Novel quantum criticality of topological phase transitions

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References

- B.-J. Yang, E. –G. Moon, H. Isobe, and N. Nagaosa, "Quantum criticality of topological phase transitions in 3D interacting electronic systems", Nature Physics (2014)
- H. Isobe, B.-J. Yang, A. Chubukov, J. Schmalian, and N. Nagaosa, "Emergent non-Fermi liquid at the quantum critical point of a topological phase transition in two dimensions", Phys. Rev. Lett. (2016).
- J. Ahn and B.-J. Yang, "Unconventional topological phase transition in two dimensional systems with space-time inversion symmetry", submitted.

Quantum states in condensed matters

" Principle of broken symmetry "

	Magnet	Superconductor
Broken symmetry	Spin rotation	Gauge
Order parameter	Magnetization	Pairing amplitude

- **Order parameter (M) : measure of broken symmetry**

$M \neq 0$
Ordered phase
(Symmetry broken)

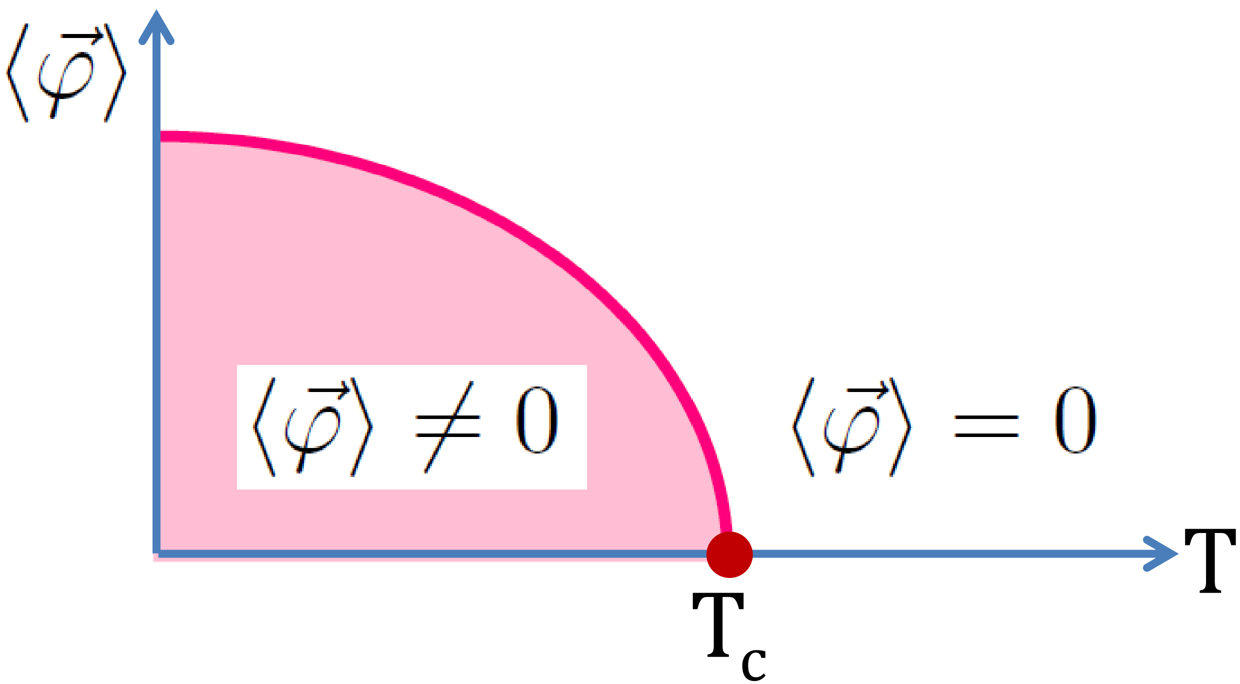
$M = 0$
Disordered phase
(Symmetric)

(Quantum) phase transition

Ginzburg-Landau theory

Effective action for 3D magnet

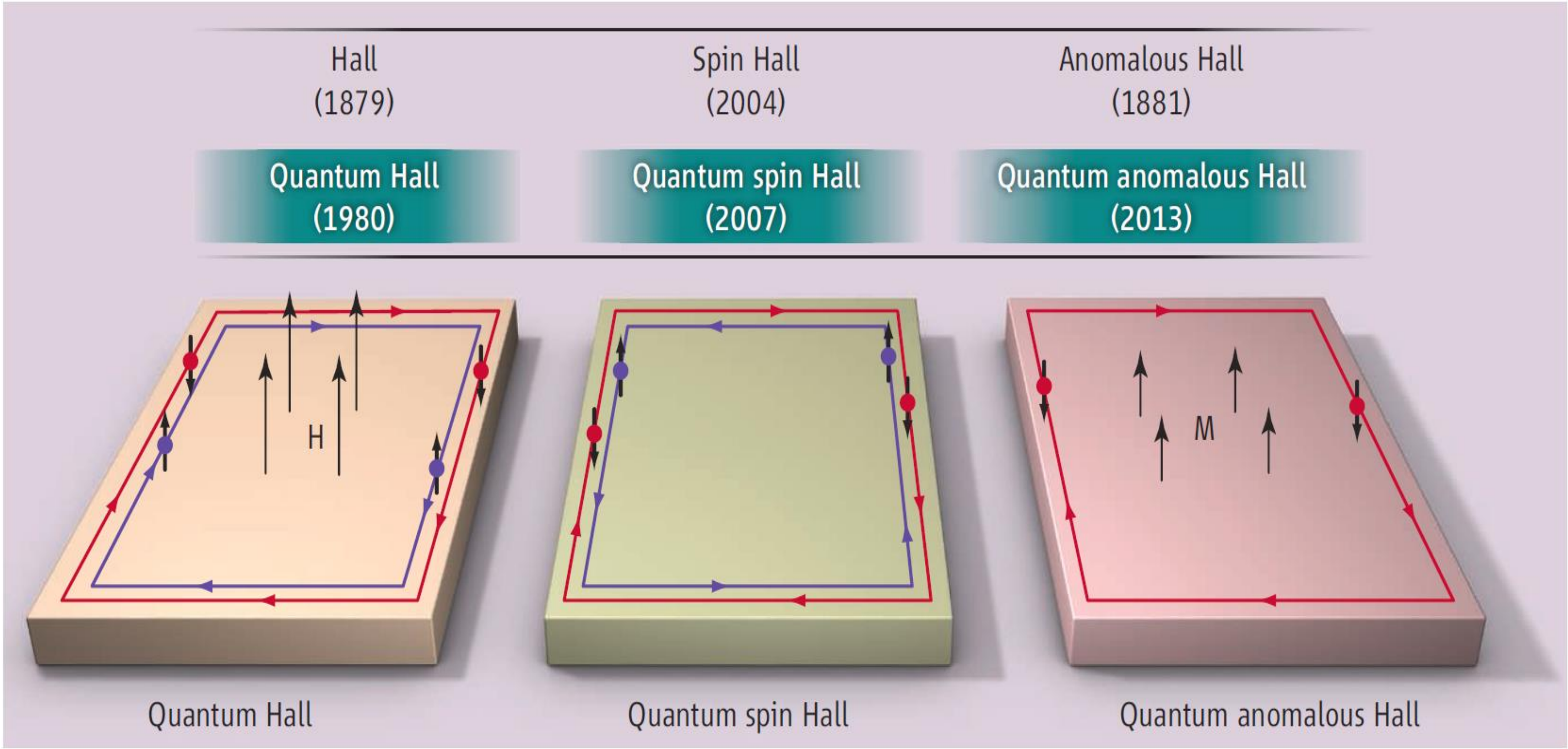
$$S_{\rm eff}(\vec{\varphi}) = \int d^d x \left[(\partial_\mu \vec{\varphi})^2 + r|\vec{\varphi}|^2 + u(|\vec{\varphi}|^2)^2 \right], \quad \vec{\varphi} \in O(3),$$



Critical point :Universality

Topological phases

- A bulk phase is characterized by a topological invariant
- Distinguish gapped phases sharing same symmetries



Seongshik Oh (Science 2013)

Classification of bulk topological phases

A. P. Schnyder, S. Ryu, A. Furusaki, A.W.W. Ludwig, A. Kitaev

	symmetry			d							
	$\mathcal{T}^2$	$\mathcal{C}^2$	$\mathcal{S}^2$	0	1	2	3	4	5	6	7
A	0	0	0	Z	0	Z	0	Z	0	Z	0
AIII	0	0	1	0	Z	0	Z	0	Z	0	Z
AI	1	0	0	Z	0	0	0	2Z	0	Z_2	Z_2
BDI	1	1	1	Z_2	Z	0	0	0	2Z	0	Z_2
D	0	1	0	Z_2	Z_2	Z	0	0	0	2Z	0
DIII	−1	1	1	0	Z_2	Z_2	Z	0	0	0	2Z
AII	−1	0	0	2Z	0	Z_2	Z_2	Z	0	0	0
CII	−1	−1	1	0	2Z	0	Z_2	Z_2	Z	0	0
C	0	−1	0	0	0	2Z	0	Z_2	Z_2	Z	0
CI	1	−1	1	0	0	0	2Z	0	Z_2	Z_2	Z

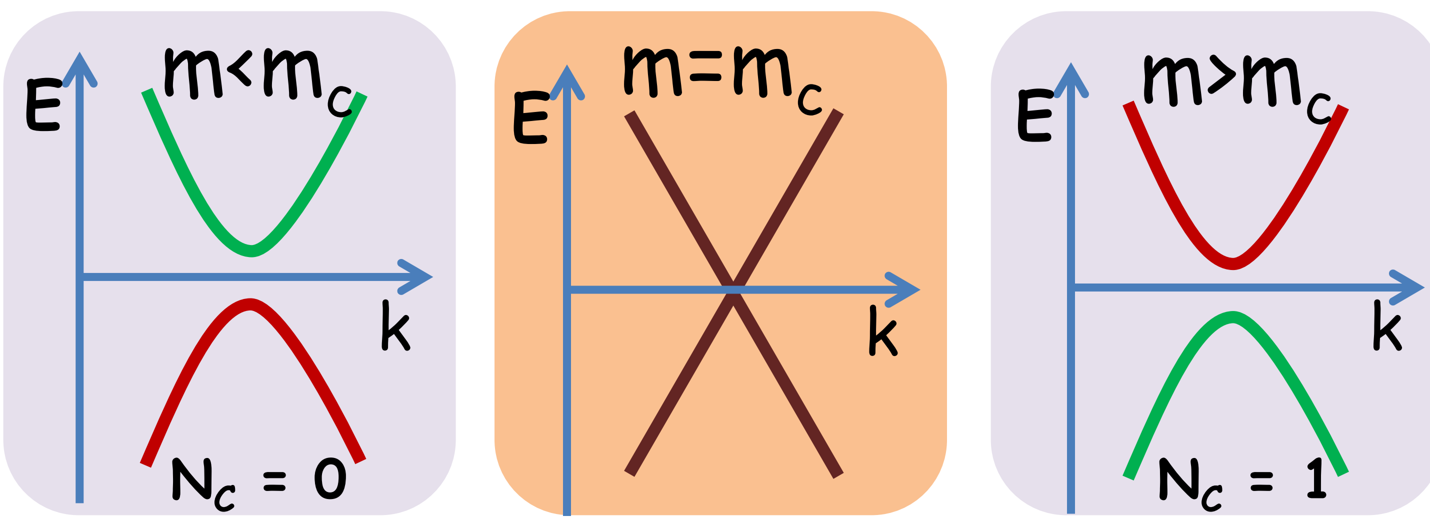
$\mathcal{T}$: Time-reversal $\mathcal{C}$: Particle-hole $\mathcal{S}$: Chiral

- Various topological insulators can exist in nature!
- Bulk properties of topological insulators are well-established

Nonzero topological invariant ⟷ (Bulk-boundary correspondence) ⟷ Metallic states on the boundary

How much do we understand topological phase transitions ?

