Anderson Localization – Looking Forward

Boris Altshuler
Physics Department, Columbia University

Collaborations: Igor Aleiner

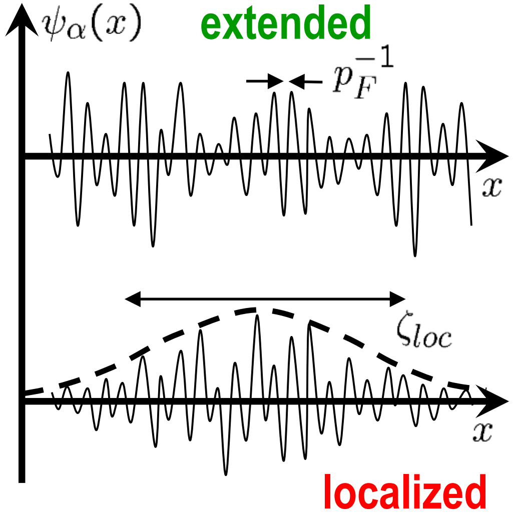
Also Denis Basko, Gora Shlyapnikov, Vincent Michal, Vladimir Kravtsov, …

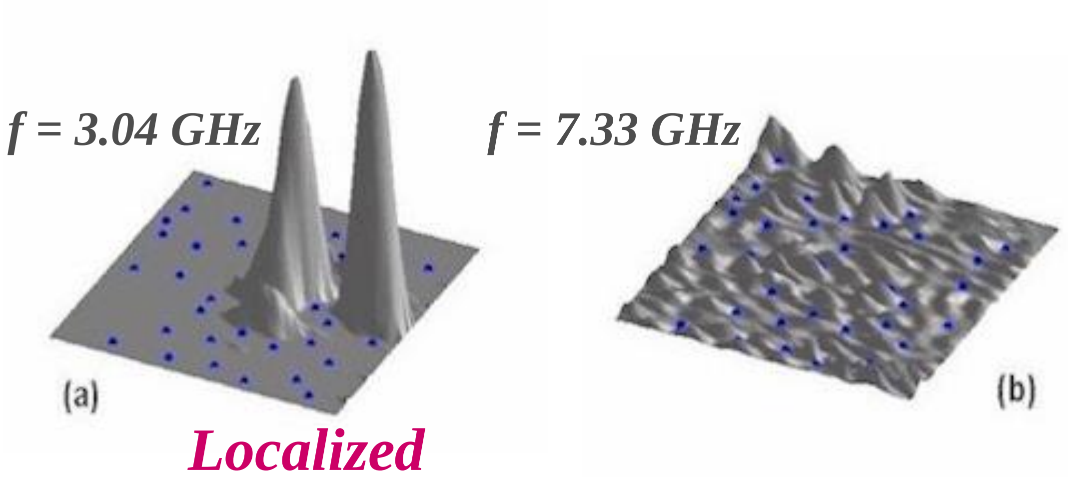
清華大學
Tsinghua University

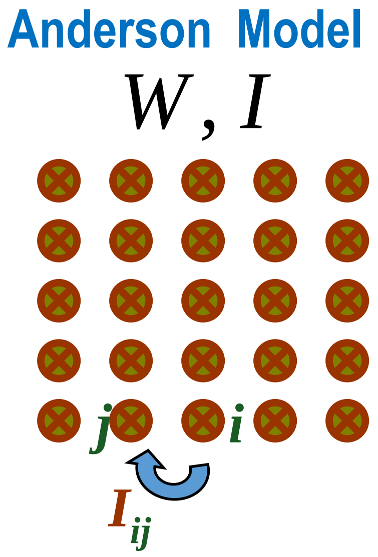
Lecture2
September, 9, 2015

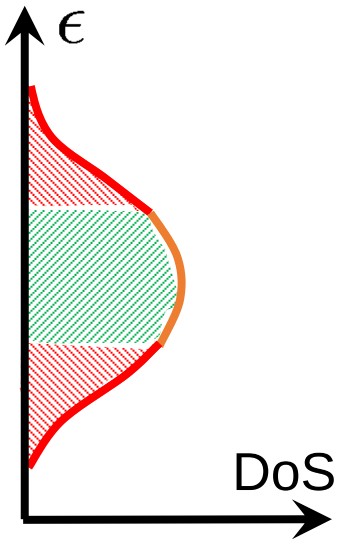
Outline

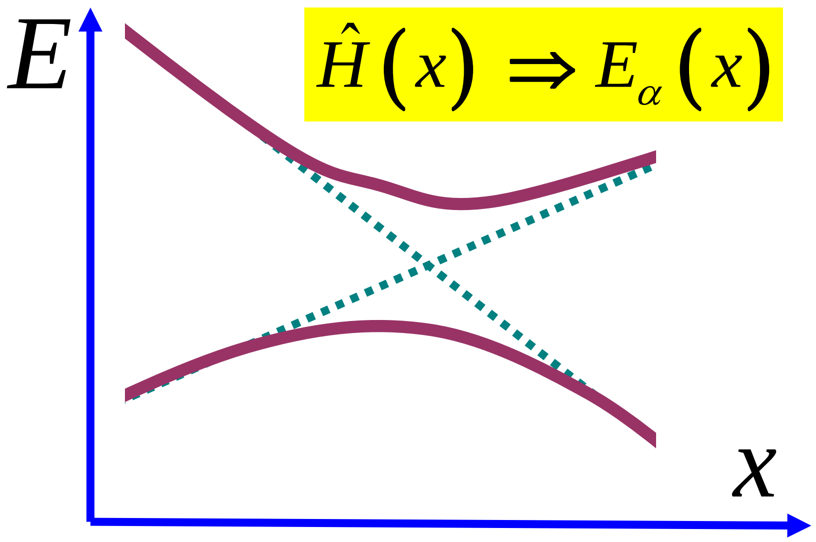
1. Introduction
2. Anderson Model; Anderson Metal and Anderson Insulator
3. Localization beyond the real space. Integrability and chaos.
4. Spectral Statistics and Localization
5. Many-Body Localization.
6. Many-Body Localization of the interacting fermions.
7. Many-Body localization of weakly interacting bosons.
8. Many-Body Localization and Ergodicity

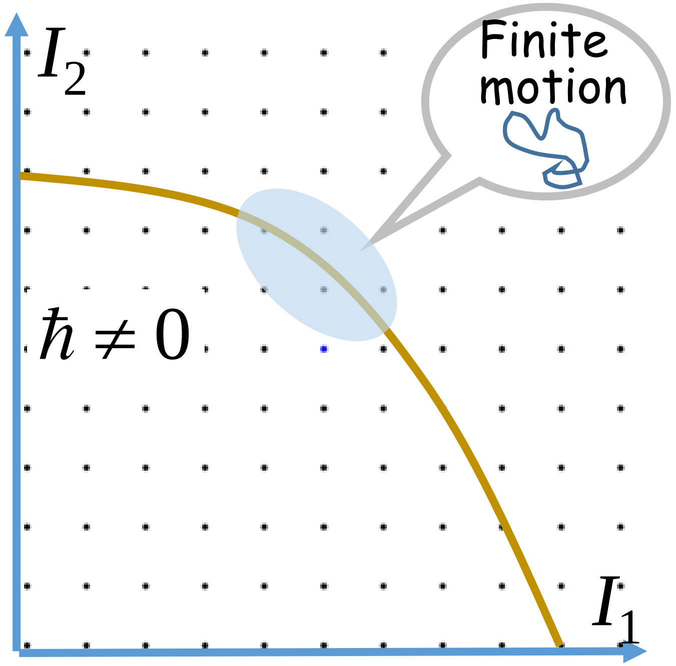

$\psi_\alpha(x)$
extended
p_F^{-1}
x
ζ_{loc}
x
localized

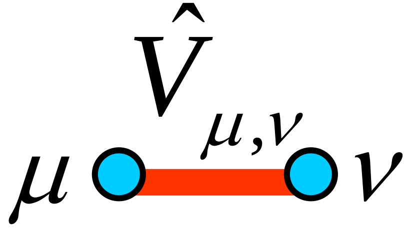

f = 3.04 GHz
f = 7.33 GHz
(a)
(b)
Localized


Anderson Model
W, I
j
i
I_{ij}


ϵ
DoS


E
$\hat{H}(x) \Rightarrow E_\alpha(x)$
X

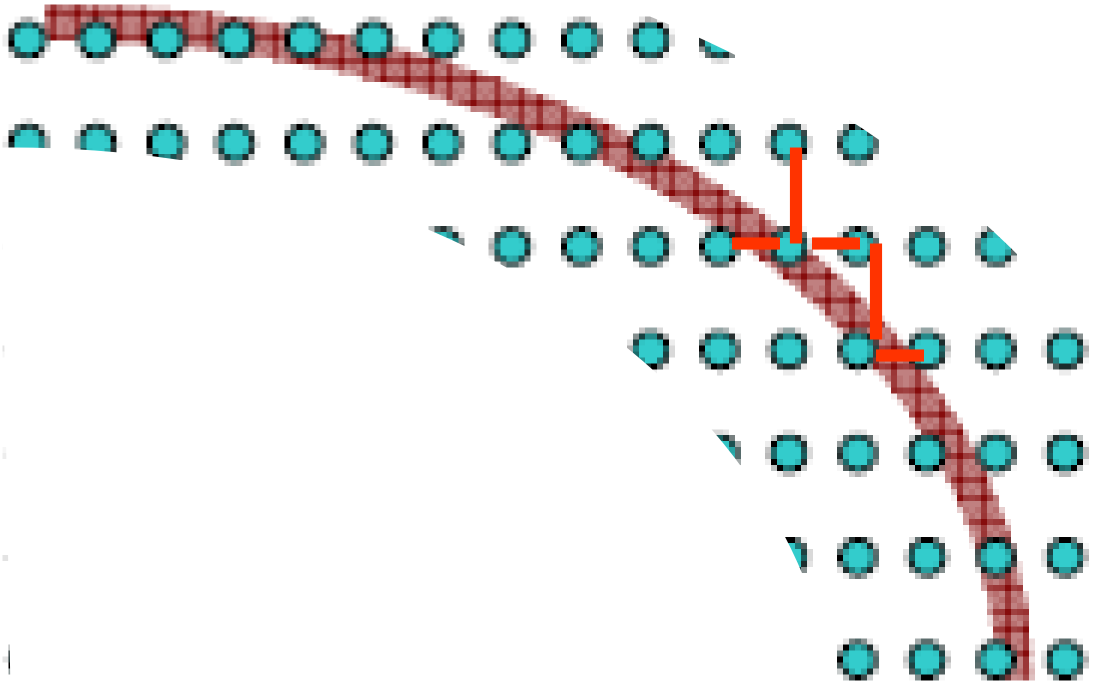

I_2
Finite motion
$\hbar \neq 0$
I_1



Matrix element of the perturbation

$$|\mu\rangle_0 = \left|\vec{I}^{(\mu)}\right\rangle$$

$$\vec{I}^{(\mu)} = \left\{I_1^{(\mu)}, \ldots, I_d^{(\mu)}\right\}$$

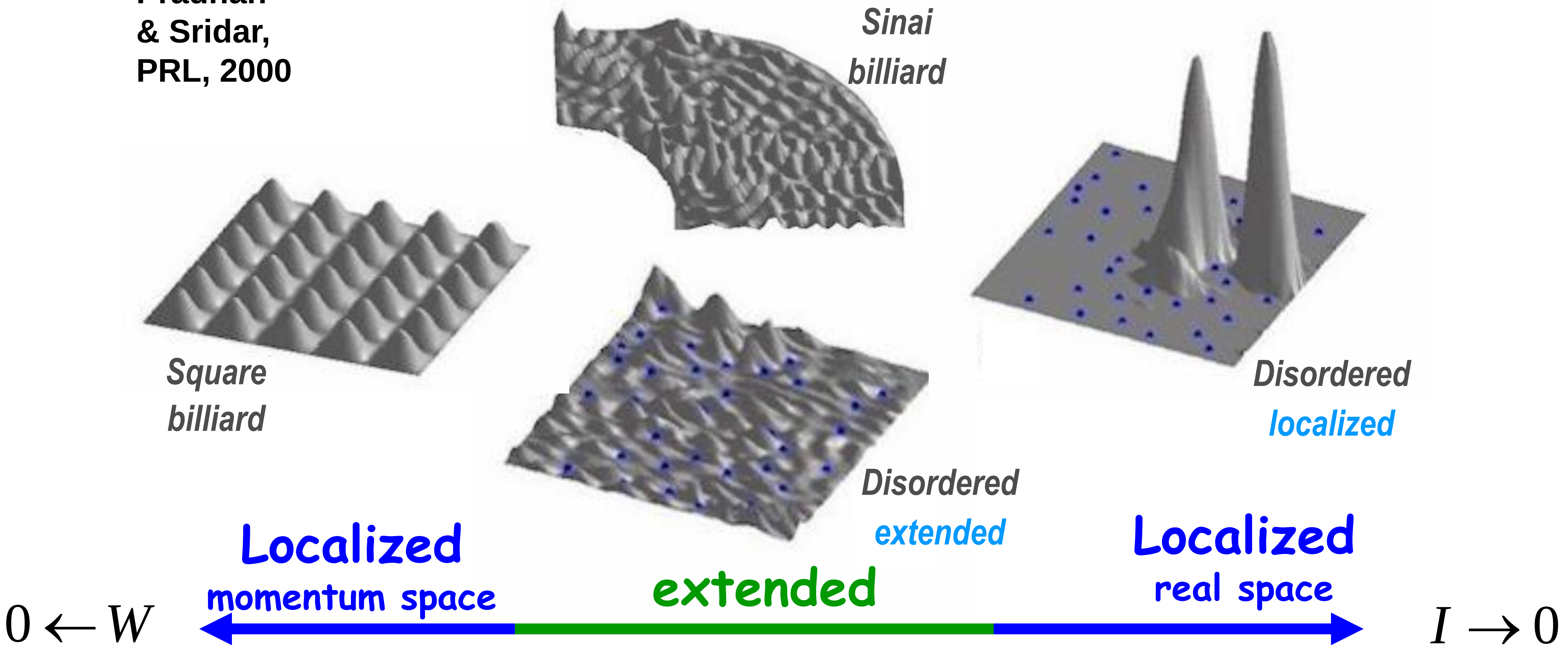


Anderson Model !

