Superfluidity in One Dimension as a Dynamical Phenomenon

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清华大学高等研究院 物理学术报告

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T. Eggel, M. A. Cazalilla, and MO,
Phys. Rev. Lett. 107, 275302 (2011)

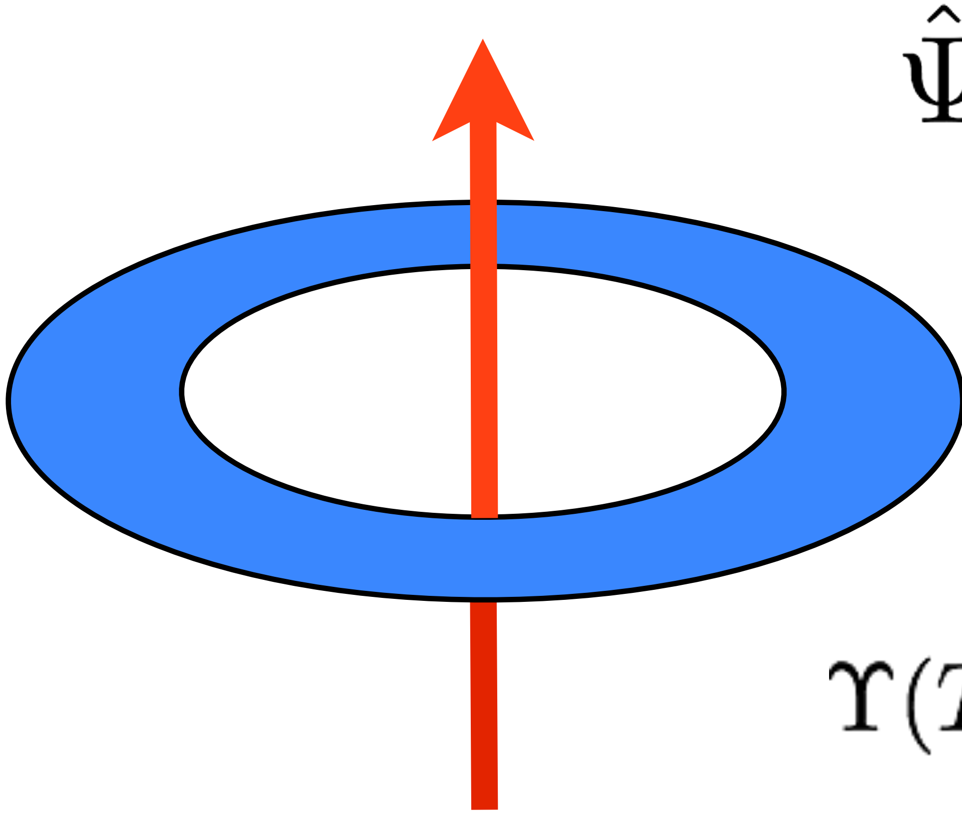
Thanks to Junko Taniguchi & Masaru Suzuki
(U. Electro-Communications, Tokyo)

Criterion for Superfluidity

Landau's criterion $$\min\left\{\frac{\epsilon(p)}{p}\right\} = v_{\mathrm{Landau}} > 0$$

$\Rightarrow$ how to understand finite T ? etc.

Helicity modulus [ME Fisher et al, 1973]



$$\hat{\Psi}(x+L,y,z) = e^{i\varphi}\hat{\Psi}(x,y,z)$$

$$\Upsilon(T) = \lim_{L\to+\infty}\frac{L}{S}\left(\frac{\partial^2 F(\varphi)}{\partial\varphi^2}\right)\Bigg|_{\varphi=0}$$

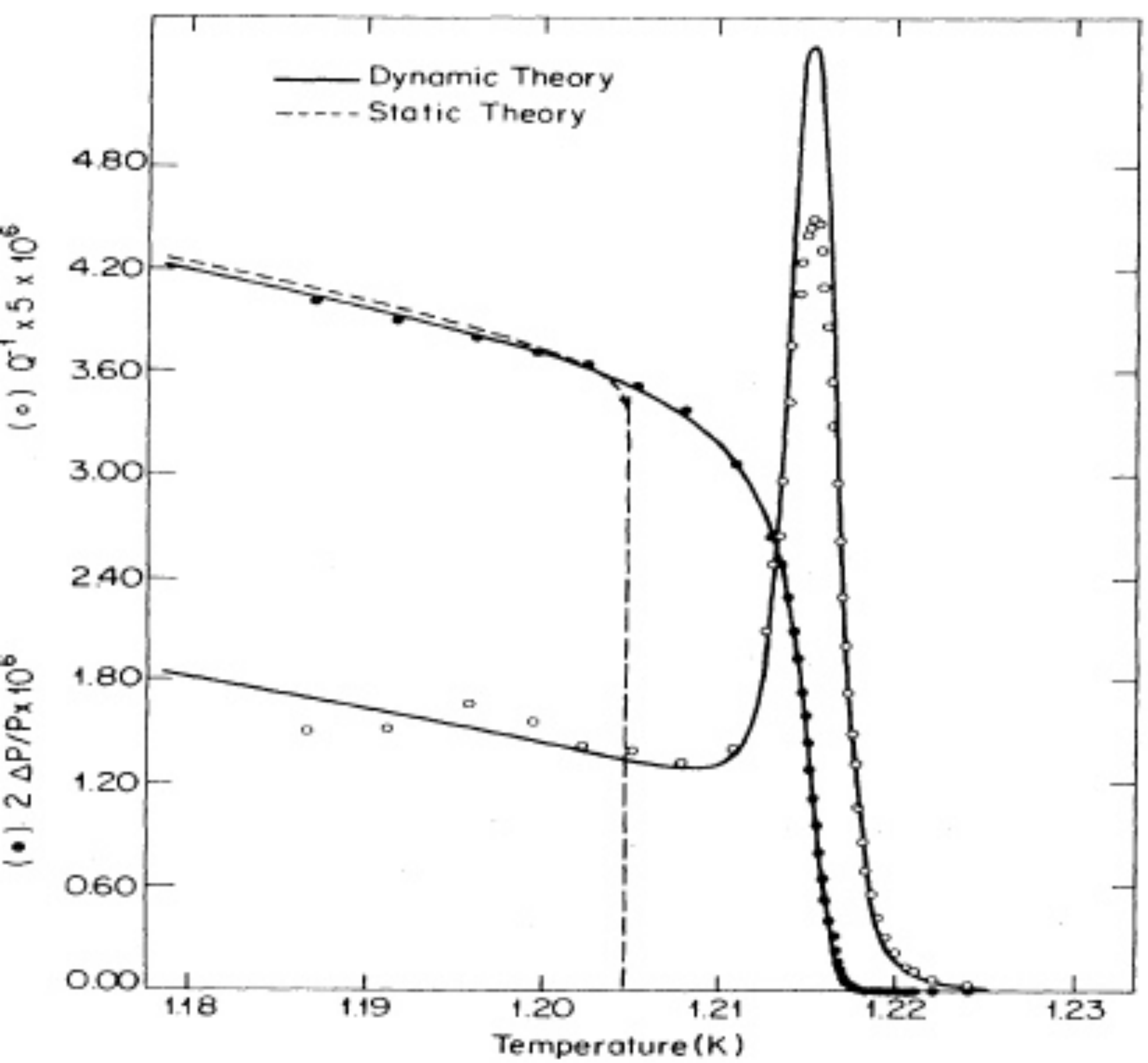
$$\Upsilon(T) = \frac{\hbar^2}{m}\rho_s(T)$$ Superfluid density

Superfluidity in 2D

No off-diagonal LRO at $T>0$ (Mermin-Wagner theorem)

But helicity modulus is finite for $T<T_{\mathrm{BKT}}$

"universal jump" at $T<T_{\mathrm{BKT}}$ [Nelson-Kosterlitz 1977]



Superfluidity is indeed observed in torsional oscillator measurements of 2D ^{4}He film [Bishop-Reppy 1978]

Dynamical effects are also important [Ambegaokar-Halperin-Nelson-Siggia 1978]

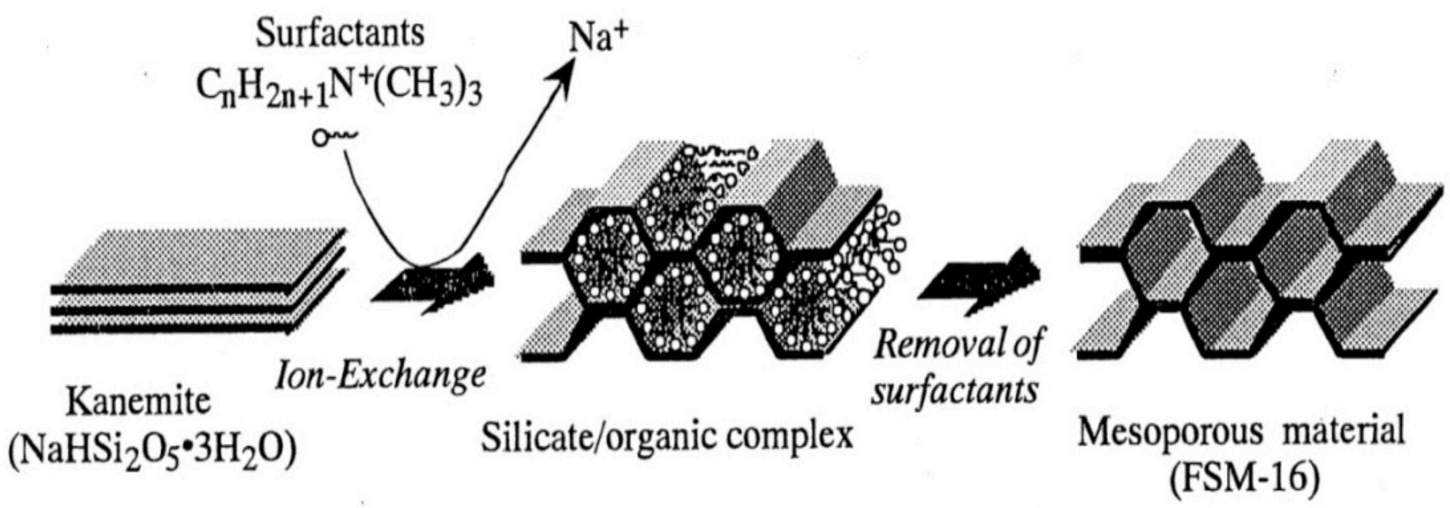
Superfluidity in 1D?

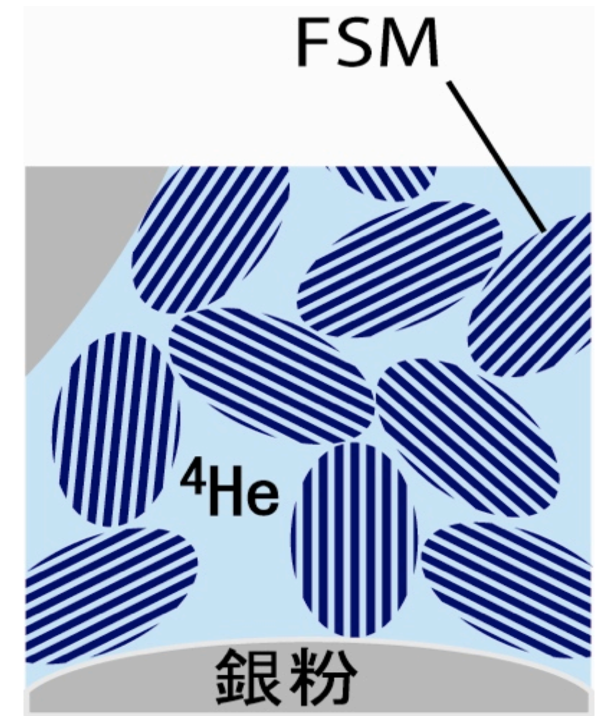
Helicity modulus vanishes in 1D (in thermodynamic limit)

$$\Upsilon_{1D}(T) = \lim_{L \to +\infty} L \left(\frac{\partial^2 F(\varphi)}{\partial \varphi^2} \right) \Bigg|_{\varphi=0} = 0$$

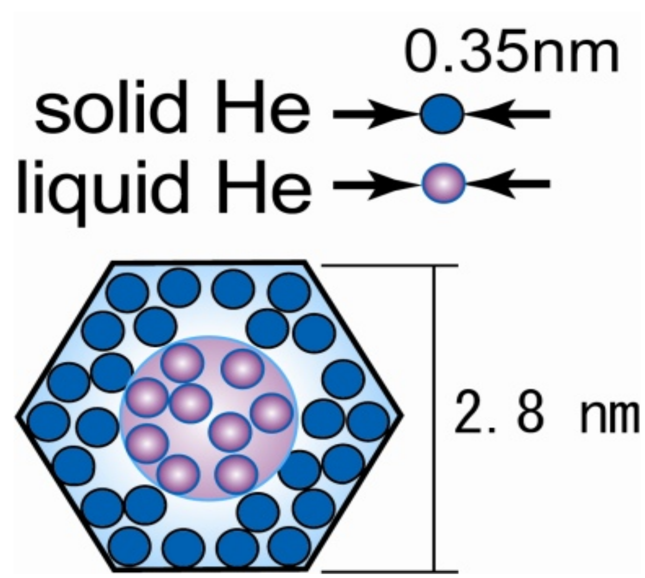
Hence, no superfluidity in 1D?

