The ups and downs of quantum criticality in cuprates.

Jan Zaanen

Condensed matter physics in action.

1. “Stoquastic” quantum criticality: an introduction.

2. The “ups”: the nearly quantum critical charge order in cuprates revealed by the RIXS machine.

3. The “downs”: ARPES demonstrates that the big deal cuprate strange metal has nothing to do with a quantum phase transition

4. Theoretical epilogue: fermion signs, dense entanglement and the demise of semiclassics.

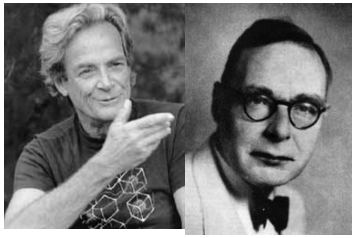
Quantum field theory = Statistical physics.

$$Z = \sum_{configs.} e^{-\frac{E_{config}}{k_B T}}$$

Path integral mapping

"Thermal QFT", Wick rotate:

$$t \to i\tau$$



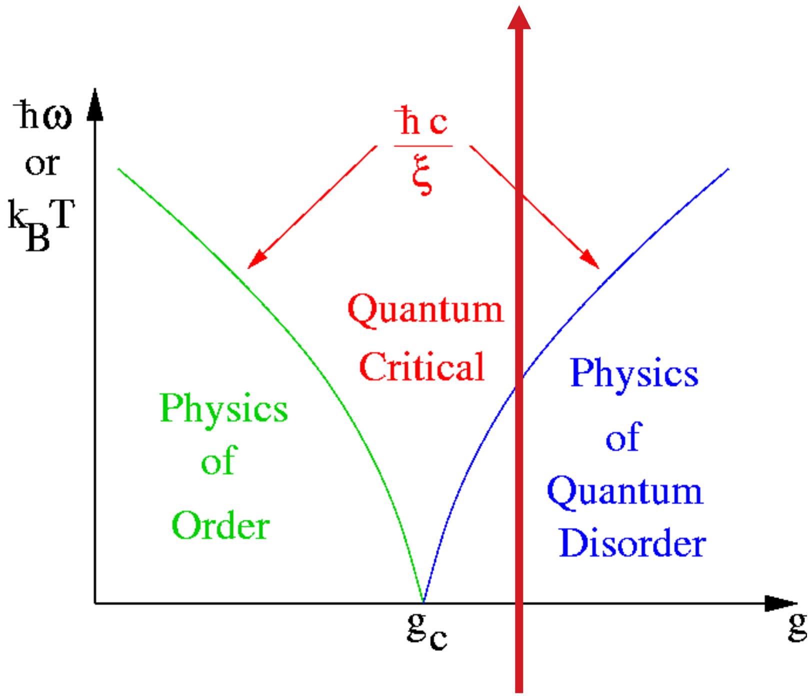
$$Z_\hbar = \sum_{worldhistories} e^{-\frac{S_{history}}{\hbar}}$$

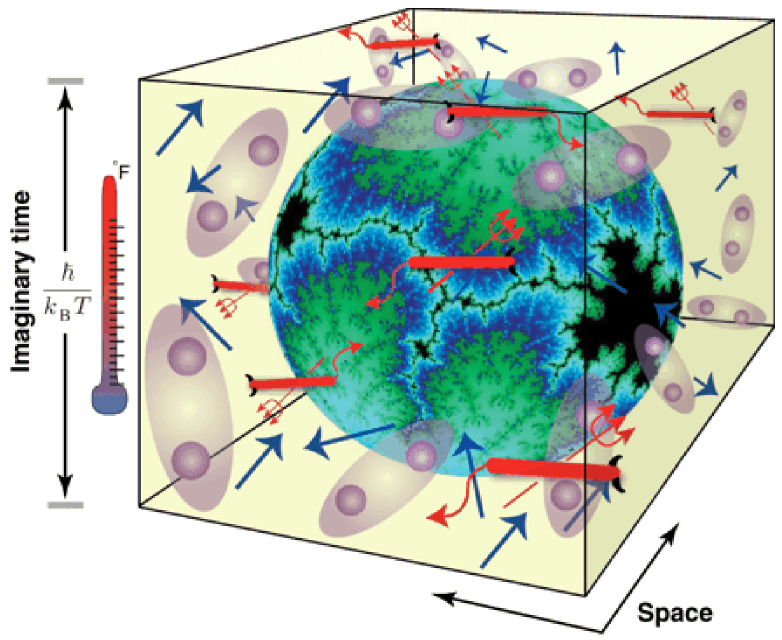
But generically: the quantum partition function is not probabilistic: "sign problem" or "non-stoquastic" = no mathematical control!

$$Z_\hbar = \sum_{worldhistories} (-1)^{history} e^{-\frac{S_{history}}{\hbar}}$$

Quantum Phase transitions

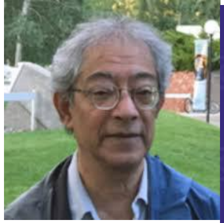
Quantum scale invariance emerges naturally at a zero temperature continuous phase transition driven by quantum fluctuations:



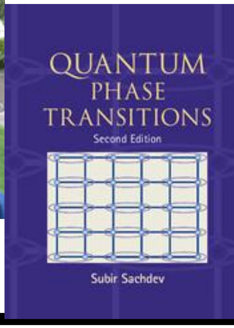


JZ, Science 319, 1205 (2008)

Stoquastic quantum phase transitions.

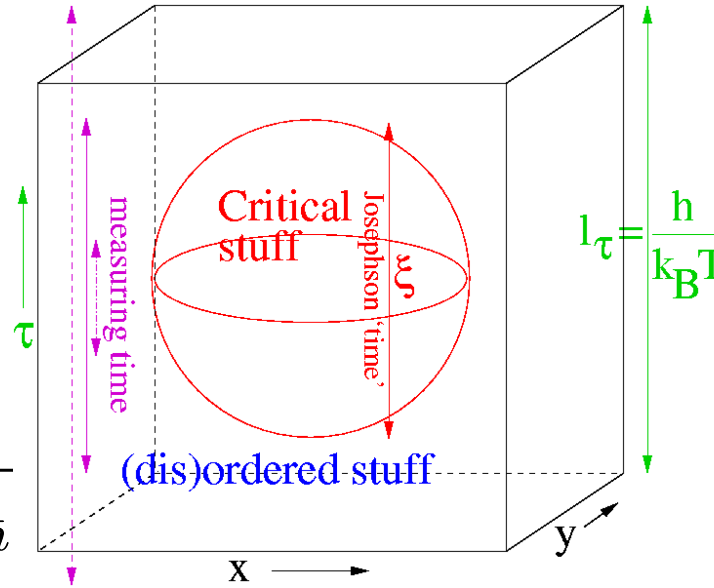


Chakravarty
1988



Quantum critical regime: $k_B T >> \frac{\hbar c}{\xi}$

$$\hbar\omega > k_B T : \quad \chi(\omega) \sim \frac{1}{(i\omega)^{2-\eta}}$$

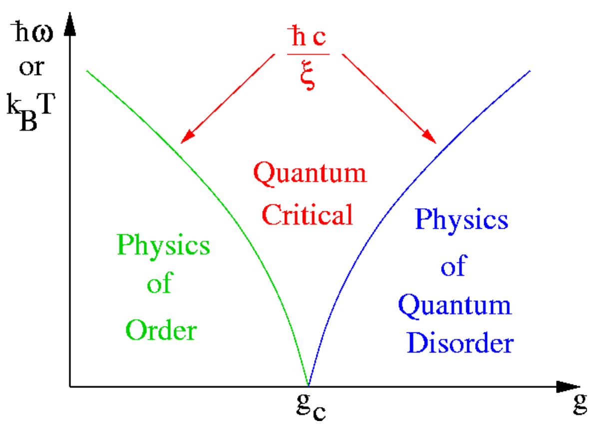


Non-conserved order parameter:

$$\hbar\omega < k_B T : \; \chi(\omega) \sim \frac{1}{T^{2-\eta}} \frac{1}{1 - i\omega\tau_\hbar}$$

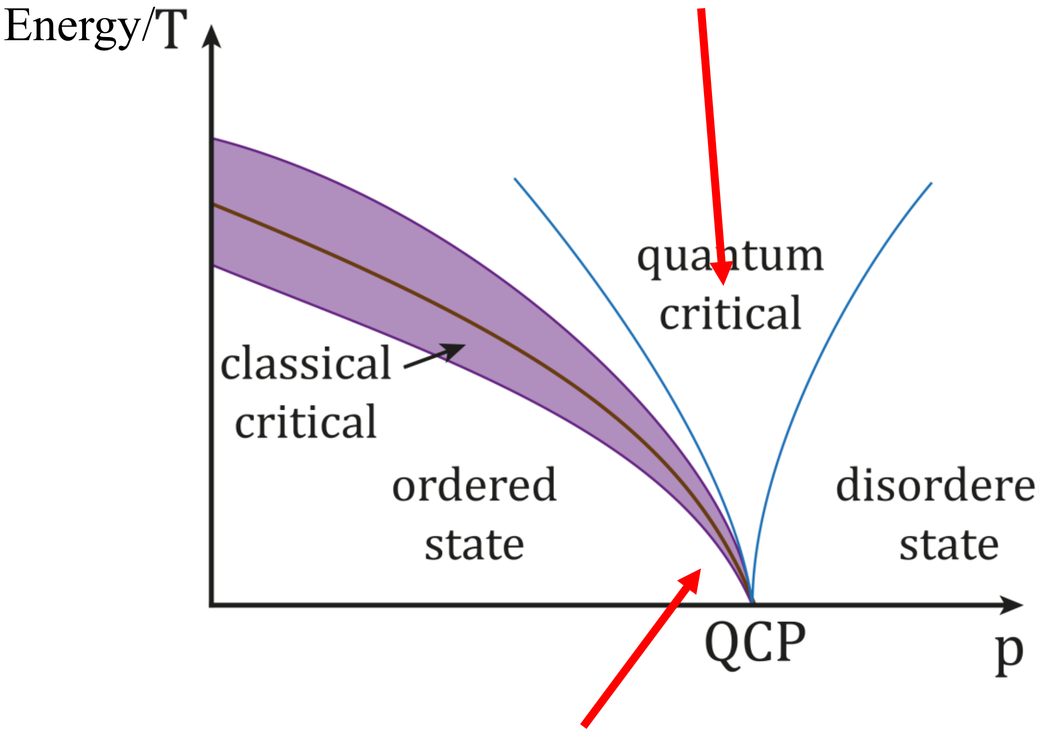
Planckian dissipation:

$$\tau_\hbar \simeq \frac{\hbar}{k_B T}$$



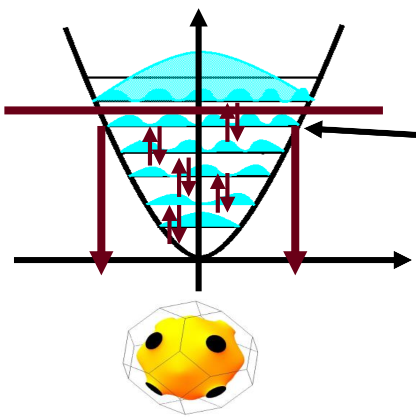
”Classical” gets renormalized .

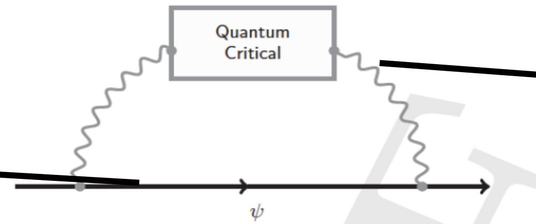
Scale invariance is dynamically generated “inside the wedge” anchored at an isolated point in coupling constant space:



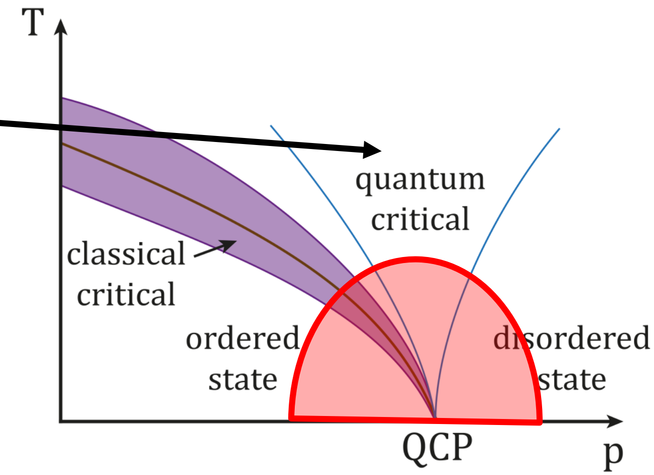
The classical order parameter and its stiffness are weakening approaching the “QCP”.

Quantum critical points in metals: "Hertz-Millis"(1975).





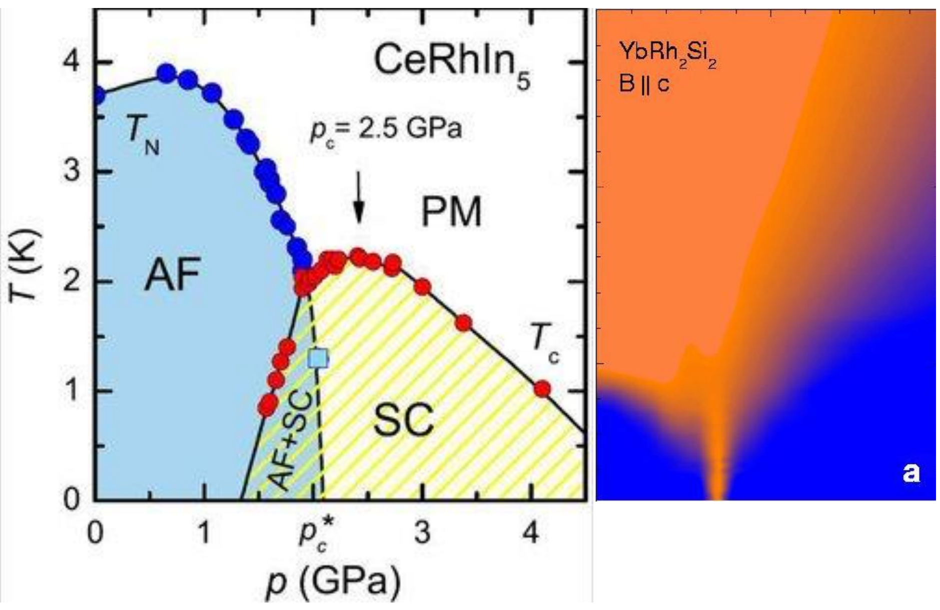
Couple quantum critical modes perturbatively to Fermi gas.

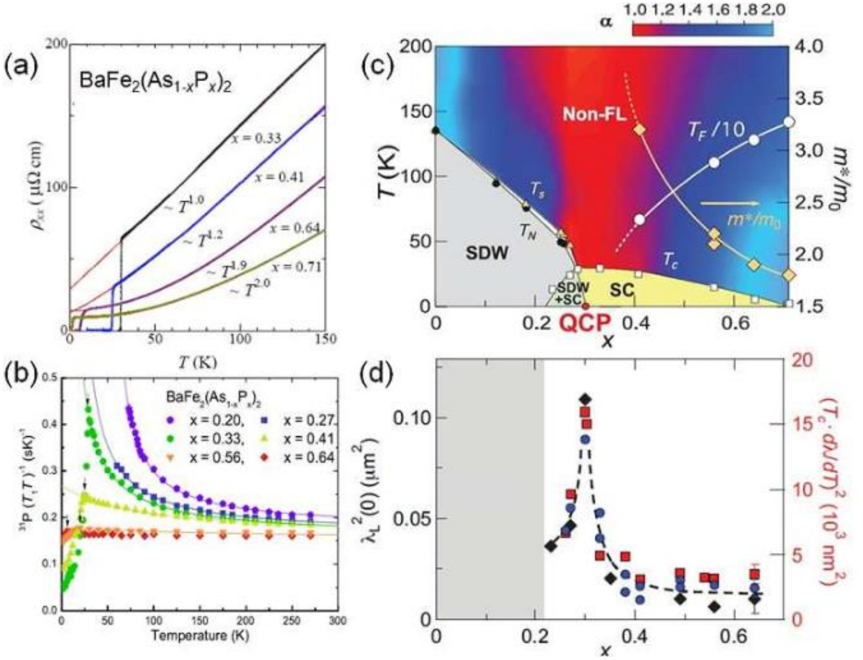


- Fermions acquire anomalous damping

- The quantum critical modes act as "pairing glue" bosons causing the superconducting dome centered at the QCP.

Quantum critical points: examples.