AL hops are local – one can distinguish "near" and "far"

KAM perturbation is smooth enough

Pradhan & Sridar, PRL, 2000



Glossary

Classical	Quantum
Integrable $H_0 = H_0(\vec{I});\quad \partial\vec{I}/\partial t = 0$	**Integrable** $\hat{H}_0 = \sum_{\mu} E_{\mu} \lvert\mu\rangle\langle\mu\rvert, \quad \lvert\mu\rangle = \lvert\vec{I}\rangle$
Perturbation $V;\quad \partial\vec{I}/\partial t \neq 0$	**Perturbation** $\hat{V} = \sum_{\mu,\nu} V_{\mu,\nu} \lvert\mu\rangle\langle\nu\rvert$
KAM	**Localized**
Ergodic (chaotic)	**Extended ?**

Question:

What is the reason to speak about localization if we in general do not know the space in which the system is localized ?

Need an invariant (basis independent) criterion of the localization

Part 4.

Spectral Statistics

and

Localization

RANDOM MATRIX THEORY

Spectral statistics

$N \times N$ *ensemble of Hermitian matrices with random matrix element* $N \to \infty$

E_α - spectrum (set of eigenvalues)

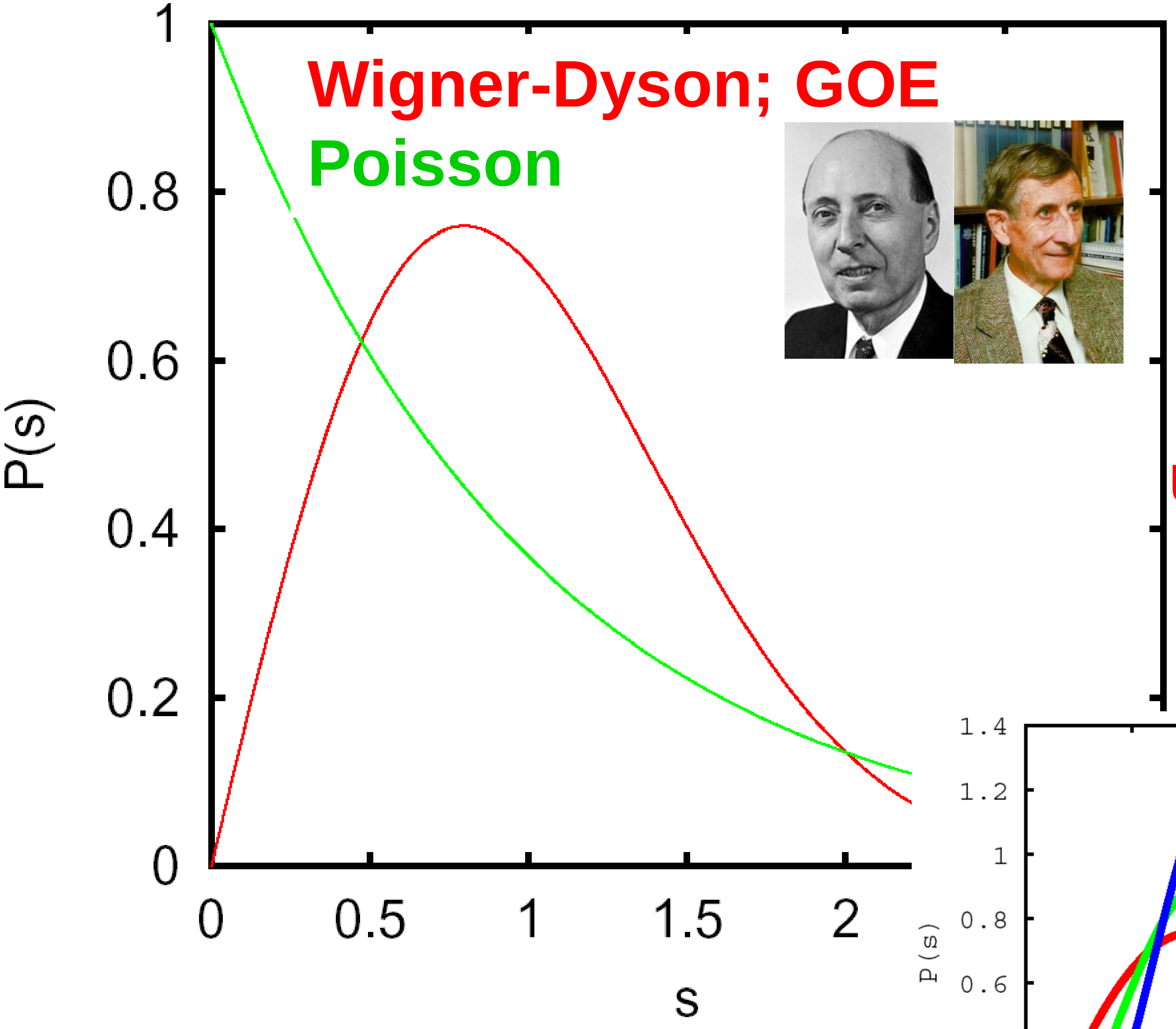
$\delta_1 \equiv \langle E_{\alpha+1} - E_\alpha \rangle$ - mean level spacing

$\langle \ldots\ldots \rangle$ - ensemble averaging

$s \equiv \dfrac{E_{\alpha+1} - E_\alpha}{\delta_1}$ - spacing between nearest neighbors

$P(s)$ - distribution function of nearest neighbors spacing between

Spectral Rigidity	$P(s=0)=0$
Level repulsion	$P(s \ll 1) \propto s^\beta \quad \beta=1,2,4$

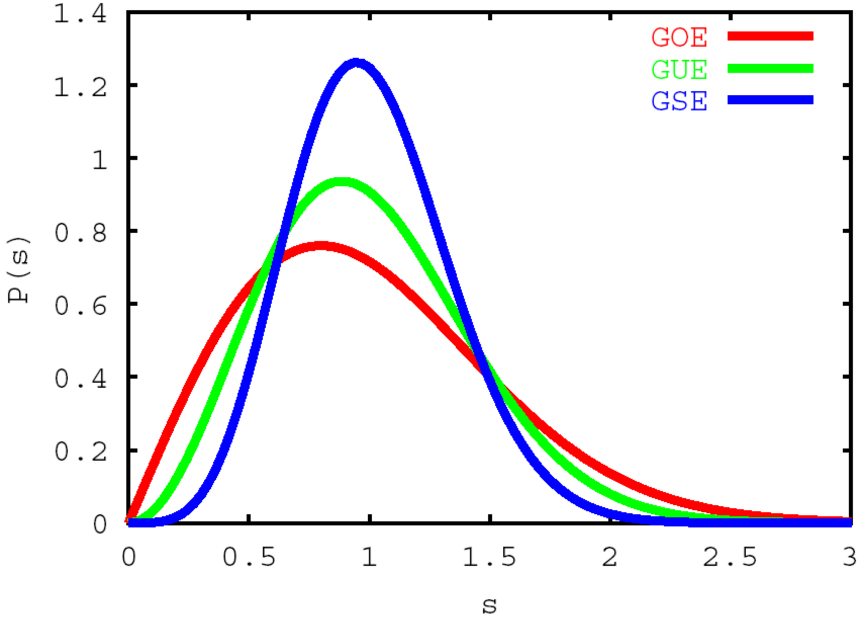


Gaussian Orthogonal Ensemble

Orthogonal
$\beta=1$

Unitary
$\beta=2$

Simplectic
$\beta=4$



Poisson – completely uncorrelated levels

RANDOM MATRICES

$N \times N$ matrices with random matrix elements. $N \to \infty$

Dyson Ensembles

Matrix elements	Ensemble	β	realization
real	orthogonal	1	T-inv potential
complex	unitary	2	broken T-invariance (e.g., by magnetic field)
2×2 matrices	simplectic	4	T-inv, but with spin-orbital coupling

Reason for $P(s) \to 0$ when $s \to 0$:

$$\hat{H} = \begin{pmatrix} H_{11} & H_{12} \\ H_{12}^{*} & H_{22} \end{pmatrix}$$

$$E_2 - E_1 = \sqrt{(H_{22} - H_{11})^2 + |H_{12}|^2}$$

small small small

Recall the Wigner – von Neumann noncrossing rule

1. The assumption is that the matrix elements are statistically independent. Therefore probability of two levels to be degenerate vanishes.

2. If H_{12} is real (orthogonal ensemble), then for s to be small two statistically independent variables ($(H_{22} - H_{11})$ and H_{12}) should be small and thus $P(s) \propto s$ $\quad \beta = 1$

$$\hat{H} = \begin{pmatrix} H_{11} & H_{12} \\ H_{12}^{*} & H_{22} \end{pmatrix}$$

$$P(E_2 - E_1) = \iint d(H_{11} - H_{22})\, dH_{12}\, \delta\left(E_2 - E_1 - \sqrt{(H_{22} - H_{11})^2 + |H_{12}|^2}\right) \times$$
$$\times\, p(H_{11} - H_{22})\, p(H_{12})$$

Distribution function of the diagonal matrix elements

Distribution function of the off-diagonal matrix elements

Distribution function of the spacing $P(s)$

Reason for $P(s)\to 0$ when $s\to 0$:

$$\hat{H}=\begin{pmatrix} H_{11} & H_{12} \\ H_{12}^{*} & H_{22} \end{pmatrix}$$

$$E_2-E_1=\sqrt{\left(H_{22}-H_{11}\right)^2+\left|H_{12}\right|^2}$$

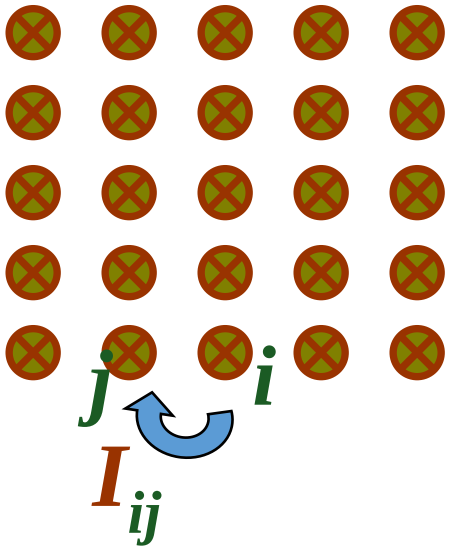
small small small

1. The assumption is that the matrix elements are statistically independent. Therefore probability of two levels to be degenerate vanishes.

2. If H_{12} is real (orthogonal ensemble), then for s to be small two statistically independent variables ($(H_{22}$- $H_{11})$ and H_{12}) should be small and thus $P(s)\propto s$ $\beta=1$

3. Complex H_{12} (unitary ensemble) ⟹ both $Re(H_{12})$ and $Im(H_{12})$ are statistically independent ⟹ three independent random variables should be small ⟹ $P(s)\propto s^2$ $\beta=2$

Anderson Model



- *Lattice - tight binding model*
- *Onsite energies* ε_i - *random*
- *Hopping matrix elements* I_{ij}

$$\hat{H} = \sum_i \varepsilon_i \hat{a}_i^+ \hat{a}_i + I \sum_{i,j=n.n.} \hat{a}_i^+ \hat{a}_j$$

$$-W < \varepsilon_i < W$$

uniformly distributed

Is there much in common between Random Matrices and Hamiltonians with random potential ?