Liquid ^{4}He in 1D nanopore



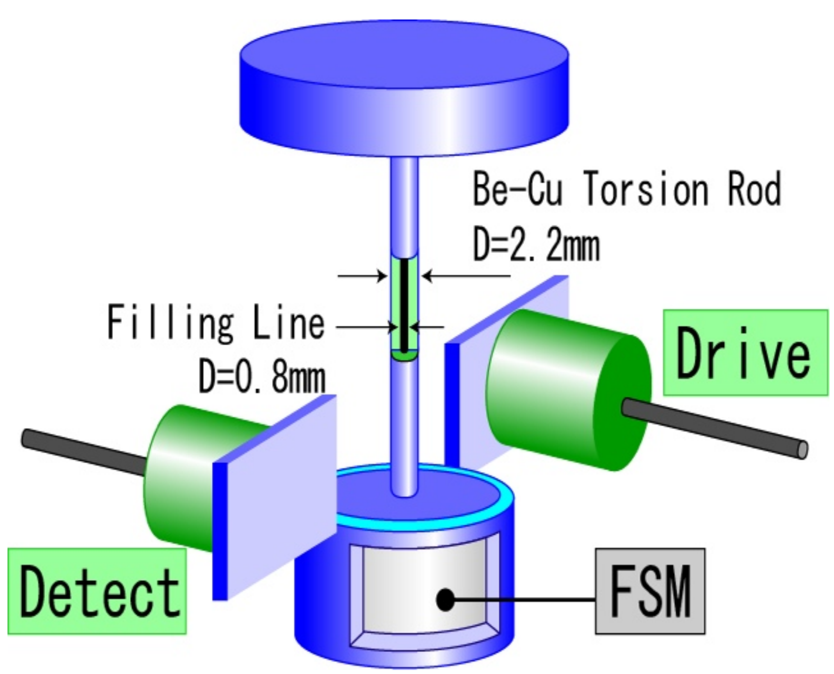


"FSM-16"



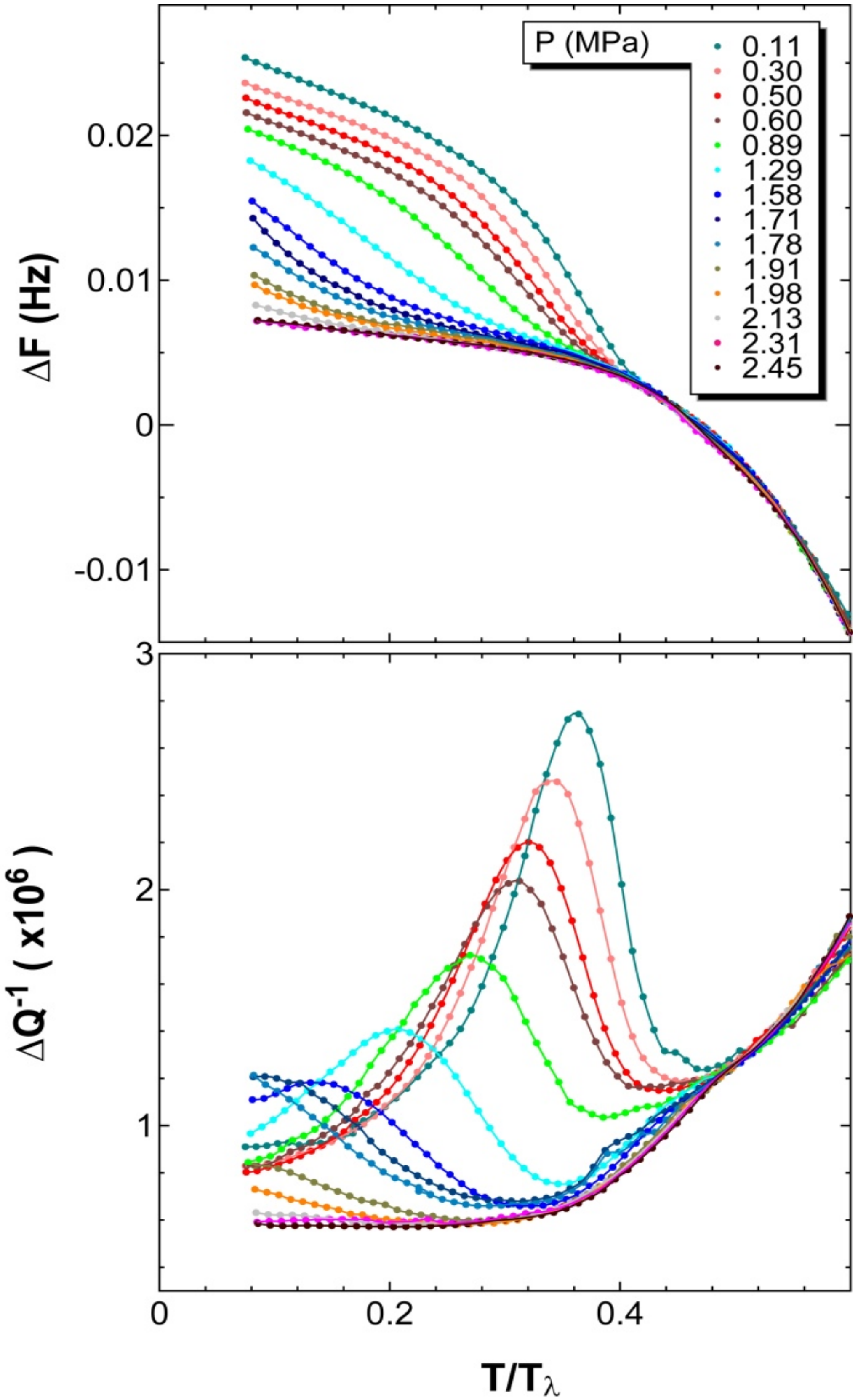
channel length:
0.2〜0.5 μm

length/diameter
〜 100



[Taniguchi-Aoki-Suzuki 2010]

Results (2.8nm diameter)

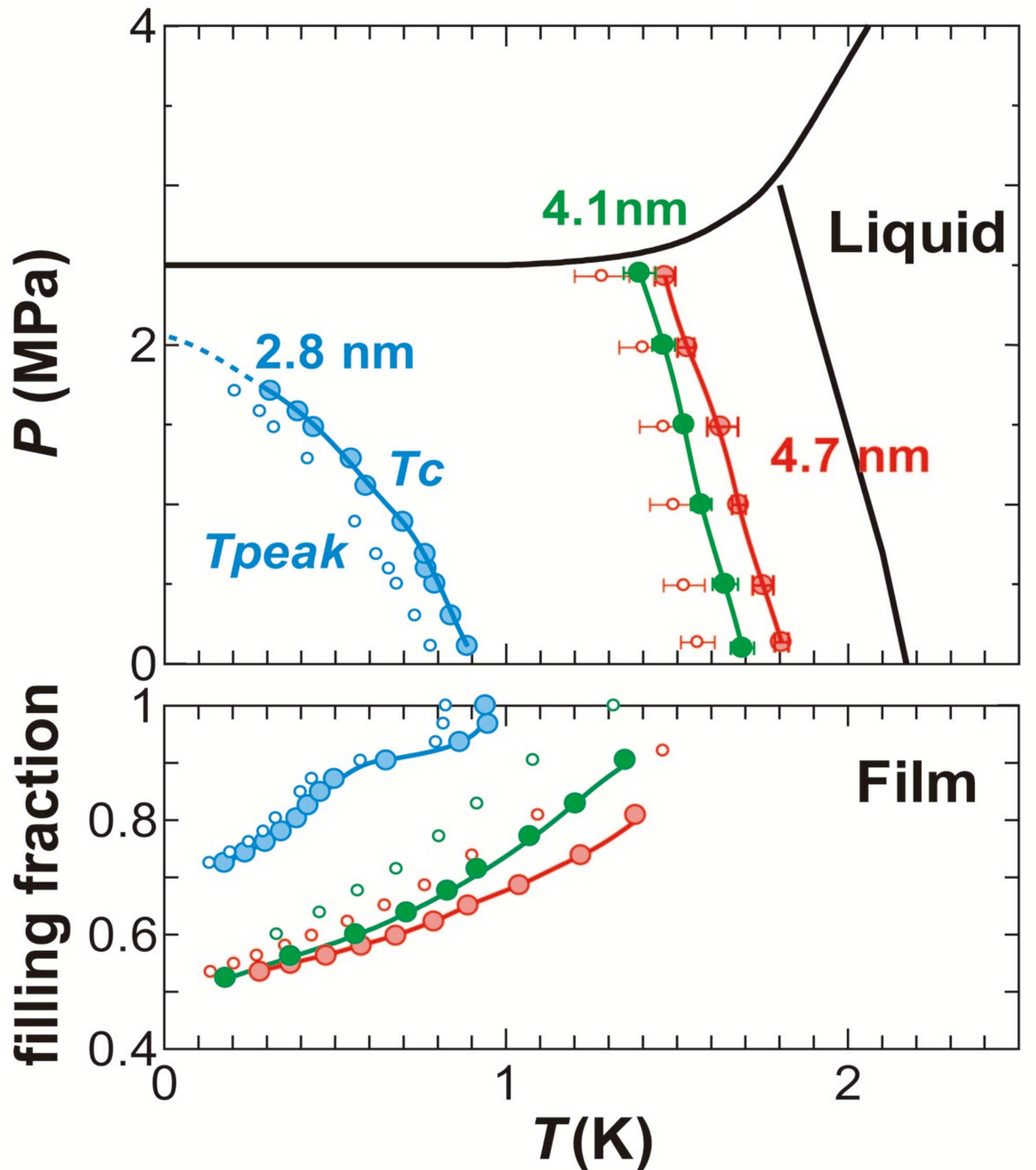


Superfluid(-like) response!

superfluidity suppressed at higher pressures

Dissipation peak at "superfluid transition temperature"