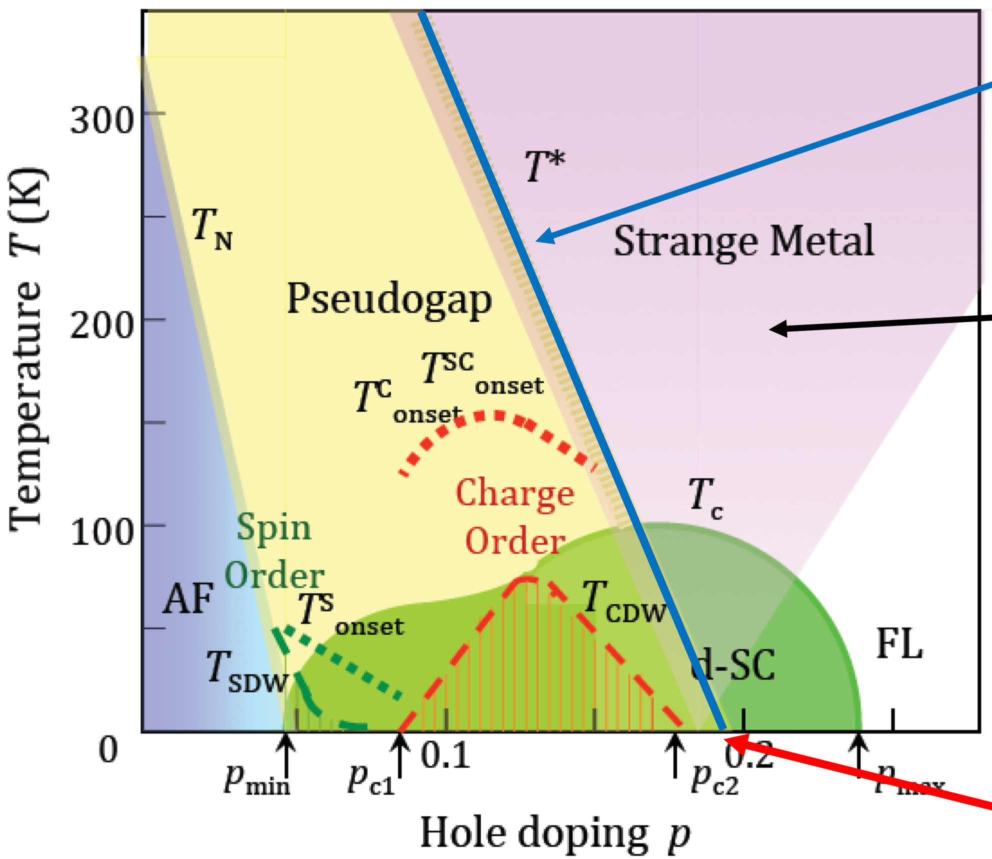




Heavy Fermion intermetallics

Iron superconductors

Cuprate high Tc superconductors?



B. Keimer et al., Nature 518, 179 (2015)

Associated with classical order parameter !?

Perfect Planckian dissipator: momentum relaxation time

$$\tau_K = \tau_\hbar = \frac{\hbar}{k_B T}$$

Putative quantum critical point !?

The need for dynamical information.

Space and time are "intertwined" in densely entangled quantum systems: the crucial information is encoded in the *dynamical* response functions $\chi_{\mathcal{O}\mathcal{O}}(\vec{q},\omega,T) = \langle \mathcal{O}\mathcal{O} \rangle_{\vec{q},\omega}$

$\mathcal{O}$ **is the spin operator: inelastic neutron scattering, in the 1990's evidences for "dynamical (fluctuating order) spin stripes".**

$\mathcal{O}$ **is the charge density: optical conductivity only at q =0, while EELS has just started to deliver!**

$\mathcal{O}$ **is the charge density: Resonant Inelastic X-ray Scattering has just started to deliver!**

$\mathcal{O}$ **is the single electron operator: plenty of high resolution data from ARPES and STS, pushing it in high resolution (un) particle detection.**

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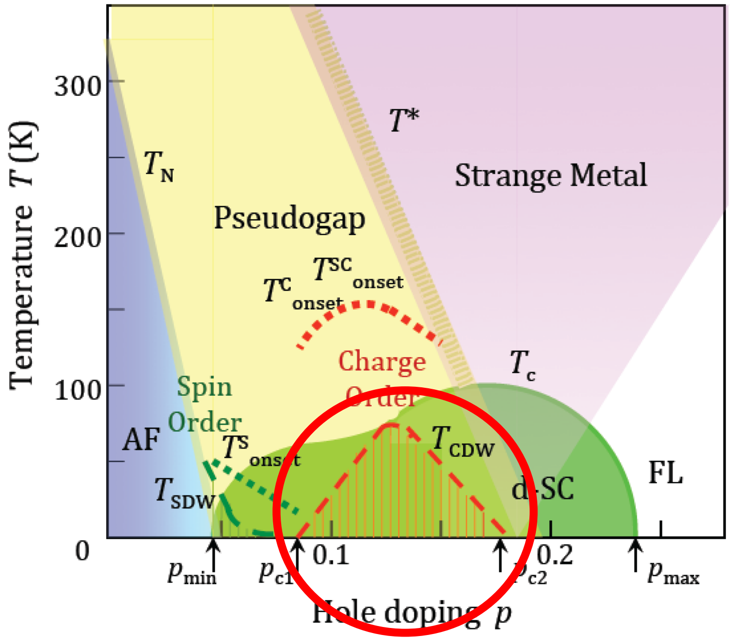
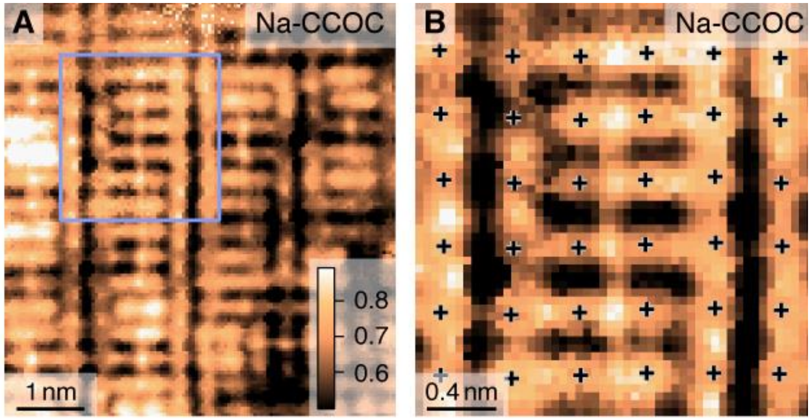
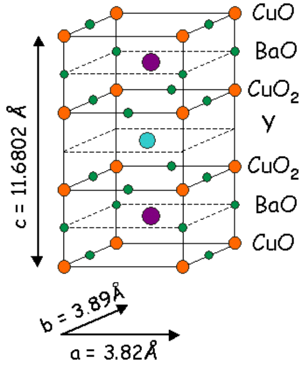
The "stripy" charge order.



Scanning Tunneling Spectroscopy: Cu 3d, O 2p electrons





Tiny Bragg peaks in hard X-ray diffraction: atoms barely move.

Big Bragg peaks in resonant X-ray diffraction: valence electrons.

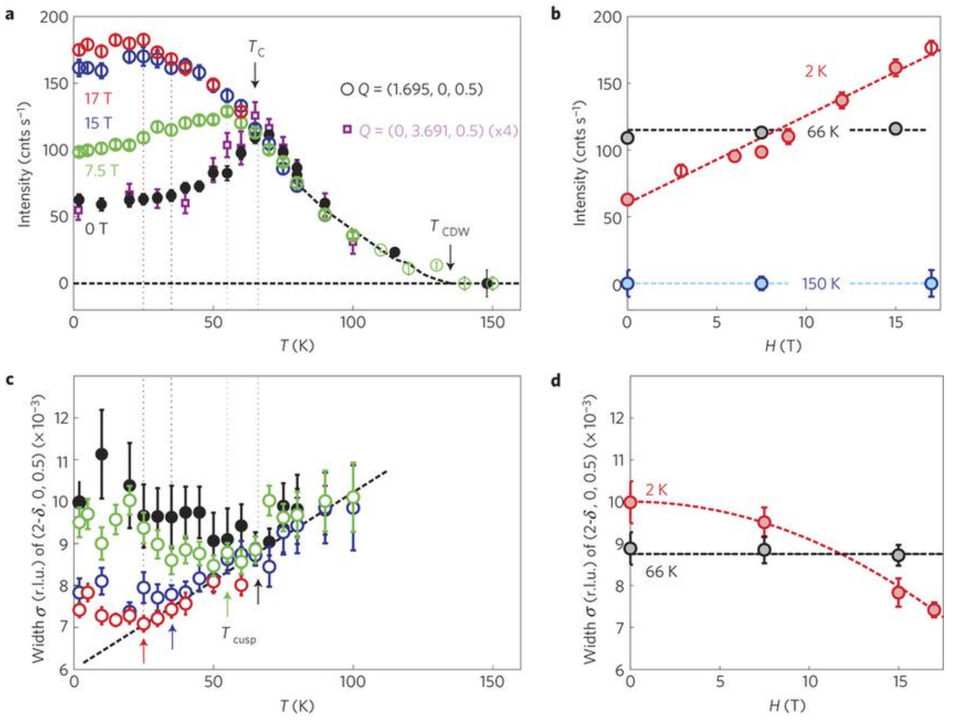
A "crystal" formed nearly entirely from electrons!

Superconductivity competing with charge order.

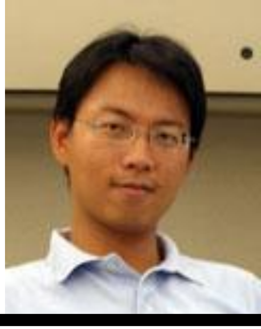
Interpreted as Landau mean field competing order:

$$F = m_\Psi^2|\Psi|^2 + w_\Psi|\Psi|^4 + m_\Phi^2|\Phi|^2 + w_\Phi|\Phi|^4 + g|\Psi|^2|\Phi|^2$$

$$F \to (-|m_\Psi^2| + g|\Phi|^2)|\Psi|^2 + \cdots$$

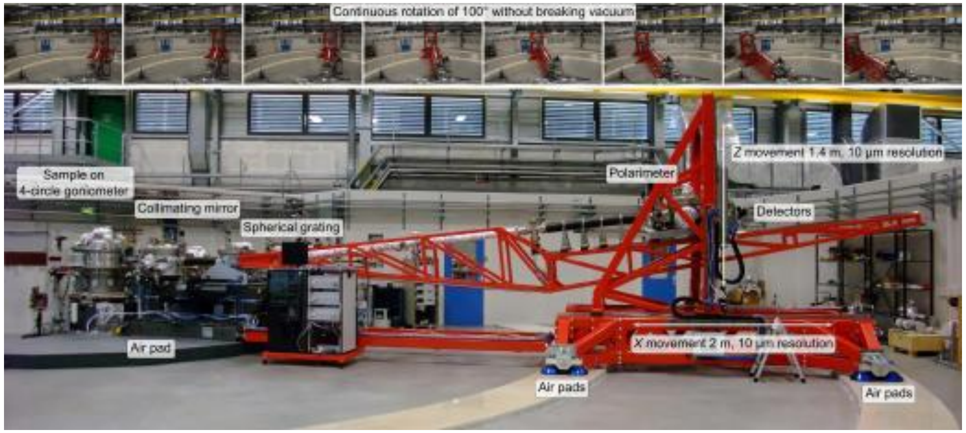


Resonant X-ray scattering.



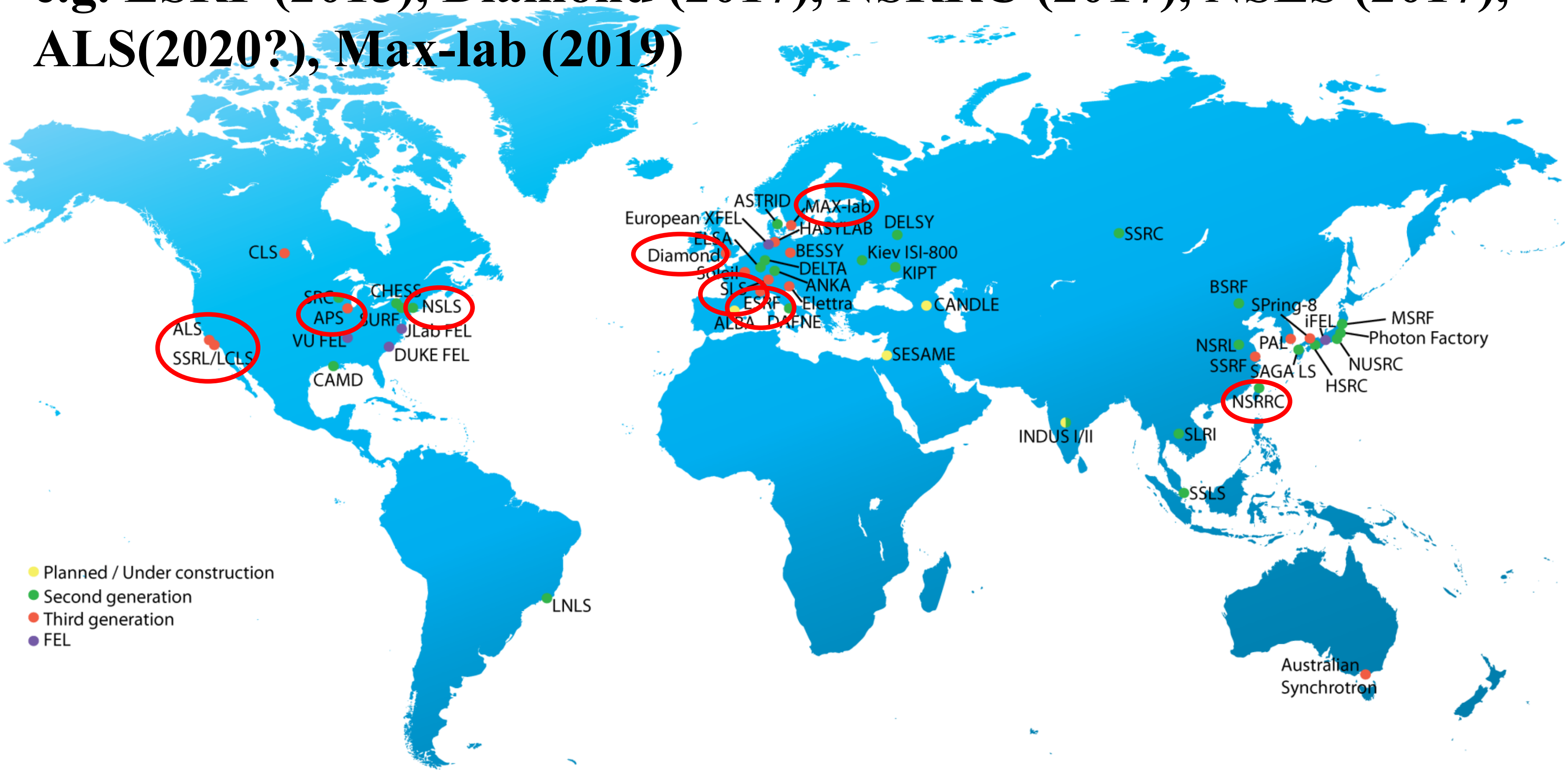
Tom
Devereaux

Wei-Sheng
Lee

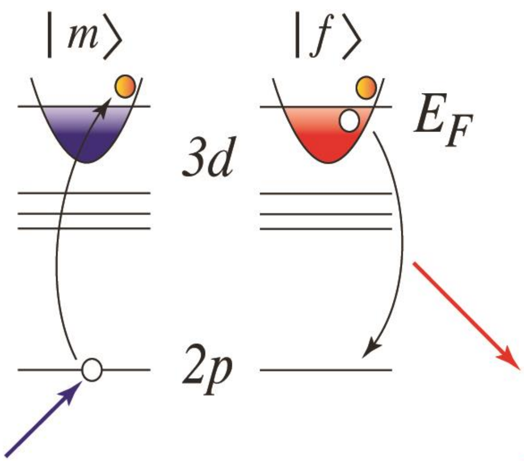


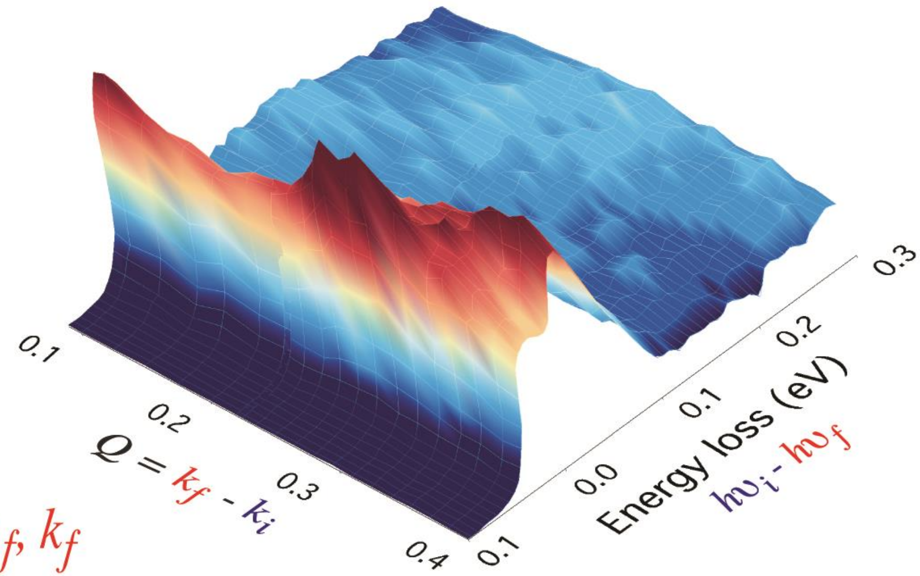
Map of the new RIXS instruments

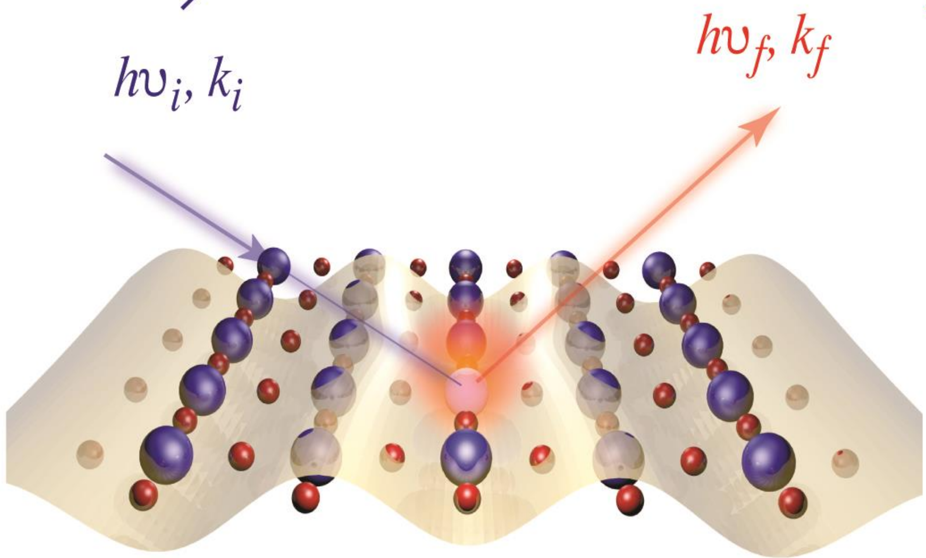
e.g. ESRF (2015), Diamond (2017), NSRRC (2017), NSLS (2017), ALS(2020?), Max-lab (2019)