Q: What are the spectral statistics of a finite size Anderson model **?**

Anderson Transition

Strong disorder

$I < I_c$

Insulator

All eigenstates are localized

Localization length ξ

The eigenstates, which are localized at different places will not repel each other

Poisson spectral statistics

Weak disorder

$I > I_c$

Metal

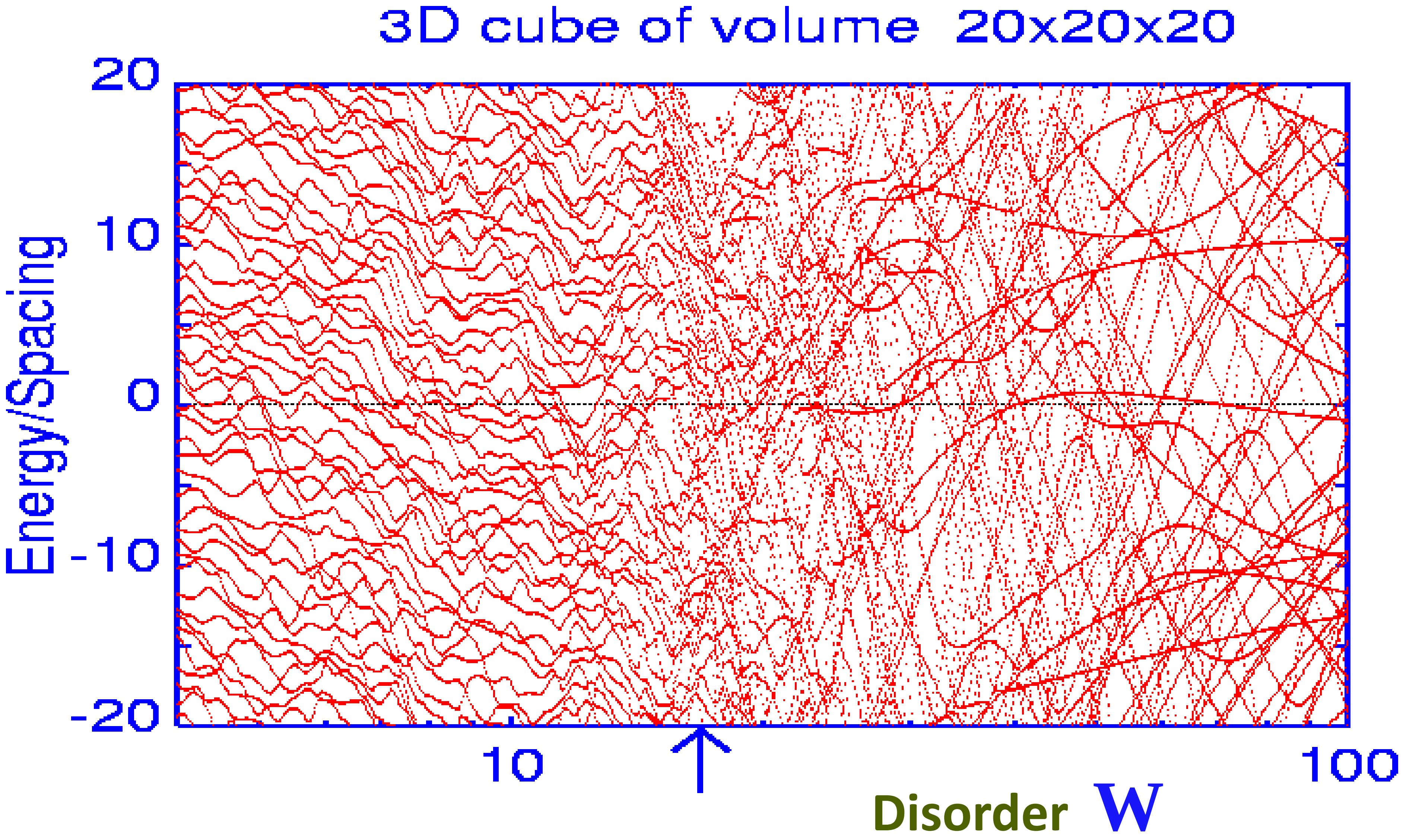
There appear states extended all over the whole system

Any two extended eigenstates repel each other

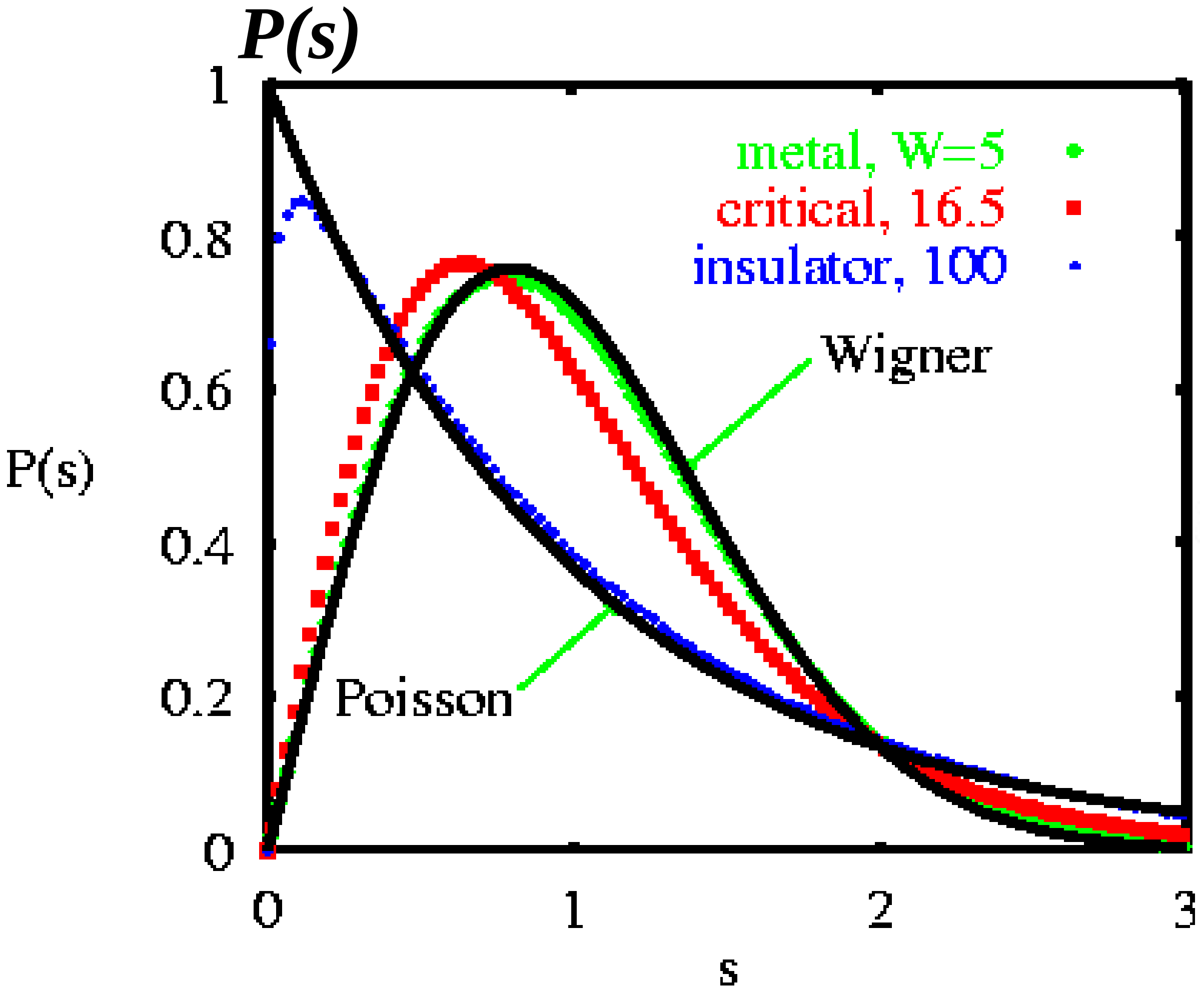
Wigner – Dyson spectral statistics

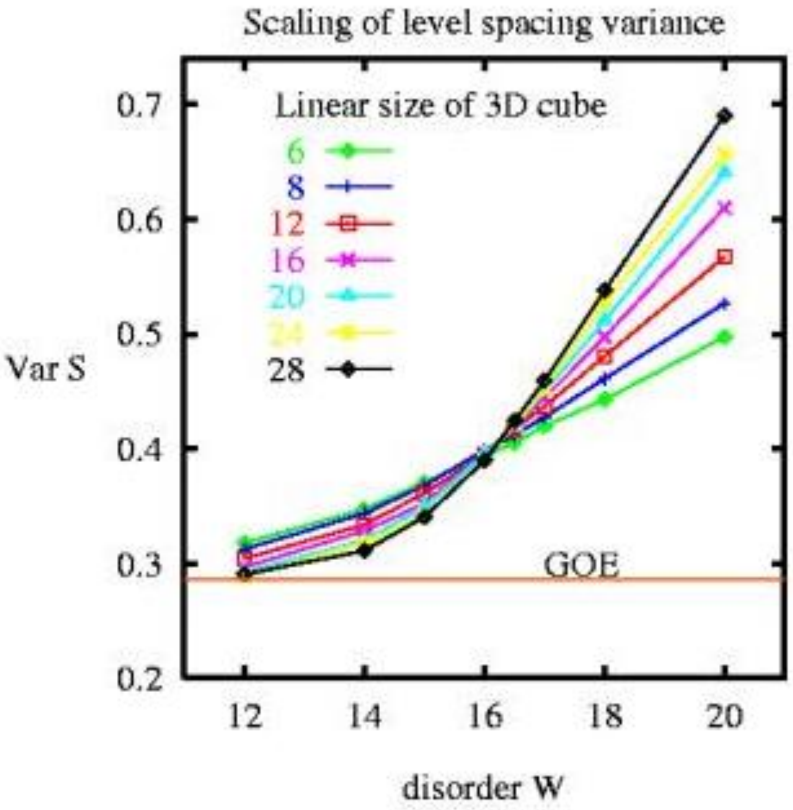
Zharekeschev & Kramer.

Exact diagonalization of the Anderson model



Anderson transition in terms of pure level statistics





Part 5.

Quantum Chaos, Integrability and Localization

ATOMS Main goal is to classify the eigenstates in terms of the quantum numbers

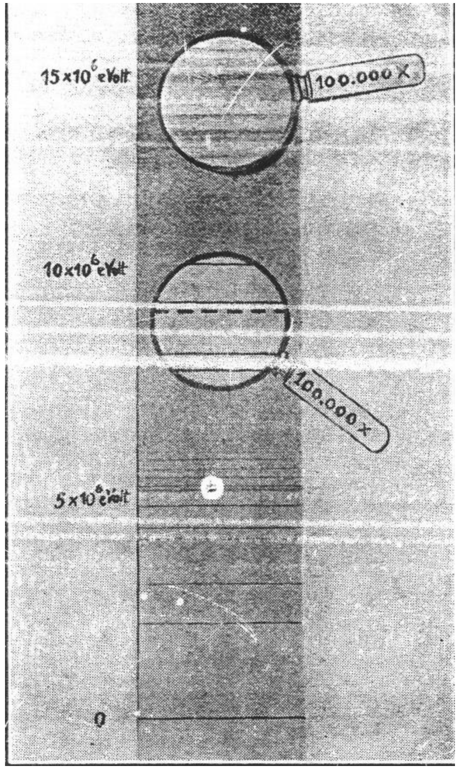
NUCLEI For the nuclear excitations this program does not work

Wigner: Study spectral **statistics** of a **particular** quantum system – a given nucleus

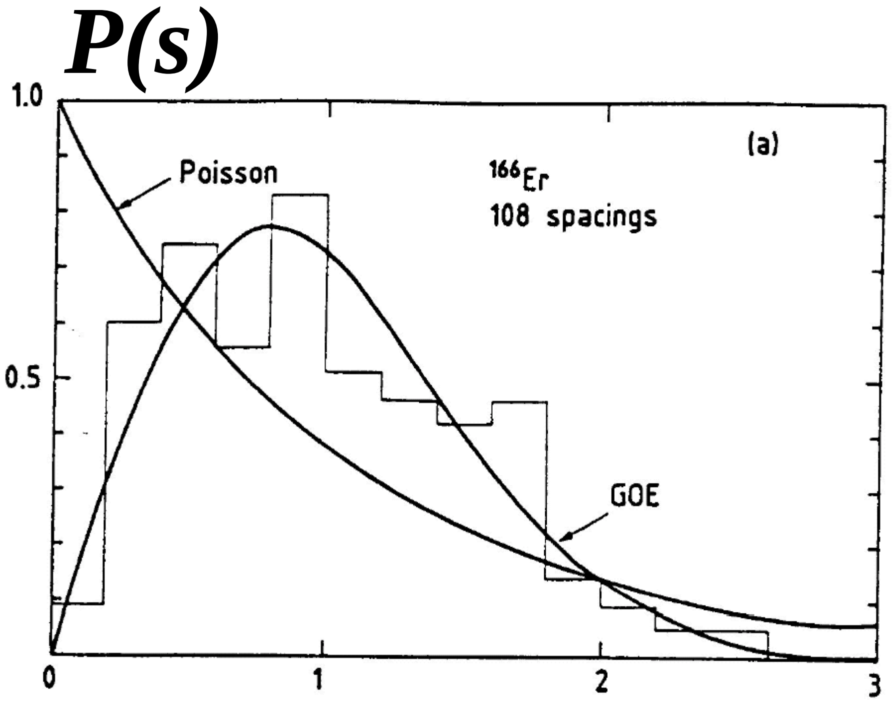
Spectra: $\{E_\alpha\}$

Random Matrices	Atomic Nuclei
• *Ensemble* • *Ensemble averaging*	•*Spectral averaging (over α)* •*Particular quantum system*

Nevertheless Statistics of the nuclear spectra are almost exactly the same as the Random Matrix Statistics

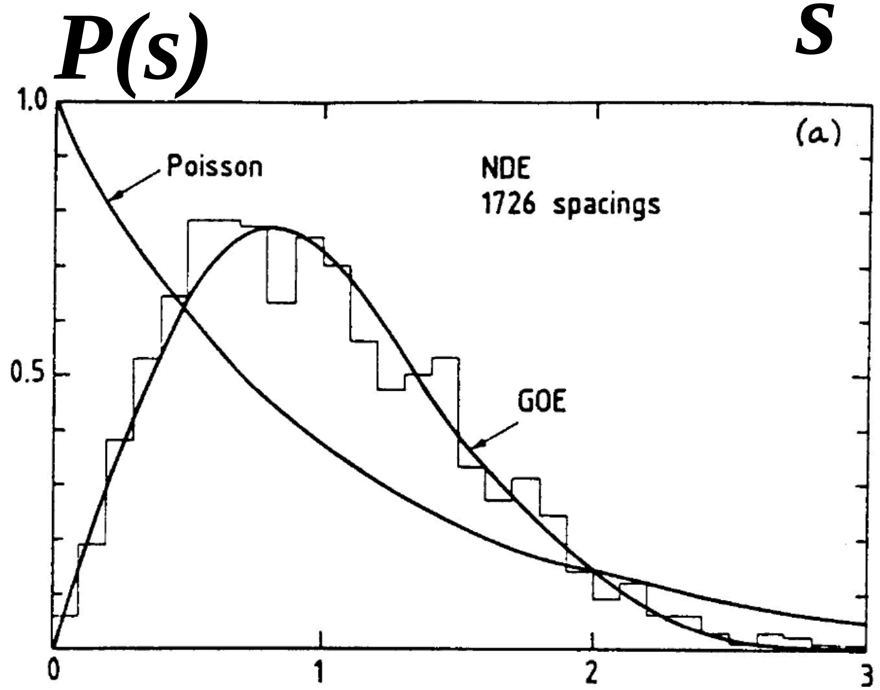


N. Bohr, Nature 137 (1936) 344.



Particular nucleus

^{166}Er



Spectra of several nuclei combined (after spacing) rescaling by the mean level

Q: Why the random matrix theory (RMT) works so well for nuclear spectra ?