Phase Diagram

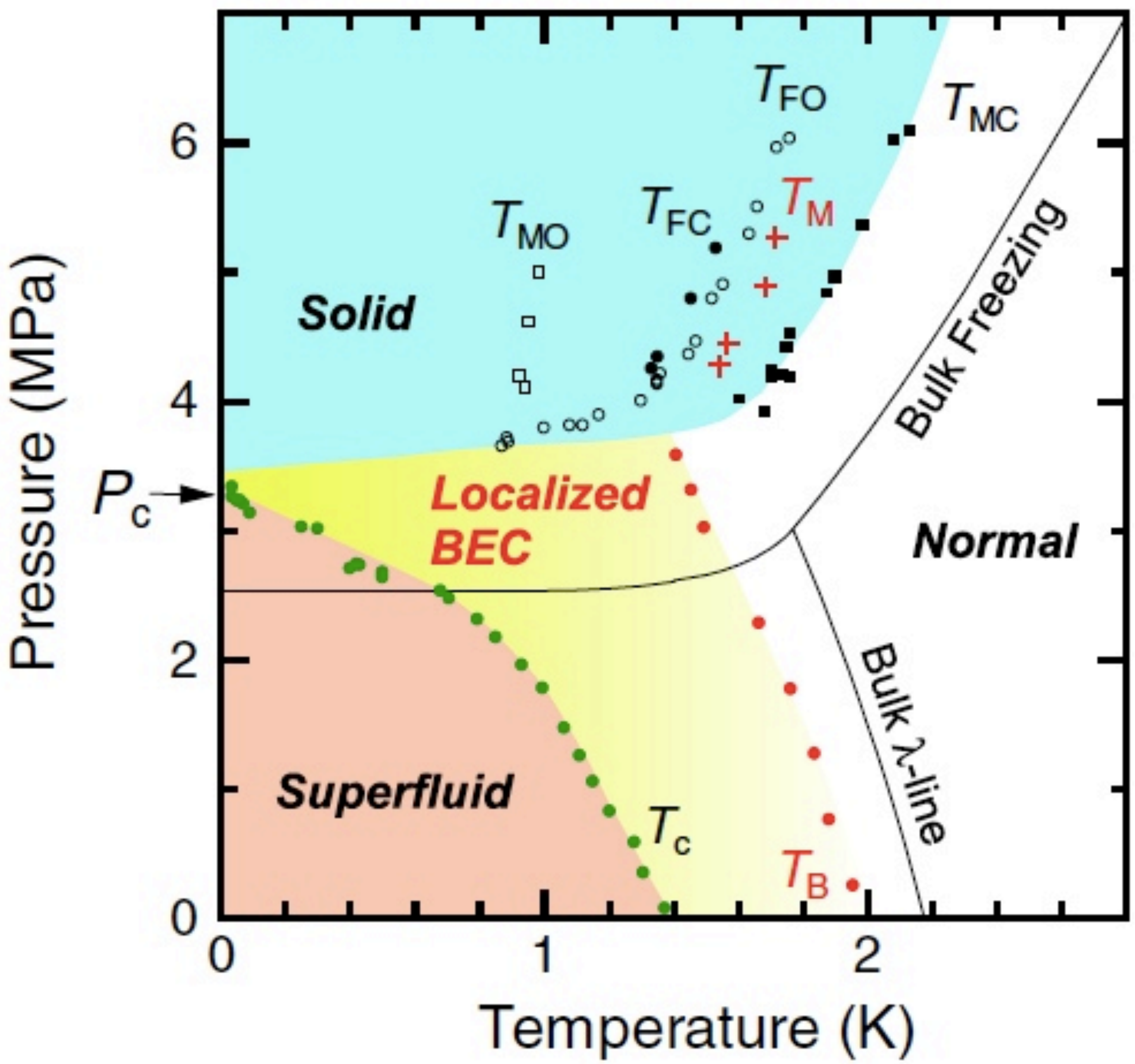


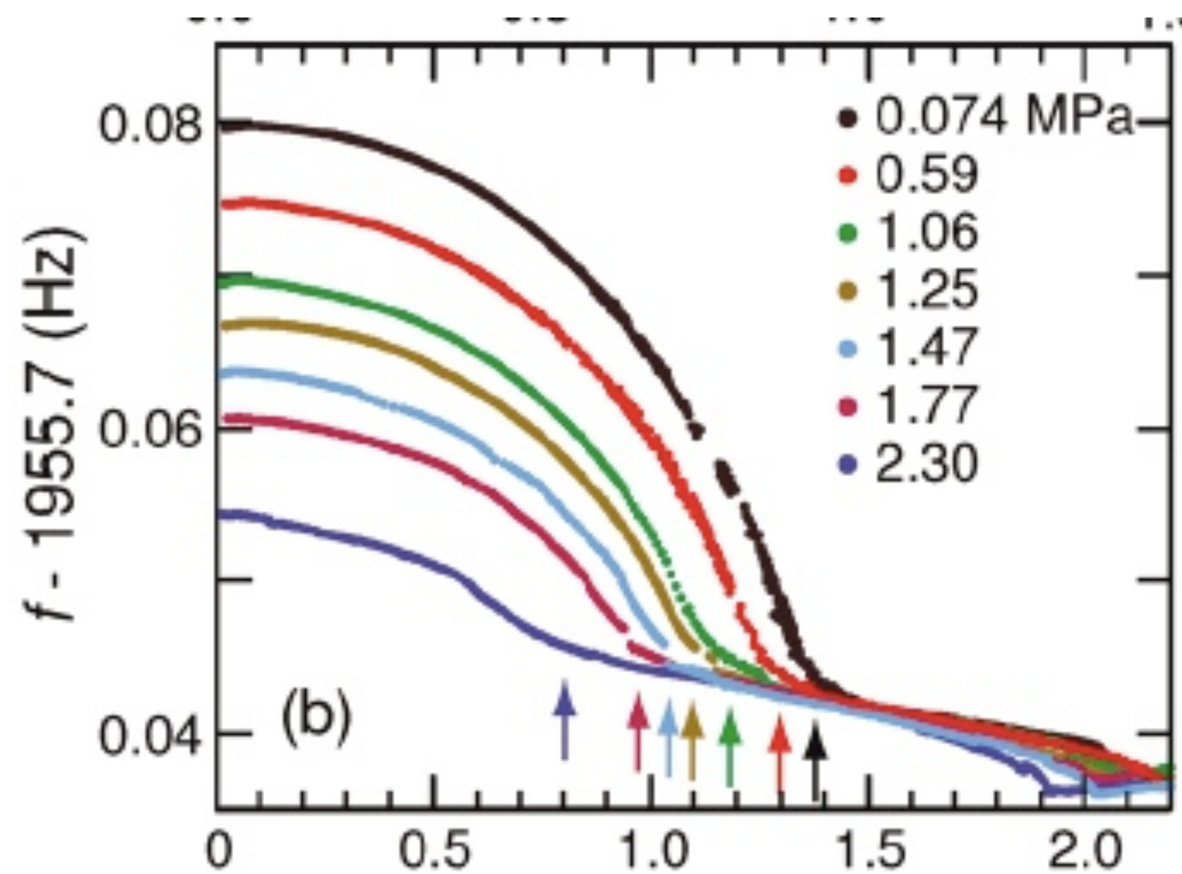
[Taniguchi-Aoki-Suzuki 2010]

cf.) ^{4}He in 3D porous media

Gelsil
(pore Φ〜25Å)

Shirahama et al. 2004〜





Similar-looking phenomena
but ***different*** physics
(3D LRO)

Eggel,-M.O. -Shirahama 2011

TLL description

Quantum Monte Carlo simulation (Worm Algorithm-Path Integral) of microscopic Hamiltonian for 4He in 1D nanopore

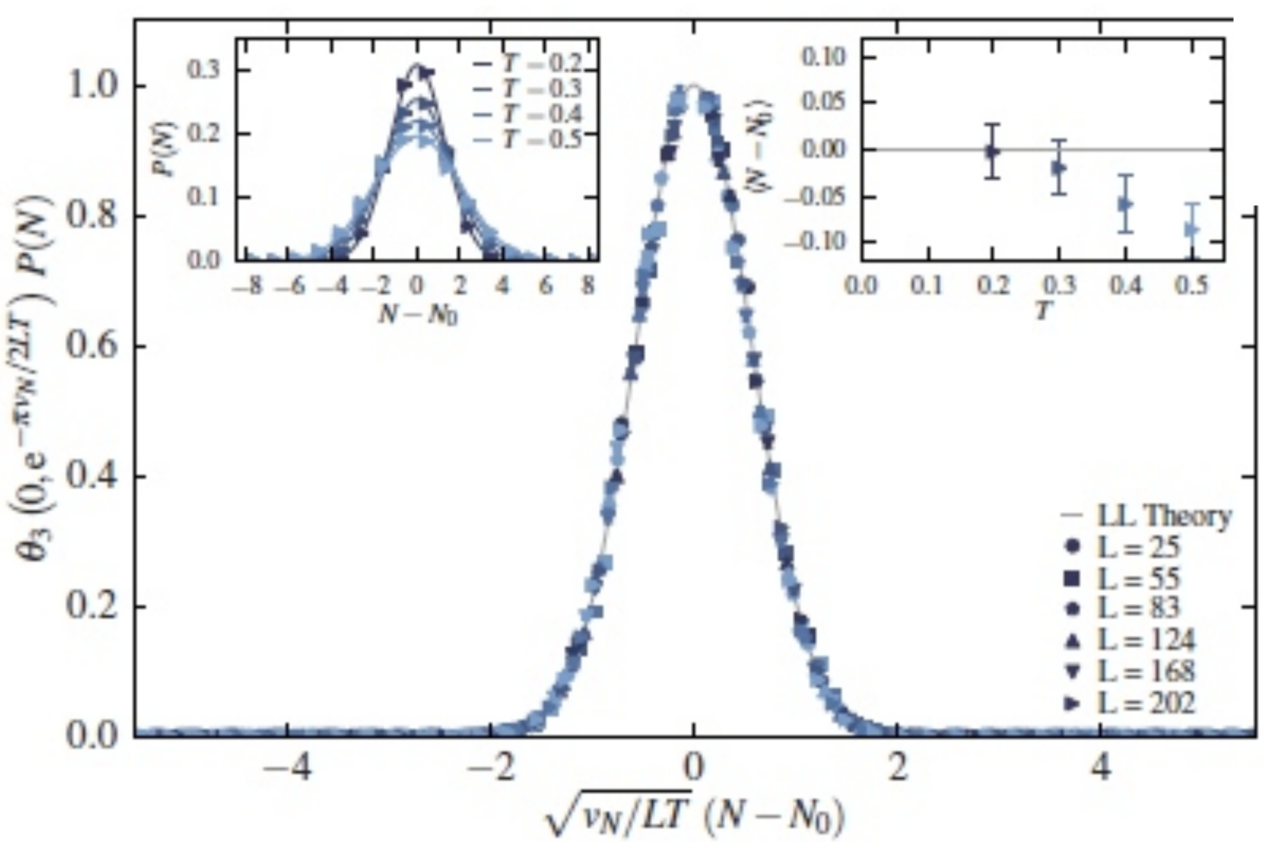


FIG. 1. (Color online) QMC data (symbols) combined with Luttinger-liquid predictions (solid lines) for the particle number probability distribution at fixed system size (upper left inset), scaling of the particle number probability distribution (main panel), and the temperature dependence of the mean number of particles (upper right inset) measured with respect to the ground-state value $N_0 = \rho_0 L$.

$$H = -\frac{1}{2m}\sum_{i=1}^{N} \nabla_i^2 + \frac{1}{2}\sum_{i,j=1}^{N} V(|\vec{r}_i - \vec{r}_j|)$$

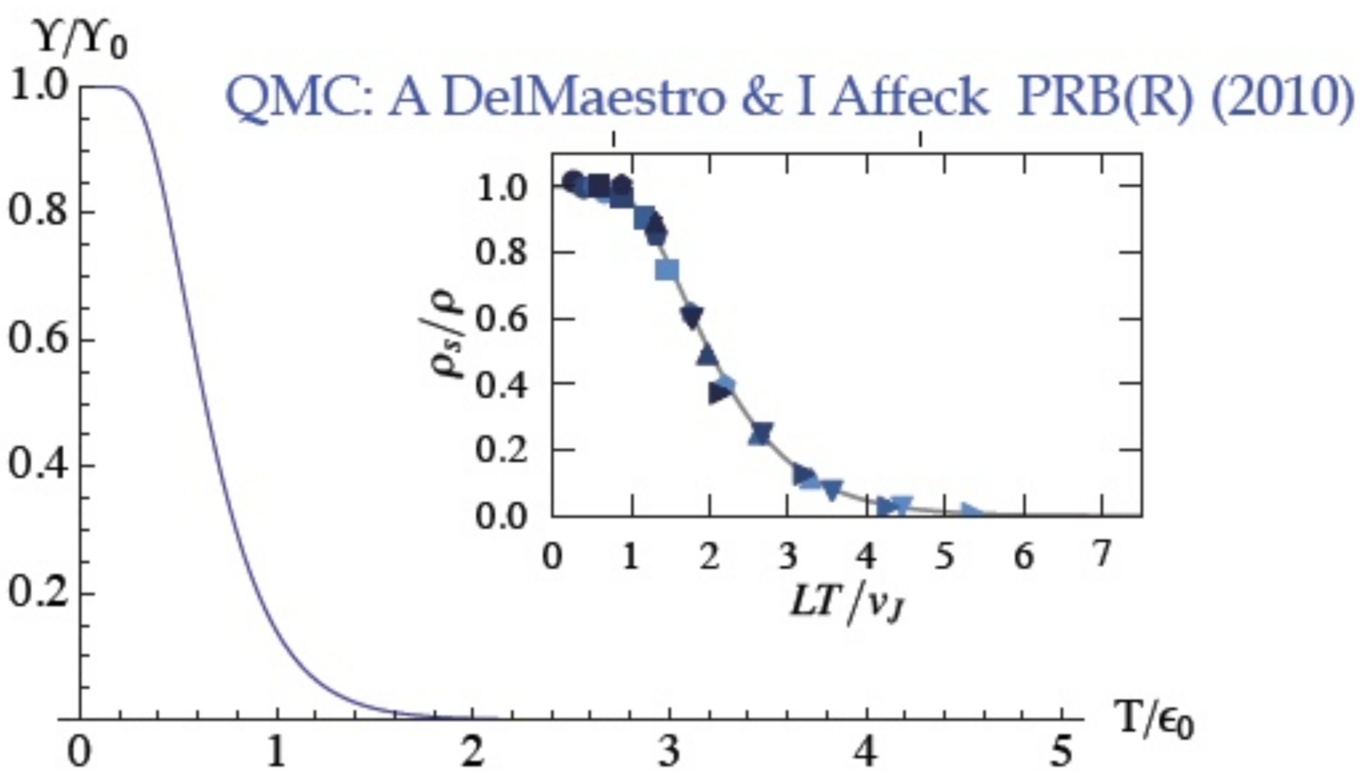
Quantitative agreement with TLL on static quantities

(Del Maestro-Affleck 2010, Del Maestro-Boninsegni-Affleck 2011)

But not (yet) for the diameter 2.8nm of Taniguchi et al. expt.