	Broken symmetry phase	**Topological phase**
Bulk phase	Order parameter	Topological invariant
Phase transition	Ginzburg-Landau theory	Band-crossing theory
Low energy excitations	Critical bosons	Emergent Dirac particles

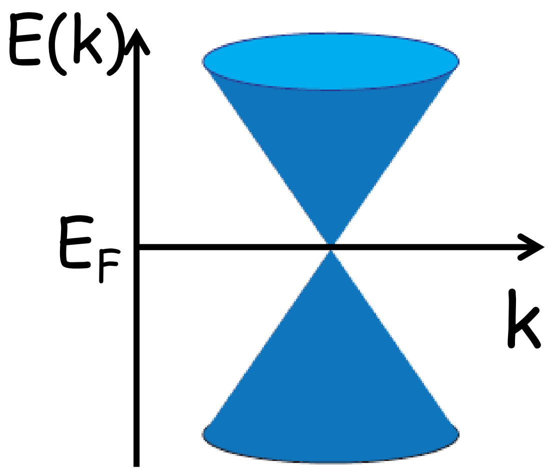


- m = an external control parameter such as pressure, doping, etc.

Quantum critical point of topological PT

"Criticality of interacting Weyl/Dirac fermions"

$$H_{QCP} = v_1 k_1 \sigma_1 + v_2 k_2 \sigma_2 + v_3 k_3 \sigma_3$$



Vanishing density of states

$$V_{sc}(r) = \frac{e^2}{\epsilon_0}\frac{1}{r}e^{-q_{TF}r} \quad , q_{TF}{}^2 \propto \mathrm{D}(E_F) = 0$$

Long-range Coulomb potential!

Fermi points (similar to 1D system)

Non-Fermi liquid (Luttinger liquid)

Outline

1. Novel quantum criticality of topological PT in 3D systems breaking P or T

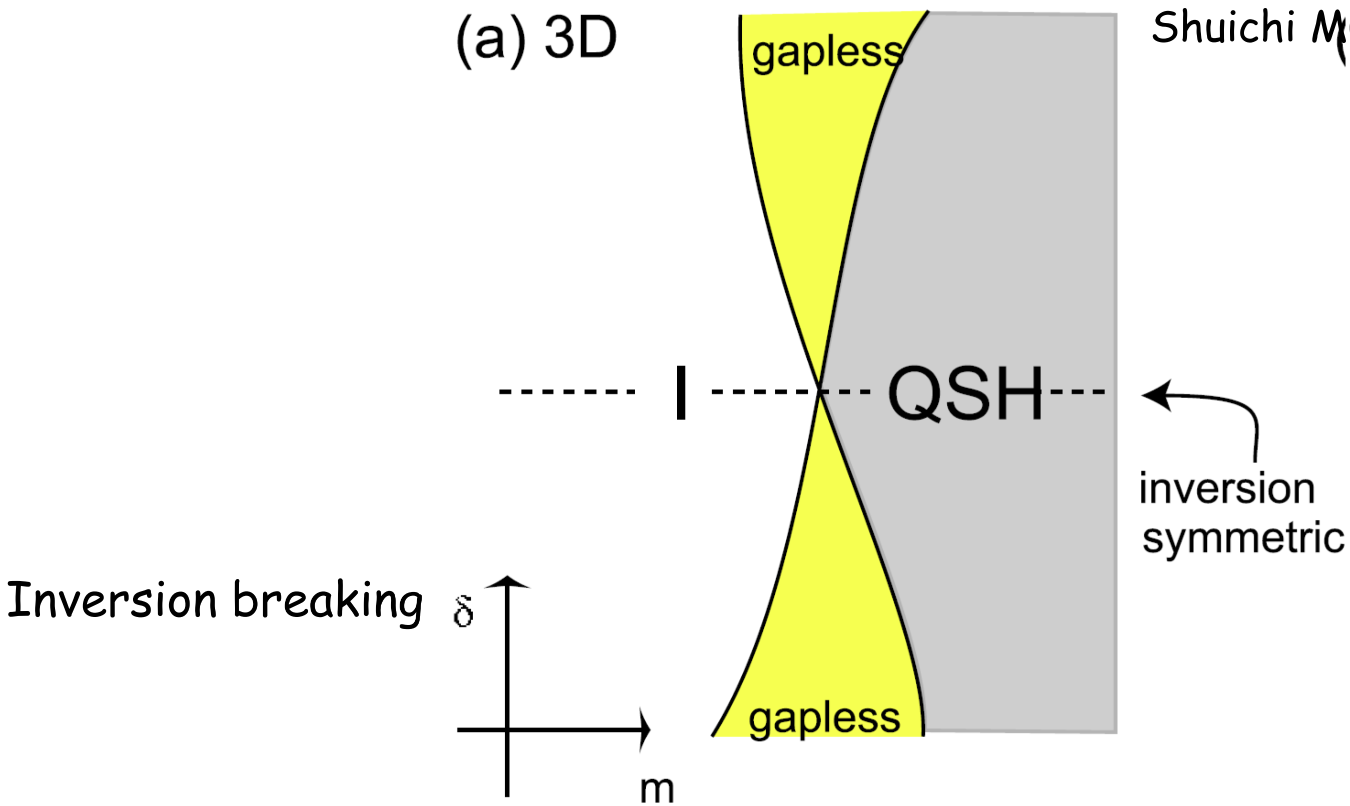
2. Novel quantum criticality of topological PT in 2D systems with PT symmetry or space-time inversion

3. Conclusion

Symmetry and topological PT

- Symmetry determines the phase diagram
- Two types of phase transitions

3D time-reversal symmetric systems

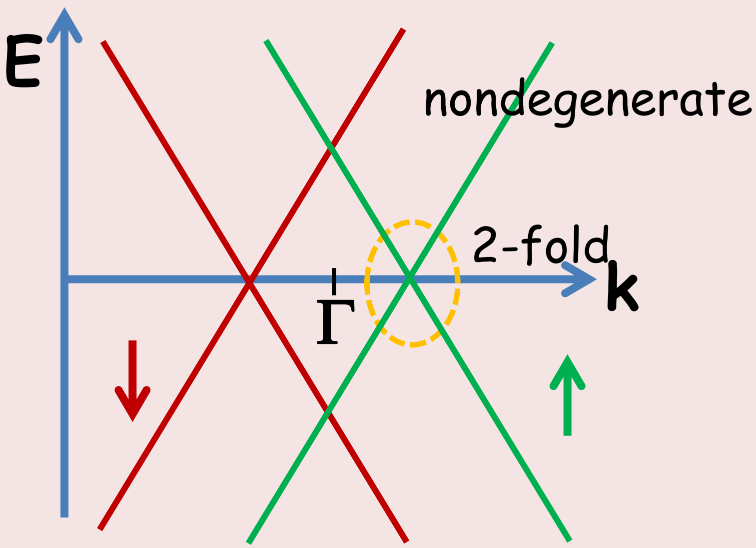


Shuichi Murakami, PRB(2008)

Tuning parameter
(electric field, doping, pressure, ..)

Symmetry and low energy excitations

Without inversion

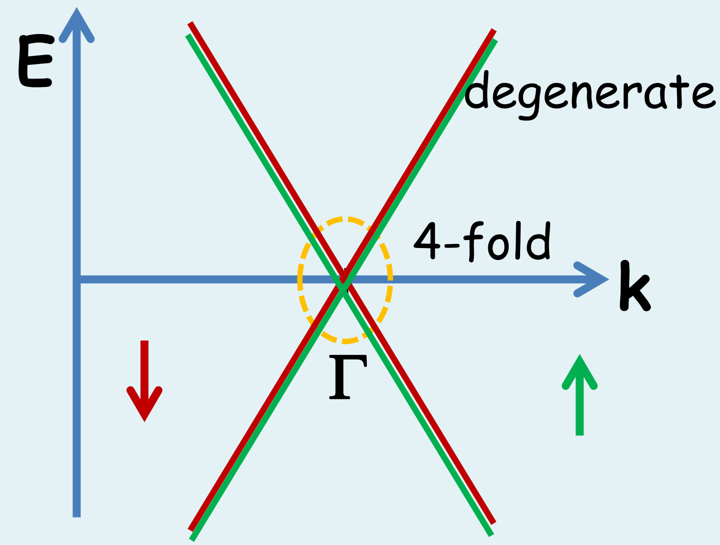


$$H(k) = k_x\sigma_x + k_y\sigma_y + k_z\,\sigma_z$$

2x2 matrix

"Weyl fermions"