Resonant Inelastic X-ray Scattering (RIXS)





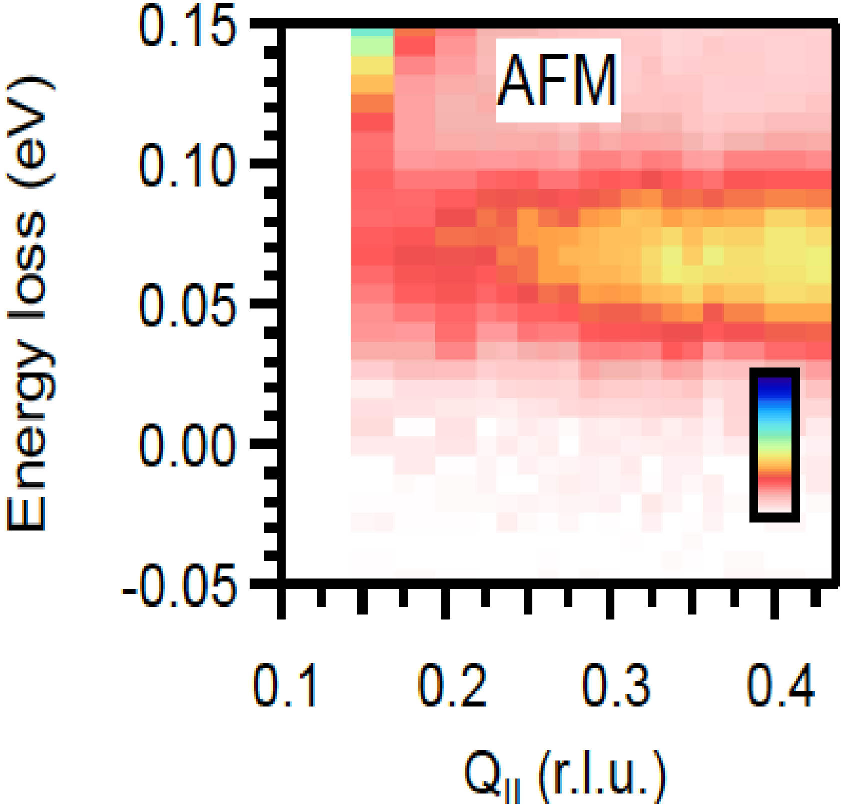


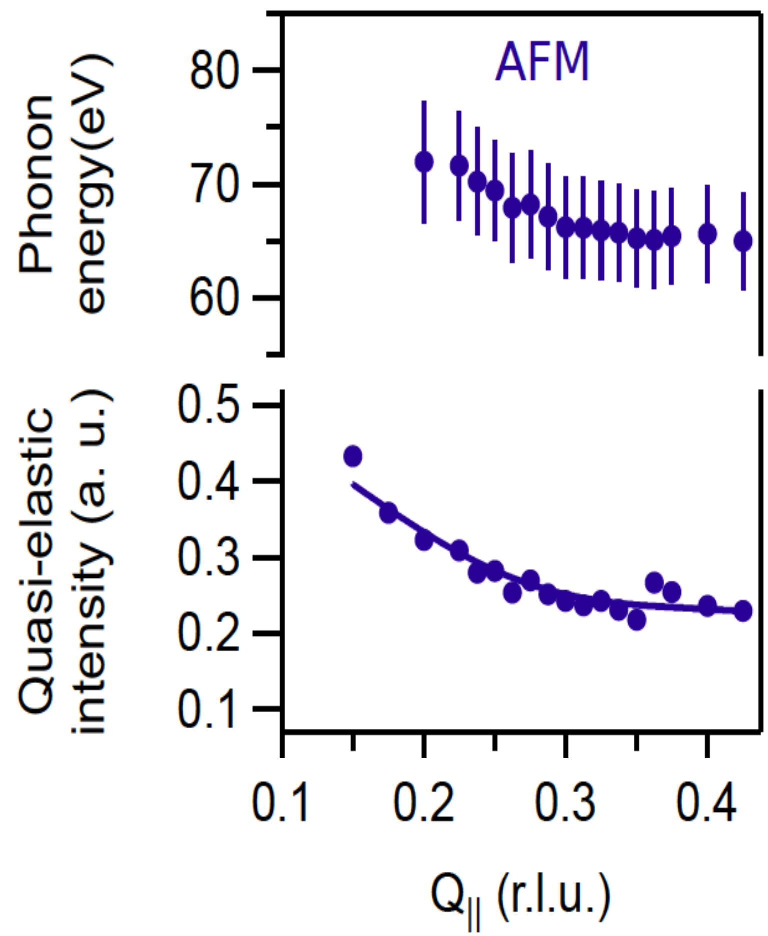
- **Element Specific.**
- **Excited Final States:**
  Elementary excitations!
- **Momentum-conserved**

$$\sum_n \frac{\langle f|\mathcal{D}'^\dagger|n\rangle\langle n|\mathcal{D}|g\rangle}{E_g + \hbar\omega_{\mathbf{k}} - E_n + i\Gamma_n},$$

Reference: a case of no CDW

AFM-Bi2212, p < 0.03

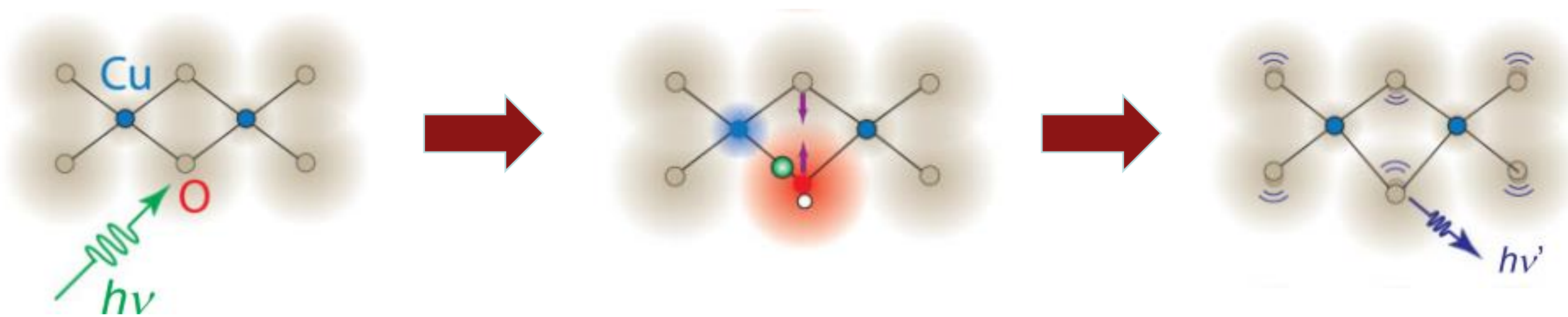


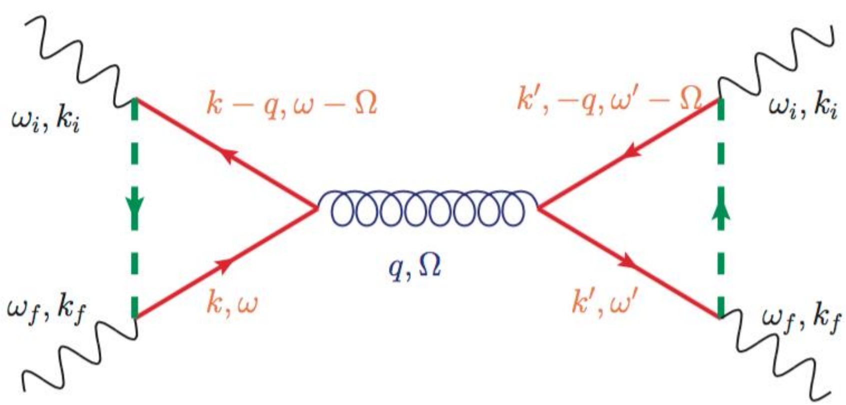


H. Lu *et al., in preparation*

E-ph coupling via RIXS

Initial state **Intermediate state** **Final state**





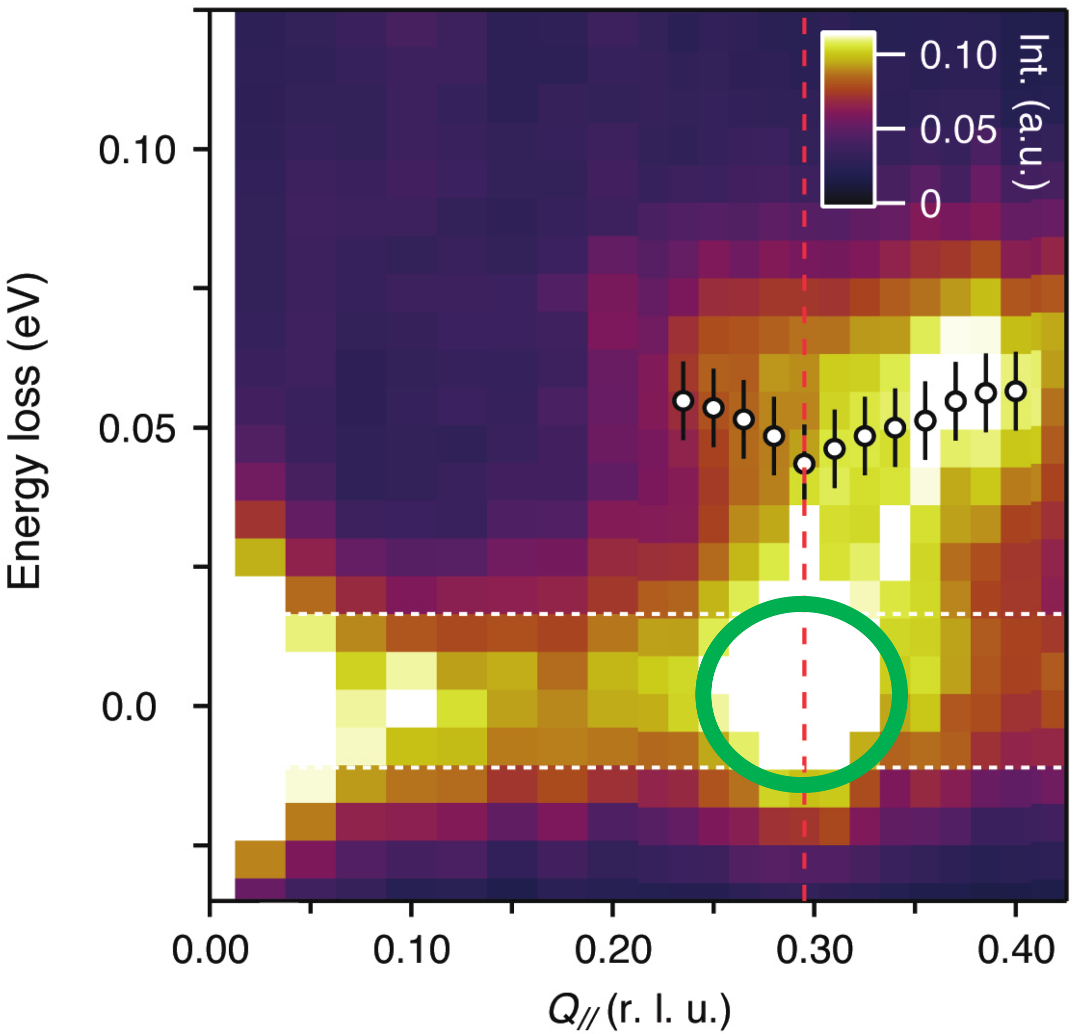
RIXS phonon cross section

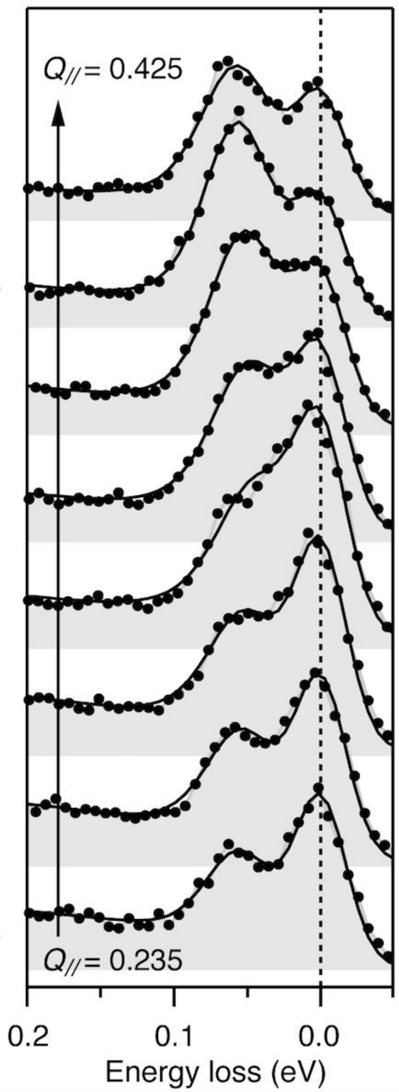
- **directly reflects the e-ph strength.**
- **Fano interference with underlying charge modes/continuum.**

L. J. P. Ament *et al*., EPL 95, 27008 (2011).
T.P. Devereaux et al., PRX 6, 041019 (2016).

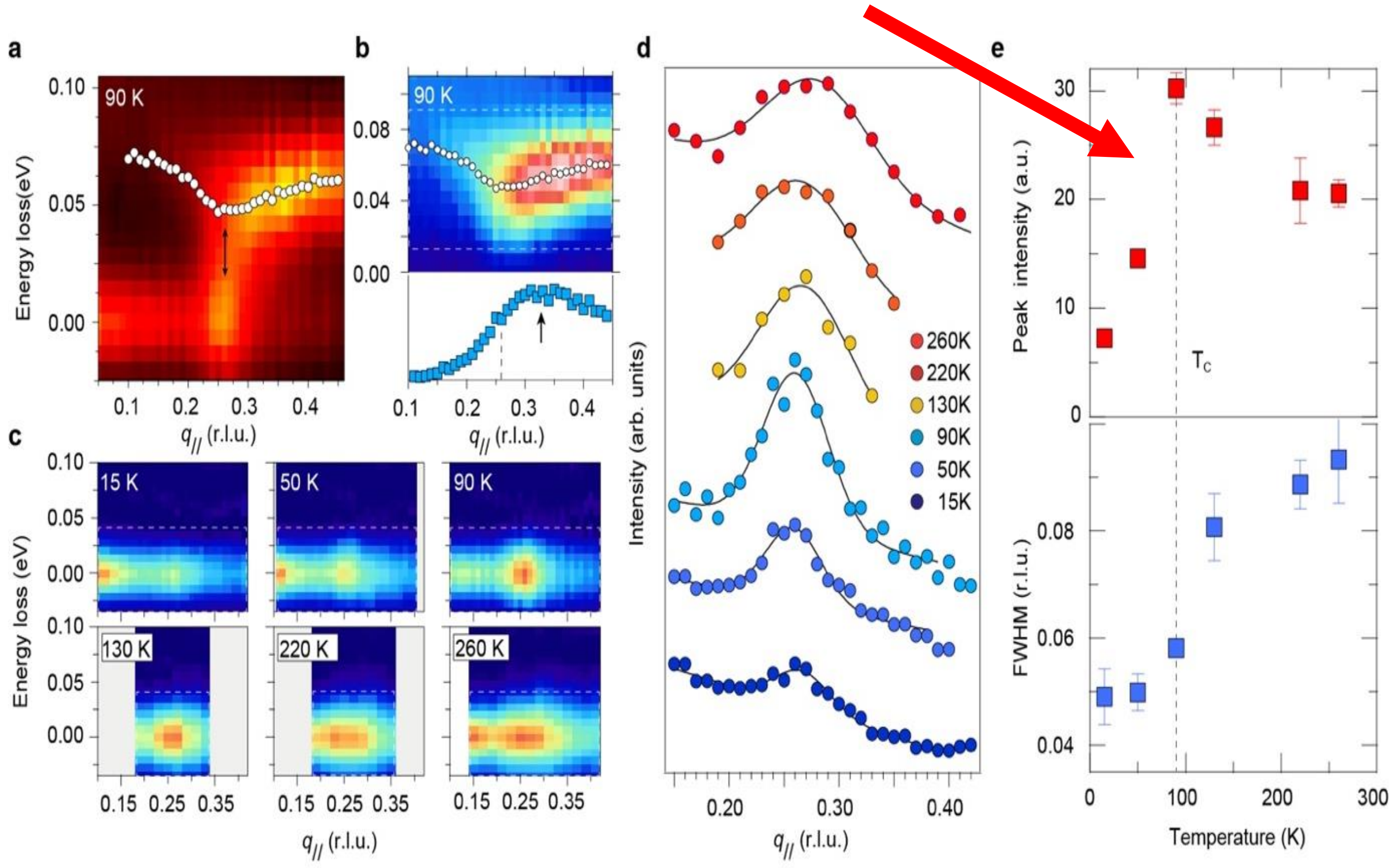
RIXS spectrum on under-doped Bi2212

Bi-2212 Tc = 45 K





RIXS: elastic peak and the charge order.