Finite-size effect?

Helicity modulus $\Upsilon(T)$ of a 1D system vanishes,
but ***only in the thermodynamic limit***



Tomonaga-Luttinger Liquid

$$\Upsilon(T,L) = \Upsilon_0 \left(1 + \frac{\epsilon_0}{T}\frac{\vartheta_3''(0, e^{-2\epsilon_0/T})}{\vartheta_3(0, e^{-2\beta\epsilon_0})}\right)$$

maximum onset temperature of helicity modulus

$$\frac{\epsilon_0}{k_B} = \frac{\hbar v K}{L} < \frac{\hbar^2 \pi \rho_0}{mL} \simeq 0.2\mathrm{K}$$

Too low to account the experimental results (onset temperature can be $\sim$ 1K or higher)

Yamashita-Hirashima 2009

Why superfluidity in 1D?

~~finite-size effect~~

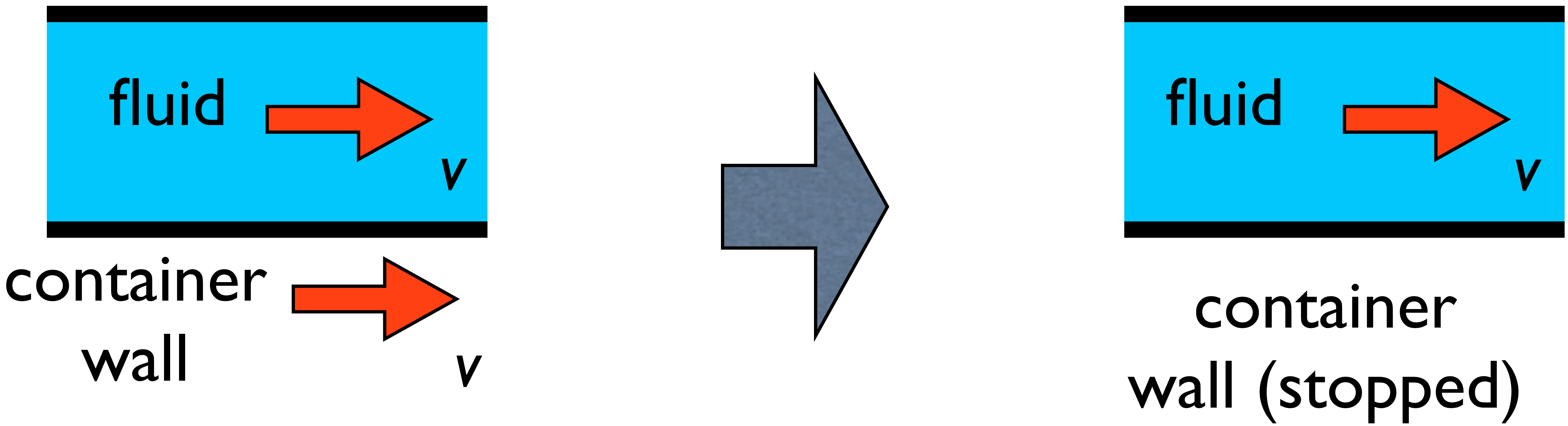
$$\Upsilon_{1D}(T) = \lim_{L\to+\infty} L \left(\frac{\partial^2 F(\varphi)}{\partial \varphi^2} \right) \Bigg|_{\varphi=0} = 0$$

$$\Upsilon(T) \overset{?}{=} \frac{\hbar^2}{m} \rho_s(T)$$

Static property in equilibrium

Dynamics

What is Superfluidity?

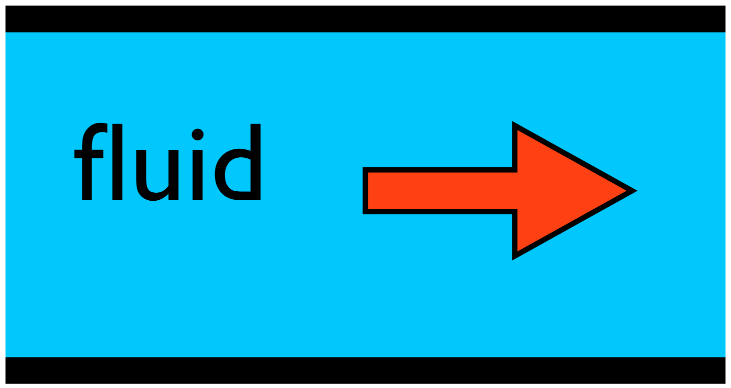


in equilibrium at velocity v

How will the fluid behave?

- eventually come to rest (normal fluid)
- move perpetually at velocity v (superfluid)

What is superfluidity?



container wall (stopped)

Initial condition of the fluid: Galilean boost of the equilibrium fluid at rest, with velocity *v*

$$\rho_{\rm ini} = e^{-i\hbar m v x} \rho_{\rm eq} e^{i\hbar m v x}$$

$$\langle \mathcal{O}(t) \rangle = \mathrm{Tr}\left(e^{i\mathcal{H}t/\hbar} \mathcal{O} e^{-i\mathcal{H}t/\hbar} \rho_{\rm ini} \right) = \mathrm{Tr}\left(e^{i\tilde{\mathcal{H}}t/\hbar} \tilde{\mathcal{O}} e^{-i\tilde{\mathcal{H}}t/\hbar} \rho_{\rm eq} \right)$$

$$\tilde{\mathcal{O}} \equiv e^{i\hbar m v x} \mathcal{O} e^{-i\hbar m v x}$$

$$\tilde{\mathcal{H}} = \sum_i \left(\frac{\hbar^2}{2m} (\vec{p}_i + m\vec{v})^2 + U(\vec{r}_i) \right) + \sum_{i>j} V(\vec{r}_i - \vec{r}_j)$$

"effective Hamiltonian" equivalent to phase twist

What happens at $t \to \infty$?

Fluid reaches equilibrium with respect to effective Hamiltonian (in the presence of static wall potential)

In a normal liquid, the resulting state should be equivalent to ρ_{eq}. But in a superfluid, a fraction of fluid is still moving at velocity *v*

Free energy density $$f(\vec{v}) \sim f(\vec{0}) + \frac{\rho_s}{2}\vec{v}^2$$

$$\Upsilon(T) = \lim_{L \to +\infty} \frac{L}{S} \left(\frac{\partial^2 F(\varphi)}{\partial \varphi^2} \right) \Bigg|_{\varphi=0} = \frac{\hbar^2}{m} \rho_s(T)$$

Helicity modulus = Superfluid density ?

What is assumed?

Fluid reaches equilibrium with respect to effective Hamiltonian (in the presence of static wall potential)

i.e. we need (hidden) assumption of **thermalization** of in order to derive $\Upsilon = \rho_s$

Integrable systems in 1D: **thermalization is absent** due to infinite # of conserved quantities, so the equivalence between Υ and ρ_s would break down

Generic Systems in 1D?

"Non-integrable models thermalize"

- common belief

This may not be always the case, but we would assume that realistic, generic non-integrable systems eventually thermalize

$\Rightarrow$ resurrection of $\Upsilon(T) = \frac{\hbar^2}{m}\rho_s(T)$

Then the superfluidity is absent in 1D in the strict sense. However, due to the anomalous dynamics in 1D, the approach to equilibrium could be very slow.

Superfluidity might be observed at experimentally relevant timescale

Phase Slips

Decay of "superflow" and thermalization caused by phase slips

Thermal Phase Slips

[Langer-Ambegaokar 1967, McCumber-Halperin 1968]

$$\Gamma_{\rm TPS} \sim \exp\left(-\frac{\Delta F}{k_B T}\right)$$

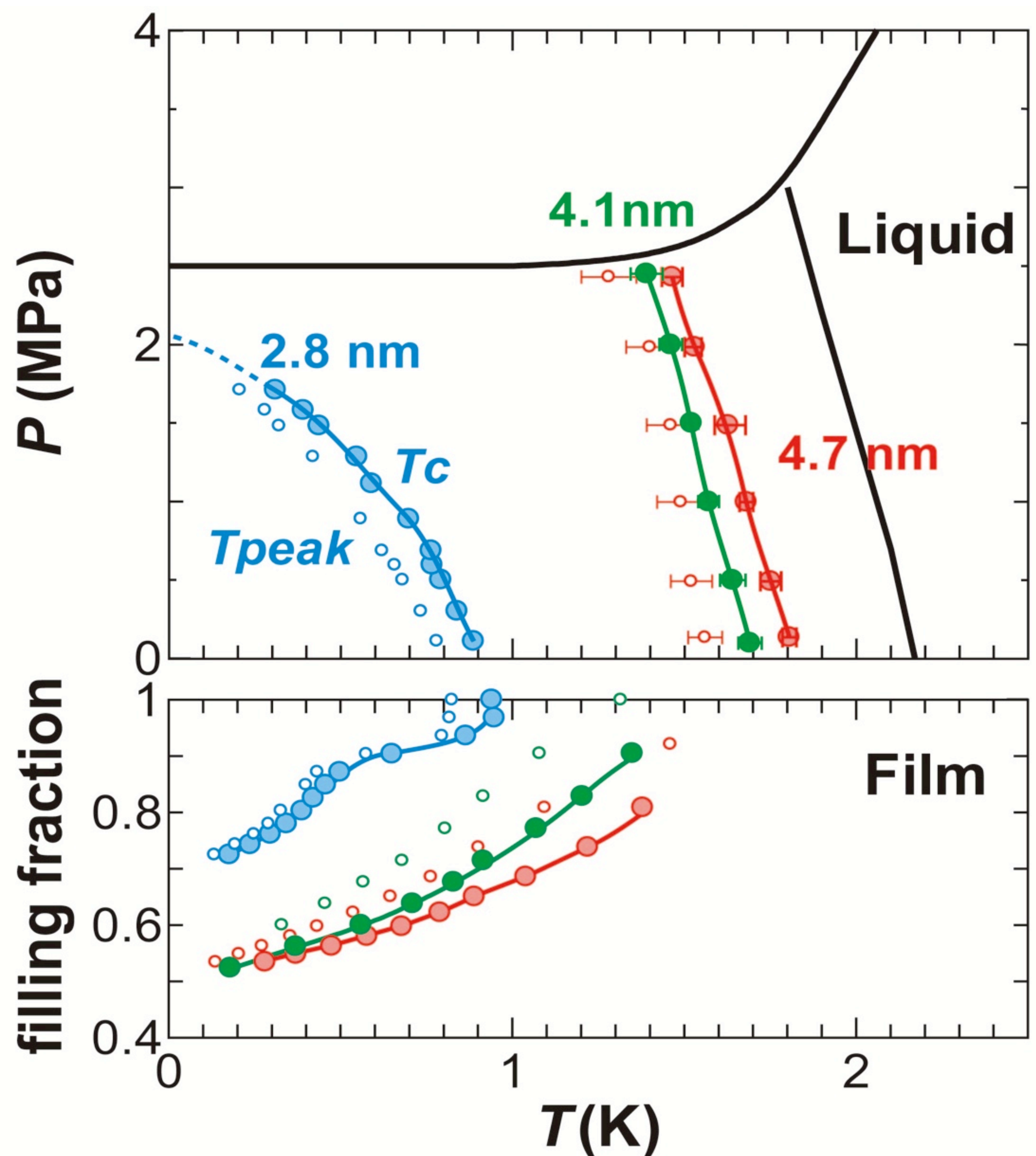
"Quantum" Phase Slips

[Khlebnikov 2005]

$$\Gamma_{\rm QPS} \sim \exp\left(-\frac{\hbar\pi v\rho_0}{k_B T}\right)$$

Exponentially suppressed PS rate at low *T*:
manifestation of constrained dynamics in 1D
but cannot account the experimental results on 1D ^{4}He

Phase Diagram



Required Formulation

- Include quantum&thermal fluctuations beyond the leading exponential
- Include explicitly the potential due to the container wall
  (in $D \geqq 2$ the wall effect can be replaced by a ***boundary condition,*** but ***NOT in 1D***)
- Include the interaction among particles (^{4}He atoms)
- Take the conserved (or nearly conserved) quantities into account properly
- Consider finite-frequency response

Memory-matrix formulation based on TL Liquid theory

cf.) conductivity [Rosch-Andrei 2000]

What to calculate?