With inversion

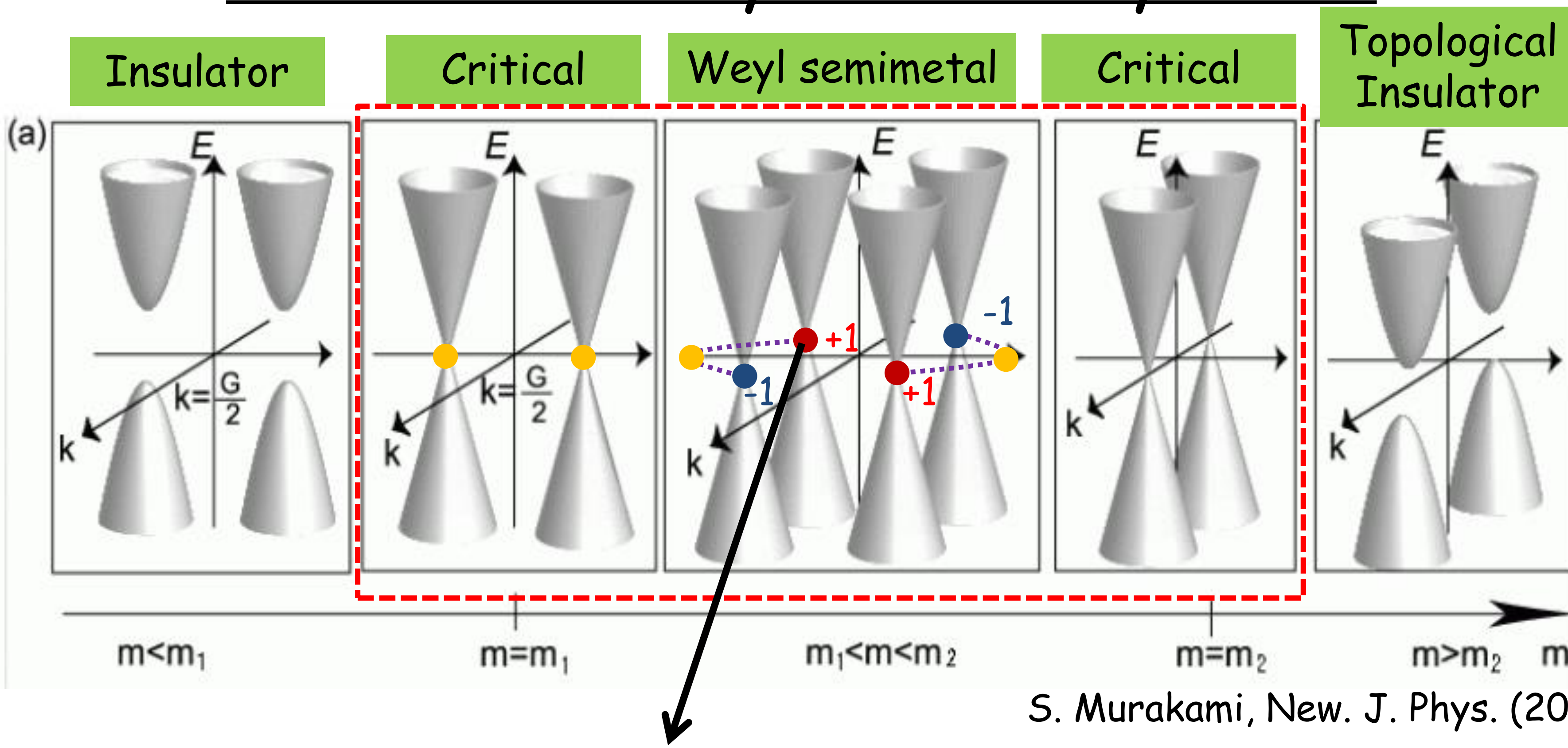


$$H(k) = \begin{bmatrix} k\cdot\sigma & 0 \\ 0 & -k\cdot\sigma \end{bmatrix}$$

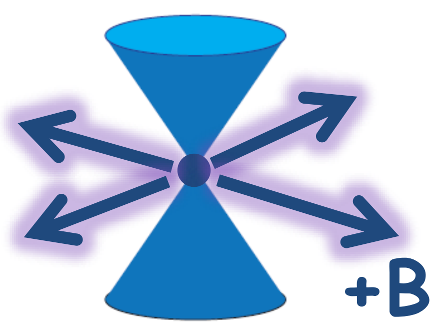
4x4 matrix

"Dirac fermions"

3D noncentrosymmetric systems



S. Murakami, New. J. Phys. (2007)



Weyl point carries a monopole charge!

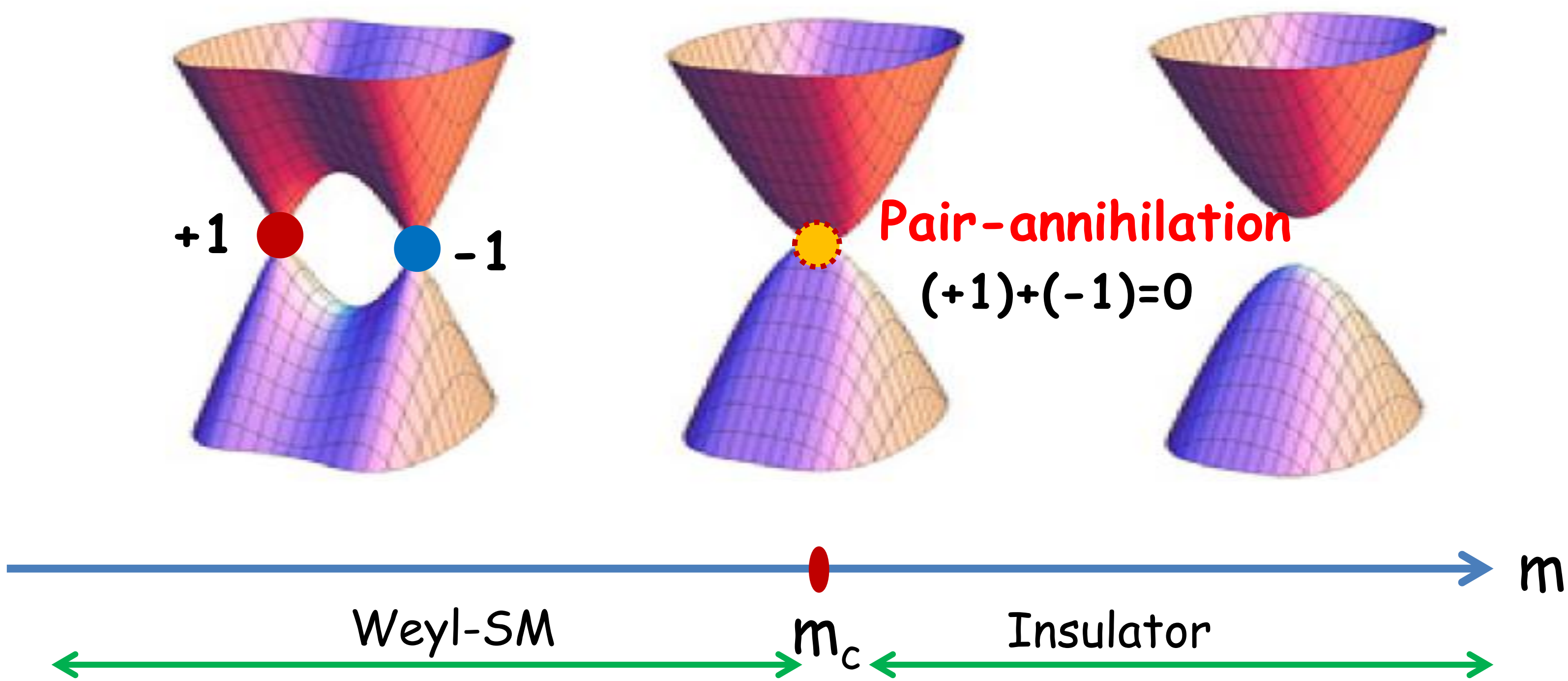
$$\mathbf{B} = \nabla \times \mathbf{A}(\mathbf{k}) \qquad \mathbf{A}(\mathbf{k}) = i\langle u_{\mathbf{k}}|\nabla_{\mathbf{k}}|u_{\mathbf{k}}\rangle$$

$$\frac{1}{2\pi}\nabla_{\mathbf{k}} \cdot \mathbf{B}(\mathbf{k}) = \pm\delta(\mathbf{k}) \qquad \left(\quad \mathbf{B}(\mathbf{k}) \propto \frac{1}{k^2}\hat{k} \right)$$

Transition from a Weyl SM to an insulator

The chiral charge of a Weyl point guarantees its stability

→ A pair-annihilation is required

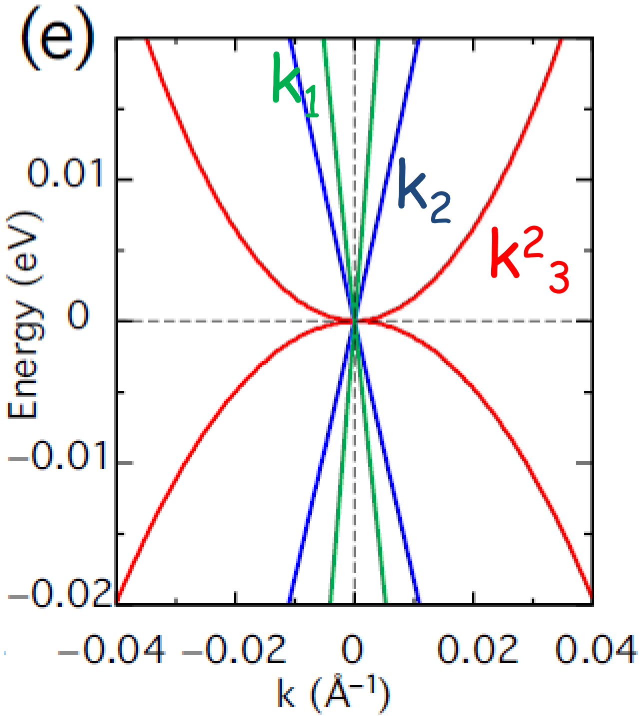


$$H = k_1 \sigma_1 + k_2 \sigma_2 + [(m-m_c) + k^2_3]\sigma_3$$

Anisotropic Weyl fermions at QCP

$$H_{QCP} = v\, k_1 \sigma_1 + v\, k_2 \sigma_2 + A\, k_3^2 \sigma_3$$

(S. Murakami)



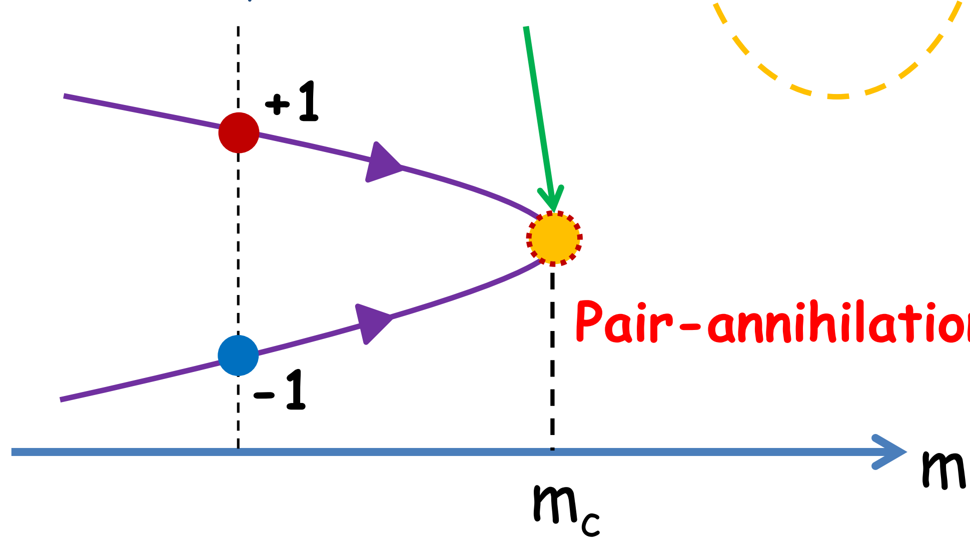
Anisotropic dispersion at QCP

: direct consequence of zero chiral charge !

$$H_{+Weyl} = v\, k_1 \sigma_1 + v\, k_2 \sigma_2 + v\, k_3 \sigma_3$$

+

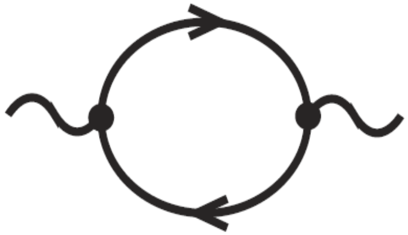
$$H_{-Weyl} = v\, k_1 \sigma_1 + v\, k_2 \sigma_2 - v\, k_3 \sigma_3$$



Screening at QCP

- Polarization

$$\Pi(\mathbf{q}) =$$



$$= -B_\perp q_\perp^{3/2} - B_3 q_3^2$$

- Screened Coulomb interaction

$$V_C(\mathbf{q}) \sim \frac{1}{q_\perp^{3/2} + \eta q_3^2}$$

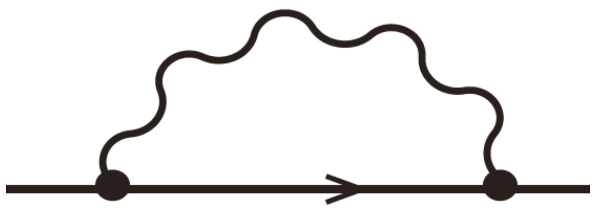
"Anisotropic partial screening!"