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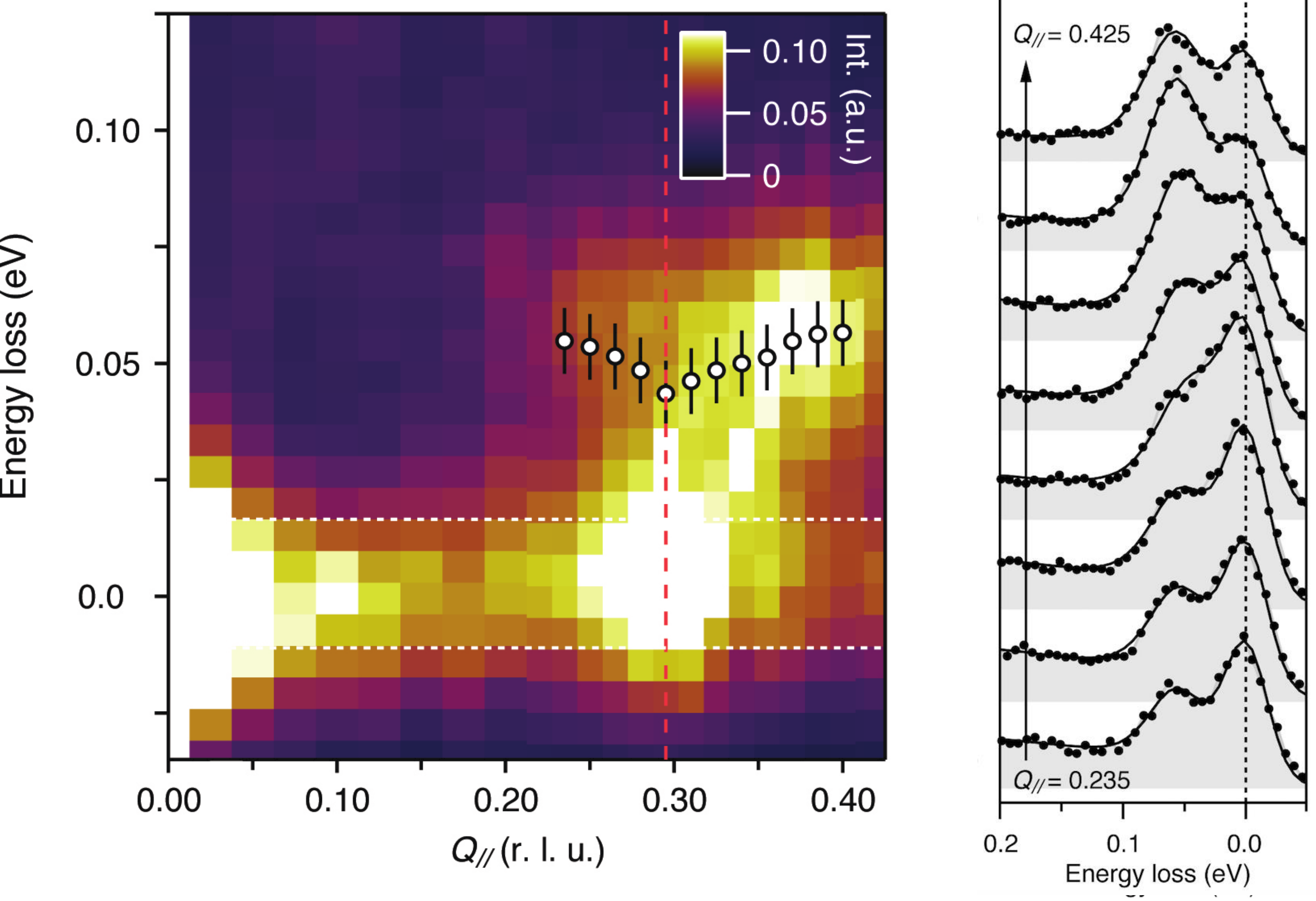
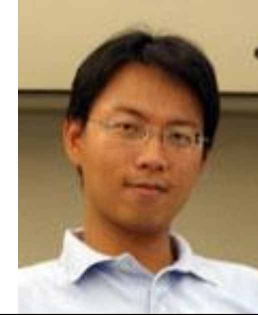
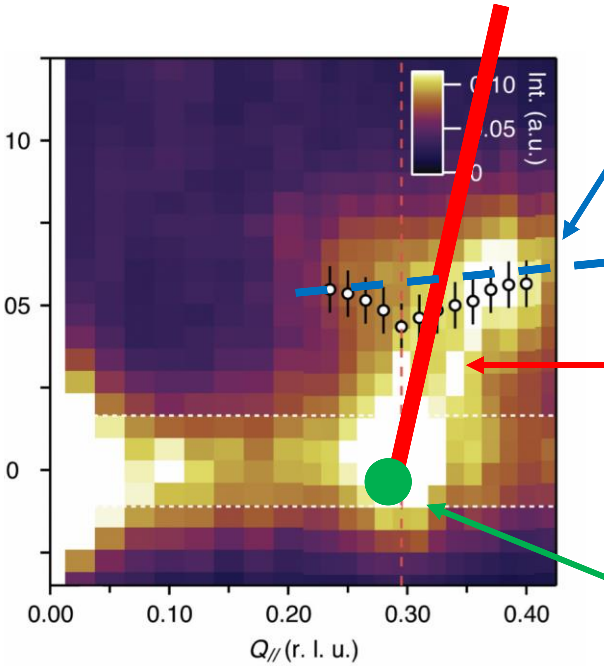


Detecting the electronic "charge mode".





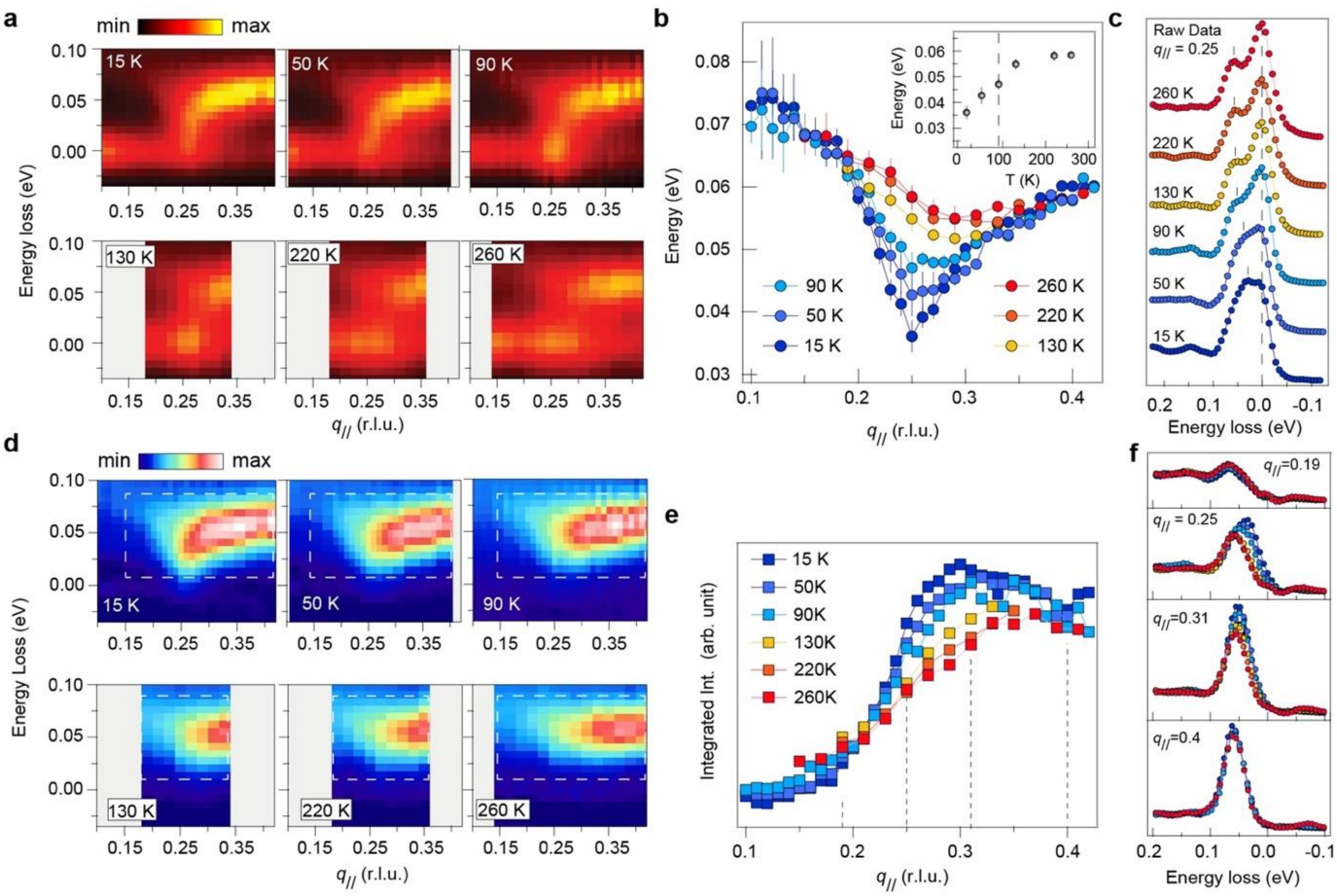
Lattice optical phonon

"Electronic phonon": steeply dispersing upward, emerging at the ordering wave vector of the charge order.

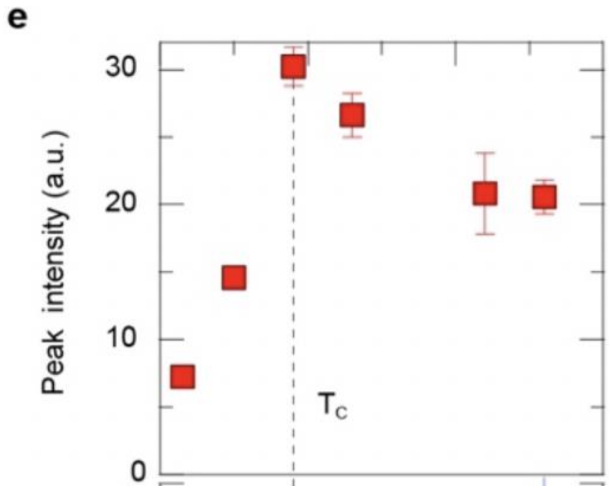
Electronic diffraction peak

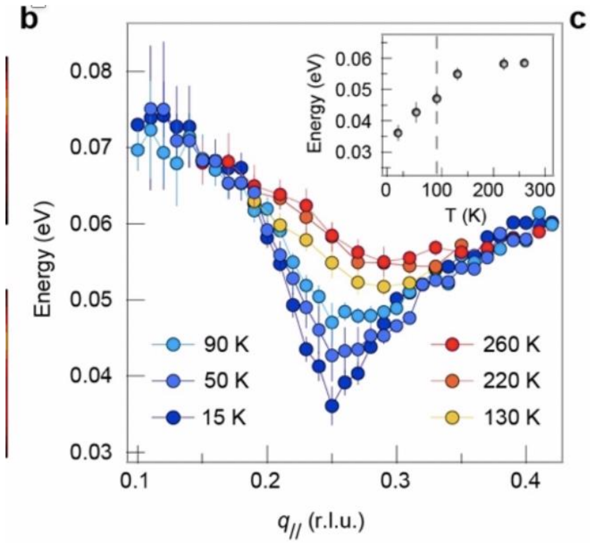
L. Chaix *et al.*, Nature Phys. 13, 952 (2017).

The "phonon dip" paradox (I).



The “phonon dip” paradox (II).

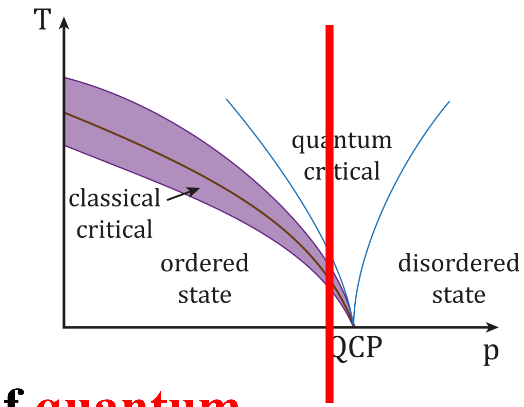




Upon cooling through the superconducting Tc the charge order parameter collapses, but its influence on the spectrum increases.

This makes no sense departing from Landau competing order: the oscillator strength of the “electronic phonon” should follow the order parameter!

The resolution of the "phonon dip" mystery (I).



Let us assume instead that the charge order is on the verge of undergoing a order-disorder quantum phase transition!

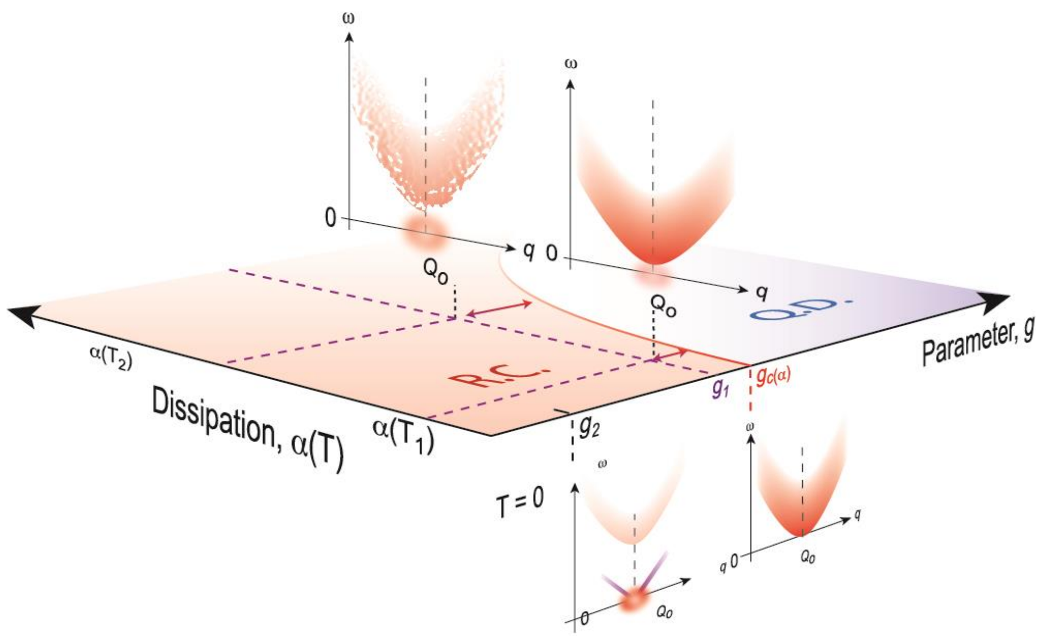
1. The "charge mode" becomes instead the continuum of quantum critical fluctuations.

2. The excitations of the metal form a heat bath and these will generically suppress the quantum fluctuations!

3. These are suppressed in the superconducting state and the charge order moves therefore closer to the QCP!

4. Finally some hassle with the RIXS process hiding the QC fluctuations to a degree by the Fano lineshape …

The resolution of the "phonon dip" mystery: the cartoon.