(Total) Momentum Response Function

$$\chi(t) = -\frac{i}{\hbar}\theta(t)\langle[\Pi(t),\Pi(0)]\rangle \qquad \Pi = \sum_j p_j$$

measures the response of the system to the perturbation in the effective Hamiltonian

$$\tilde{\mathcal{H}} = \sum_i \left(\frac{\hbar^2}{2m}(\vec{p}_i + m\vec{v})^2 + U(\vec{r}_i) \right) + \sum_{i>j} V(\vec{r}_i - \vec{r}_j)$$

normal fluid density $\qquad \rho_n = -\frac{1}{m}\lim_{\omega\to 0}\chi(\omega)$

Tomonaga-Luttinger Liquid

$$\mathcal{H}_* = \frac{\hbar v}{2\pi}\int dx \;\left[\frac{1}{K}(\partial_x\phi)^2 + K(\partial_x\theta)^2\right]$$

Low-energy fixed point with ∞ number of conserved qtys

$$J = \frac{mvK}{\pi}\int dx\, \partial_x\theta(x,t)$$

particle mass current

$$P = \frac{\hbar}{\pi}\int dx\, \partial_x\phi\partial_x\theta$$

energy current

Due to the curvature of the dispersion, total momentum is

$$\Pi = J + \frac{vKm}{\hbar\pi\rho_0}P$$

Wall Potential

We assume periodic potential due to the wall (reasonable for FEM-16 expt)

$$H_{PS} = \sum_{n>0,m} \frac{\hbar v g_{nm}}{\pi a_0^2} \int dx \cos\left(2n\phi(x) + \Delta k_{nm} x\right) .$$

"irrelevant" in the RG sense, but is important since it causes phase slips

J and *P* (and thus Π) are exactly conserved in pure TLL (= fixed point Hamiltonian H_*), but not conserved in the presence of H_{PS}

Nevertheless, the decay is slow due to constrained dynamics in 1D -- how to describe?

Memory Matrix Formalism

$$\chi(\omega) = \mathrm{Tr}\left\{V[\omega\hat{1} + i\hat{M}(\omega)]^{-1} i\hat{M}(\omega)\hat{\chi}(\omega)\right\}$$

$$\hat{\chi} \sim \mathrm{diag}\{\chi_{JJ}, \chi_{PP}\}$$

$\hat{M}$: 2x2 matrix describing the decay rates of two currents

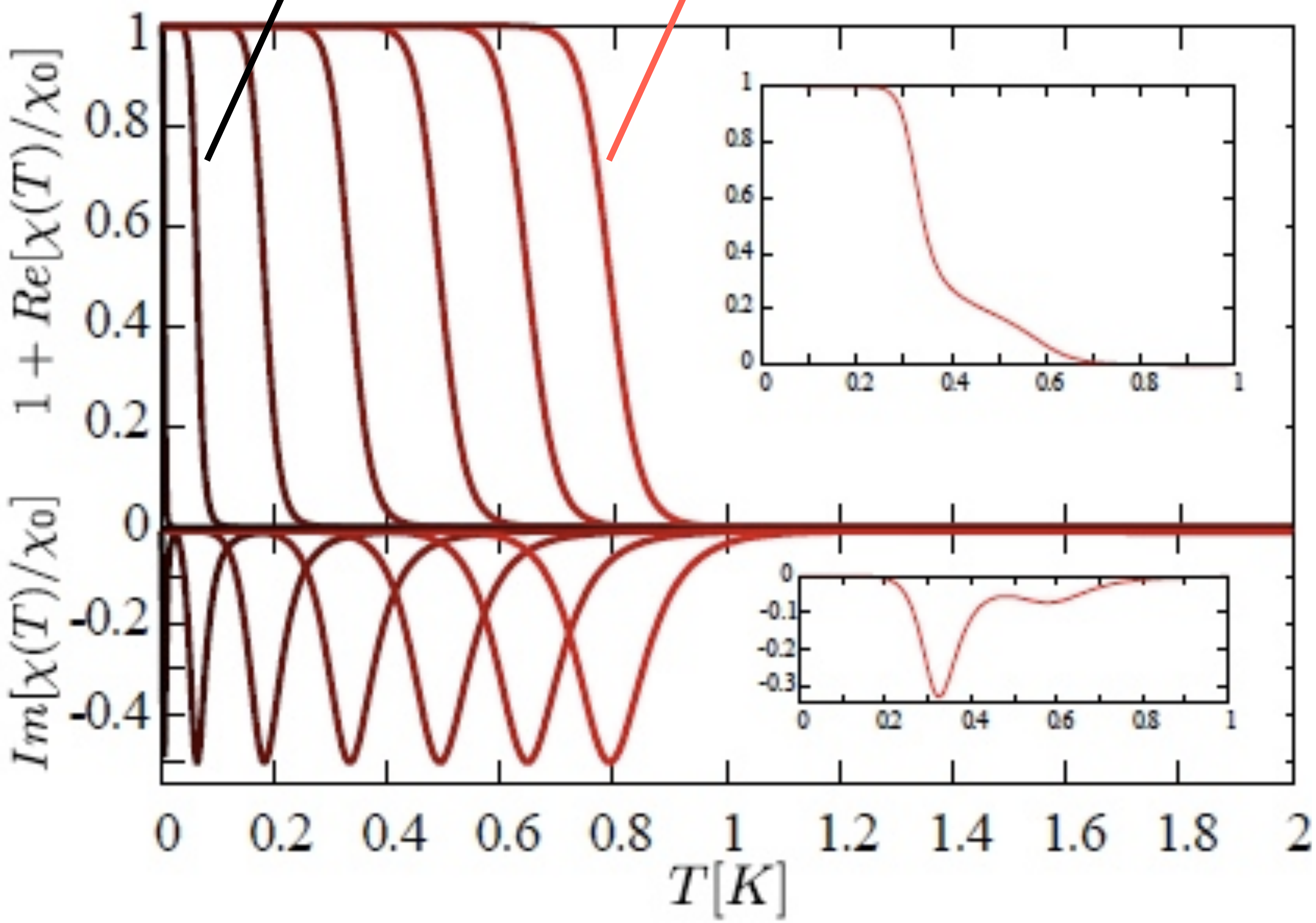
Perturbative evaluation in H_{PS}

cf.) D. Forster "Hydrodynamic fluctuations,...." (1975), Rosch-Andrei (2000)

