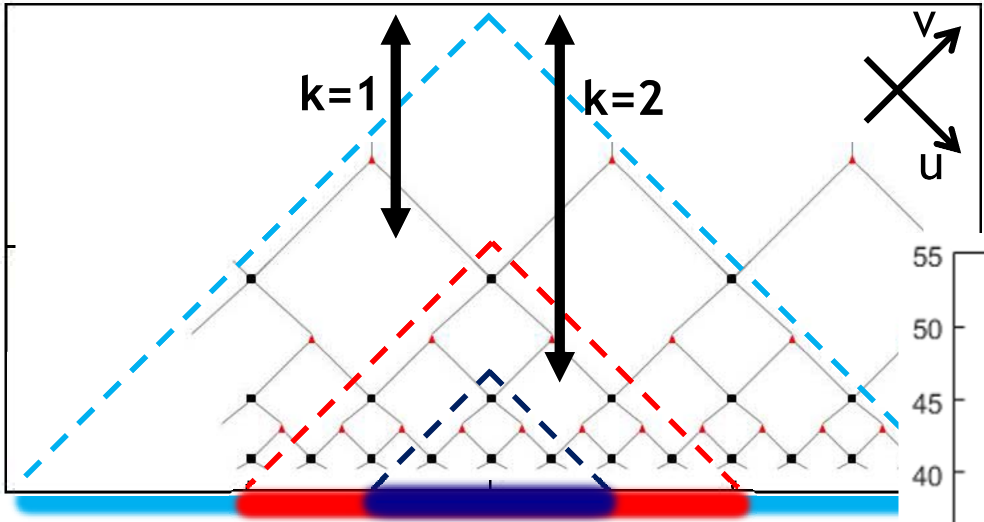
- therefore we expect:

$$\log(\lambda_i/\lambda_0) = -2\pi^2\Delta_i/k(\log 2)$$

MERA quotient prepares thermal state

Czech et al., 2015

KINEMATIC SPACE / MERA



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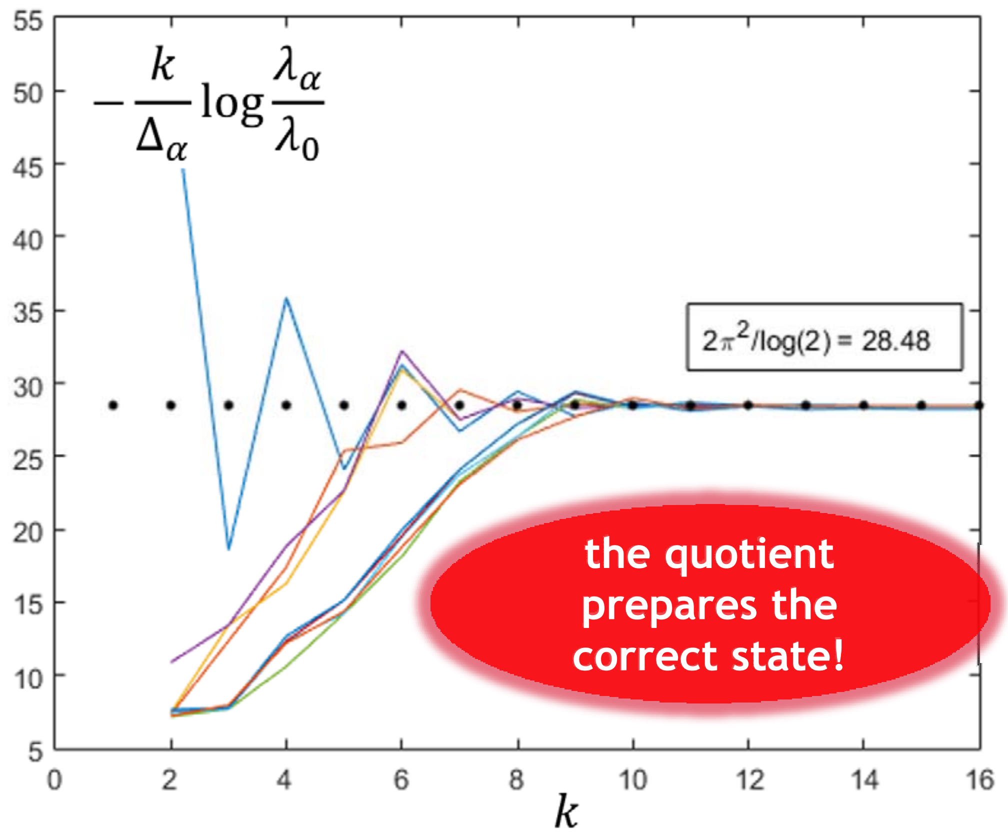
- Test in the critical 1+1d Ising model
- Substitute the known critical dimensions Δ_i and plot:



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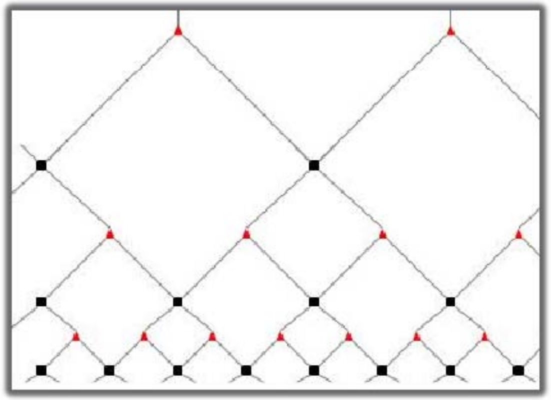


CFT ground state

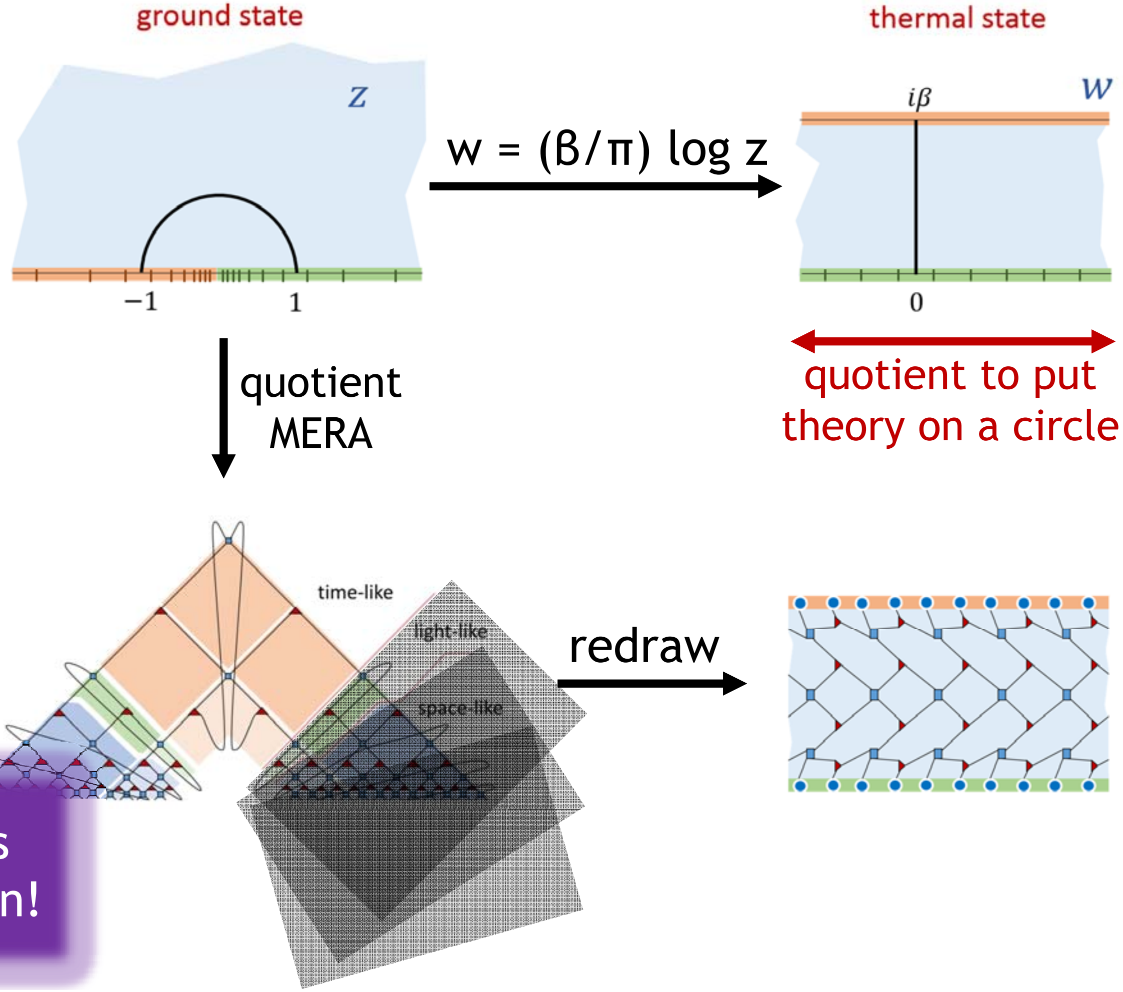
→ thermal density operator

?

Why did it work?

Czech et al., 2015

- We could also construct the thermal state by a local conformal transformation of the Euclidean path integral
- To get the state on a circle, quotient by translation
- We did our quotient directly in the MERA representation of the ground state
- The quotient selected a set of indices, which become uniformly distributed after the conformal transformation

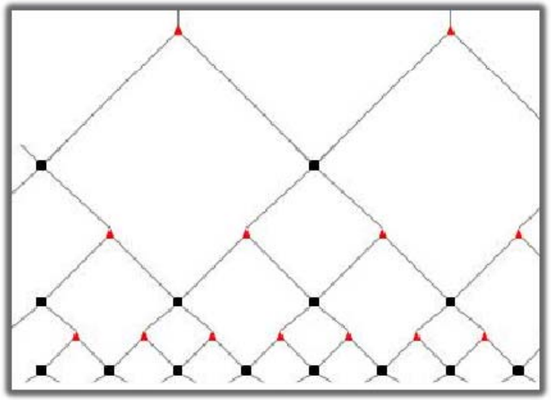


Erasing these tensors performs a local conformal transformation!