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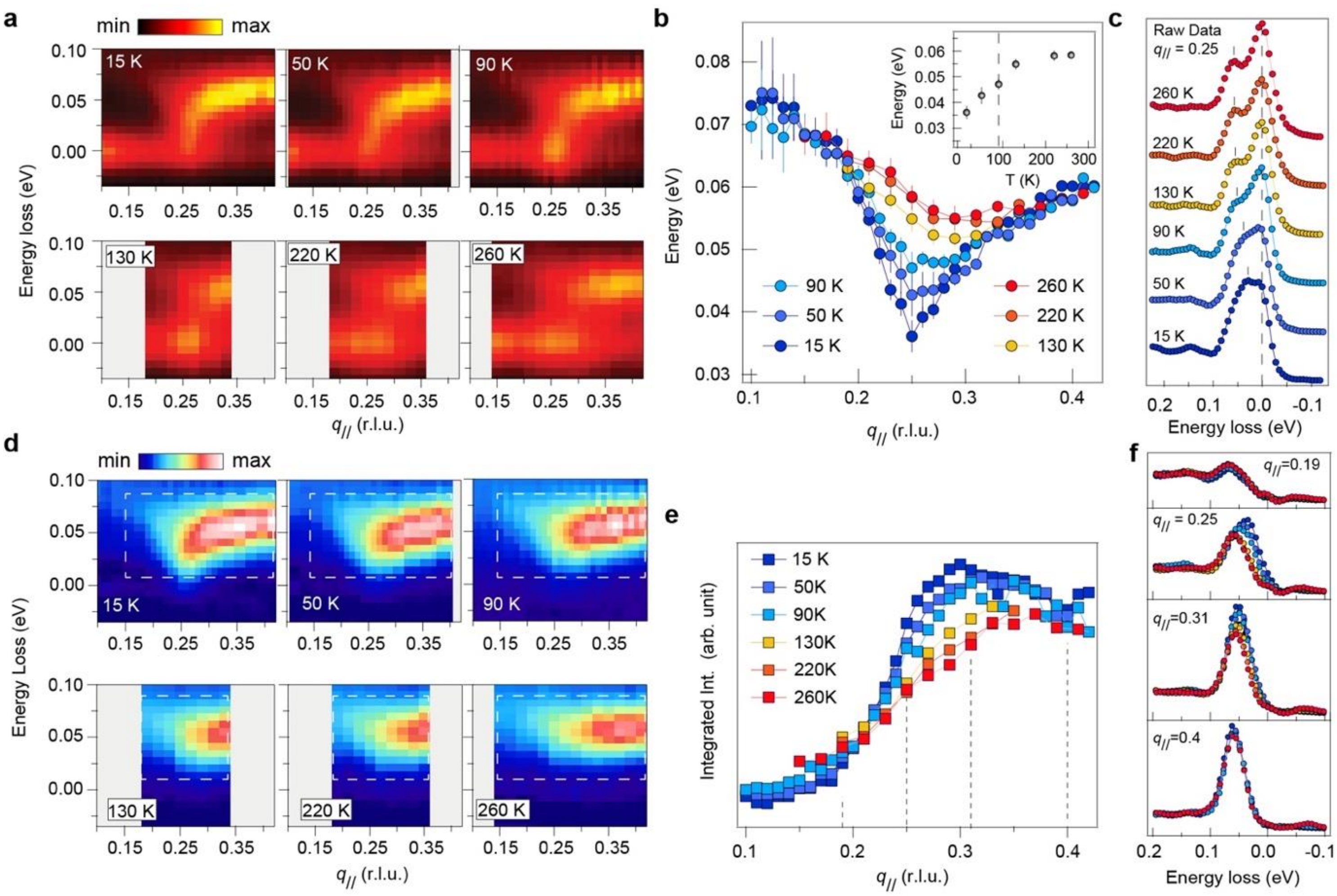
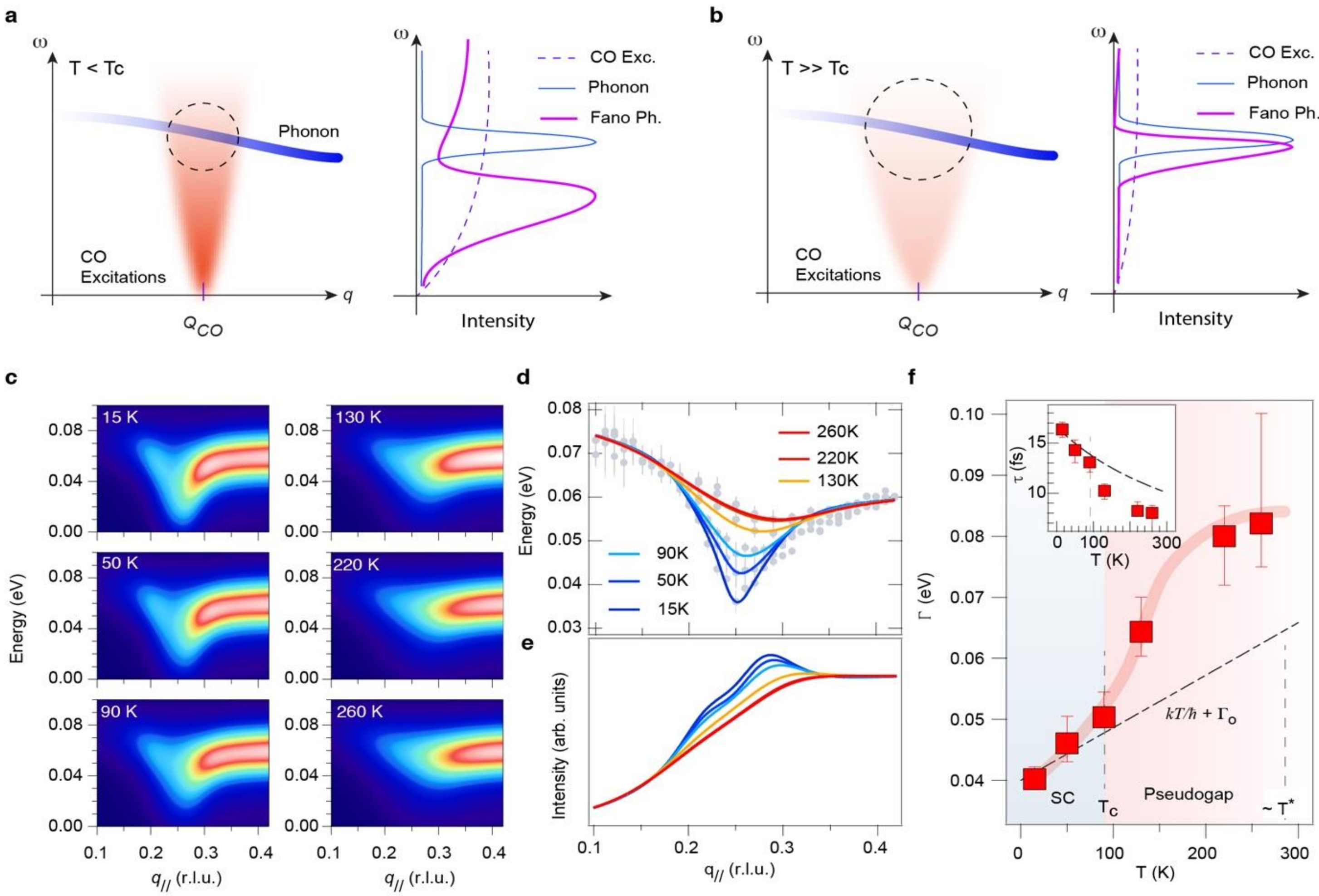


The resolution of the "phonon dip": a glorious fit!



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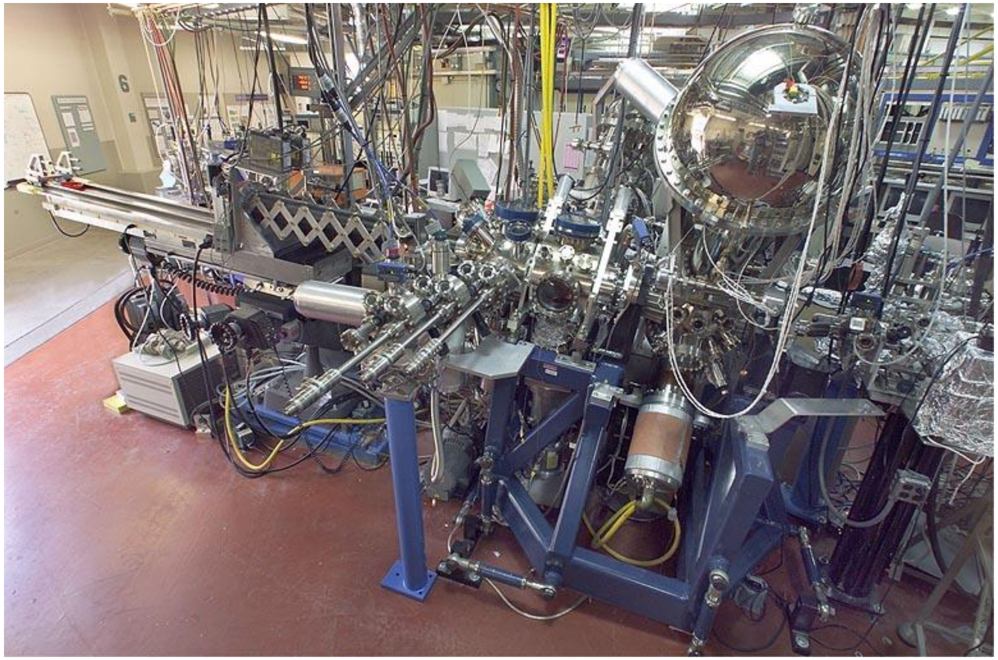
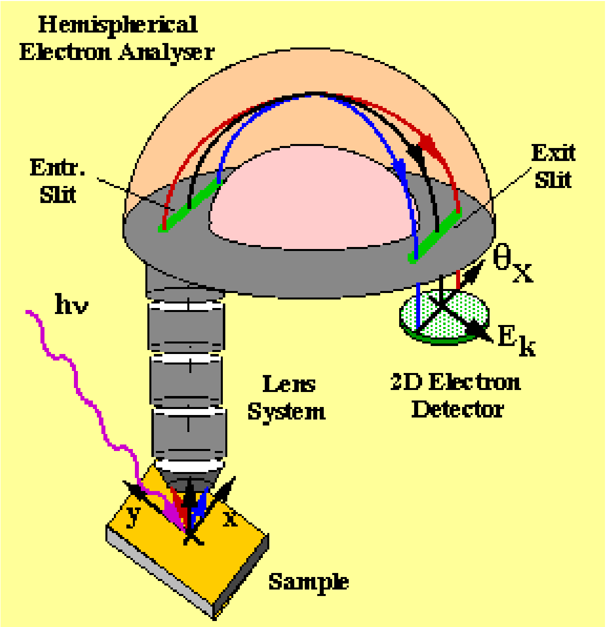
Anugular Resolved Photoemission: the fermionic precision telescope.

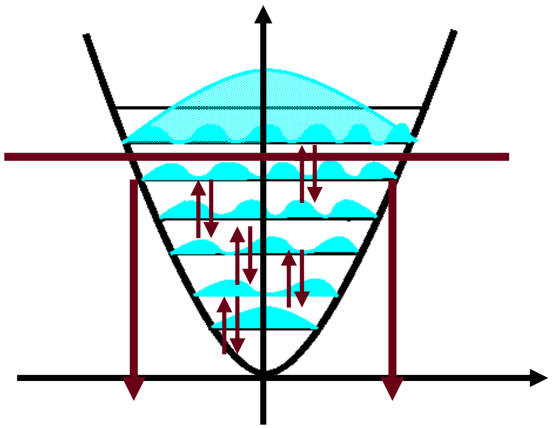


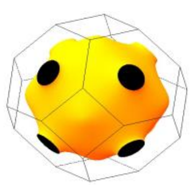


Measures the electron spectral function: the probability to remove an electron at a given energy and momentum (Kelvin energy and 10 nm spatial resolution).

Fermions: the tiny repertoire …

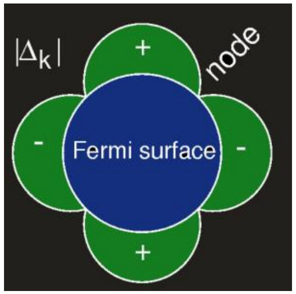
Fermiology

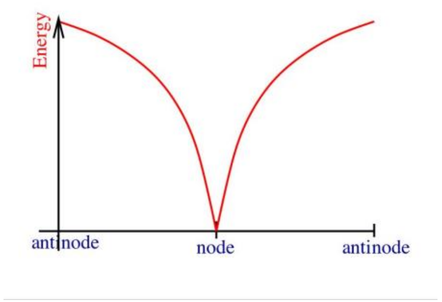




BCS superconductivity

$$\Psi_{BCS} = \Pi_k \left(u_k + v_k c^+_{k\uparrow} c^+_{-k\downarrow} \right) \left| vac. \right\rangle$$



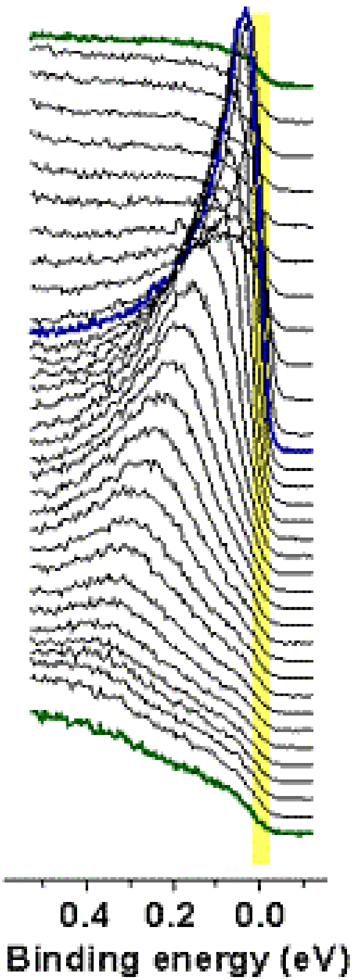


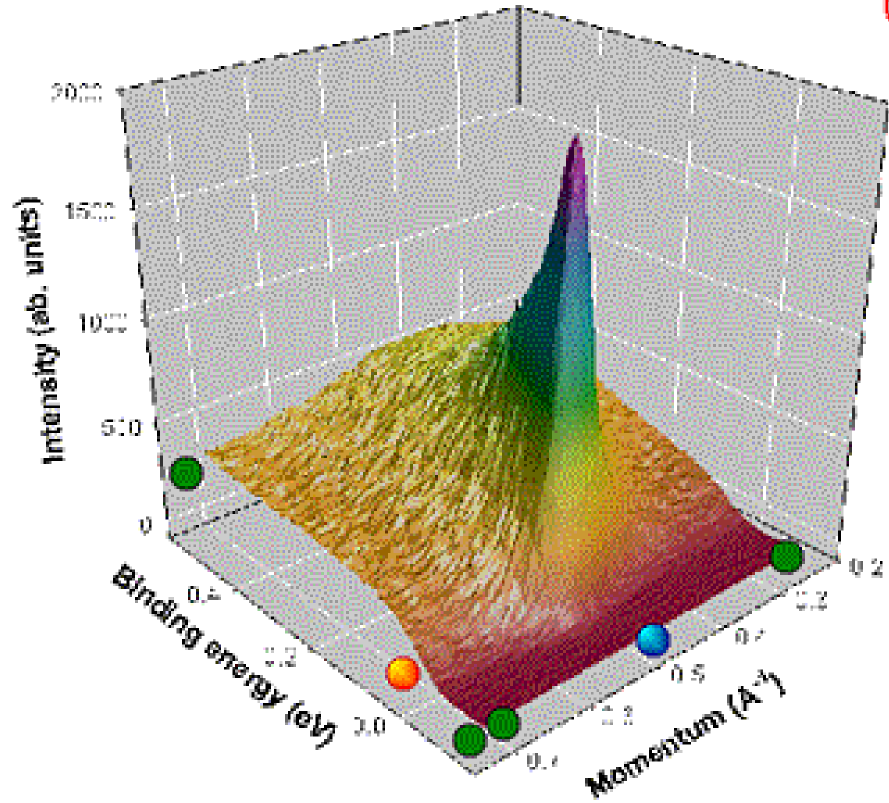
ARPES with an angle multiplexing analyser

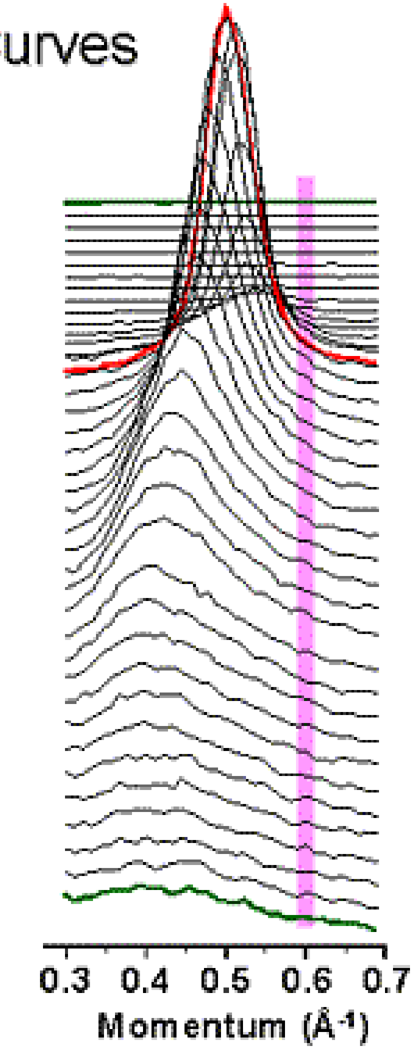
Energy Distribution Curves

I (k,E)

Momentum Distributi Curves







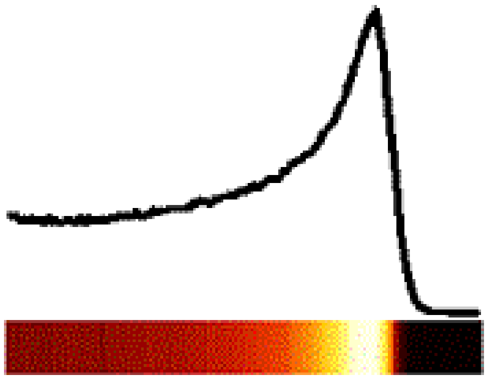
....this all lands on the detector in 5 mins. !