Original answer:

These are systems with a **large number of degrees of freedom**, and therefore the "**complexity**" is high

Later it became clear that

there exist very "**simple**" **systems** with as many as 2 degrees of freedom ($d=2$), which demonstrate **RMT - like spectral statistics**

Classical Dynamical Systems with d degrees of freedom

Integrable Systems

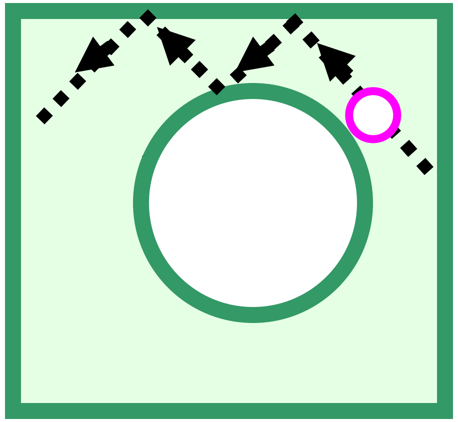
The variables can be separated ⇨ d one-dimensional problems ⇨ d integrals of motion

Rectangular and circular billiard, Kepler problem, . . . ,
1d Hubbard model and other exactly solvable models, . .

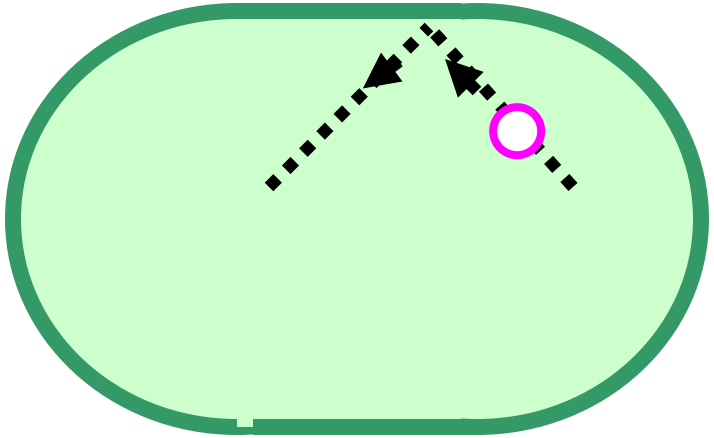
Chaotic Systems

The variables can not be separated ⇨ there is only one integral of motion - energy

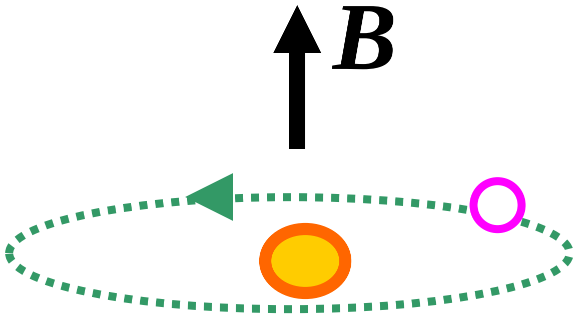
Examples



Sinai billiard



Stadium



Kepler problem in magnetic field

$\hbar \neq 0$ Bohigas – Giannoni – Schmit conjecture

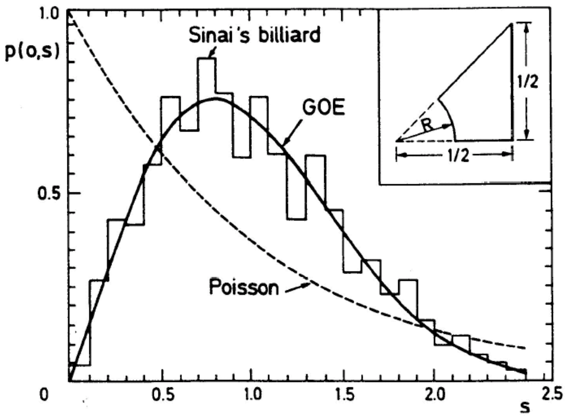
VOLUME 52 2 JANUARY 1984 NUMBER 1

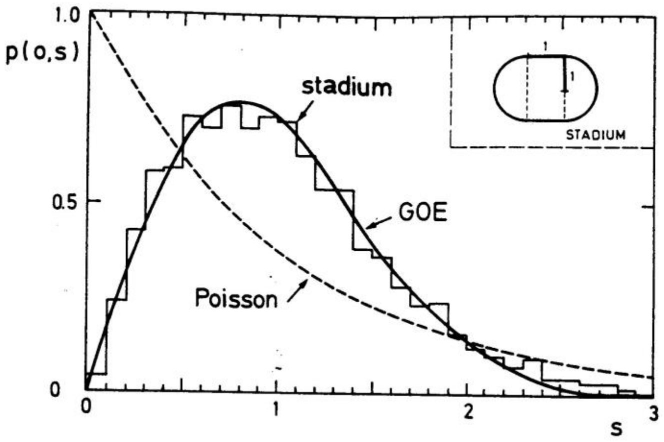
Characterization of Chaotic Quantum Spectra and Universality of Level Fluctuation Laws

O. Bohigas, M. J. Giannoni, and C. Schmit
Division de Physique Théorique, Institut de Physique Nucléaire, F-91406 Orsay Cedex, France
(Received 2 August 1983)

It is found that the level fluctuations of the quantum Sinai's billiard are consistent with the predictions of the Gaussian orthogonal ensemble of random matrices. This reinforces the belief that level fluctuation laws are universal.

In summary, the question at issue is to prove or disprove the following conjecture: Spectra of time-reversal-invariant systems whose classical analogs are K systems show the same fluctuation properties as predicted by GOE





Chaotic classical analog

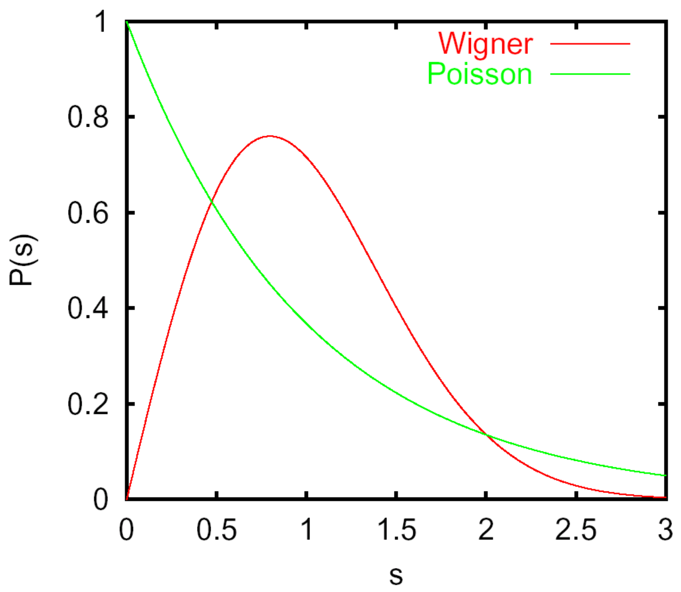
Wigner- Dyson spectral statistics

No quantum numbers except energy

Classical **Quantum**

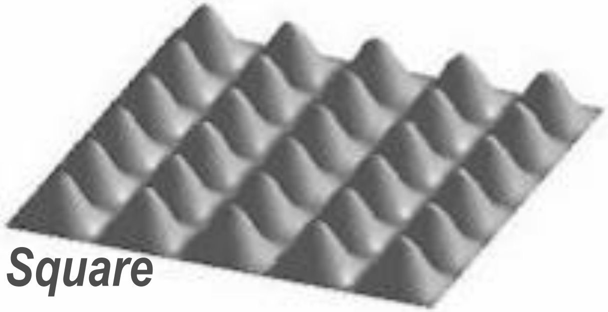
Integrable ? ⟷ **Poisson**

Chaotic ? ⟷ **Wigner-Dyson**



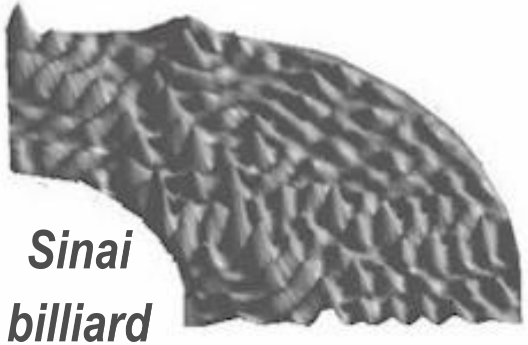
Integrable

Chaotic

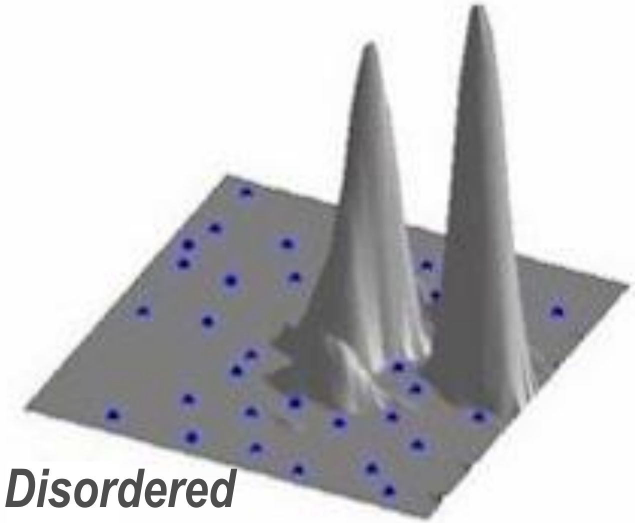


Square billiard

All chaotic systems resemble each other.