In real space : $V_\mathrm{C}(r_\perp, z=0) \sim \frac{1}{r_\perp^{5/4}}, \quad V_\mathrm{C}(r_\perp = 0, z) \sim \frac{1}{|z|^{5/3}},$

Effective Coulomb interaction between fermions became weaker !
(Screened Coulomb interaction is irrelevant)

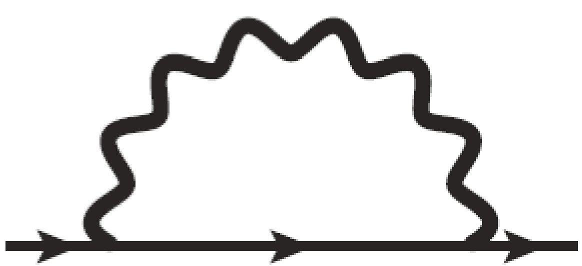
Irrelevance of screened Coulomb potenital

Bare Coulomb potential



~ Log(Λ/E)

Screened Coulomb potential



~ finite

New quantum criticality in 3D

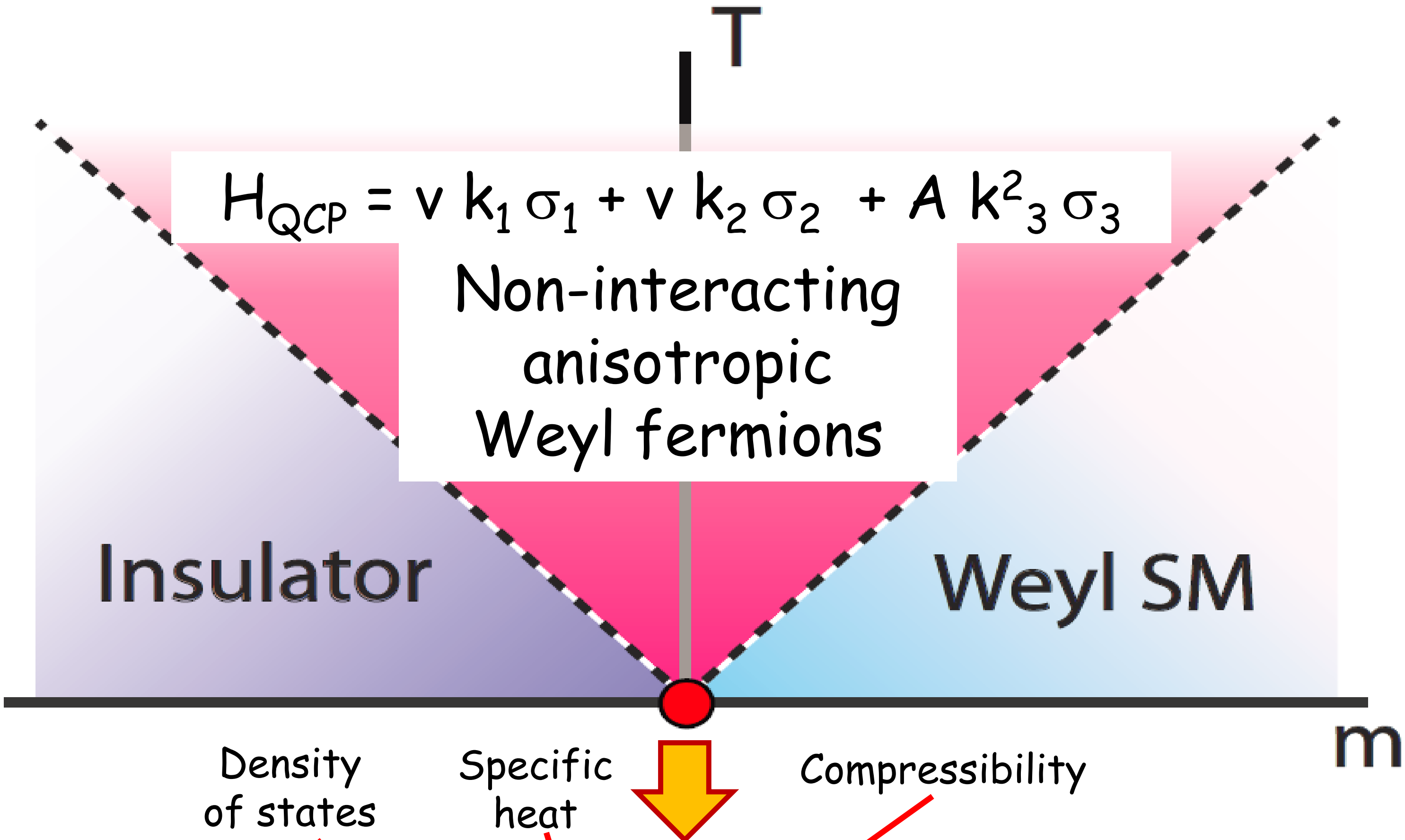
Unique metallic properties at the QCP!

- Novel screening effect
- New emergent fermions

	Screened Coulomb potential $V_C(q)$	Effective interaction between fermions
Conventional 3D Metal	$\frac{1}{q^2 + {q_{TF}}^2}$	Marginal
3D isotropic Weyl/Dirac SM	$\frac{1}{q^2}$	Marginally irrelevant
Anisotropic QCP	$\frac{1}{{q_\perp}^{3/2} + {q_3}^2}$	Irrelevant

(**B.-J. Yang,** E. G. Moon, H. Isobe, N. Nagaosa, Nature Physics, 2014)

Physical quantities at QCP



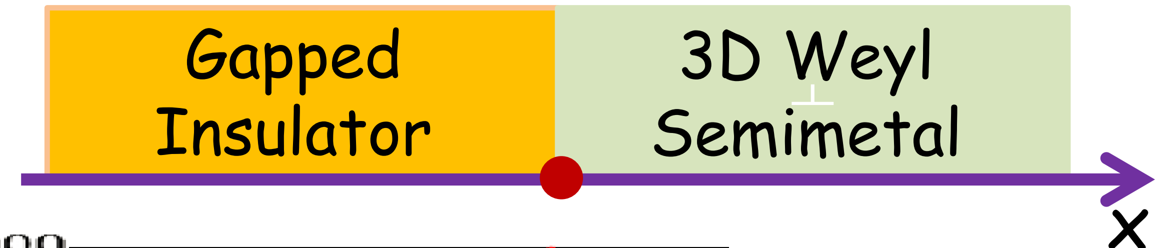
	$D(\varepsilon)$	$C_V(T)$	$\kappa(T)$	$\chi_D(T)$
Weyl semi-metal	ε^2	$T^3(\ln T)^3$	$T^2(\ln T)^3$	$(\ln T)^2$
At the QCP	$\varepsilon^{3/2}$	$T^{5/2}$	$T^{3/2}$	$T^{-1/2}$

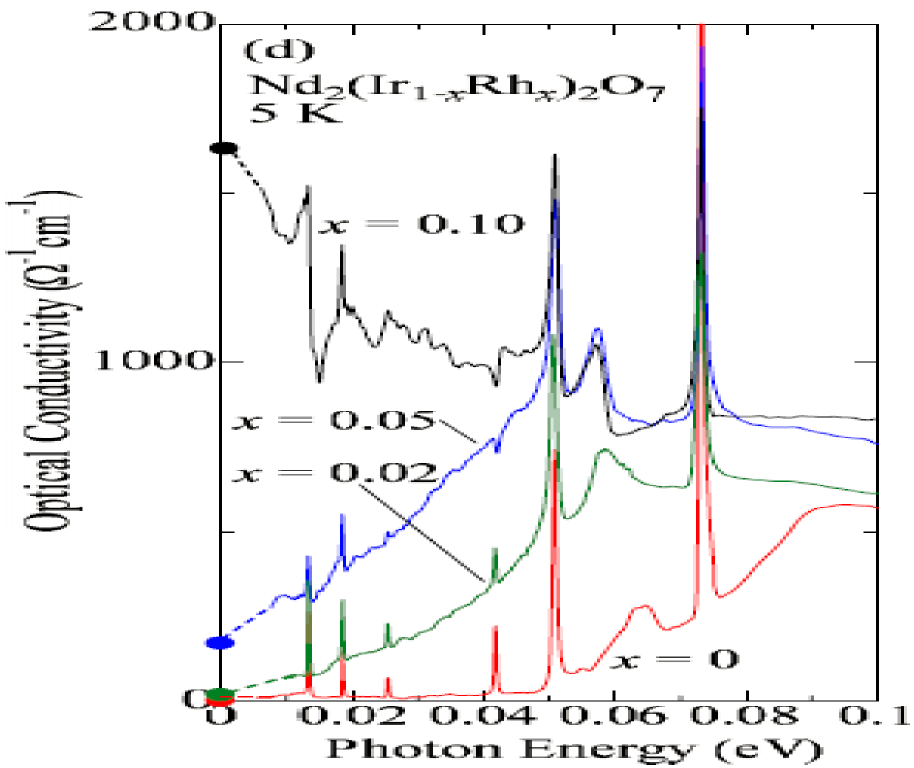
(**B.-J. Yang**, N. Nagaosa et al., PRL, 2013)

Candidate 1: pyrochlore iridates

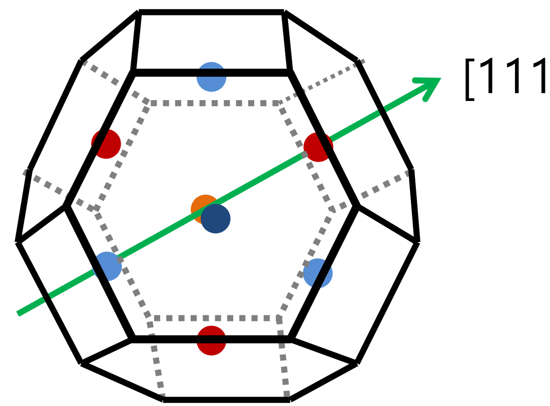
Insulator-semimetal transition is achieved