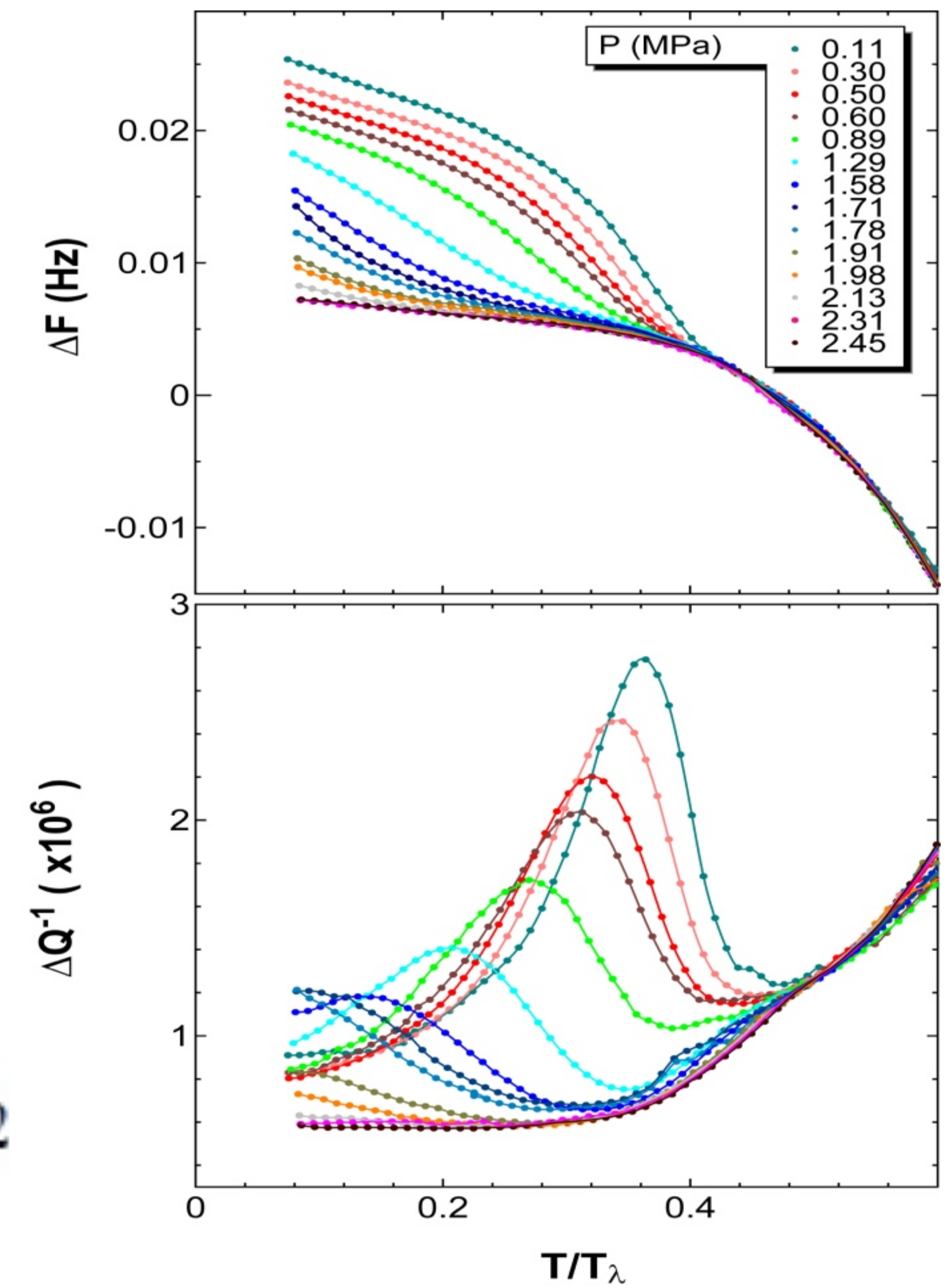
Results

$\omega = \omega_0 = 2\ \text{kHz}$

K=4.2 *K*=9.2



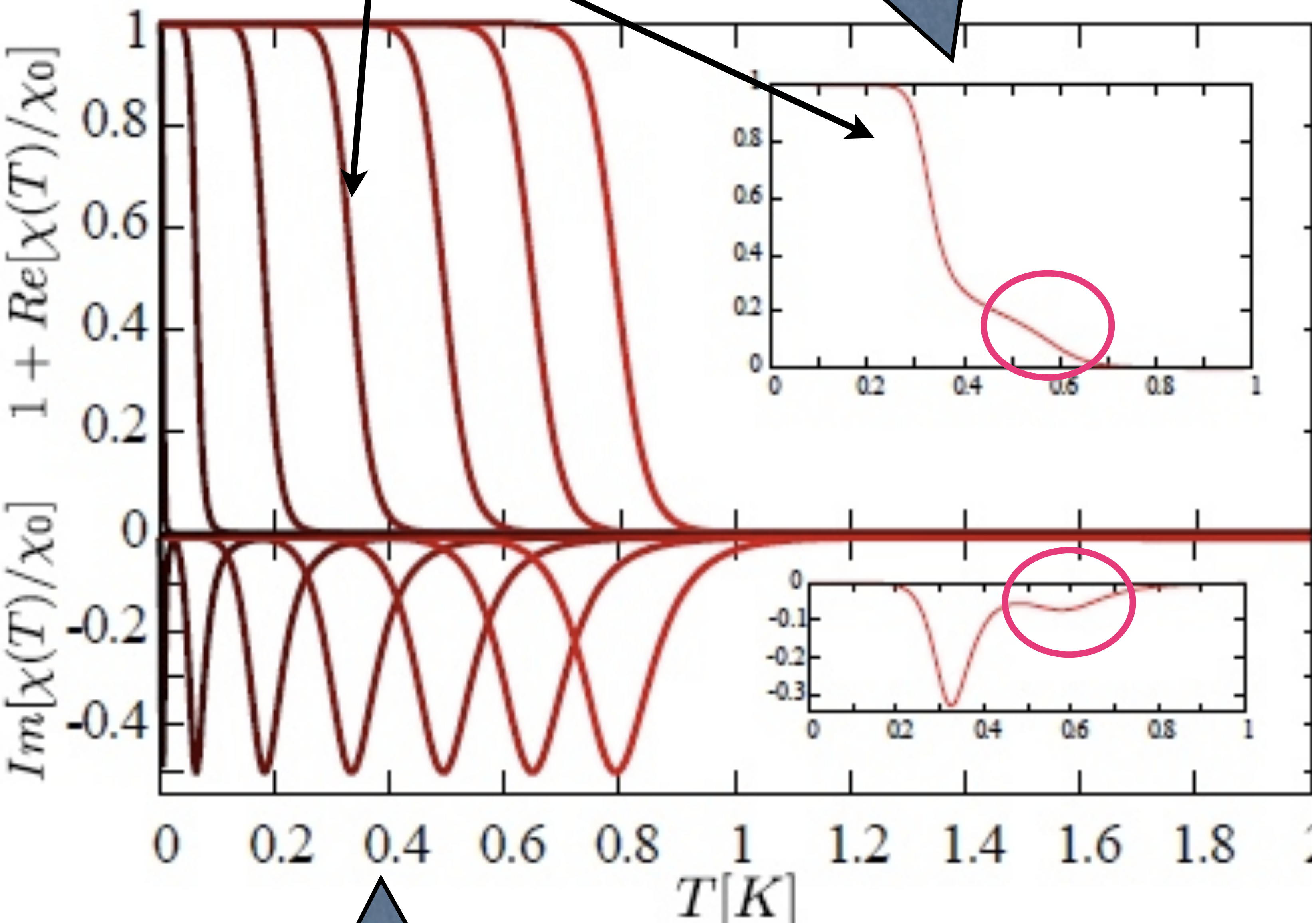
Expt. [Taniguchi et al. 2010]