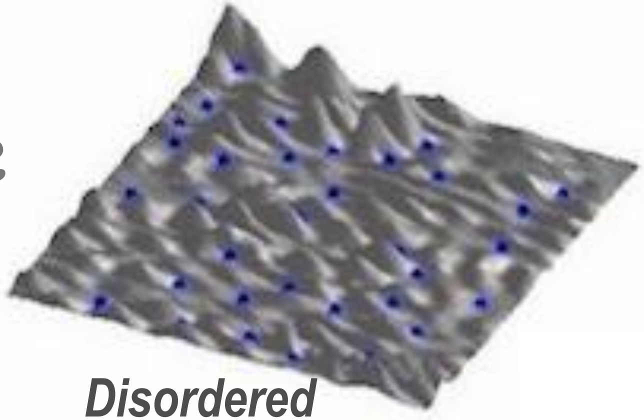


Sinai billiard

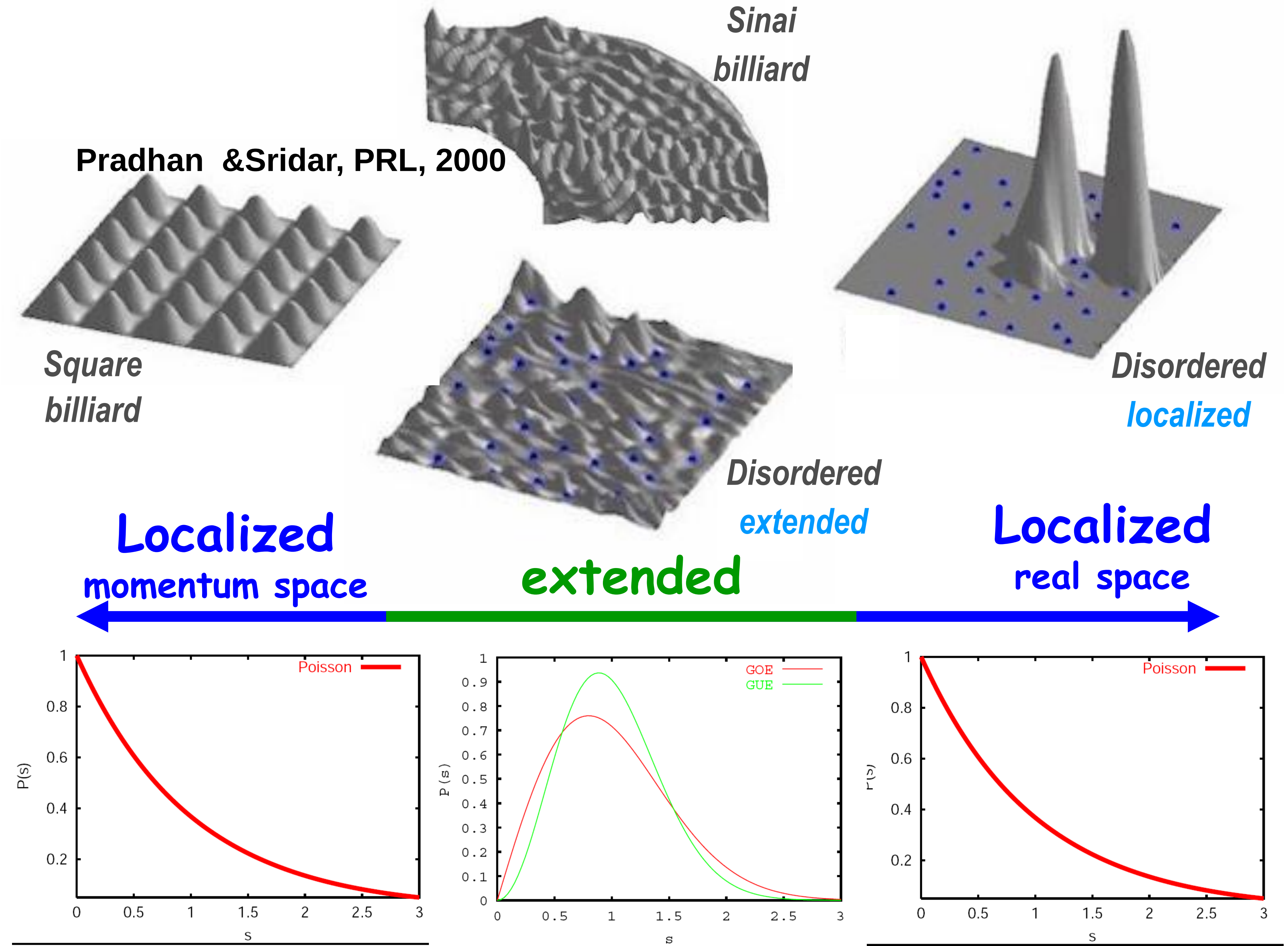


Disordered localized

All integrable systems are integrable in their own way



Disordered extended

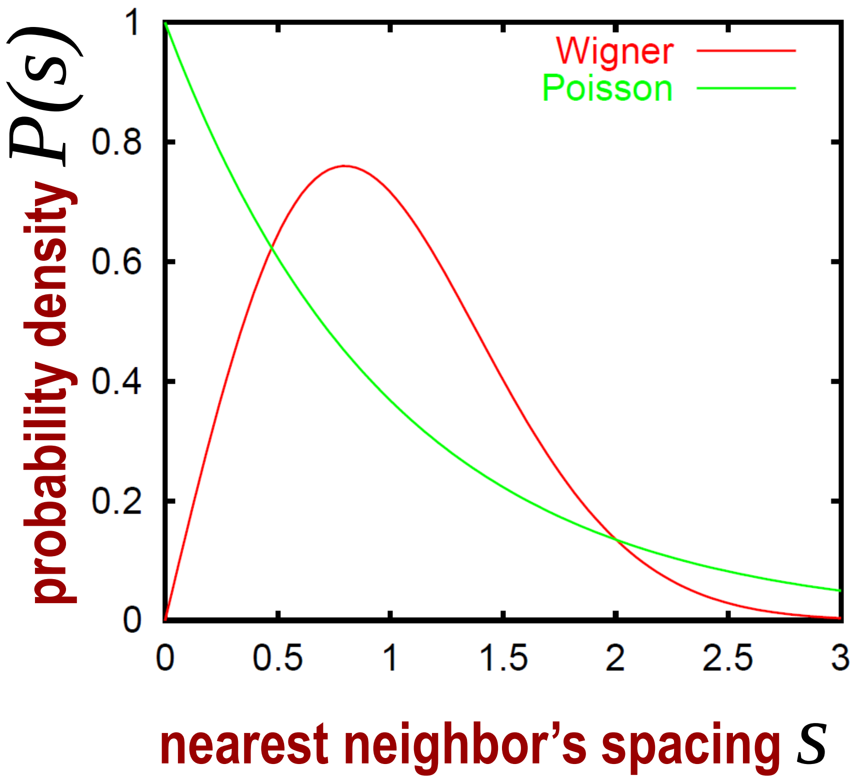


Sinai
billiard
Pradhan &Sridar, PRL, 2000
Square
billiard
Disordered
localized
Disordered
extended
Localized
momentum space
extended
Localized
real space
Poisson
P(s)
s
GOE
GUE
P(s)
s
Poisson
s

Spectral statistics

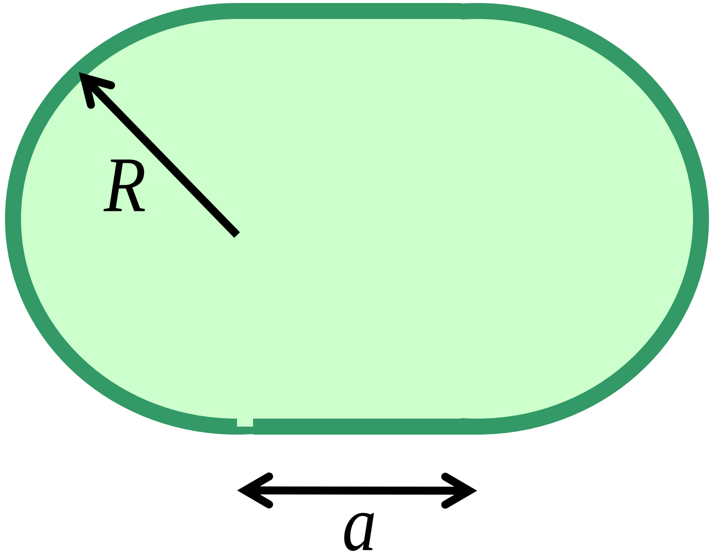
Extended states: Level repulsion, avoided crossings, Wigner-Dyson spectral statistics (random matrices)

Localized states: Poisson spectral statistics No level repulsion



Invariant (basis independent) definition

Example:
Stadium - Localization in the angular momentum space.



$$\varepsilon \equiv \frac{a}{R}$$ - parameter

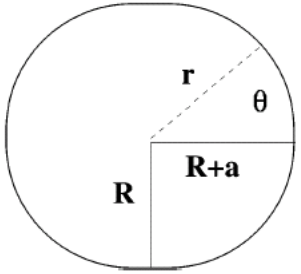
Localization and diffusion in the angular momentum space

VOLUME 77, NUMBER 23 PHYSICAL REVIEW LETTERS 2 DECEMBER 1996

Diffusion and Localization in Chaotic Billiards

Fausto Borgonovi,[1,3,4] Giulio Casati,[2,3,5] and Baowen Li[6,7]
[1]Dipartimento di Matematica, Università Cattolica, via Trieste 17, 25121 Brescia, Italy
[2]Università di Milano, sede di Como, Via Lucini 3, Como, Italy
[3]Istituto Nazionale di Fisica della Materia, Unità di Milano, via Celoria 16, 22100, Milano, Italy
[4]Instituto Nazionale di Fisica Nucleare, Sezione di Pavia, Pavia, Italy
[5]Instituto Nazionale di Fisica Nucleare, Sezione di Milano, Milano, Italy
[6]Department of Physics and Centre for Nonlinear and Complex Systems, Hong Kong Baptist University, Hong Kong
[a7]Center for Applied Mathematics and Theoretical Physics, University of Maribor, Krekova 2, 2000 Maribor, Slovenia
(Received 29 July 1996)

$$\varepsilon \equiv \frac{a}{R}$$



$\varepsilon > 0$ Chaotic stadium

$\varepsilon \to 0$ Integrable circular billiard

Angular momentum is the integral of motion

$$\hbar = 0; \quad \varepsilon << 1$$

Diffusion in the angular momentum space

$$D \propto \varepsilon^{5/2}$$

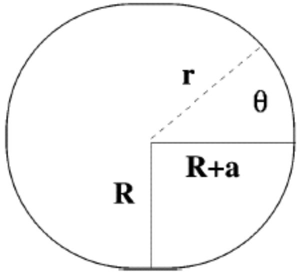
VOLUME 77, NUMBER 23 PHYSICAL REVIEW LETTERS 2 DECEMBER 1996

Diffusion and Localization in Chaotic Billiards

Fausto Borgonovi,[1,3,4] Giulio Casati,[2,3,5] and Baowen Li[6,7]
[1]Dipartimento di Matematica, Università Cattolica, via Trieste 17, 25121 Brescia, Italy
[2]Università di Milano, sede di Como, Via Lucini 3, Como, Italy
[3]Istituto Nazionale di Fisica della Materia, Unità di Milano, via Celoria 16, 22100, Milano, Italy
[4]Instituto Nazionale di Fisica Nucleare, Sezione di Pavia, Pavia, Italy
[5]Instituto Nazionale di Fisica Nucleare, Sezione di Milano, Milano, Italy
[6]Department of Physics and Centre for Nonlinear and Complex Systems, Hong Kong Baptist University, Hong Kong
[7]Center for Applied Mathematics and Theoretical Physics, University of Maribor, Krekova 2, 2000 Maribor, Slovenia
(Received 29 July 1996)

Localization and diffusion in the angular momentum space

$$\varepsilon \equiv \frac{a}{R}$$



$\varepsilon > 0$ **Chaotic stadium**

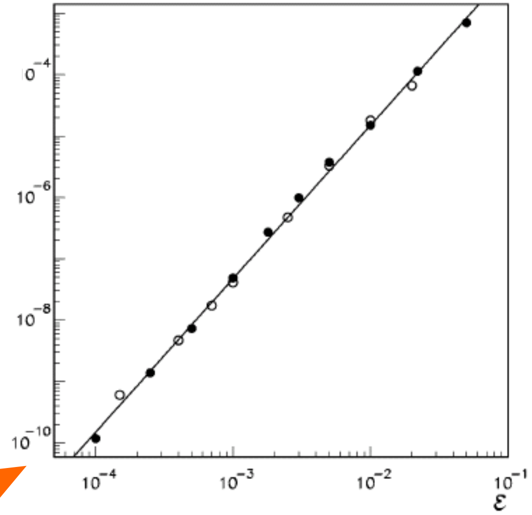
$\varepsilon \to 0$ **Integrable circular billiard**

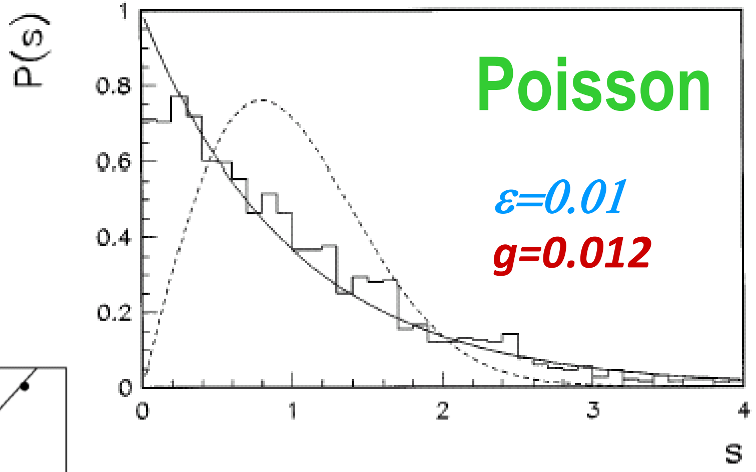
Angular momentum is the integral of motion

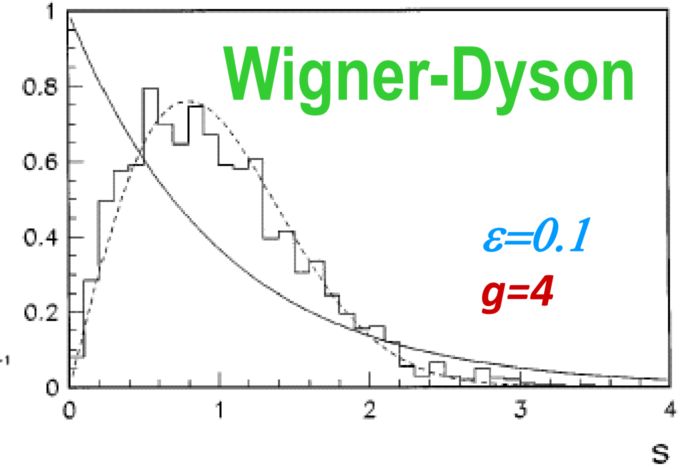
$\hbar = 0; \quad \varepsilon << 1$

Diffusion in the angular momentum space

$$D \propto \varepsilon^{5/2}$$







Part 6.