$Nd_2(Ir_{1-x}Rh_x)_2O_7$





Ueda, Fujioka,Tokura, PRL (2012)



$$\sigma^{(N)}(\omega) \approx N \frac{e^2}{12h} \frac{\omega}{v_F}$$

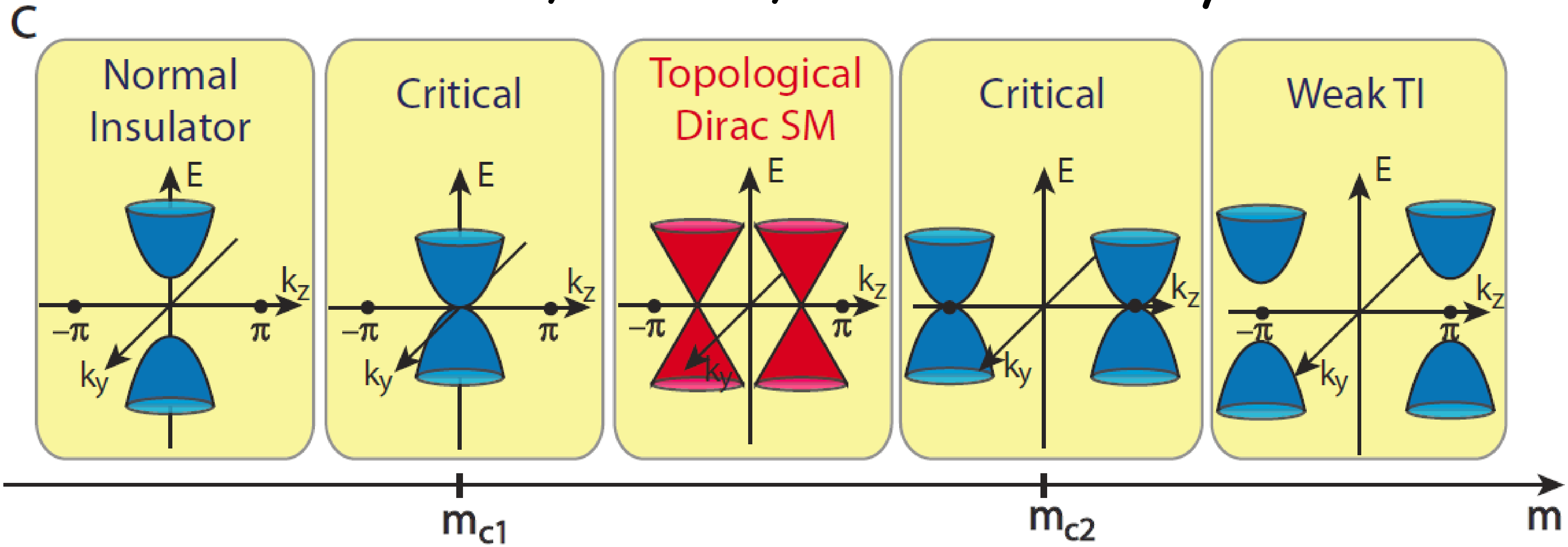
Hosur, Vishwanath

Candidate 2: 3D Dirac semimetals

Cd_3As_2, Na_3Bi, $ZrTe_5$ (Q.Li's and I. Pletikosie's talks)

Liu, Shen, Fang, Dai, Chen (Science,2014); Xu, Bansil, Cava, Hasan (arXiv:1312.7624); Neupane, Hasan (arXiv:1309.7892); Borisenko, Cava (arXiv:1309.7978);

- **Time-reversal, inversion, uniaxial rotation symmetries**



B.J.Yang and N. Nagaosa, Nature Comm.2014

- A single anisotropic Weyl fermion appears at QCP
- Quadratic dispersion along kz direction at QCP

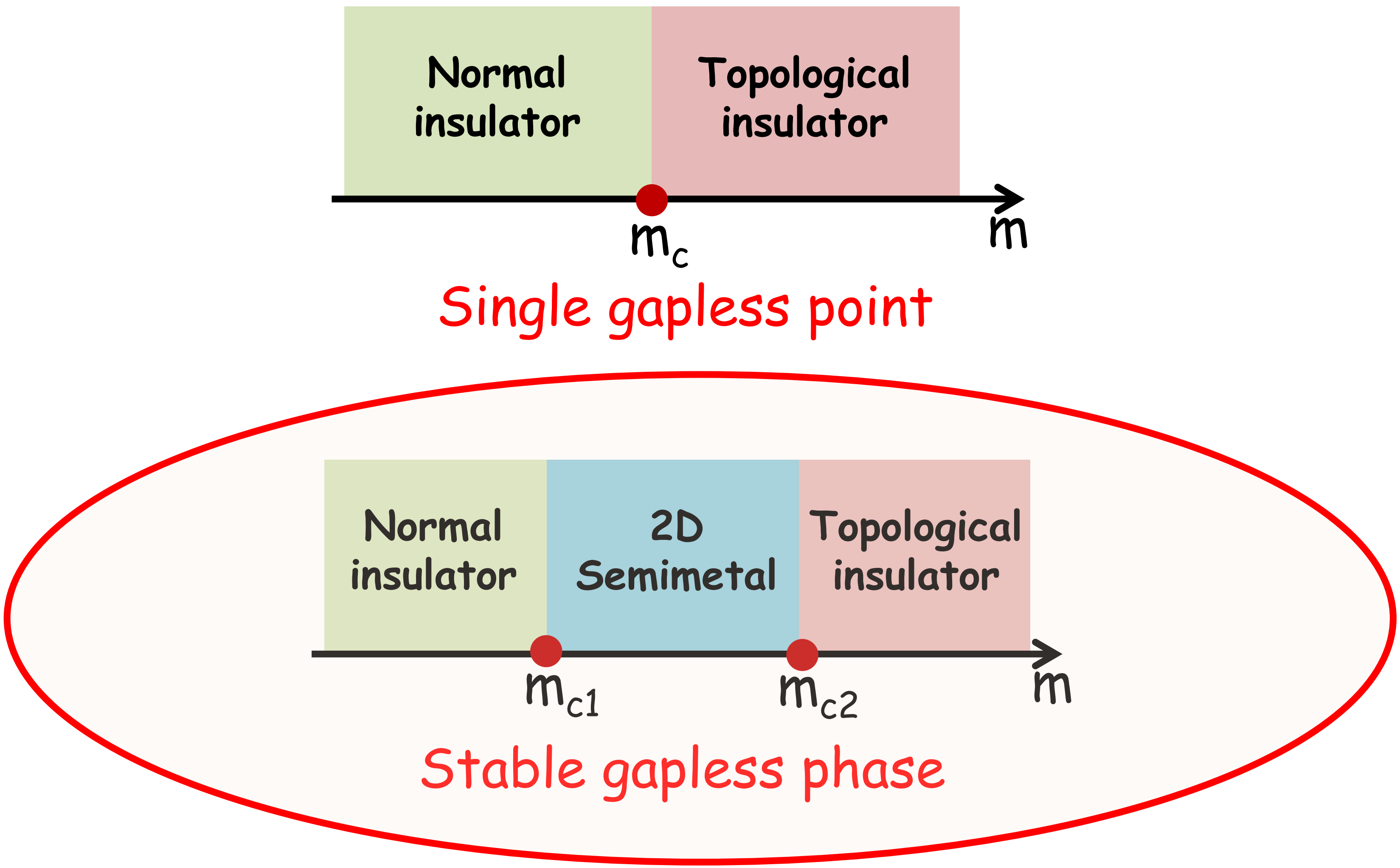
Outline

1. Novel quantum criticality of topological PT in 3D systems breaking P or T

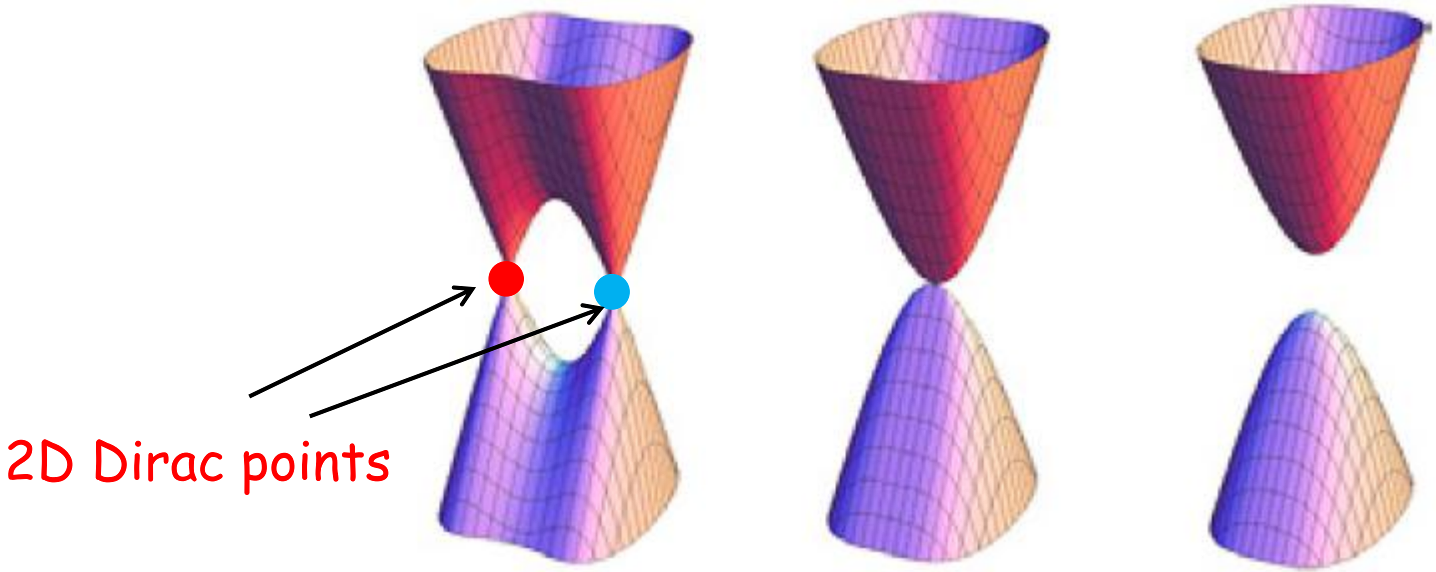
2. Novel quantum criticality of topological PT in 2D systems

3. Conclusion

Topological phase transition in 2D

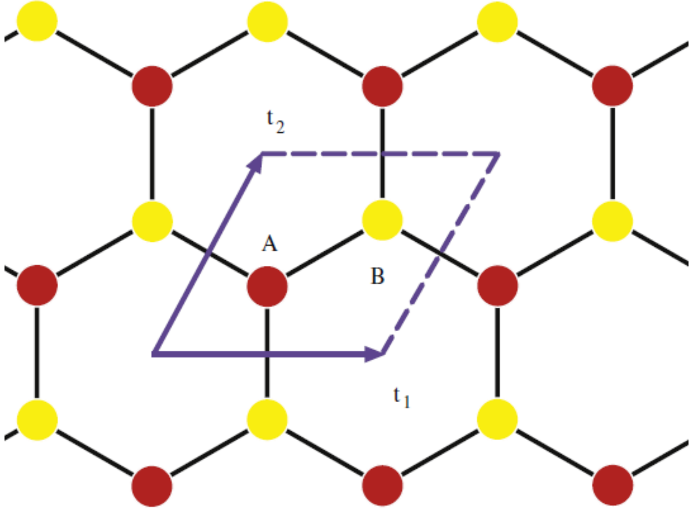


Merging transitions in 2D ?