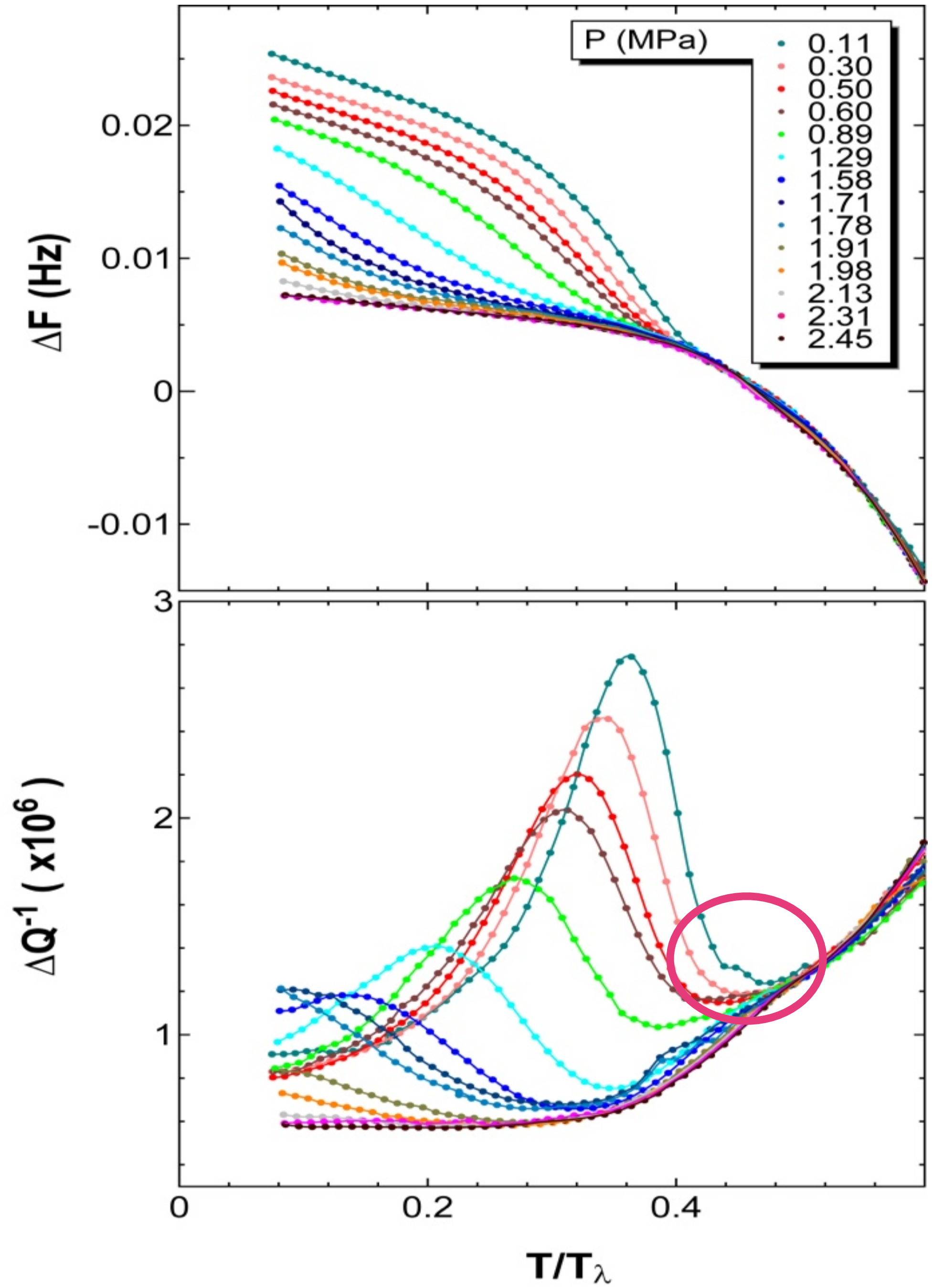


Double onset

Large incommensurability: $\Delta k_{10} = 0.5{a_0}^{-1}$

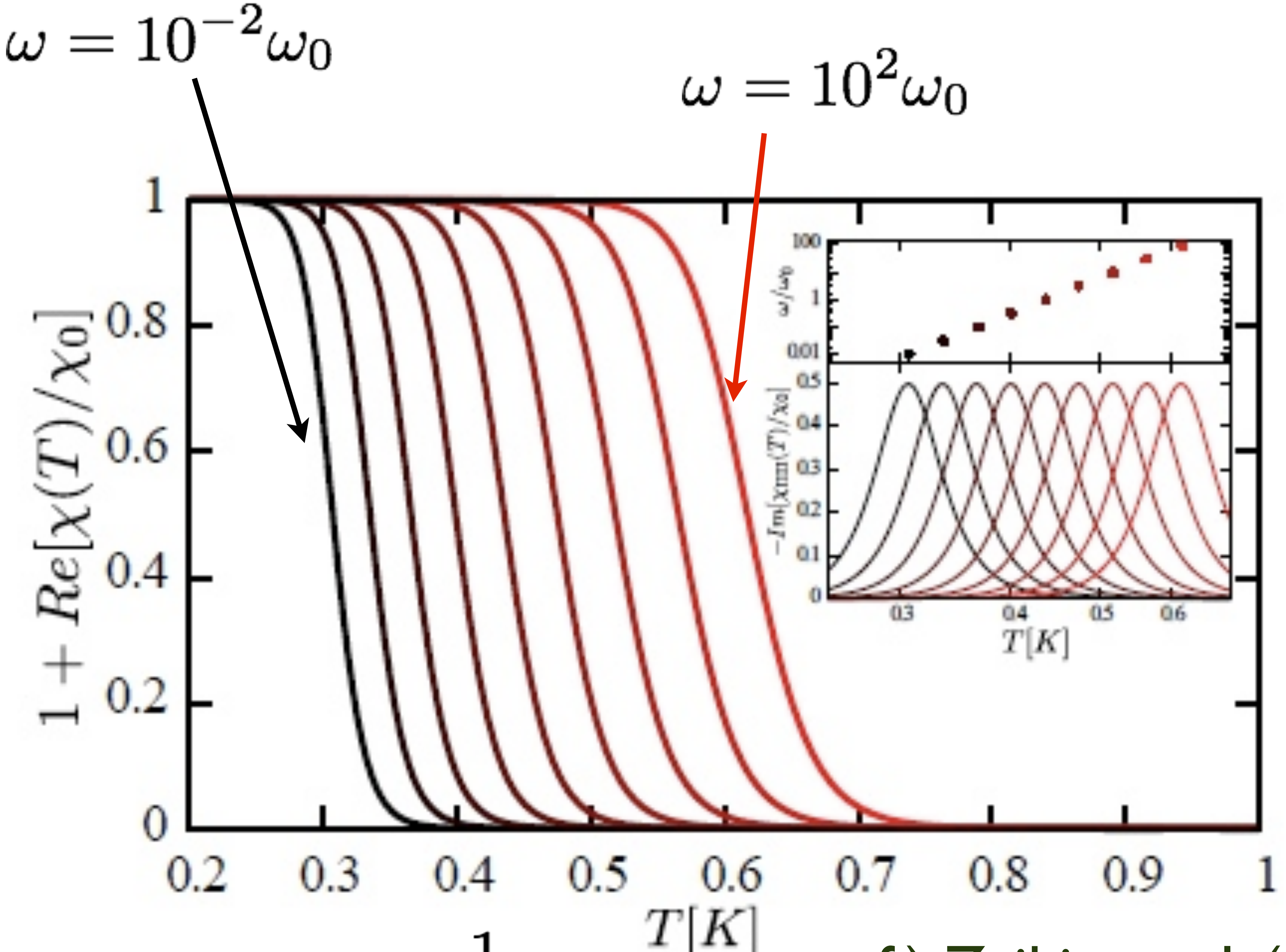
K=6.2





$\Delta k_{10} = 0.001{a_0}^{-1}$

Frequency Dependence



K=8.1

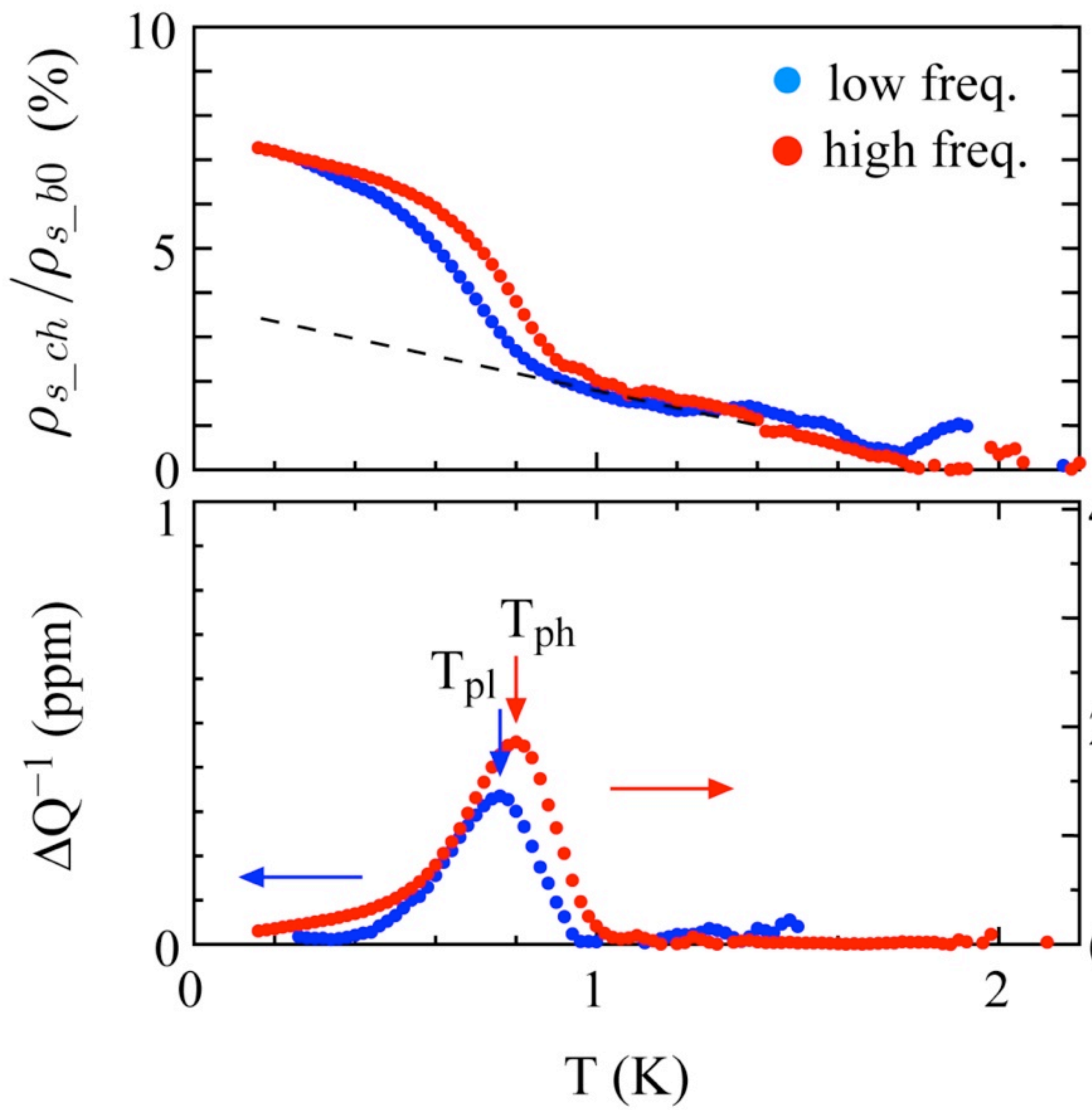
$$T_p \sim \omega^{\frac{1}{2K-3}}$$

cf.) Zaikin et al. (1997)
Lobos-Giamarchi (2005)
Danshita-Polkovnikov (2011)

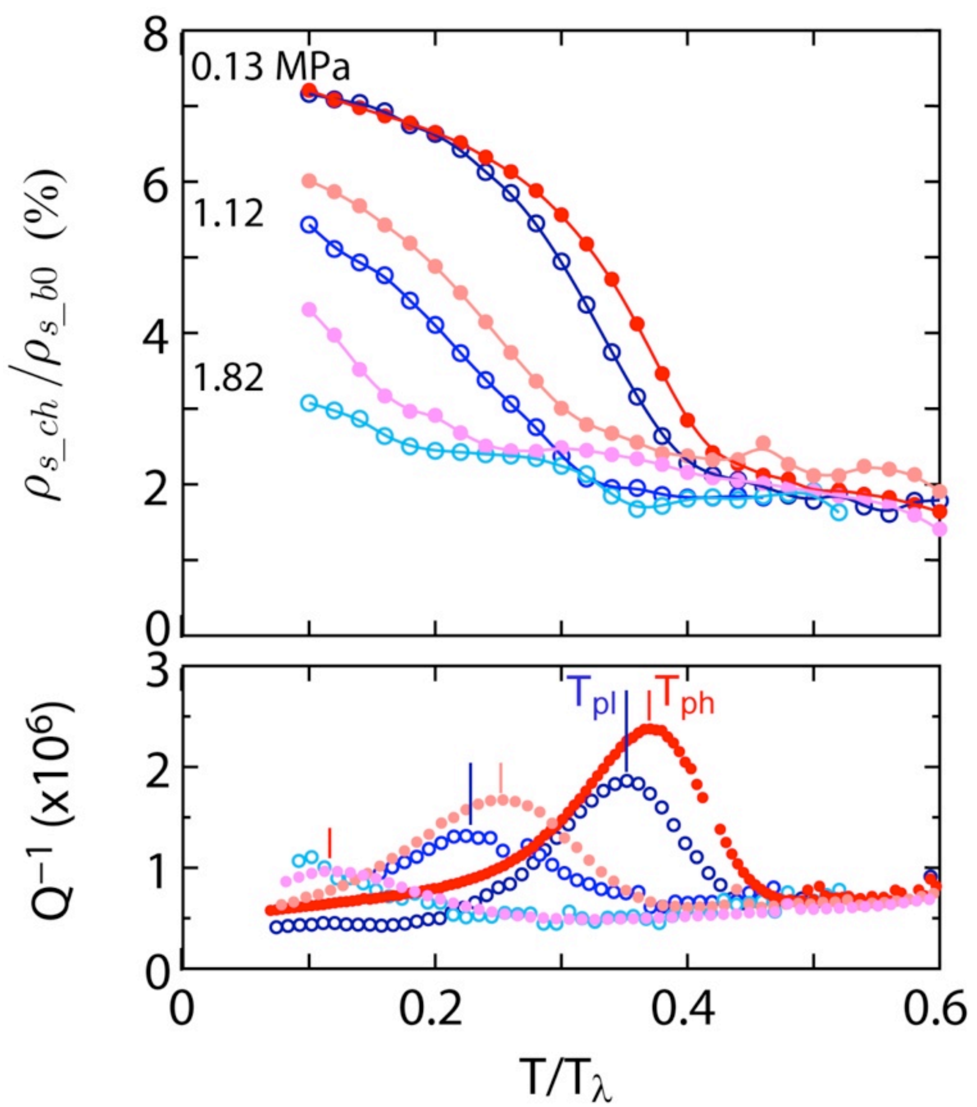
Frequency dependence (expt.)

J. Taniguchi et al. (private communications)

500Hz vs. 2000Hz



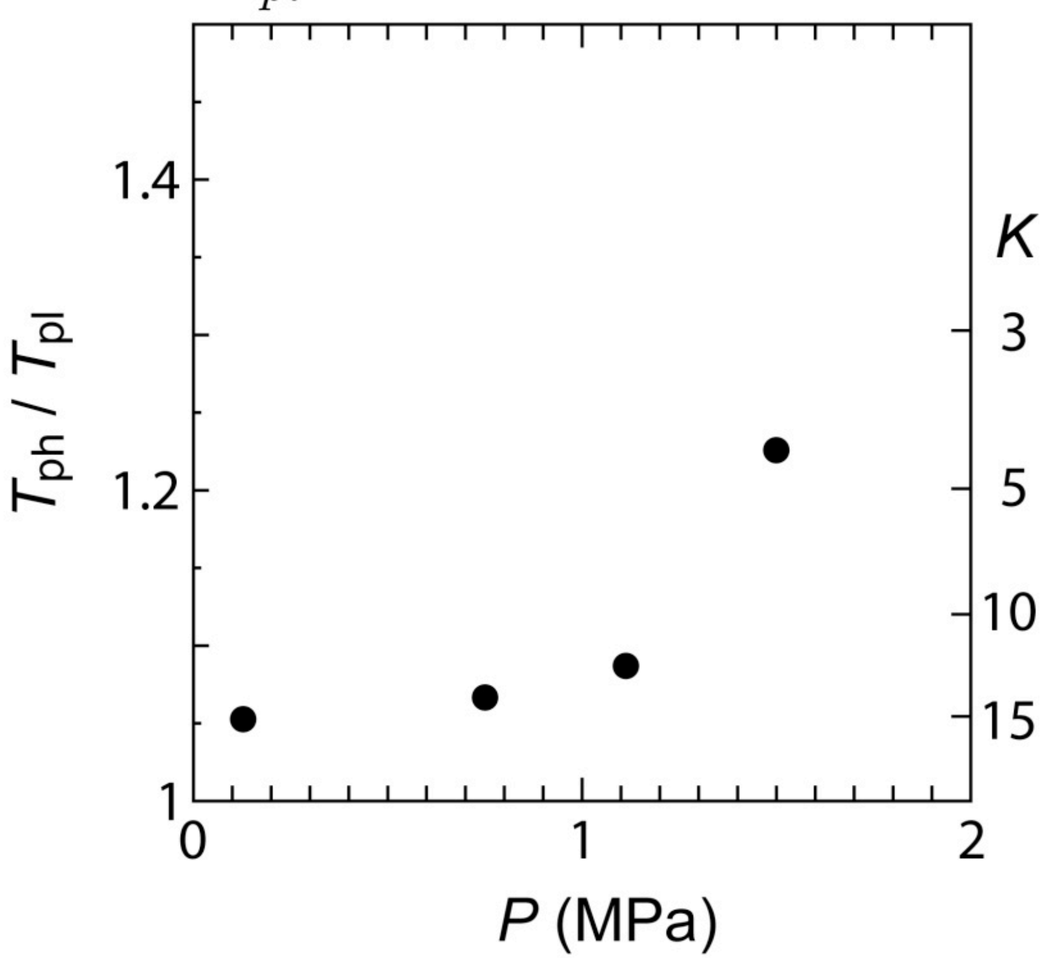
pressure effect



Frequency dependence (expt.)

J. Taniguchi et al. (private communications)

$$\frac{T_{ph}}{T_{pl}} \propto \left(\frac{2000\ \mathrm{Hz}}{500\ \mathrm{Hz}}\right)^{\frac{1}{2K-3}}$$



$$T_p \sim \omega^{\frac{1}{2K-3}}$$

may be explained by
the pressure dependence of
the Luttinger parameter?