Many-Body Localization

a) Spin systems; Quantum Computer

Example: Random Ising model in the perpendicular field

Will not discuss today in detail

$$\hat{H} = \sum_{i=1}^{N} B_i \hat{\sigma}_i^z + \sum_{i \neq j} J_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z + I \sum_{i=1}^{N} \hat{\sigma}_i^x \equiv \hat{H}_0 + I \sum_{i=1}^{N} \hat{\sigma}_i^x$$

Random Ising model in a parallel field

Perpendicular field

$\vec{\sigma}_i$ - Pauli matrices, $\sigma_i^z = \pm \frac{1}{2}$

$i = 1, 2, \ldots, N; \quad N >> 1$

Without perpendicular field all σ_i^z commute with the Hamiltonian, i.e. they are integrals of motion

$$\hat{H} = \sum_{i=1}^{N} B_i \hat{\sigma}_i^z + \sum_{i \neq j} J_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z + I \sum_{i=1}^{N} \hat{\sigma}_i^x \equiv \hat{H}_0 + I \sum_{i=1}^{N} \hat{\sigma}_i^x$$

Random Ising model in a parallel field

Perpendicular field

$\vec{\sigma}_i$ - Pauli matrices

$i = 1, 2, \ldots, N; \quad N >> 1$

Without perpendicular field all σ_i^z commute with the Hamiltonian, i.e. they are integrals of motion

$\{\sigma_i^z\}$ **determines a site of an N-dimensional hypercube**

$$\hat{H} = \sum_{i=1}^{N} B_i \hat{\sigma}_i^z + \sum_{i \neq j} J_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z + I \sum_{i=1}^{N} \hat{\sigma}_i^x \equiv \hat{H}_0 + I \sum_{i=1}^{N} \hat{\sigma}_i^x$$

Random Ising model in a parallel field

Perpendicular field

$\vec{\sigma}_i$ - Pauli matrices

$i = 1, 2, \ldots, N; \quad N >> 1$

Without perpendicular field all σ_i^z commute with the Hamiltonian, i.e. they are integrals of motion

Anderson Model on N-dimensional cube

$\{\sigma_i^z\}$ determines a site

$H_0(\{\sigma_i\})$

onsite energy

$$\hat{\sigma}^x = \hat{\sigma}^+ + \hat{\sigma}^-$$

perp. field ⇨ hoping between nearest neighbors

$$\hat{H} = \sum_{i=1}^{N} B_i \hat{\sigma}_i^z + \sum_{i \neq j} J_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z + I \sum_{i=1}^{N} \hat{\sigma}_i^x \equiv \hat{H}_0 + I \sum_{i=1}^{N} \hat{\sigma}_i^x$$

Anderson Model on *N*-dimensional cube

Usually:

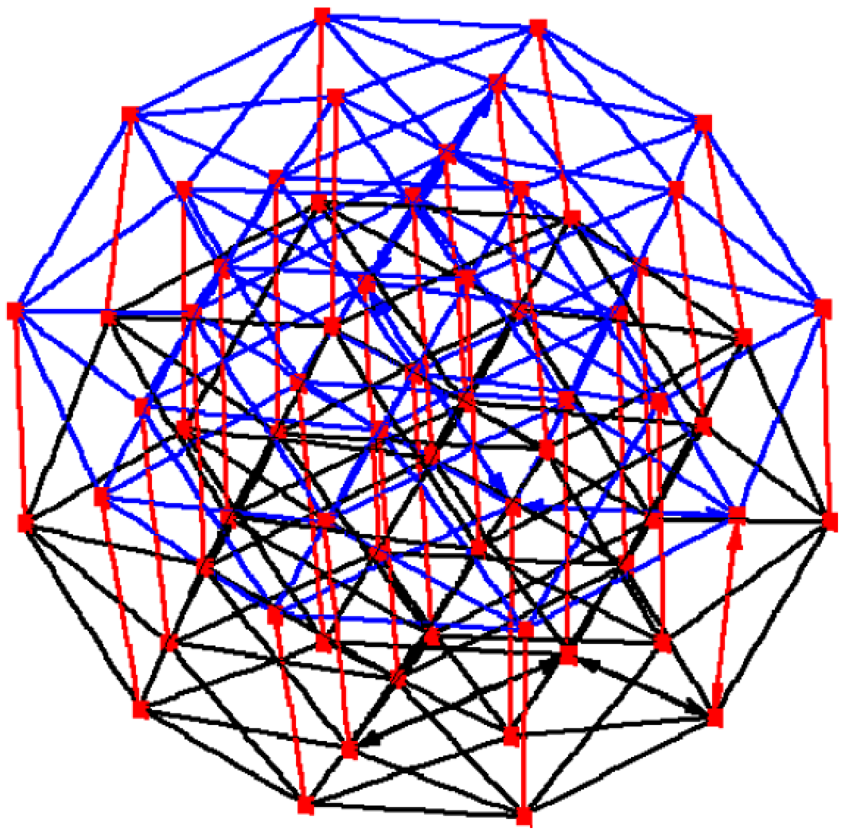
# of dimensions $d \to const$

system linear size $L \to \infty$

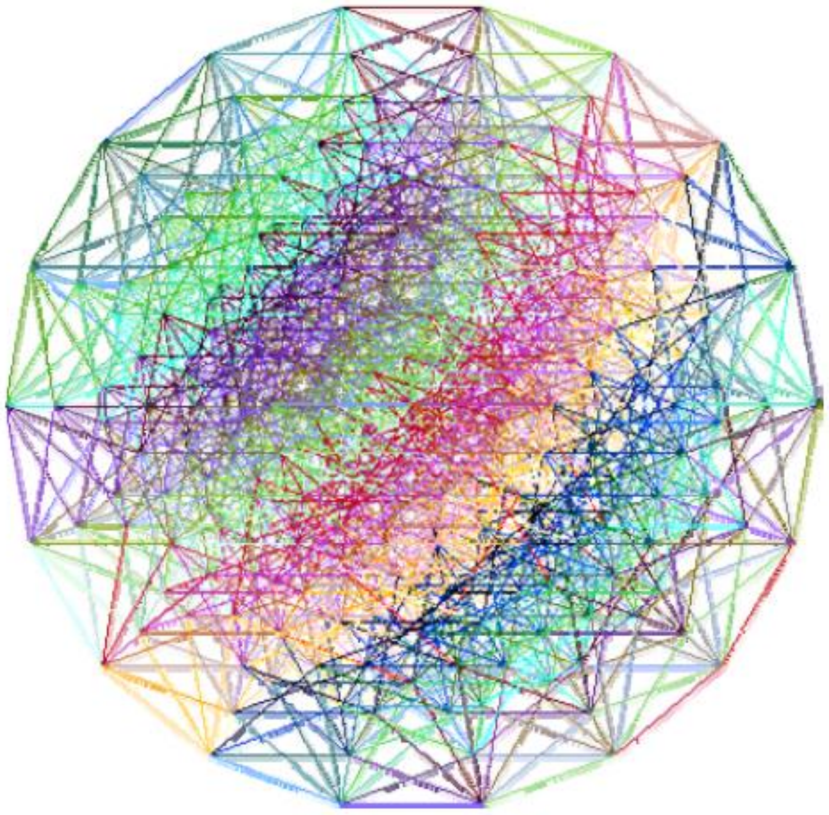
Here:

# of dimensions $d = N \to \infty$

system linear size $L = 1$



6-dimensional cube



9-dimensional cube

Part 6.

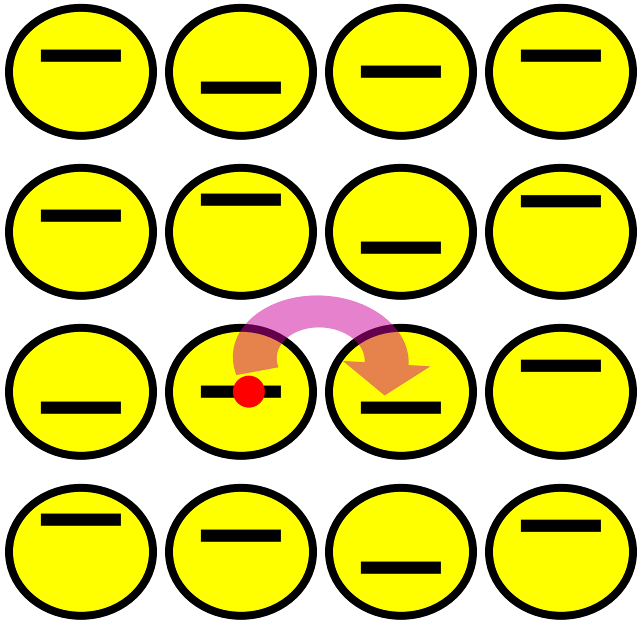
Many-Body Localization

b) Interacting particles

Conventional Anderson Model

- one particle,
- one level per site,
- onsite disorder
- nearest neighbor hoping

Basis: $|i\rangle$, i labels sites



Hamiltonian: $\hat{H} = \hat{H}_0 + \hat{V}$

$$\hat{H}_0 = \sum_i \varepsilon_i |i\rangle\langle i|$$

$$\hat{V} = \sum_{i,j=n.n.} I|i\rangle\langle j|$$

many (N) particles no interaction. Individual energies $\varepsilon_k, k = 1,\ldots N$ are conserved → N conservation laws "integrable system" →

$$\hat{H} = \sum_\mu E_\mu |\mu\rangle\langle\mu|$$

$$E_\mu = \sum_k \varepsilon_k = \sum_\alpha \varepsilon_\alpha n_\alpha$$

α labels one-particle eigenstates; n_α - occupation numbers; $\mu = \{n_\alpha\}$

many (N) particles no interaction. Individual energies $\varepsilon_k, k = 1,\ldots N$ are conserved

$\Rightarrow$ N conservation laws "integrable system" $\Rightarrow$

$$\hat{H} = \sum_{\mu} E_{\mu} |\mu\rangle\langle\mu|$$

$$E_{\mu} = \sum_{k} \varepsilon_k = \sum_{\alpha} \varepsilon_{\alpha} n_{\alpha}$$

α labels one-particle eigenstates; n_{α} - occupation numbers; $\mu = \{n_{\alpha}\}$

Role of the interaction: $|\mu\rangle \to |\mu'\rangle$

Transitions between the "ideal gas" states

Basis: $|\mu\rangle$ Hamiltonian: $\hat{H} = \hat{H}_0 + \hat{V}, \quad \hat{H}_0 \equiv \sum_{\mu} E_{\mu} |\mu\rangle\langle\mu|$

Interaction: $\hat{V} \equiv \sum_{\mu,\mu'=n.n?} I_{\mu,\mu'} |\mu\rangle\langle\mu'|$

Localization in Fock space

Disorder + Weak Interaction

Basis: $|\mu\rangle$

Hamiltonian: $\hat{H} = \hat{H}_0 + \hat{V},$

$$\hat{H}_0 \equiv \sum_{\mu} E_{\mu} |\mu\rangle\langle\mu|$$

Interaction: $\hat{V} \equiv \sum_{\mu,\mu'=n.n?} I_{\mu,\mu'} |\mu\rangle\langle\mu'|$

Anderson model

Q: What is the lattice ?

Part 6.

Many-Body Localization

c) Statistical mechanics

Main postulate of the Gibbs StatMech- equipartition (microcanonical distribution):

In the equilibrium all states with the same energy are realized with the same probability.

Without interaction between particles the equilibrium would never be reached – each one-particle energy is conserved.

Common believe: Even weak interaction should drive the system to the equilibrium.

It might be not true for many-body localized states !!!

Localization in Fock space

What does it mean?

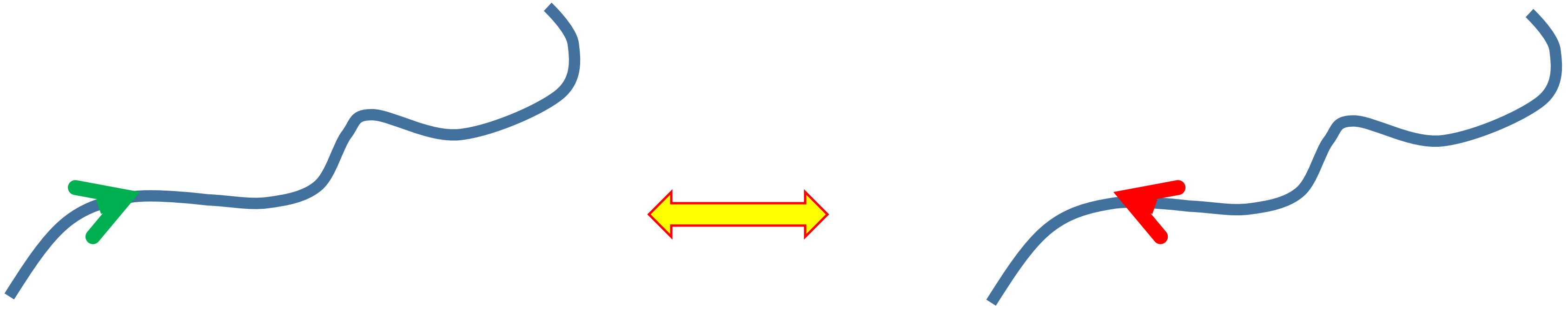
- No two-body operator can cause transitions between many-body states that are close in energy.
- No dissipation due to the external field – ideal insulator (glass)
- The concept of the equilibrium looses its meaning – no relaxation to a thermal state.
- No entropy production