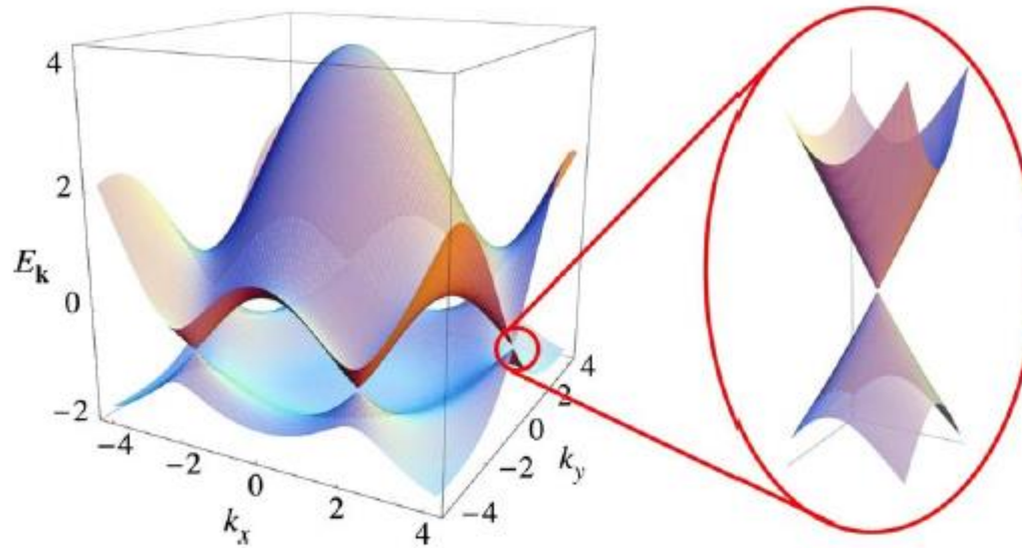


- Each gap-closing point should be stable (quantized topological charge)
- The location of gap-closing point should be tunable

Stable Dirac points in graphene



Vozmediano et al. Phys. Rep. (2010)



Castro Neto et al. Rev. Mod. Phys. (2009)

Quantized Berry phase: $\exp i \oint_C \mathbf{A} \cdot d\mathbf{k} = -1$

i) Time-reversal and inversion symmetries
ii) Vanishing spin orbit coupling

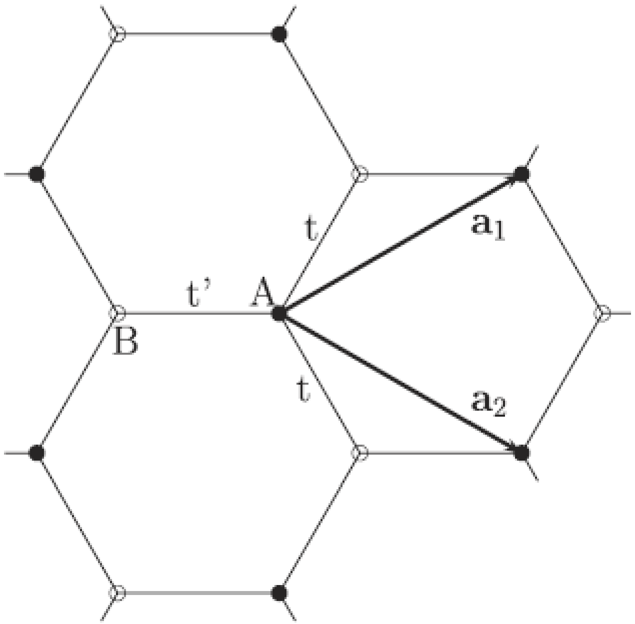
Time-reversal: F(k)=-F(-k)
Inversion: F(k)=F(-k)

F(k)=0

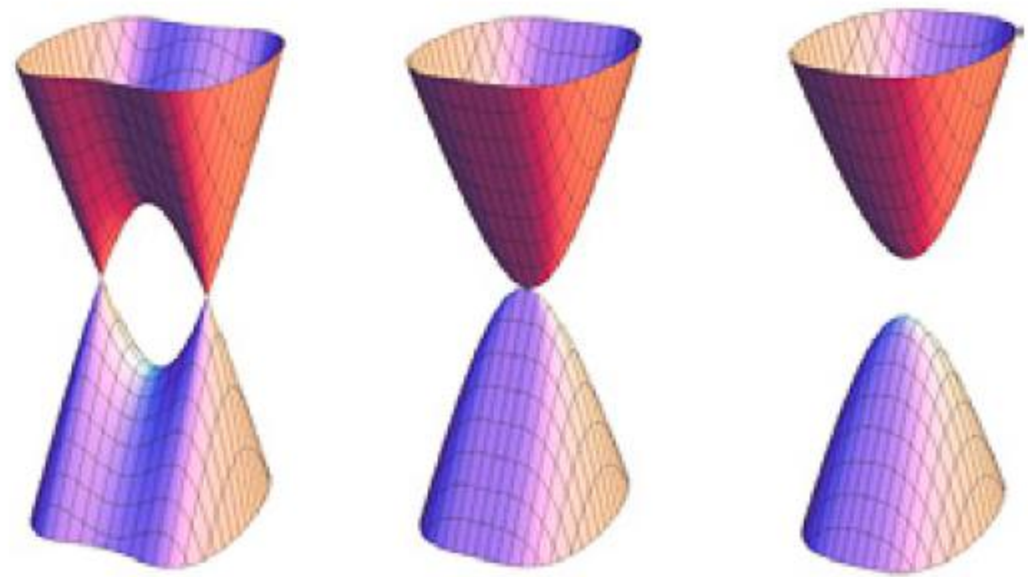
How to move the Dirac points

- Modulate n.n. hopping amplitudes

(Hasegawa , Konno, Nakano, Kohmoto (2006))



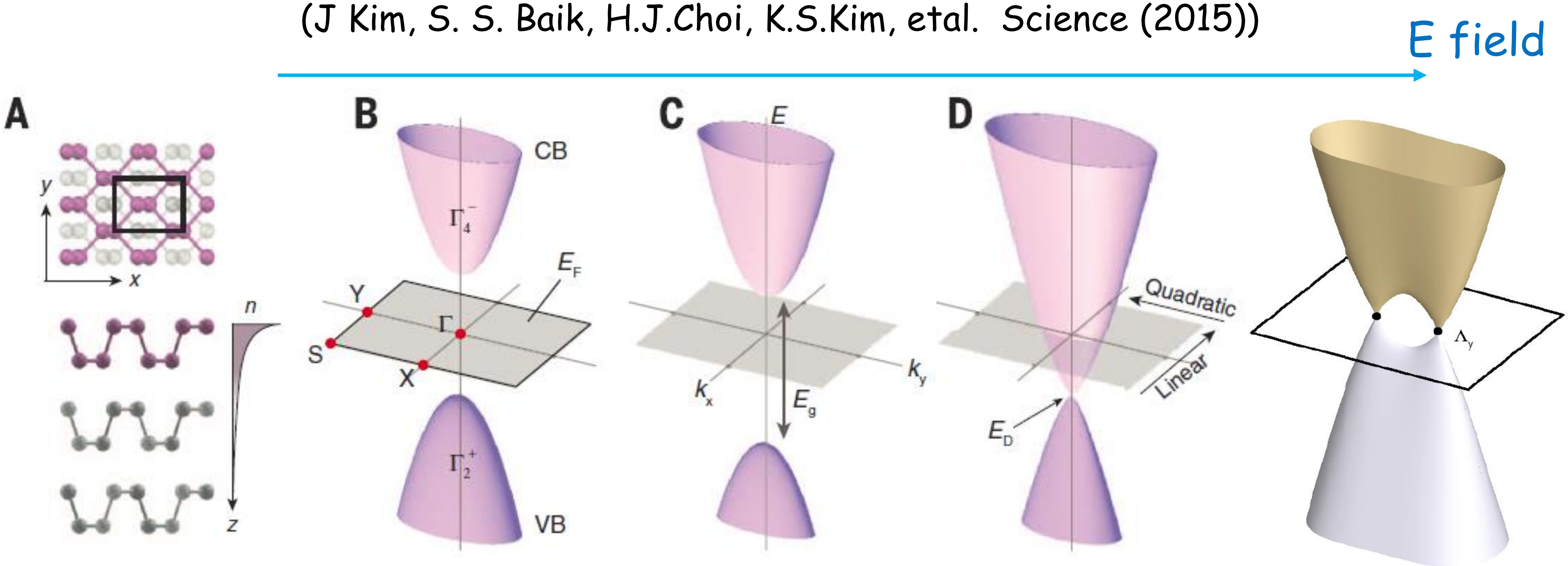
$t<t'<2t$ $t'=2t$ $t'>2t$

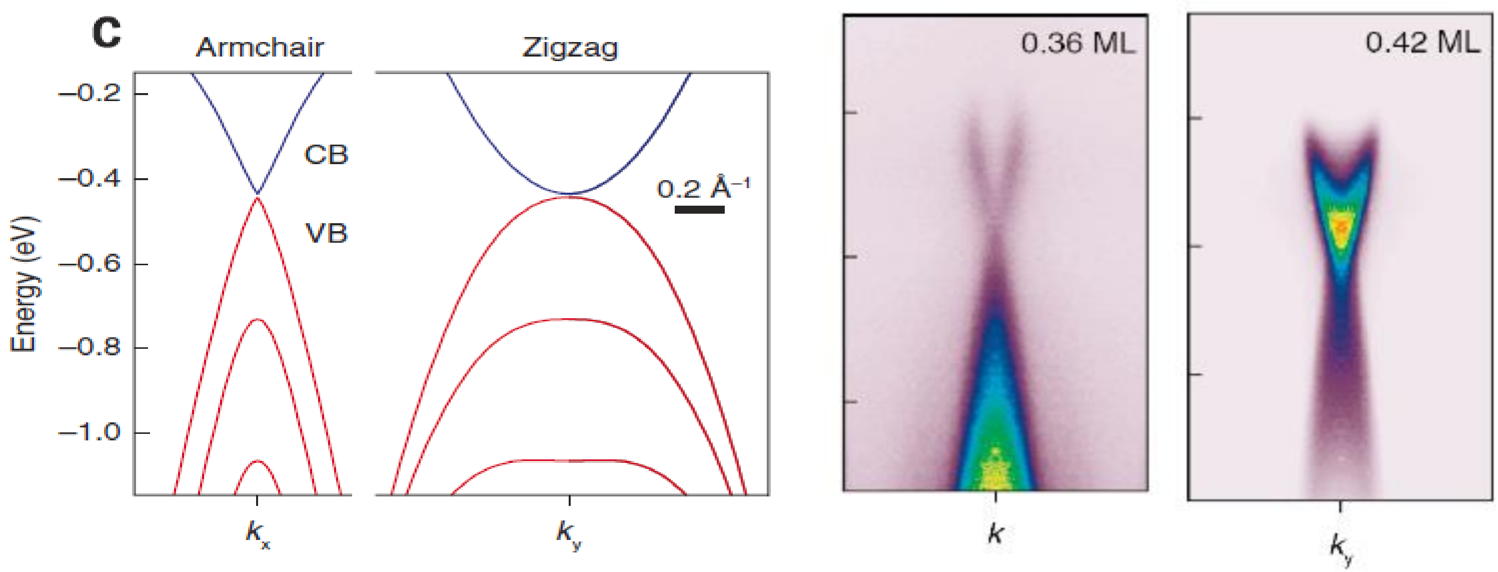


$$H = [(t'-2t)+ k^2_1]\sigma_1 + k_2 \sigma_2$$

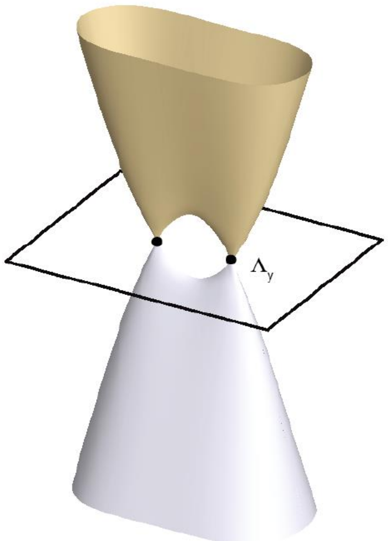
2D anisotropic Weyl in black phosphorus

(J Kim, S. S. Baik, H.J.Choi, K.S.Kim, etal. Science (2015))

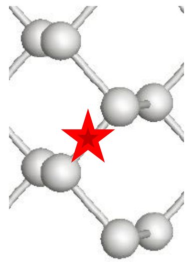




Symmetry protection of Dirac points?