MDC: I(k) for E = const.

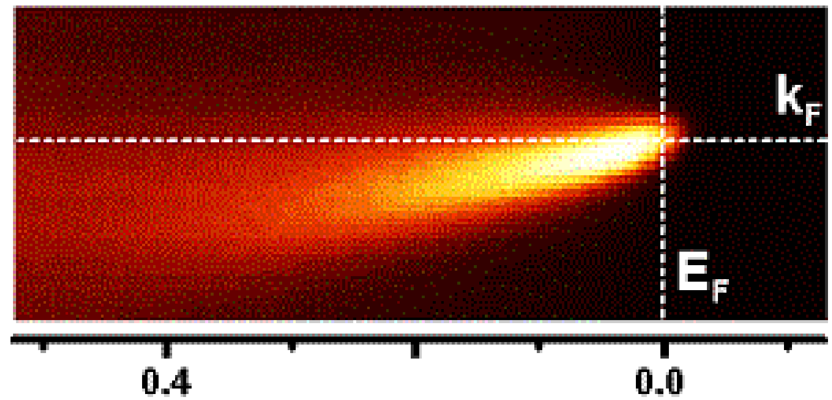
EDC: I(E) for approx. const. k

Representing ARPES data in 2D

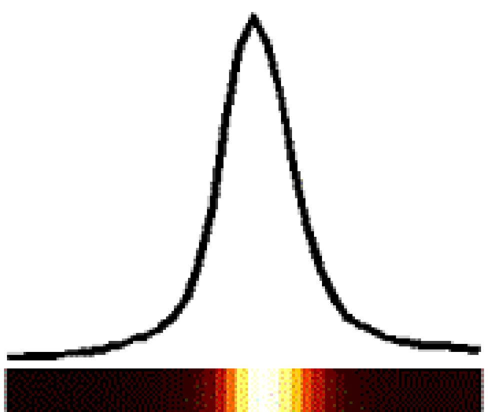
EDC - $I(k_x,k_y,\omega)$



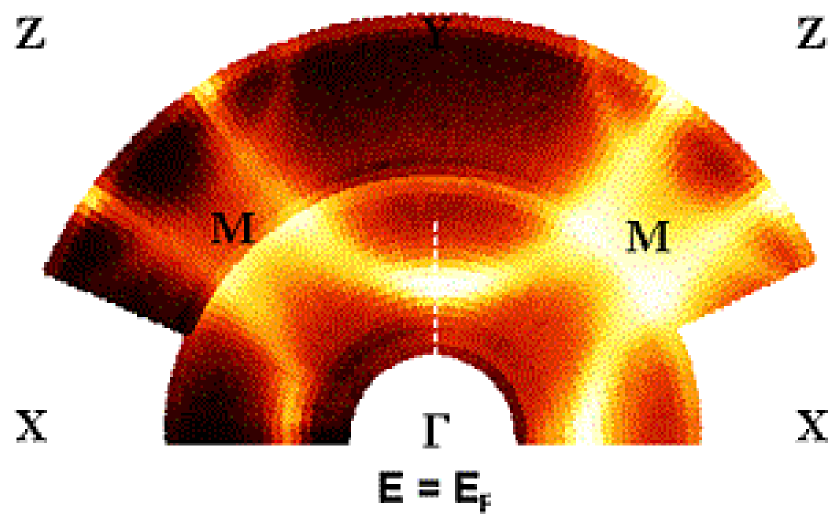
EDM - $I(k_x,k_y,\omega)$



MDC - $I(k_x,k_y,\omega)$

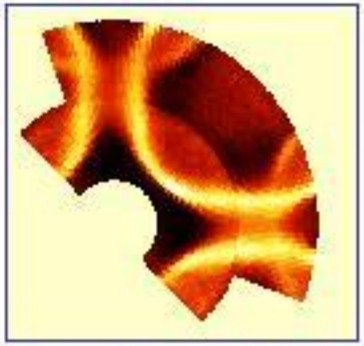


MDM - $I(k_x,k_y,\omega)$

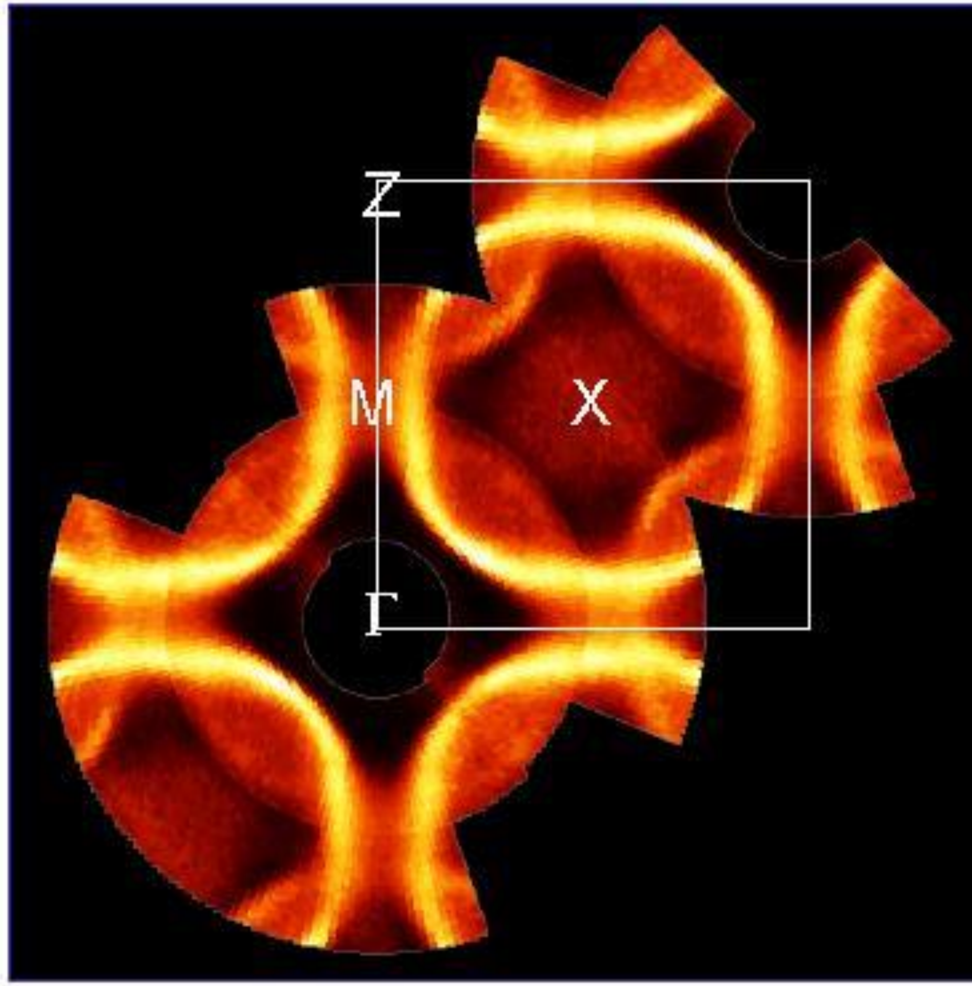


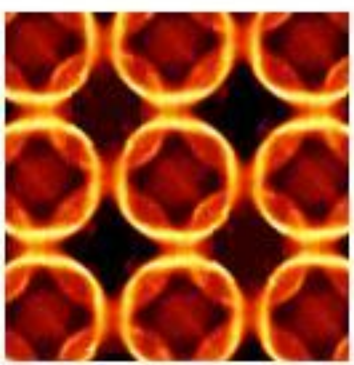
Borisenko et al., PRB**64**, 094513 (2001)

The Fermi surface of Pb-doped Bi-2212



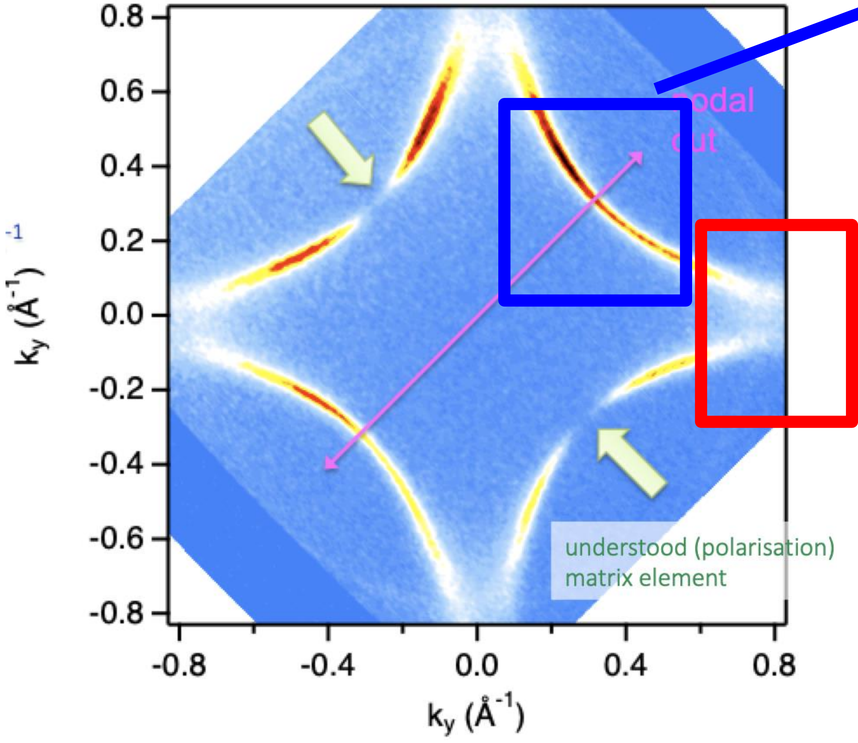
measured data





T = 300 K (OD72K)

The nodal-antinodal dichotomy.

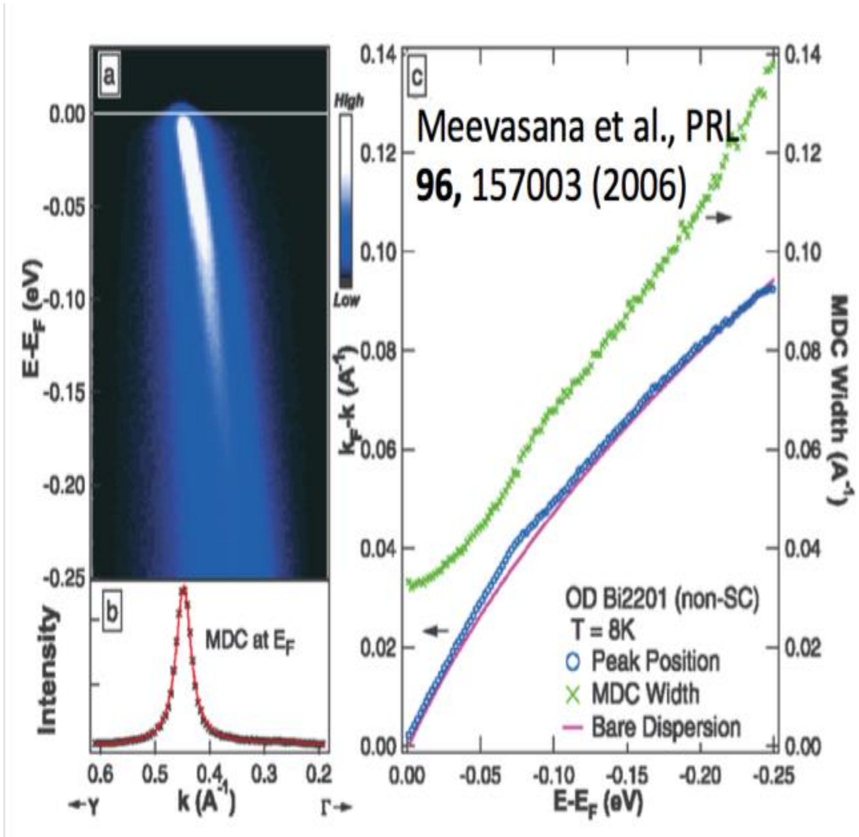


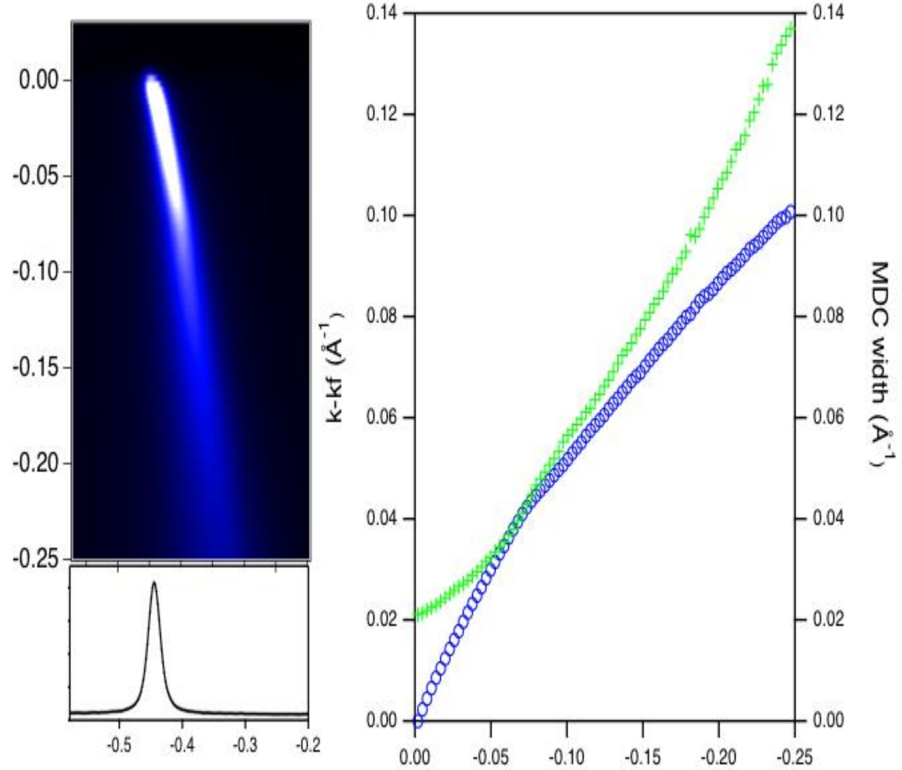
Along the nodes we used to see quasi-particles.

But up to optimal doping this used to be less obvious near the anti-nodes

Benchmark to the best data

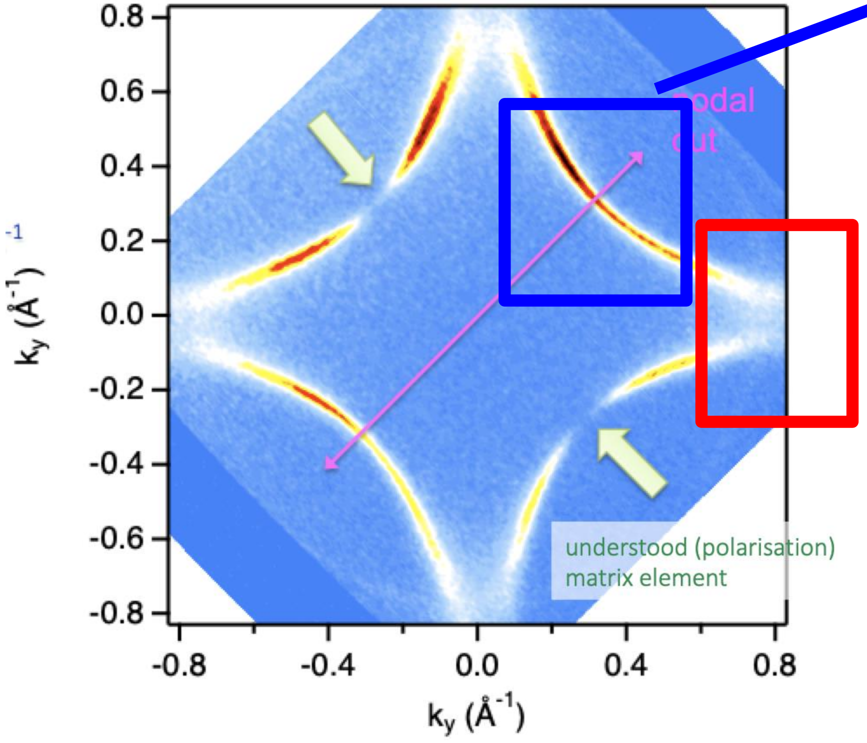
- nodal direction, Pb,Bi-2201





- minimal MDC width = 0.02 $Å^{-1}$

The nodal-antinodal dichotomy.