Temperature ⟺ Energy

Time-reversal symmetry = T-invariance

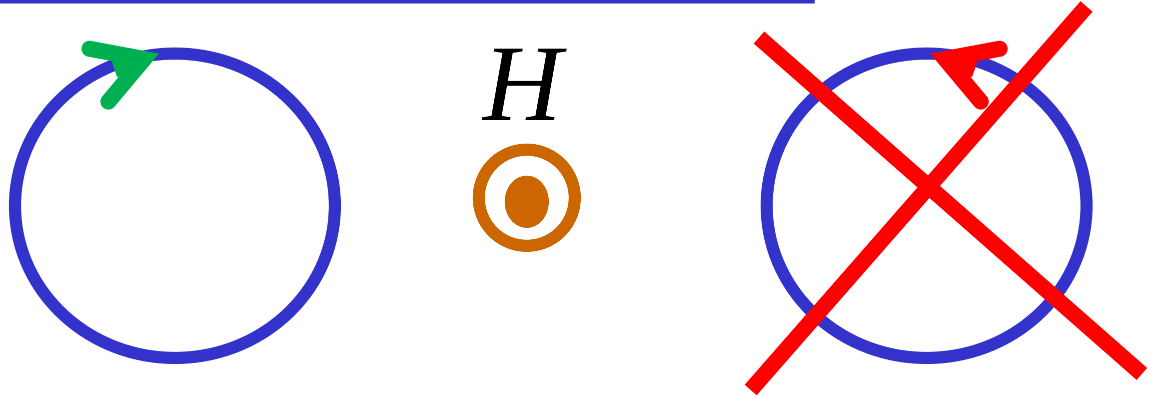
Equations of motion are invariant under $t \leftarrow -t$

For each classical trajectory there is another trajectory, which is its inversion in time

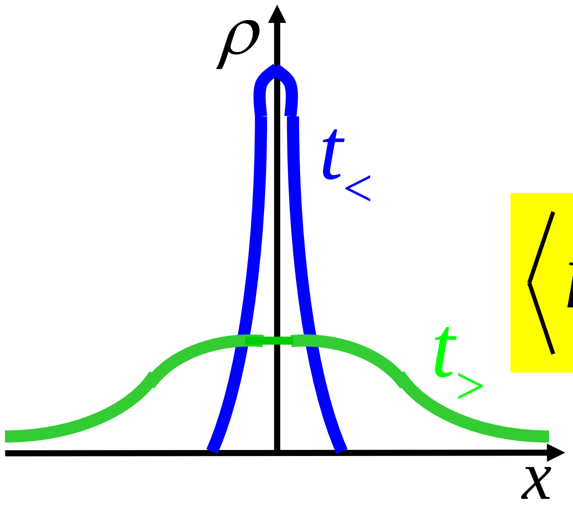


Violation of the T-invariance

e.g. by magnetic field



Statistical mechanics – Irreversibility - arrow of time



$$\langle r^2 \rangle \propto t$$

$$\langle r^2(t_>) \rangle > \langle r^2(t_<) \rangle$$

Has **nothing** to do with the violation of the T-invariance

Has everything to do with the delocalization

Extended states – **irreversible** dynamics

Localized states – dynamics is to some extent **reversible**

The same is true for many body systems

Heat Theorem

Nerns, Einstein, Planck, Polany,…

"[Nernst is] not open to reason, because he is not enough of a logician" ("der Vernunft nich zuga¨nglich (zu wenig Logiker))". Einstein, letter to Ehrenfest

Is it possible to reach zero temperature?

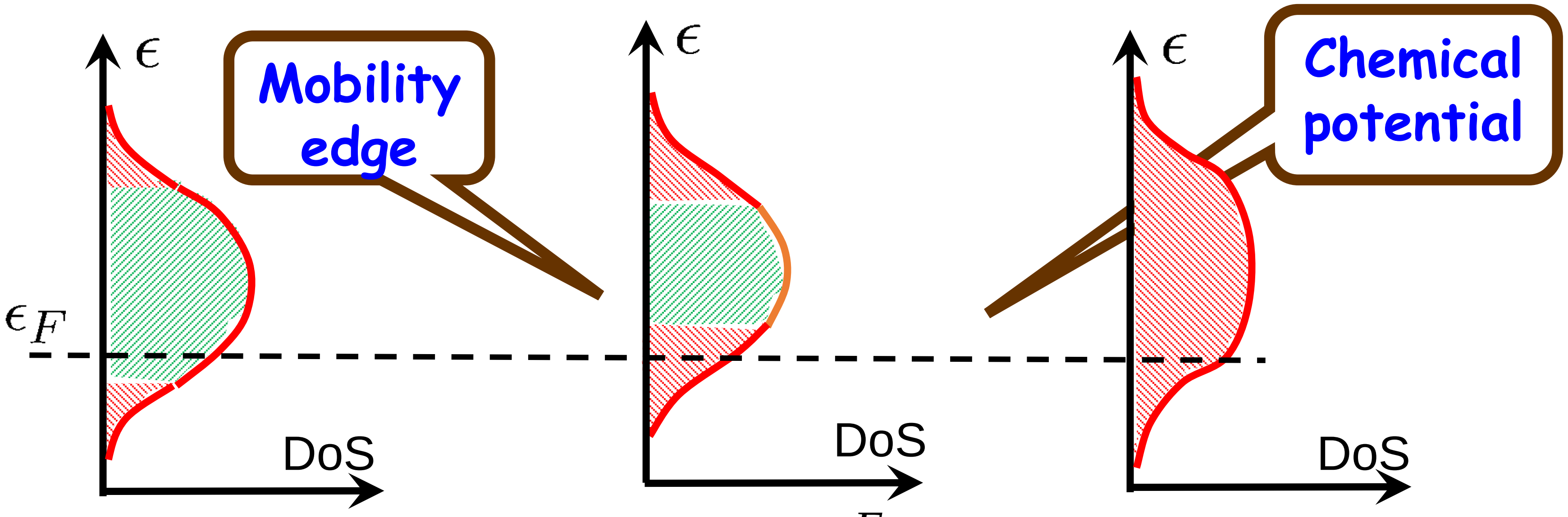
Is it possible to reach thermal equilibrium close to the ground state ?

Part 6.

Many-Body Localization

d) Interacting fermions; phononless transport

Temperature dependence of the conductivity
one-electron picture



$$\sigma(T \to 0) > 0 \qquad \sigma(T) \propto e^{-\frac{E_c - \varepsilon_F}{T}} \qquad \sigma(T) = 0 \;\; \forall T$$

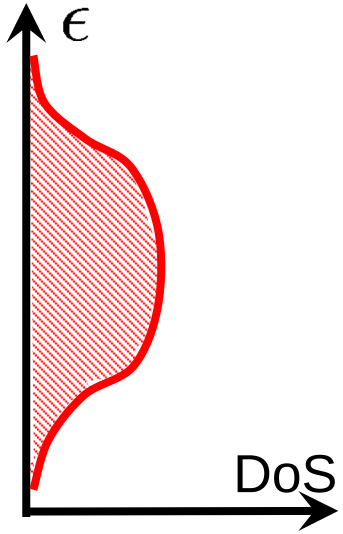
there are extended states — $I > I_c$

all states are localized — $I < I_c$

Temperature dependence of the conductivity
one-electron picture

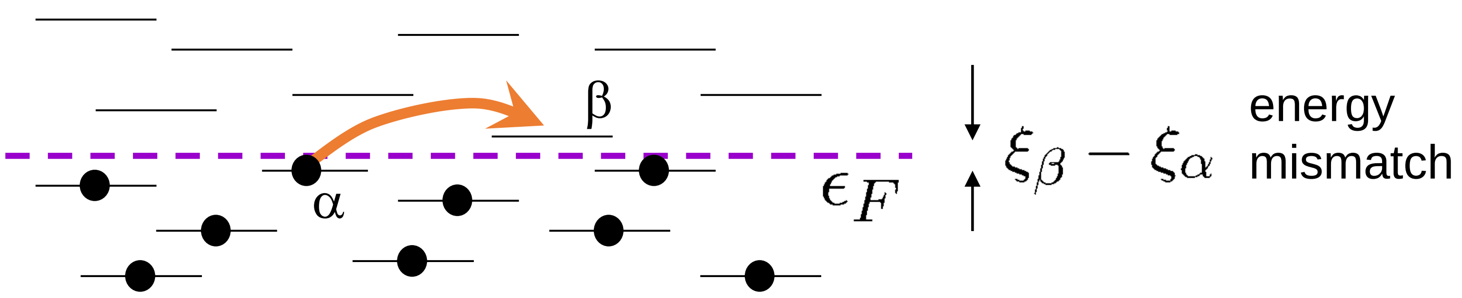
Assume that all the states are **localized**;

e.g. $d = 1,2$



$$\sigma(T) = 0 \quad \forall T$$

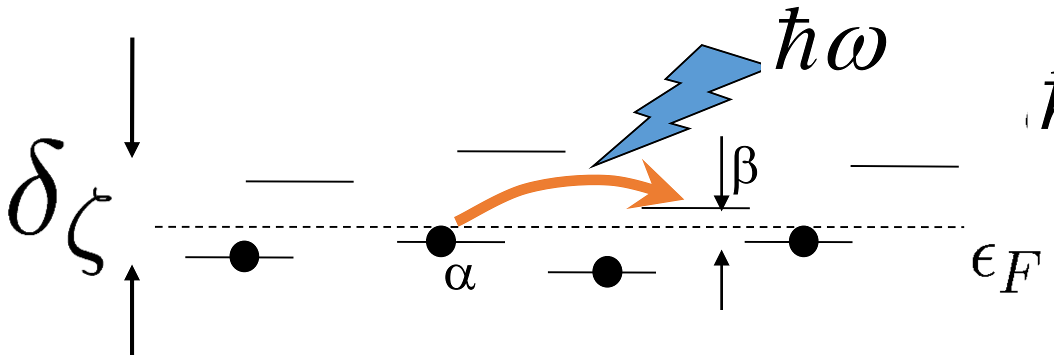
Inelastic processes
transitions between localized states



$$T = 0 \quad \Rightarrow \quad \sigma = 0$$

(any mechanism)

Phonon-assisted hopping



$$\hbar\omega = \varepsilon_\alpha - \varepsilon_\beta$$

$$\sigma(T=0) = 0$$

Variable Range Hopping

N.F. Mott (1968)

$$\sigma(T) \propto T^\gamma \exp\left[-\left(\frac{\delta_\zeta}{T}\right)^{\frac{1}{d+1}}\right]$$

Mechanism-dependent prefactor

Optimized phase volume

Any bath with a continuous spectrum of delocalized excitations down to $\omega = 0$ will give the same exponential

Common belief: Anderson Insulator weak e-e interactions → Phonon assisted hopping transport

Can hopping conductivity exist without phonons ?

Given:

1. All one-electron states are localized
2. Electrons interact with each other
3. The system is closed (no phonons)
4. Temperature is low but finite

Find: DC conductivity $\sigma(T, \omega=0)$ (zero or finite?)

Q: Can e-h pairs lead to phonon-less variable range hopping in the same way as phonons do ?

A#1: Sure

1. Recall phonon-less *AC conductivity:*
   Sir N.F. Mott (1970)

$$\sigma(\omega) = \frac{e^2 \zeta_{loc}^{d-2}}{\hbar} \left(\frac{\hbar\omega}{\delta_\zeta}\right)^2 \ln^{d+1}\left|\frac{\delta_\zeta}{\hbar\omega}\right|$$

2. Fluctuation Dissipation Theorem:
   there should be Johnson-Nyquist noise

3. Use this noise as a bath instead of phonons

4. Self-consistency (whatever it means)

Q: Can e-h pairs lead to phonon-less variable range hopping in the same way as phonons do ?

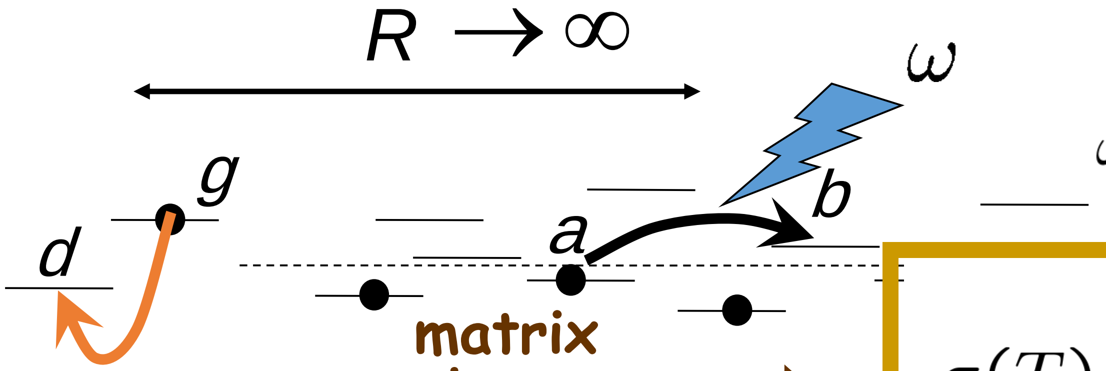
A#1: **Sure**

A#2: **No way** (L. Fleishman. P.W. Anderson (1980))

Except maybe Coulomb interaction in 3D

$$\sigma(\omega) \simeq \frac{e^2 \zeta_{loc}^{d-2}}{\hbar} \left(\frac{\hbar\omega}{\delta_\zeta}\right)^2 \ln^{d+1}\left|\frac{\delta_\zeta}{\hbar\omega}\right|$$

is contributed by rare resonances



$$\omega = \xi_\beta - \xi_\alpha = \xi_\gamma - \xi_\delta$$

$R \to \infty \Rightarrow$ matrix element vanishes

$$\sigma(T) \propto 0 \; \exp\left[-\left(\frac{\delta_\zeta}{T}\right)^{\frac{1}{d+1}}\right]$$

No phonons → ??? → No transport $\forall T$

Problem:

- If the localization length exceeds L_φ, then – metal.
- In a metal e-e interaction leads to a finite L_φ

⇒ At high enough temperatures conductivity should be **finite** even **without phonons**

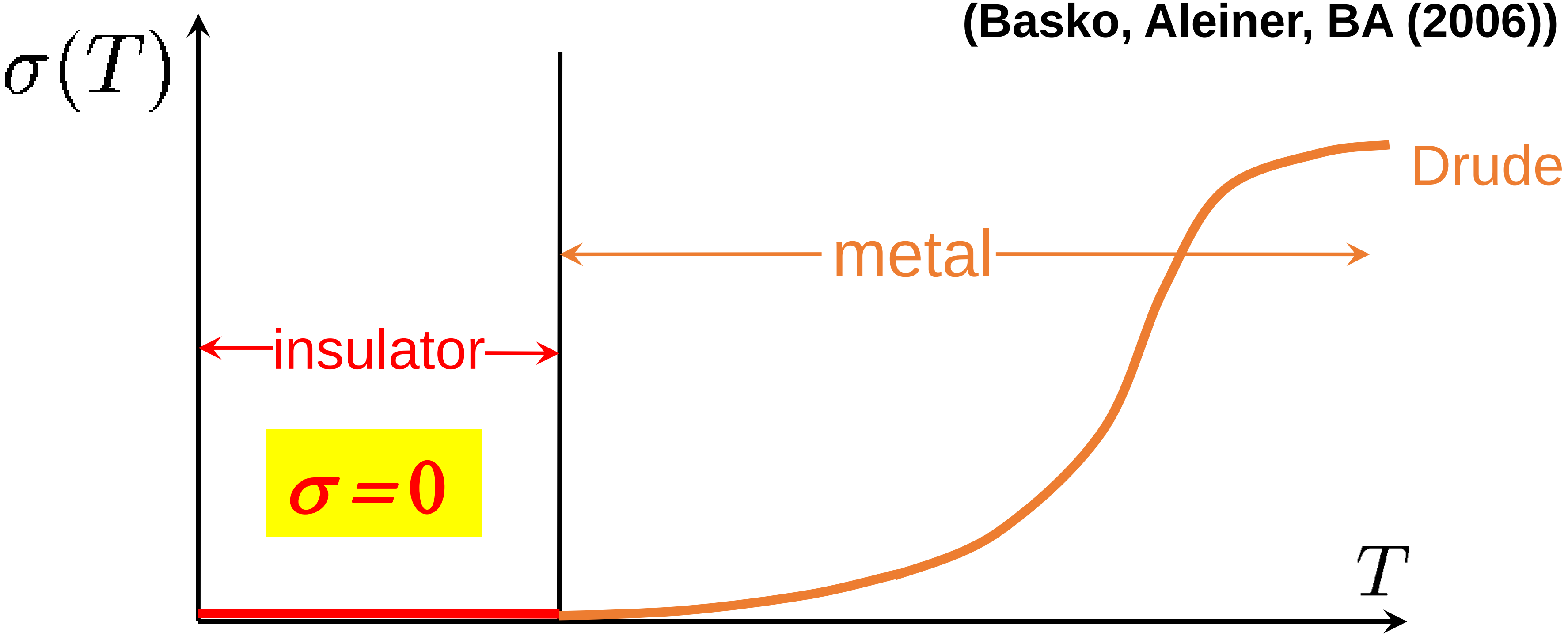
Q: Can e-h pairs lead to phonon-less variable range hopping in the same way as phonons do ?

A#1: Sure

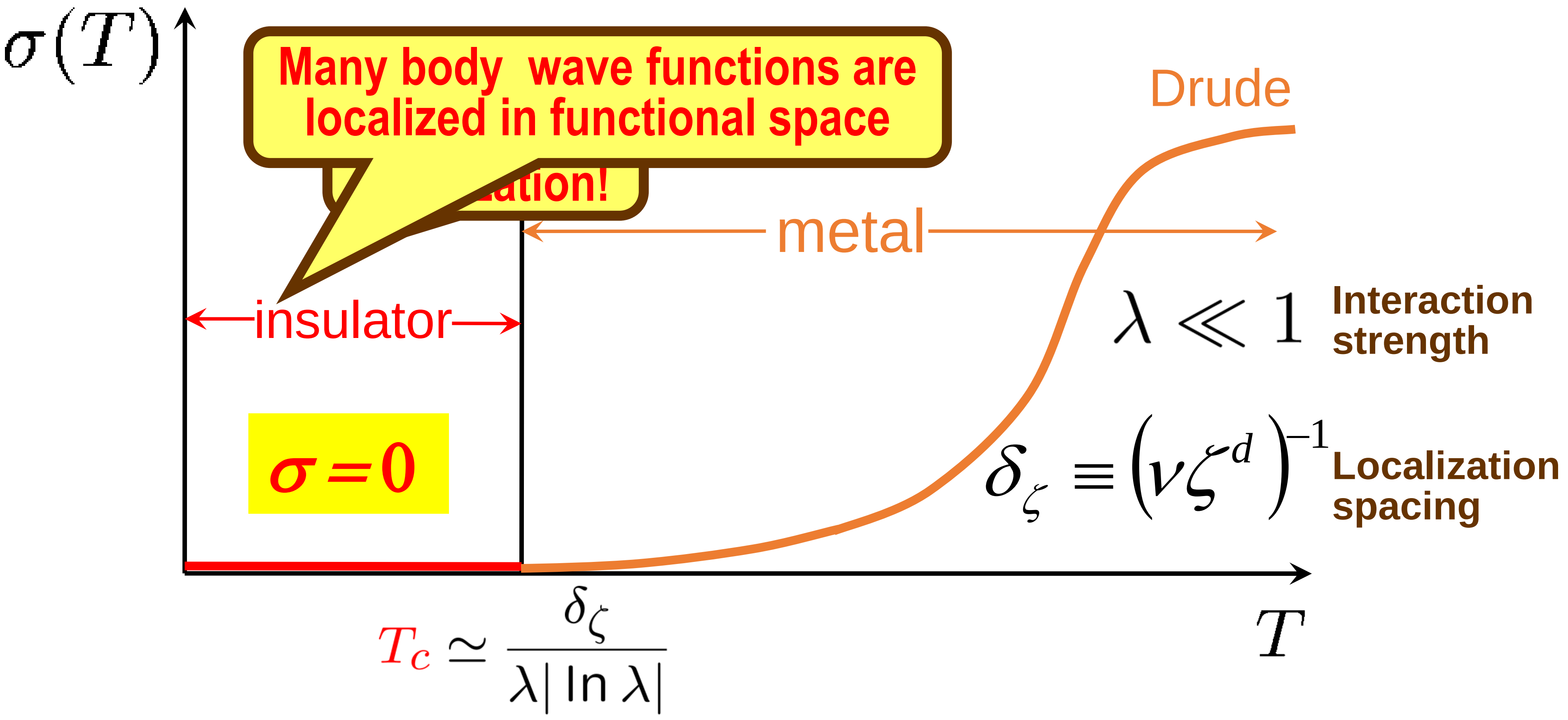
A#2: No way (L. Fleishman. P.W. Anderson (1980))

A#3: Finite temperature Metal-Insulator Transition

(Basko, Aleiner, BA (2006))



Finite temperature Metal-Insulator Transition



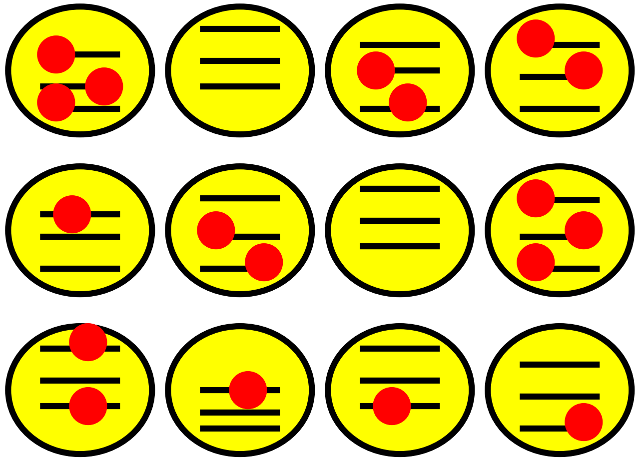
Definitions:

Insulator $\sigma = 0$ **Metal** $\sigma \neq 0$

not $d\sigma/dT < 0$ **not** $d\sigma/dT > 0$

Many body Anderson-like Model

- many particles,
- several levels per site,
- onsite disorder
- local interaction



Basis: $|\mu\rangle$

$$\mu = \left\{ n_i^\alpha \right\}$$

i labels sites α labels levels

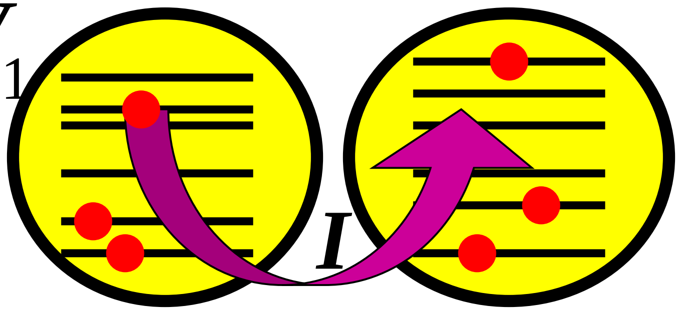
$$n_i^\alpha = 0,1$$ occupation numbers

Hamiltonian:

$$\hat{H} = \hat{H}_0 + \hat{V}_1 + \hat{V}_2$$

$$\hat{H}_0 = \sum_\mu E_\mu |\mu\rangle\langle\mu|$$

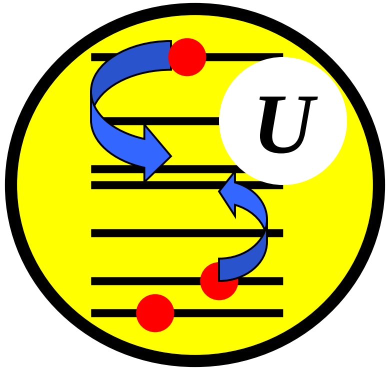
$\hat{V}_1$



$$\hat{V}_1 = \sum_{\mu,\nu(\mu)} I |\mu\rangle\langle\nu(\mu)|$$

$$|\nu(\mu)\rangle = |.., n_i^\alpha - 1, .., n_j^\beta + 1, ..\rangle, \quad i, j = n.n.$$

$\hat{V}_2$



$$\hat{V}_2 = \sum_{\mu,\eta(\mu)} U |\mu\rangle\langle\eta(\mu)|$$

$$|\nu(\mu)\rangle = |.., n_i^\alpha - 1, .., n_i^\beta - 1, .., n_i^\gamma + 1, .., n_i^\delta + 1, ..\rangle$$

Stability of the insulating phase: NO spontaneous generation of broadening

$$\Gamma_\alpha(\varepsilon)=0$$

is always a solution

$$\varepsilon \to \varepsilon + i\eta$$

linear stability analysis

$$\frac{\Gamma}{(\varepsilon-\xi_\alpha)^2+\Gamma^2} \to \pi\delta(\varepsilon-\xi_\alpha)+\frac{\Gamma}{(\varepsilon-\xi_\alpha)^2}$$

After ***n*** iterations of the equations of the Self Consistent Born Approximation

$$P_n(\Gamma) \propto \frac{\eta}{\Gamma^{3/2}}\left(const\,\frac{\lambda T}{\delta_\zeta}\ln\frac{1}{\lambda}\right)^n$$

first $n \to \infty$

then $\eta \to 0$

$(\ldots) < 1$ – insulator is stable !

Physics of the transition: cascades

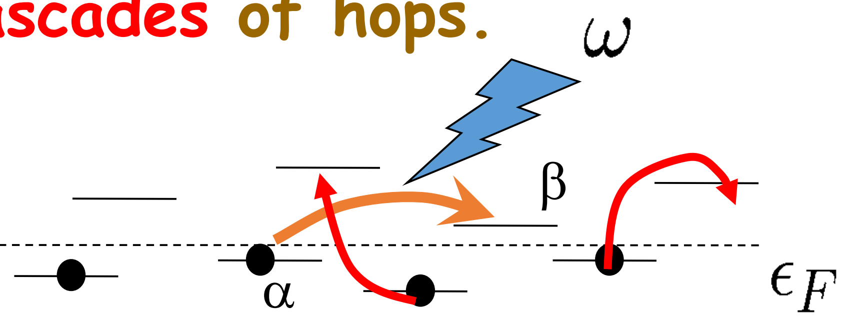
Conventional wisdom:

For phonon assisted hopping one phonon – one electron hop

It is maybe correct at low temperatures, but the higher the temperature the easier it becomes to create e-h pairs.

Therefore with increasing the temperature the typical number of pairs created n_c (i.e. the number of hops) increases. Thus phonons create cascades of hops.

Typical size of the cascade $\approx$ Localization length



Physics of the transition: cascades

Conventional wisdom:

For phonon assisted hopping one phonon – one electron hop

It is maybe correct at low temperatures, but the higher the temperature the easier it becomes to create e-h pairs.

Therefore with increasing the temperature the typical number of pairs created n_c (i.e. the number of hops) increases. Thus phonons create cascades of hops.

At some temperature $T = T_c \quad n_c(T) \to \infty.$

This is the critical temperature.

Above T_c one phonon creates infinitely many pairs, i.e., phonons are not needed for charge transport.