- Inversion is broken due to vertical electric field
- However, C_{2z} is effectively an inversion in 2D



$C_{2z} : (x,y,z) \rightarrow (-x,-y,z)$

$F(k_x,k_y)=F(-k_x,-k_y)$

- Space-time inversion $I_{ST} = C_{2z}\,T : (x,y,t) \rightarrow (-x,-y,-t)$

(C. Fang and L. Fu, PRB)

$F(k_x,k_y)=0$

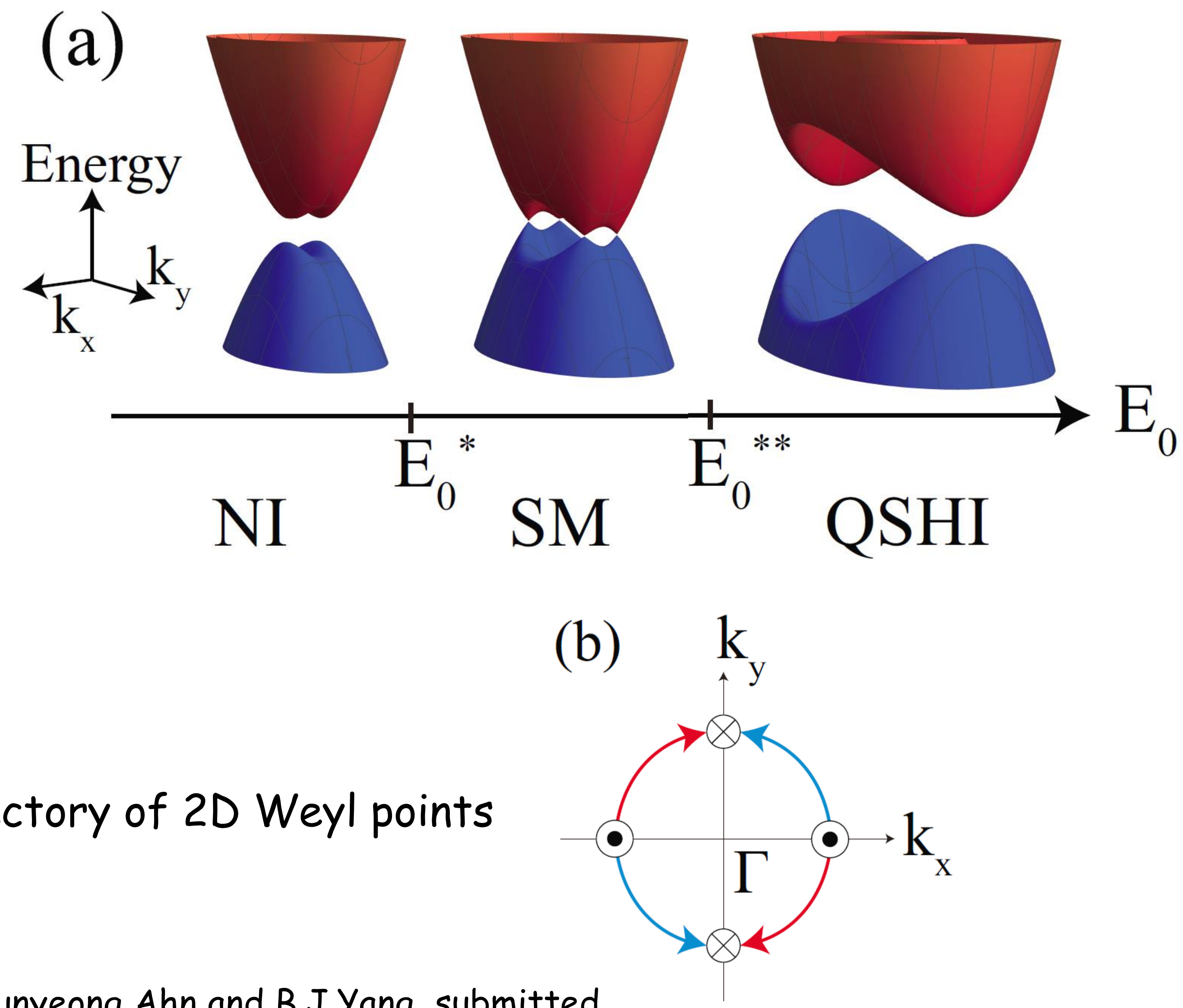
"Quantized Berry phase"

- $(I_{ST})^2 = 1$ with/without spin-orbit coupling (No Kramers degeneracy)

  cf) $(PT)^2 = -1$ (+1) with (without) spin-orbit coupling in graphene
- Berry phase is also quantized in the presence of spin-orbit coupling

Unique property of black phosphorus system!

Band crossing in the presence of SOC



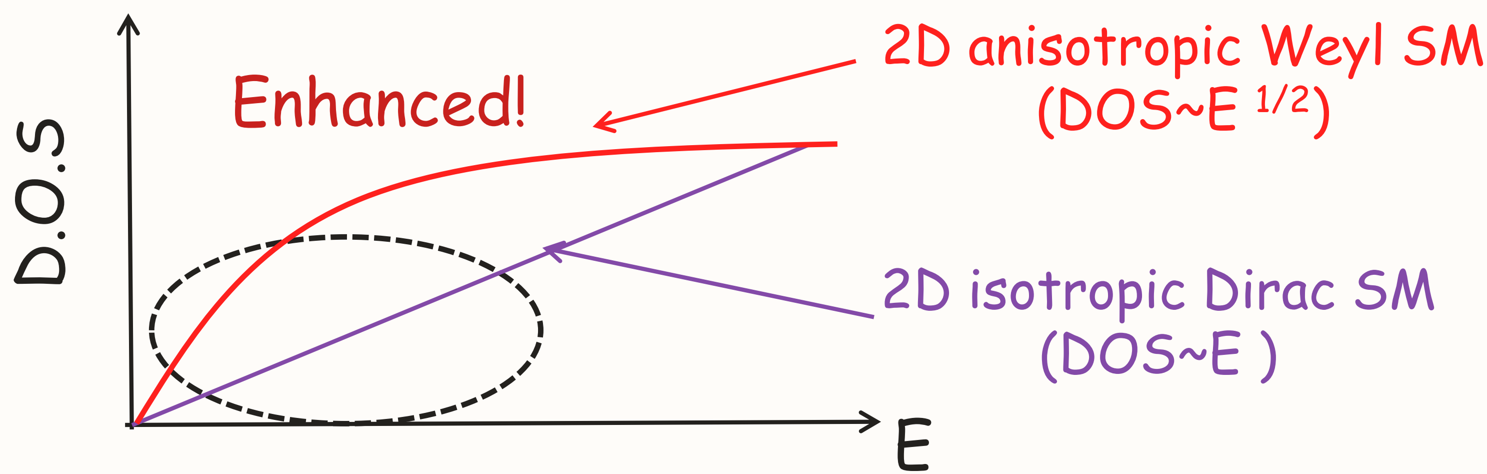
Trajectory of 2D Weyl points

Junyeong Ahn and B.J.Yang, submitted

Interacting 2D anisotropic Weyl fermion

$$\mathcal{S} = \int_{r,\tau} \psi^\dagger \{\partial_\tau - A\sigma_x\partial_x^2 - iv\sigma_y\partial_y\}\psi + \frac{e^2}{2\varepsilon}\int_{r,r',\tau} \frac{\psi^\dagger(\mathbf{r})\psi(\mathbf{r})\psi^\dagger(\mathbf{r}')\psi(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

Enhanced density of states



Anisotropic screening

- Static polarization: $\Pi(\mathbf{q}) = -b_x|q_x| - b_y\sqrt{|q_y|}$

$$V_C(\mathbf{q}) = \frac{1}{\sqrt{q_x^2 + q_y^2} + b_x|q_x| + b_y\sqrt{|q_y|}} \sim \frac{1}{|q_x| + \sqrt{|q_y|}}$$

(See also Gil-Young Cho and Eun-Gook Moon, arXiv:1508.03777)

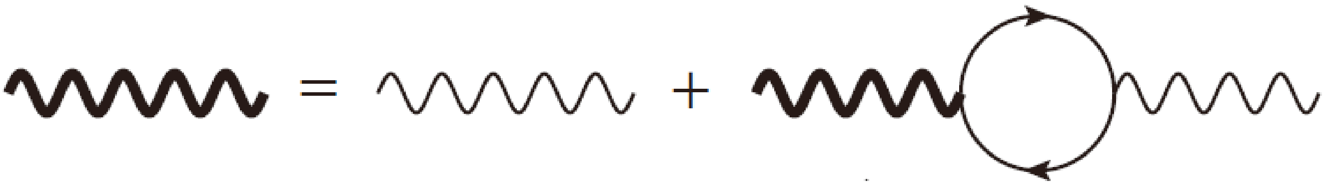
- Anisotropic Coulomb potential

$$V_C(x, y=0) \sim \frac{1}{x^2}, \qquad V_C(x=0, y) \sim \frac{1}{|y|},$$

cf) 3D QCP: $V_{\mathrm{C}}(r_\perp, z=0) \sim \frac{1}{r_\perp^{5/4}}, \quad V_{\mathrm{C}}(r_\perp=0, z) \sim \frac{1}{|z|^{5/3}},$

One loop RG with large-N expansion

$$S = \int d\tau d^2 x \psi_a^\dagger [(\partial_\tau + ig\phi) + H_0]\psi_a + \frac{1}{2}\int d\tau d^3 x (\partial_i \phi)^2,$$

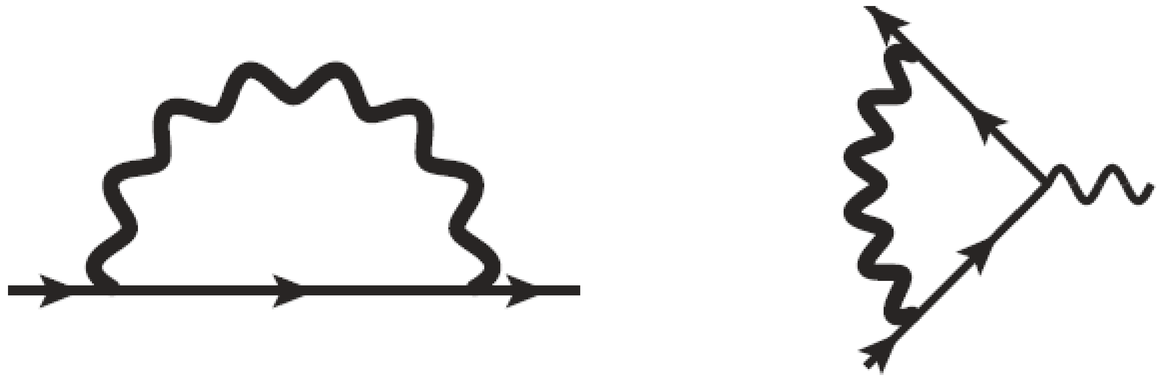


$$D^{-1}(\Omega, \mathbf{q}) = |\mathbf{q}| - N\alpha\Pi(\Omega, \mathbf{q}),$$

- Coupling constant: $N\alpha = N\, e^2/v$
- Both weak coupling ($N\alpha \ll 1$) and strong coupling ($N\alpha \gg 1$) can be studied
- Dynamics of polarization is fully considered:

Quasi-particle residue: $$Z = \frac{1}{1 + \frac{\partial \Sigma(\omega)}{\partial (i\omega)}}$$

One loop RG with large-N expansion



$H_{QCP} = A\, k_1^2\, \sigma_1 + v\, k_2\, \sigma_2$

- RG equations for quasiparticle residue (Z), velocity(v), and inverse mass(A)

$$\dot{Z}(l) = -\gamma_z(l) Z(l),\ \dot{v}(l) = \gamma_v v(l),\ \dot{A}(l) = \gamma_A A(l)$$

Strong coupling limit $N\alpha \gg 1$

$$\frac{v(E)}{v} = \left(\frac{\Lambda}{E}\right)^{\gamma_v}, \frac{A(E)}{A} = \left(\frac{\Lambda}{E}\right)^{\gamma_A}, Z(E) = \left(\frac{\Lambda}{E}\right)^{-\gamma_z + \frac{\sqrt{15}}{\pi^{3/2}} \frac{\gamma_v}{N} l}$$

$$\gamma_v = \frac{0.3625}{N}, \quad \gamma_A = \frac{0.1261}{N}, \quad \gamma_z = \frac{\sqrt{15}}{\pi^{3/2}} \frac{\log N}{N}$$

- v, A all acquire finite anomalous dimension

→ Reduced dynamical exponent, enhanced anisotropy

$$\omega(k_x) \sim A k_x^2 \sim k_x^{2-2\gamma_A} \quad \omega(k_y) \sim v k_y \sim k_y^{1-\gamma_v}$$

- Fermion propagator acquires a non-Fermi liquid form

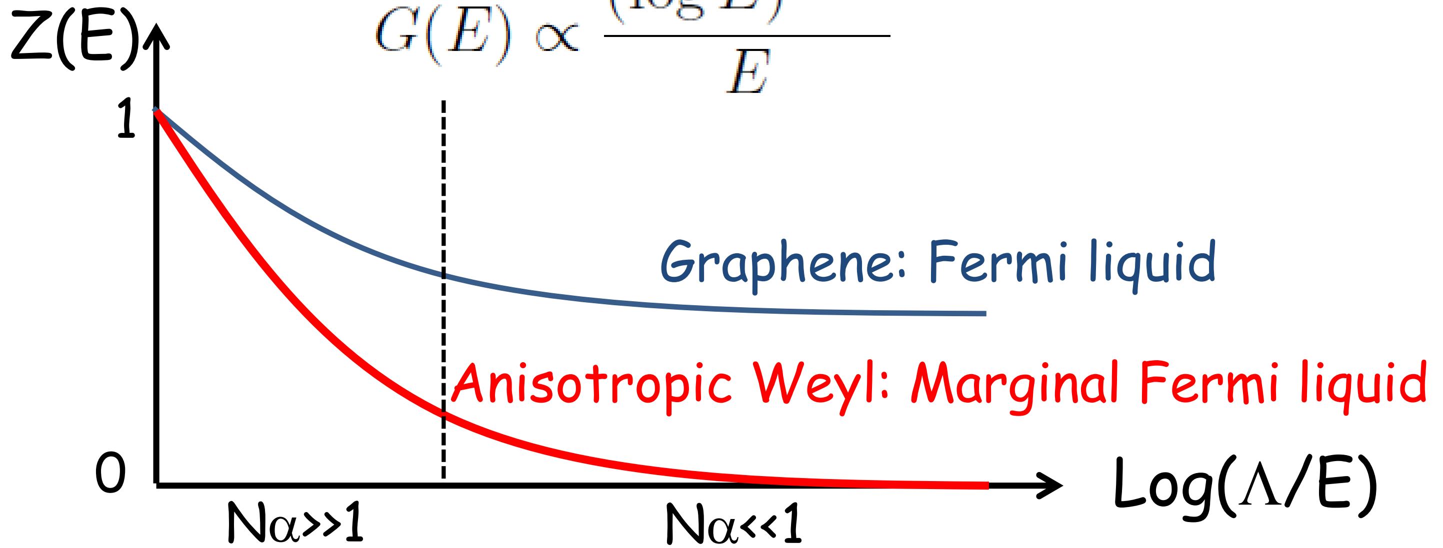
$$G(E) \propto \frac{1}{E^{1-\gamma_z}}$$

- Similar to the strong coupling behavior in graphene (D.T. Son, 2007)

Weak coupling limit $N\alpha \ll 1$

- Fermion propagator acquires a marginal-Fermi liquid form

$$G(E) \propto \frac{(\log E)^{-3/2}}{E}$$

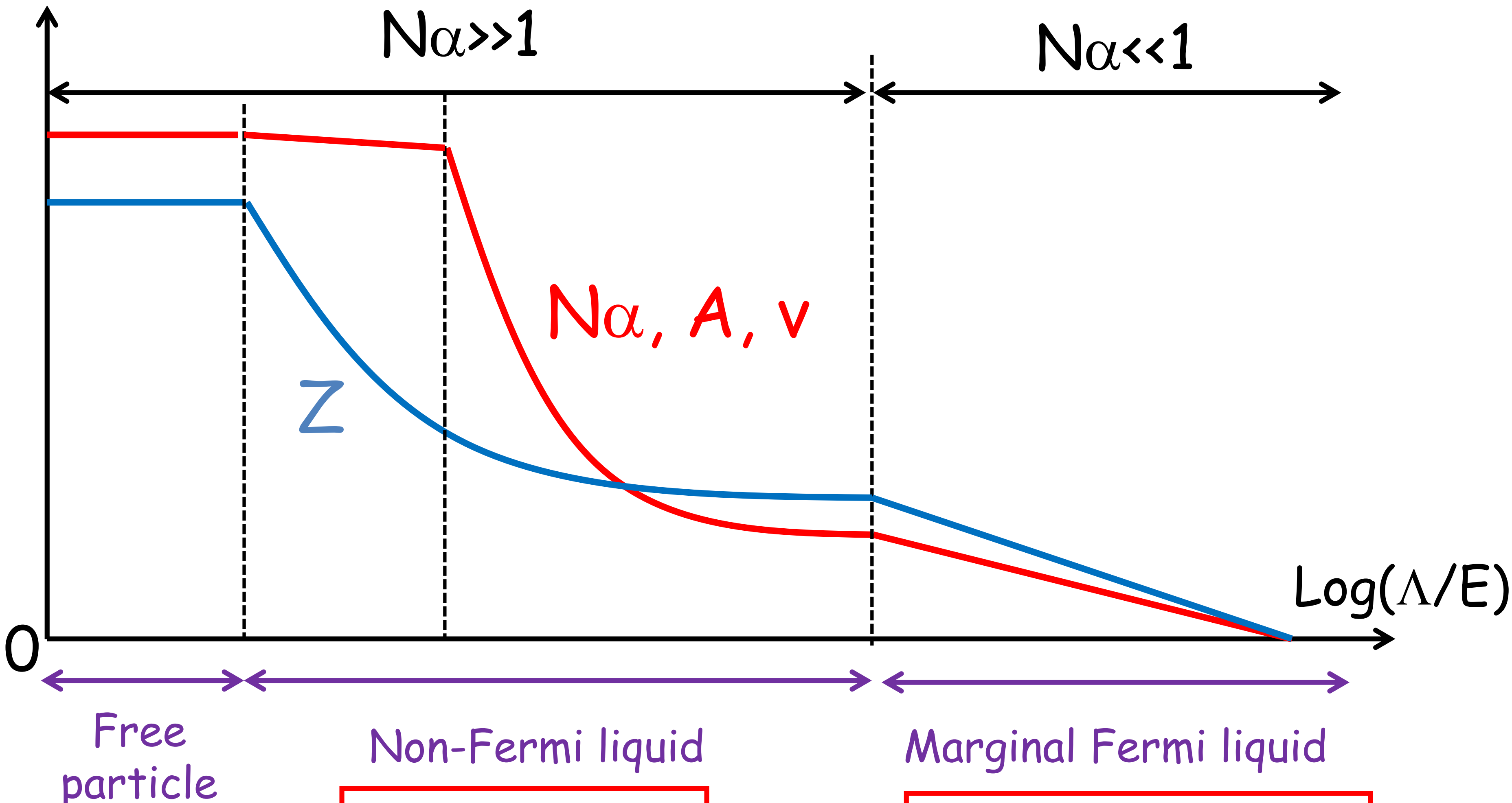


$$D^{-1}(\Omega = 0, \mathbf{q}) = |\mathbf{q}| + N\alpha \left[|q_x| + \sqrt{|q_y|} \right]$$

cf) in graphene: $D^{-1}(\Omega = 0, \mathbf{q}) = |\mathbf{q}| + N\alpha|\mathbf{q}|,$

$$v(l) = \frac{g^2}{4\pi^2} l, \; Z(l) = l^{-3/2}, \; A(l) = A e^{\log^2 l}$$

Evolution of quasi-particles properties



$$G(E) \propto \frac{1}{E^{1-\gamma_z}}$$

$$G(E) \propto \frac{(\log E)^{-3/2}}{E}$$

Outline

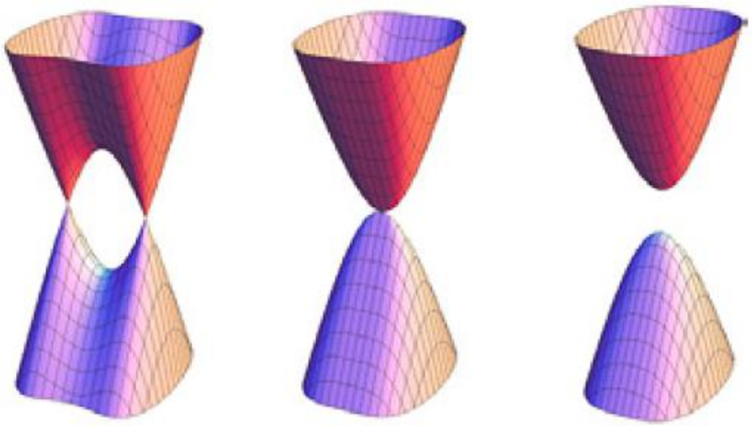
1. Novel quantum criticality of topological PT in 3D systems breaking P or T

2. Novel quantum criticality of topological PT in 2D systems with PT symmetry

3. Conclusion

Novel quantum criticality of TPT

- Critical point of semimetal-insulator transition



	Screened Coulomb potential $V_C(q)$	Quasi-particle
2D anisotropic QCP	$\frac{1}{\lvert q_x\rvert + \lvert q_y\rvert^{1/2}}$	Marginal Fermi liquid
3D Anisotropic QCP	$\frac{1}{{q_\perp}^{3/2} + {q_3}^{2}}$	Free fermions