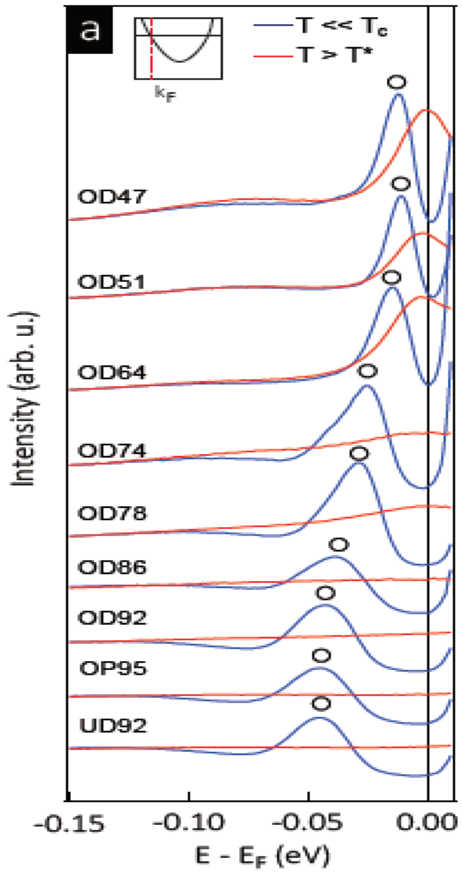


Along the nodes we used to see quasi-particles.

But up to optimal doping this used to be less obvious near the anti-nodes

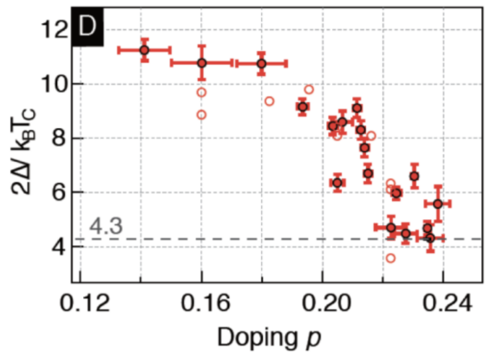
Unparticle physics in high Tc superconductors.

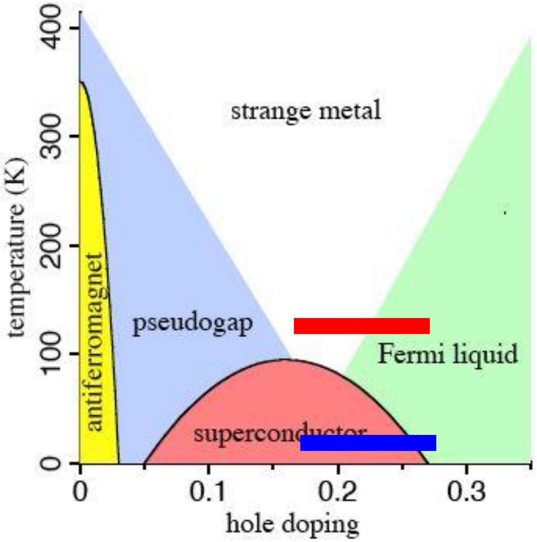
Photoemission spectra at antinodes:



Blue: Bogoliubov quasiparticles deep in the superconducting state.

Red: birth of Fermi-liquid quasiparticles in the normal state



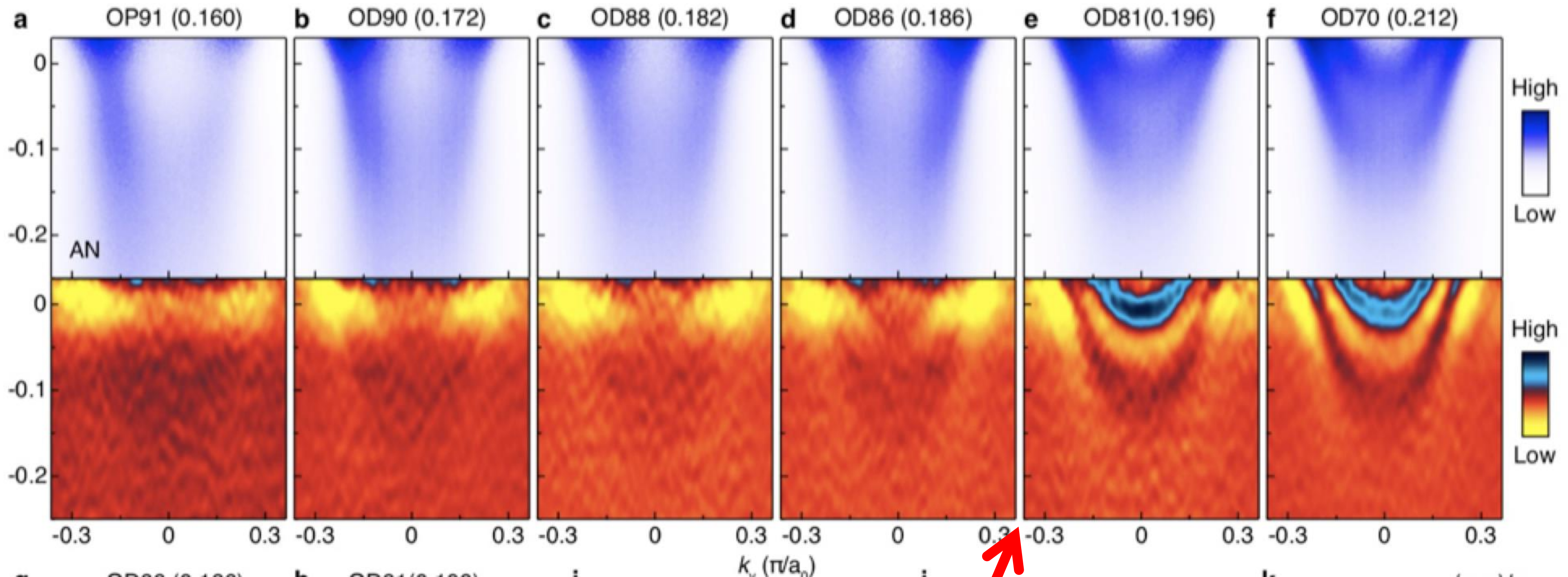


Shen group Stanford: Science 362, 62 (2018)

Sudi's surprise ...

Sudi Chen ZX Shen

Normal state 2212 antinodal ARPES as function of doping.



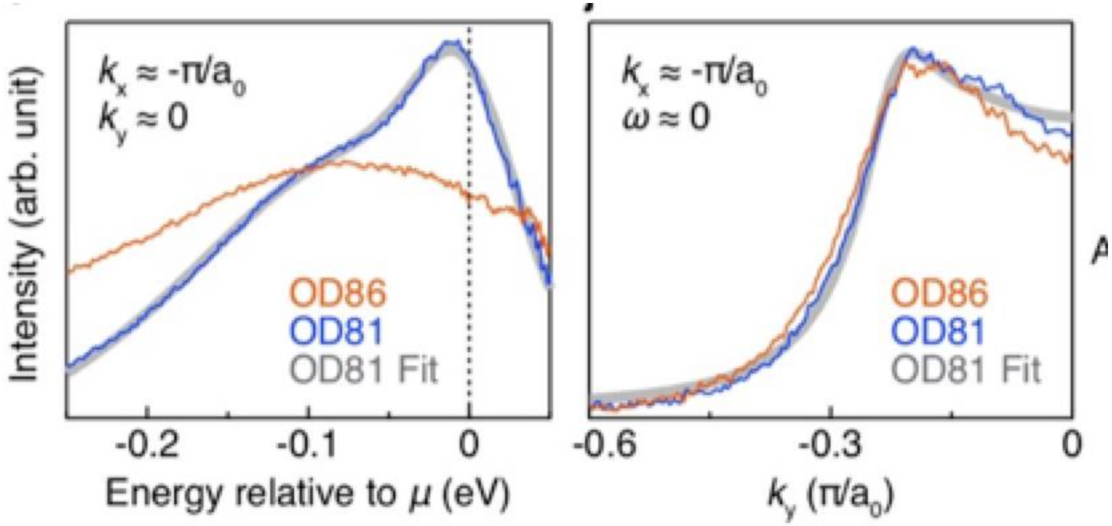
Strange metal: **incoherent**

Critical doping
p_c =0.19

Reasonable quasiparticles !

The "EDC-MDC dichotomy" on military power

Sudi Chen ZX Shen

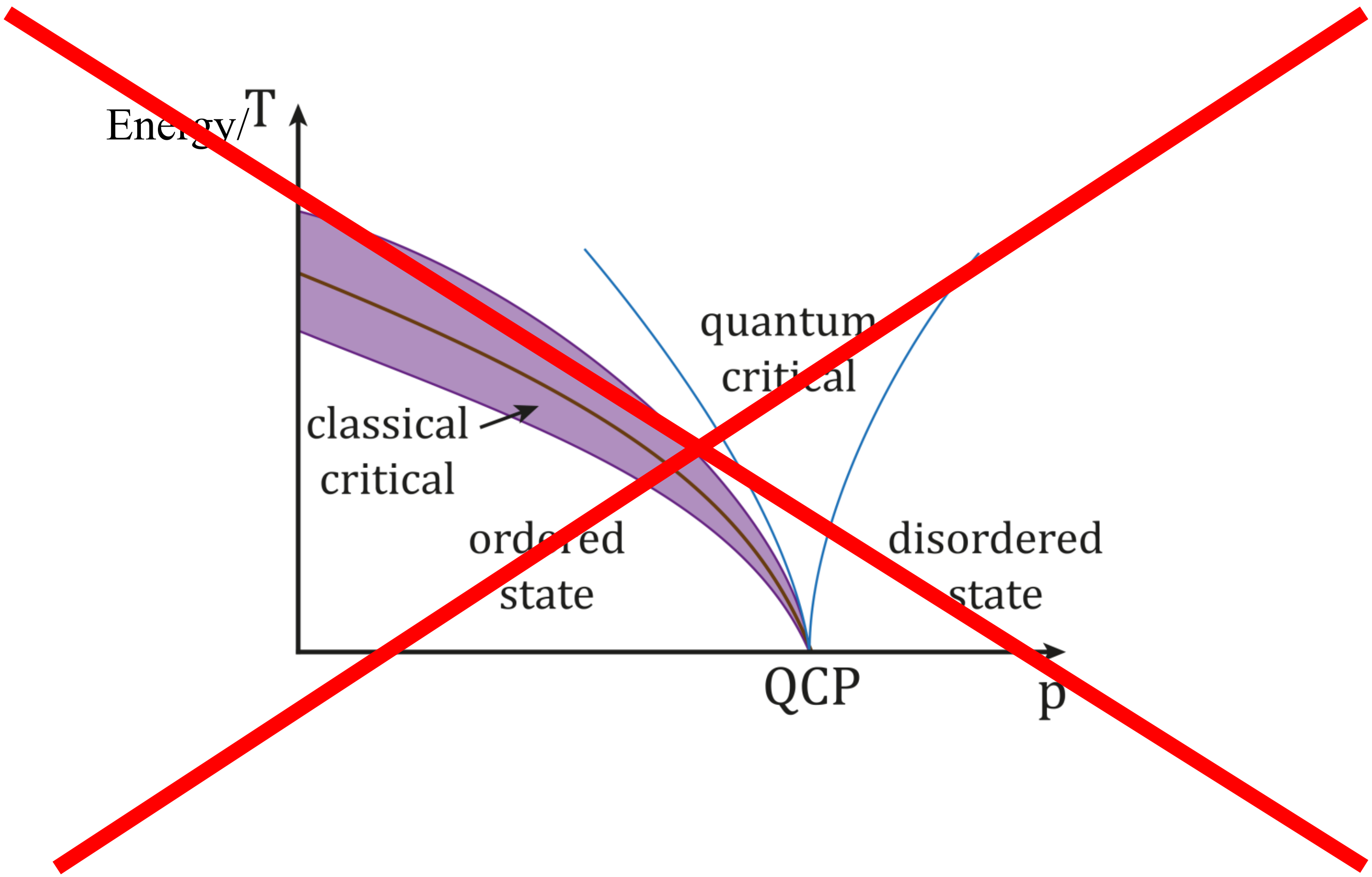


EDC's: perfect fit obtained using the industry standard "nodal" self energy for overdoped (Tc = 81 K) metal while the Tc= 86 K metal is completely incoherent.

MDC's: industry standard self-energies fit well on both sides of p_c.

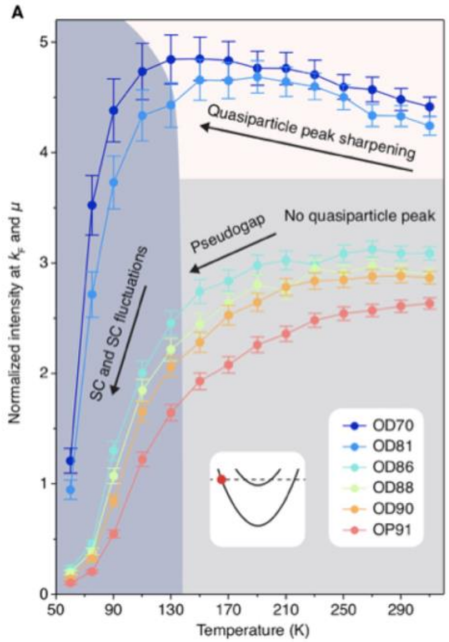
At the critical doping the EDC's turn from extreme fat tail to a reasonable perturbative self energy form.

The stoquastic view on scale invariance.



Challenging classical matter thermodynamical principle.


Sudi Chen ZX Shen



With the strange metal also the "pseudogap precursor" disappears suddenly: claimed to be seen in the specific heat by Tallon.

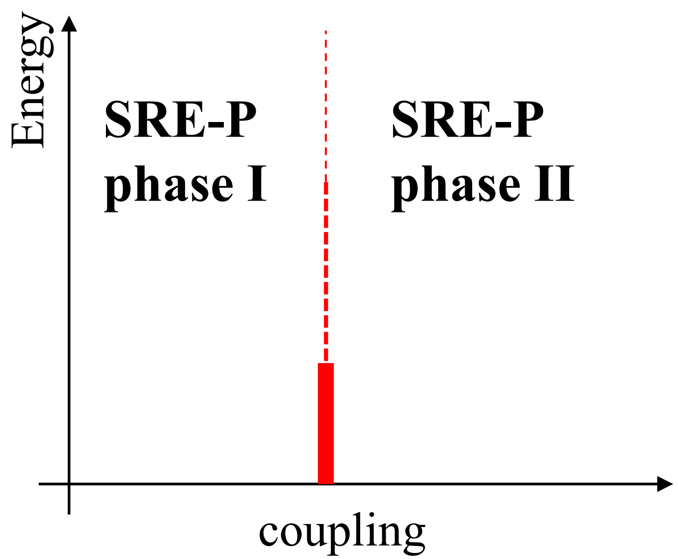
All other macroscopic properties vary smoothly through p_c (e.g. T_c).

This is surely not a continuous quantum phase transition: 10^4 papers to the trash can!

This defies general principle applying to all forms of classical matter: a discontinuous microscopic change should turn into a first order phase transition. The loop hole: the strange metal is a state of quantum supreme matter undergoing a "transition" to a classical state

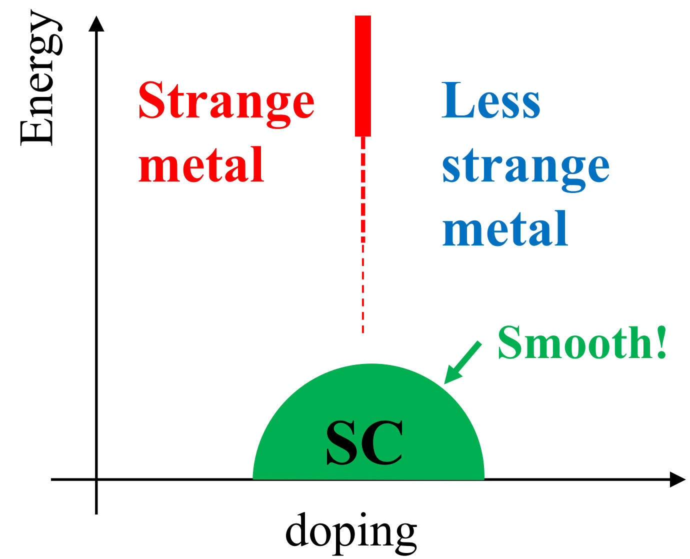
The "failed first order" transition in cuprates.

Stoquastic first order QPT



Wilsonian RG principle: discontinuity always amplifies towards IR ("runaway flow")

Cuprates



The only loophole: it is non-stoquastic and thereby quantum supreme!

Condensed matter physics in action.

1. "Stoquastic" quantum criticality: an introduction.

2. The "ups": the nearly quantum critical charge order in cuprates revealed by the RIXS machine.

3. The "downs": ARPES demonstrates that the big deal cuprate strange metal has nothing to do with a quantum phase transition

4. Theoretical epilogue: fermion signs, dense entanglement and the demise of semiclassics.

Many body/bit Hilbert space.