Application to many-body physics

Czech et al., 2015

- If MERA ≈ Kinematic Space then…
- Facts about Kinematic Space must carry over to MERA
- We will use one such fact to learn two new things about MERA in CFT_{1+1}:



CFT ground state

→ thermal density operator

We learned how to perform local conformal transformations in MERA

Summary

- We made precise the statement that connectedness = entanglement in AdS_3:

Density of Geodesics = Density of Correlations

- We explained how this program relates to tensor networks

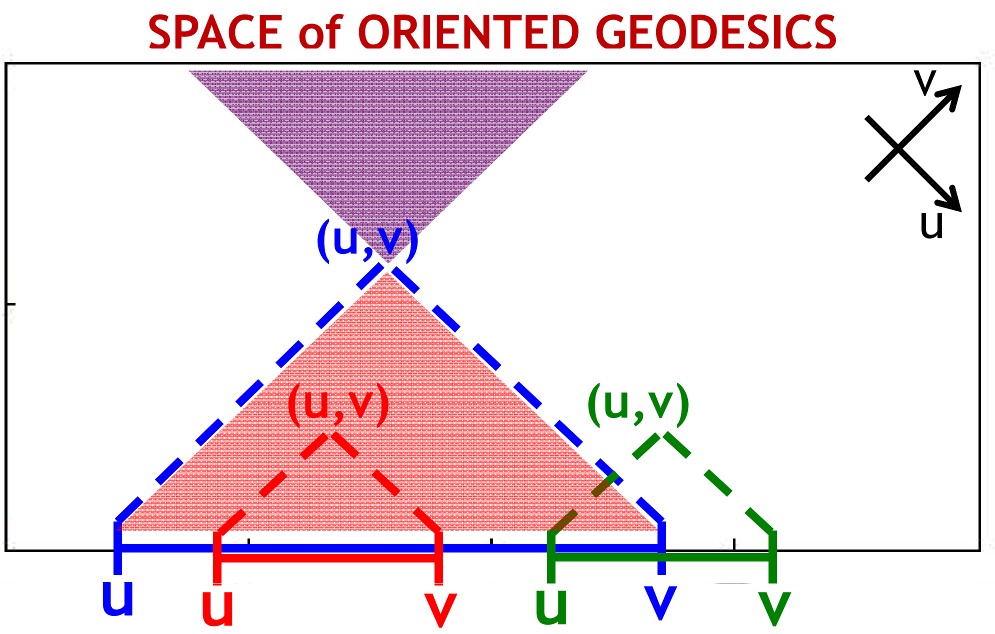
Kinematic Space (of Geodesics) = MERA

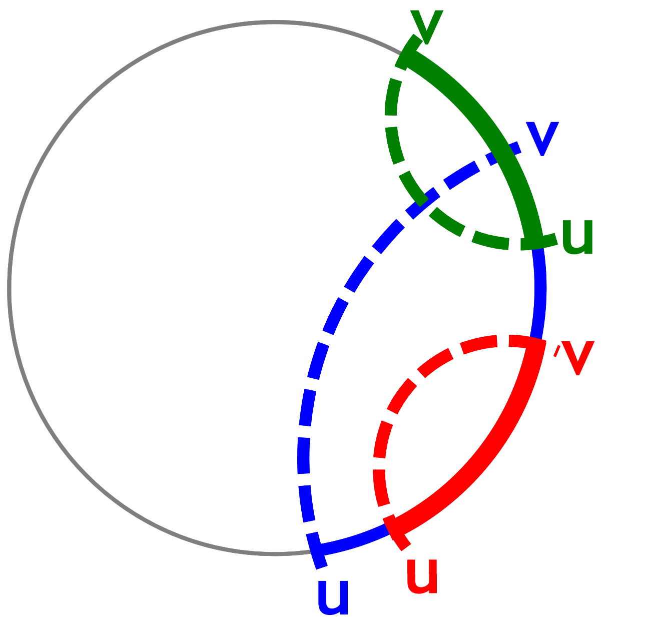
- We learned new things about the MERA network:
  - to extract the thermal density operator from the vacuum MERA
  - to perform local conformal transformations in MERA

Future directions

- How does $AdS_{>3}$ emerge?
- How to include time dependence?
- What is time?

Space of Geodesics has a "causal structure"





- **Timelike** separated (u,v): interval (u,v) contains (u,v)
- **Spacelike** separated (u,v): neither interval contains the other
- **Lightlike** separated: common endpoint left (**u** = u) or right (**v** = v)

- **Past:** all intervals contained in (u,v)
- **Future:** all intervals containing (u,v)

Space of Geodesics is also the Space of Intervals

Endpoint coordinates u,v are lightlike

Future directions

- How does $AdS_{>3}$ emerge?
- How to include time dependence?

Is Kinematic Space a model of the **emergence of time?**

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