Relevance to Cold Atoms

RL **94,** 120403 (2005) PHYSICAL REVIEW LETTERS week ending 1 APRIL 2005

Strongly Inhibited Transport of a Degenerate 1D Bose Gas in a Lattice

C. D. Fertig,[1,2] K. M. O'Hara,[1,*] J. H. Huckans,[1,2] S. L. Rolston,[1,2] W. D. Phillips,[1,2] and J. V. Porto[1]
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(Received 15 November 2004; published 1 April 2005)

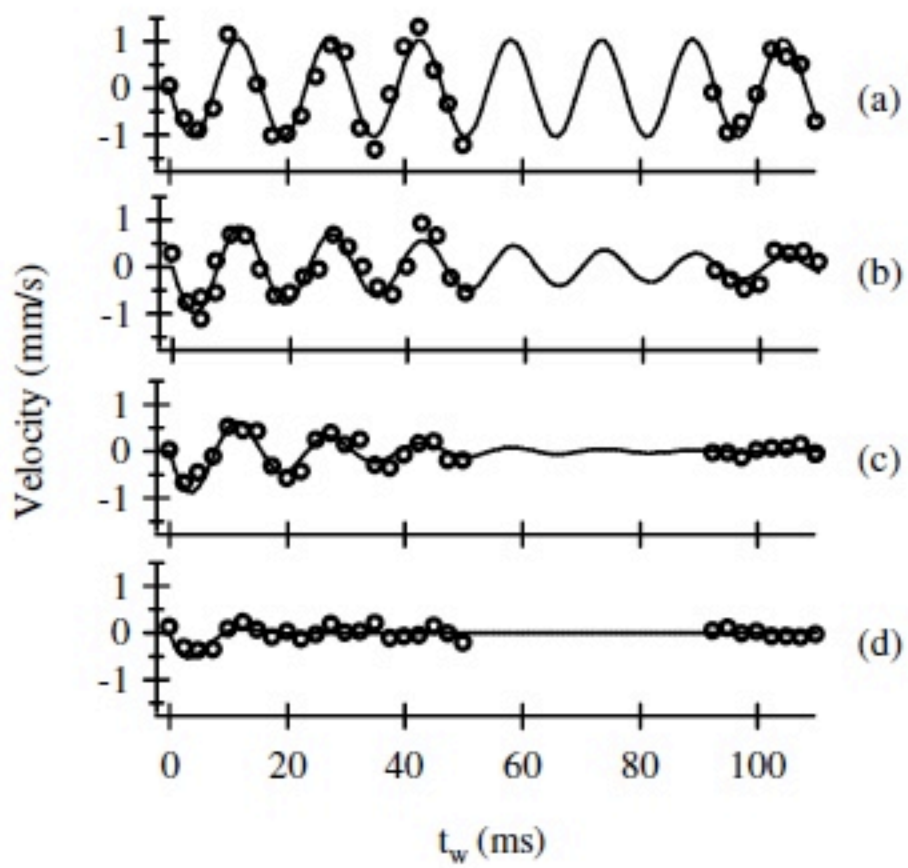
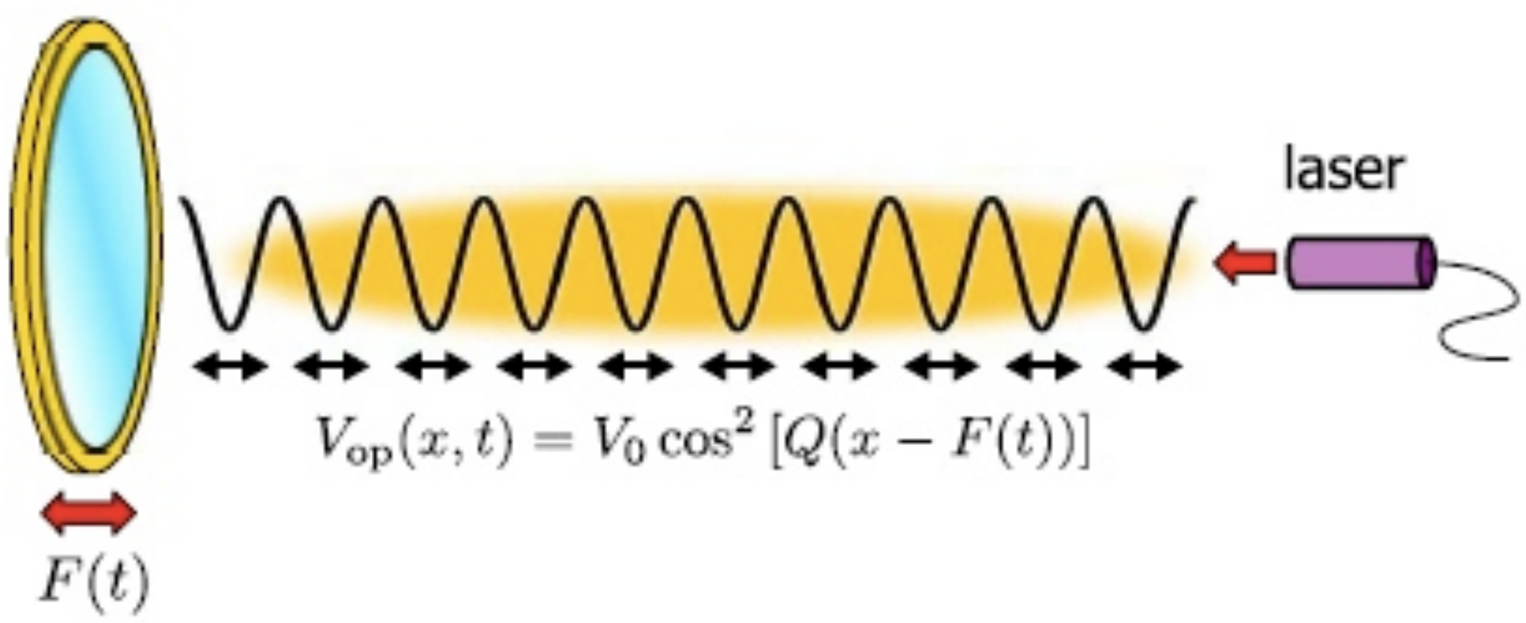


FIG. 1. Damped oscillations of a 1D Bose gas in an optical lattice. Shown are plots of velocity versus wait time t_w from $t_w = 0$ to 110 ms, and for axial lattice depths of (a) $0E_R$,

Relevance to cold atoms

[Tokuno-Giamarchi 2011]

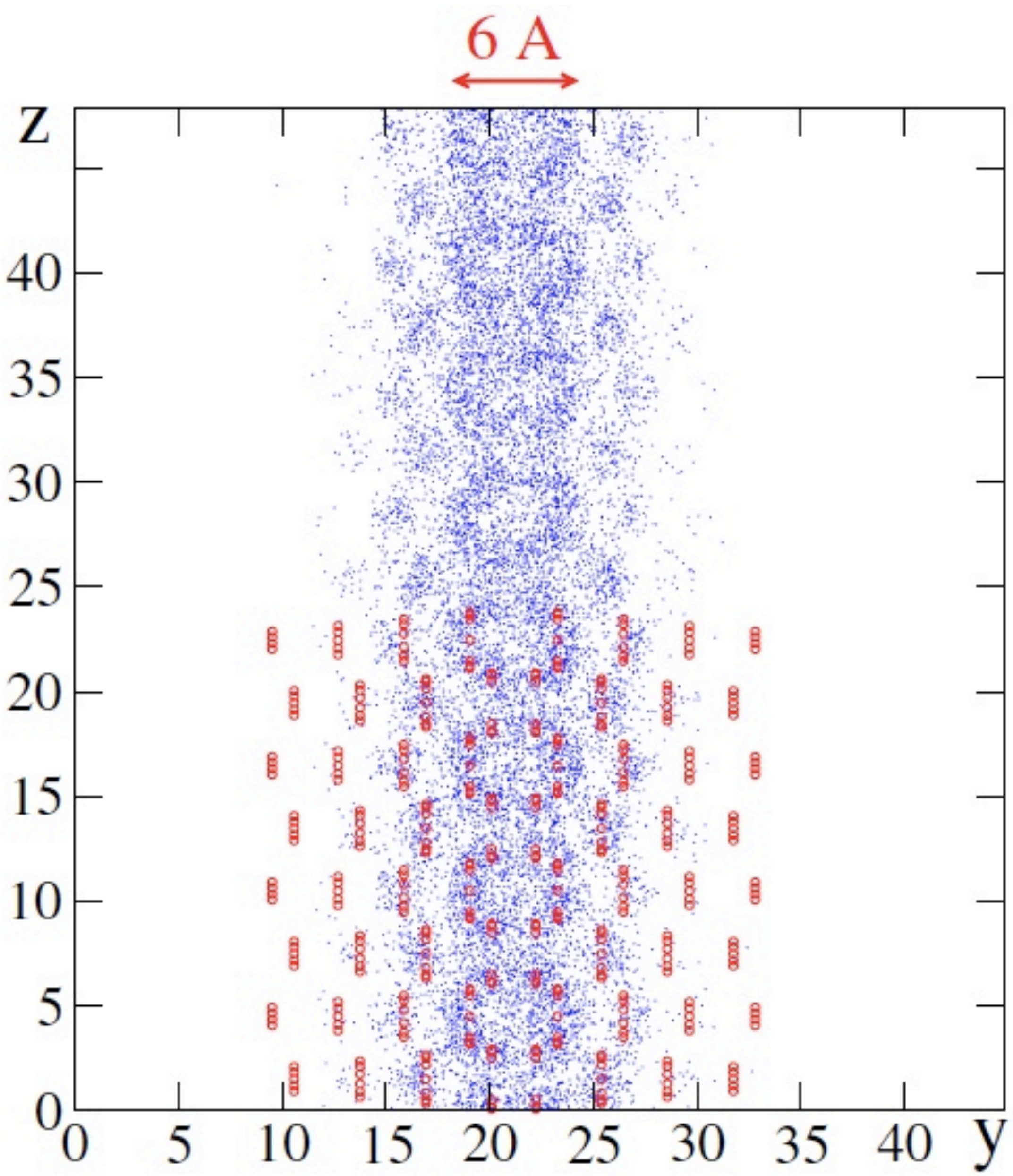


$$\overline{\dot{E}(\omega, T)} \propto -\omega \, \mathrm{Im}\, \chi(\omega, T)$$

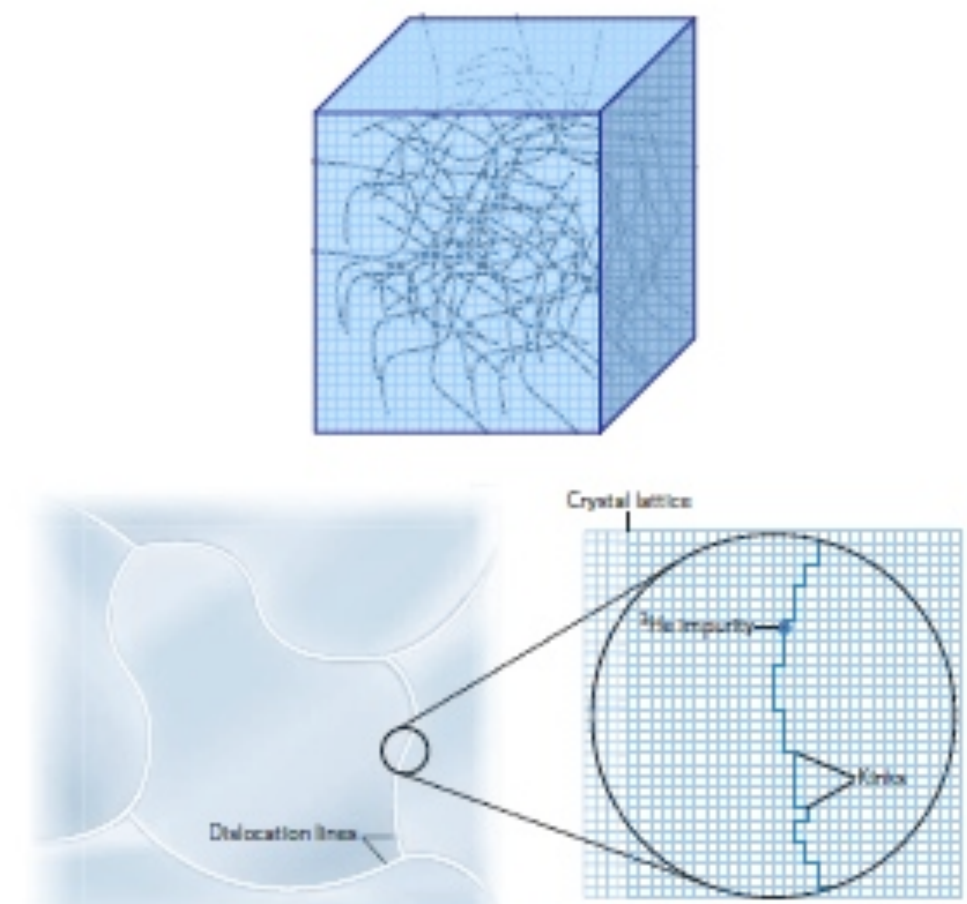
Frequency dependence may be probed over a wider range, than in torsional oscillator measurements of ^{4}He

Relevance to “supersolid”?

Skew dislocations in solid 4He behaves as TLL
[Boninsegni et al. 2007]



Dislocation network
(“Shevchenko state”)



[Balibar 2010]

Conclusions

- Helicity modulus in 1D vanishes (in thermodynamic limit)
- Superfluidity in 1D is essentially dynamical phenomenon, related to absence of (or anomalously slow) thermalization
- "Superfluid density" dependence on probe frequency is predicted
- Momentum response couples to 2 conserved currents in TLL / conservation broken by wall potential
- Qualitative agreement with 4He in 1D nanopore
- Possible relevance to dislocations in solid 4He, and to cold atoms