Two qubits: Hilbert space dimension $2^2 = 4$

$$|00\rangle, |01\rangle, |10\rangle, |11\rangle$$

Three qubits: Hilbert space dimension $2^3 = 8$

$$|000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |101\rangle, |110\rangle, |111\rangle,$$

Physical world 10^{23} "qubits": Hilbert space dimension $2^{10^{23}}$

$$|\Psi_n\rangle = \sum_{config.i} C_i^n |config.i\rangle$$

Overwhelming amount of quantum information.

"The classical condensates: from crystals to Fermi-liquids."

States of matter that we understand are short ranged entangled product!

$$\left|\Psi_0\{\Omega_i\}\right\rangle = \Pi_i \hat{X}_i^+(\Omega_i)\left|vac\right\rangle$$

- Crystals: put atoms in real space wave packets

$$X_i^+(R_i^0) \,\mu\, e^{(R_i^0 - r)\wedge 2 / s^2} y^+(r)$$

- Magnets: put spins in generalized coherent state

$$X_i^+(\vec{n}_i) \propto e^{i\varphi_i/2}\cos(\theta_i/2)c_{i\uparrow}^+ + e^{-i\varphi_i/2}\sin(\theta_i/2)c_{i\downarrow}^+$$

- Superconductors/superfluids: put bosons/Cooper pairs in coherent superposition

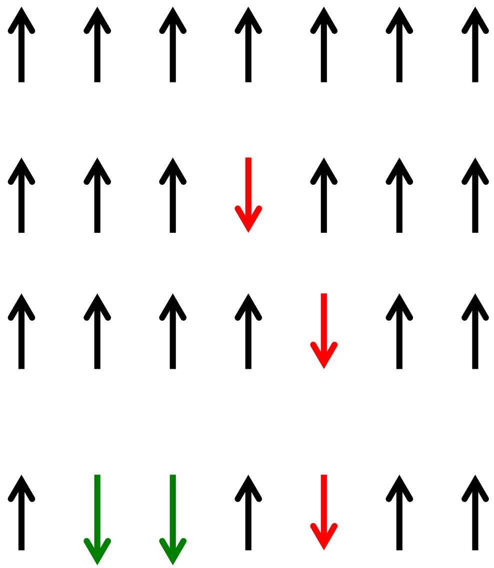
$$X_{k/i}^{\ +} \,\mu\, u_k + v_k c_{k-}^+ c_{-k-}^+, \quad u_i + v_i e^{ij_i} b_i^+$$

- Fermi gas/liquid: product state in momentum space (mod Pauli principle).

$$\left|Y_{FL}\right\rangle = P_k^{k_F} c_k^+ \left|vac\right\rangle$$

What is a "particle"?

E.g. transversal field Ising model: $H = -J \sum_{<ij>} \sigma_i^z \sigma_j^z + B \sum_i (\sigma_i^+ + \sigma_i^-)$



J << B: product state vacuum

Excitation = inject quantum number(s) (S=1)

"Particle" delocalizes: $G = \frac{1}{\varepsilon_k - \omega}$

$$A(k,\omega) = Im\ G = \delta(\varepsilon_k - \omega)$$

J < B, SRE product state = perturbative corrections:

$$|\Psi_0\rangle = A|product\rangle + \sum_i a_i |config, i\rangle \qquad G = \frac{1}{\varepsilon_k - \omega + \Sigma(\omega, k)}$$

"Good" self-energy, at the bottom of the spectrum: $G = \frac{A^2}{\hat{\varepsilon}_k - \omega} + G_{incoh}$

Quantum matter and unparticle physics.

Given: The vacuum state is infinite party entangled

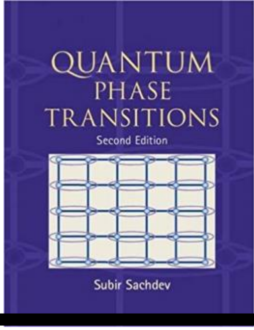
$$|\Psi\rangle = \sum_{configs} A_{configs} |configs\rangle$$

Inject a quantum-number: this information is now “dispersed” in the whole 2^N many body Hilbert space.

The quantum info is no longer “localizable”: there are no particle poles in the spectrum.

Spectral functions are fully incoherent: “unparticle physics” as quantum matter diagnostic!

Strongly interacting "stoquastic" quantum critical states.



$$S = \int d^d x d\tau \left[(\partial_\tau \Phi)^2 + (\nabla \Phi)^2 + m^2 \Phi^2 + w \Phi^4 \right]$$

$D = d + z < D_{u.c.} (= 4) : \quad w \neq 0$ at the IR fixed point

"strongly interacting" = NP-hard (critical slowing down in QMC) = densely entangled quantum critical state.

$$\langle \Phi \Phi \rangle \sim \frac{1}{\sqrt{k^2 - \omega^2}^{2-\eta}}$$

$D \geq D_{u.c.} \quad w = 0$ at the IR fixed point

Mean-field fixed point: SRE product state characterized by particles in its spectrum:

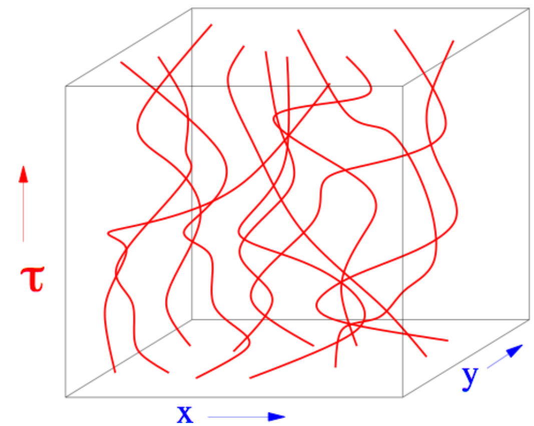
$$\langle \Phi \Phi \rangle \sim \frac{1}{k^2 - \omega^2}$$

Fermions at a finite density: the „non-stoquastic“ sign problem.

Imaginary time first quantized path-integral formulation

$$\mathcal{Z} = \mathrm{Tr}\exp(-\beta\hat{\mathcal{H}}) = \int \mathrm{d}\mathbf{R}\,\rho(\mathbf{R},\mathbf{R};\beta)$$

$$\mathbf{R} = (\mathbf{r}_1,\ldots,\mathbf{r}_N) \in \mathbb{R}^{Nd}$$



$$\rho_{B/F}(\mathbf{R},\mathbf{R};\beta) = \frac{1}{N!}\sum_{\mathcal{P}}(\pm 1)^{\mathcal{P}}\rho_D(\mathbf{R},\mathcal{P}\mathbf{R};\beta) = \frac{1}{N!}\sum_{\mathcal{P}}(\pm 1)^{\mathcal{P}}\int_{\mathbf{R}\to\mathcal{P}\mathbf{R}}\mathcal{D}\mathbf{R}(\tau)\exp\left\{-\frac{1}{\hbar}\int_0^{\hbar/T}\mathrm{d}\tau\left(\frac{m}{2}\dot{\mathbf{R}}^2(\tau)+V(\mathbf{R}(\tau))\right)\right\}$$

Boltzmannons or Bosons:

- integrand non-negative = soquastic
- probability of equivalent classical system: (crosslinked) ringpolymers

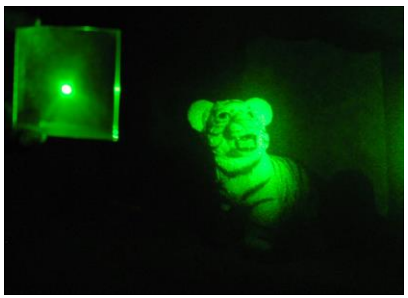
Fermions:

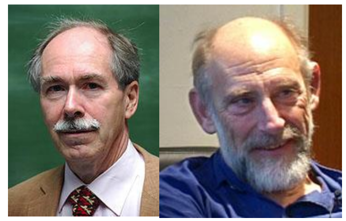
- negative Boltzmann weights
- „non-stoquastic“: NP-hard problem (Troyer, Wiese)!!!

Holographic gauge-gravity duality

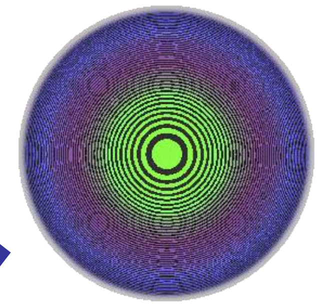
Einstein Universe "AdS"

Quantum world "CFT"



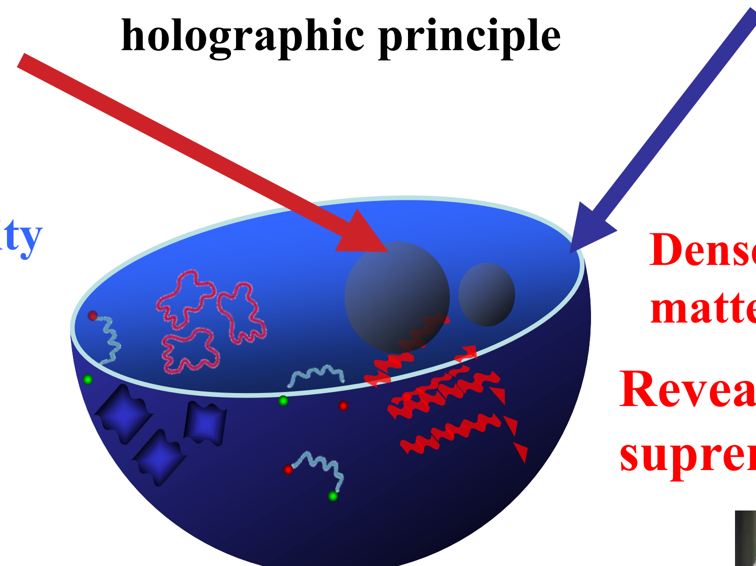


't Hooft-Susskind holographic principle



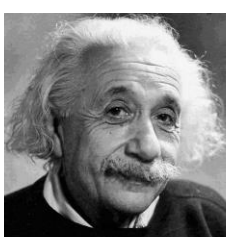
Classical general relativity

Uniqueness of GR solutions



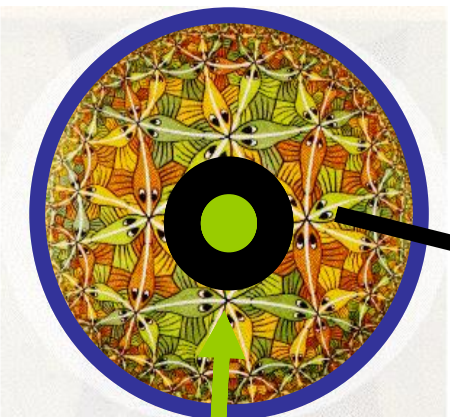
Densely entangled quantum matter

Revealing "quantum supremacy" general principle ?

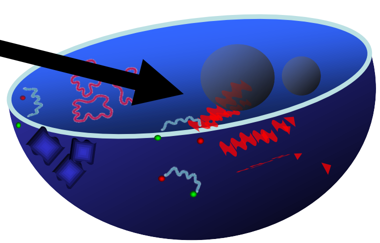


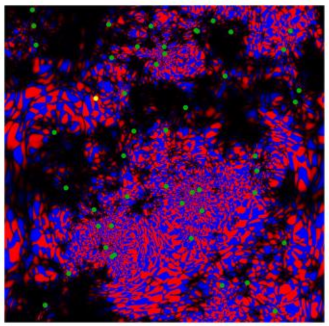
The charged back hole encoding for finite density (2008 - ????)

Anti de Sitter universe.



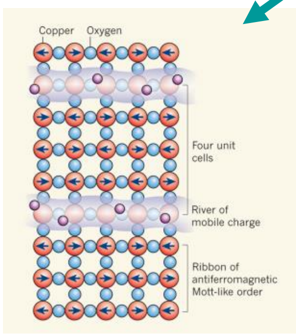
Finite density quantum matter:





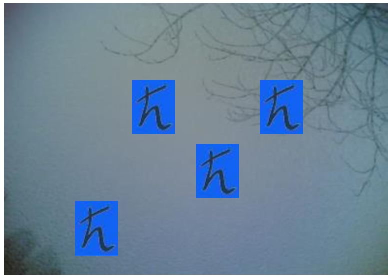
Holographic strange metals

Charged black hole in the middle



Stripy pseudogap orders

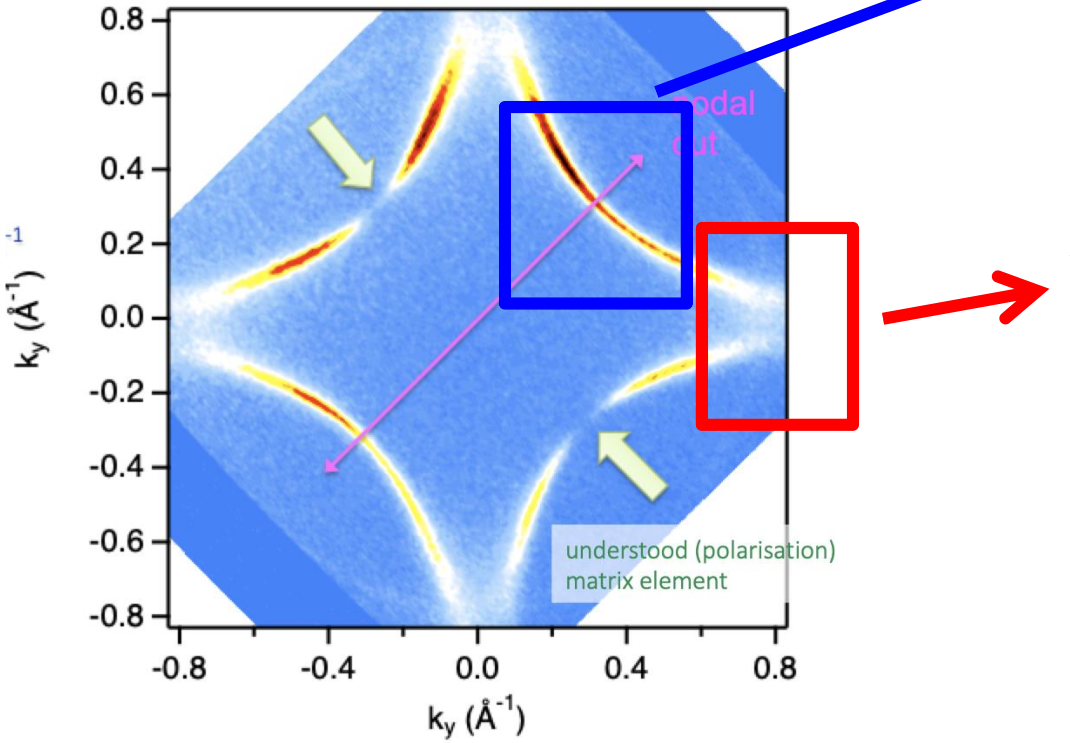
High Tc superconductors



Emergent Fermi liquids

The nodal-antinodal dichotomy.

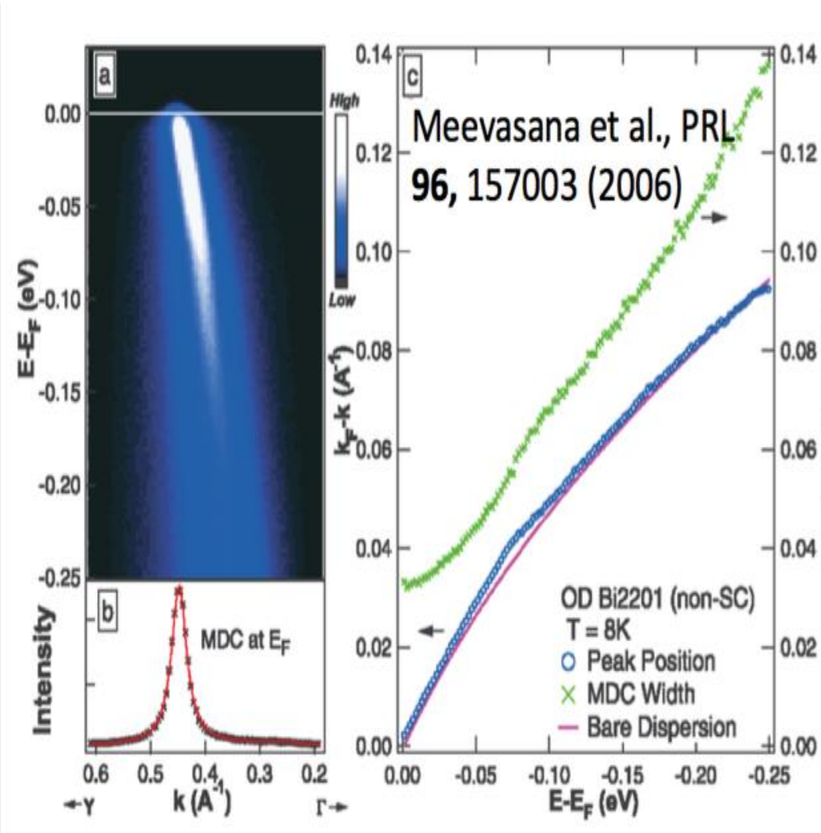
Along the nodes we used to see quasi-particles.

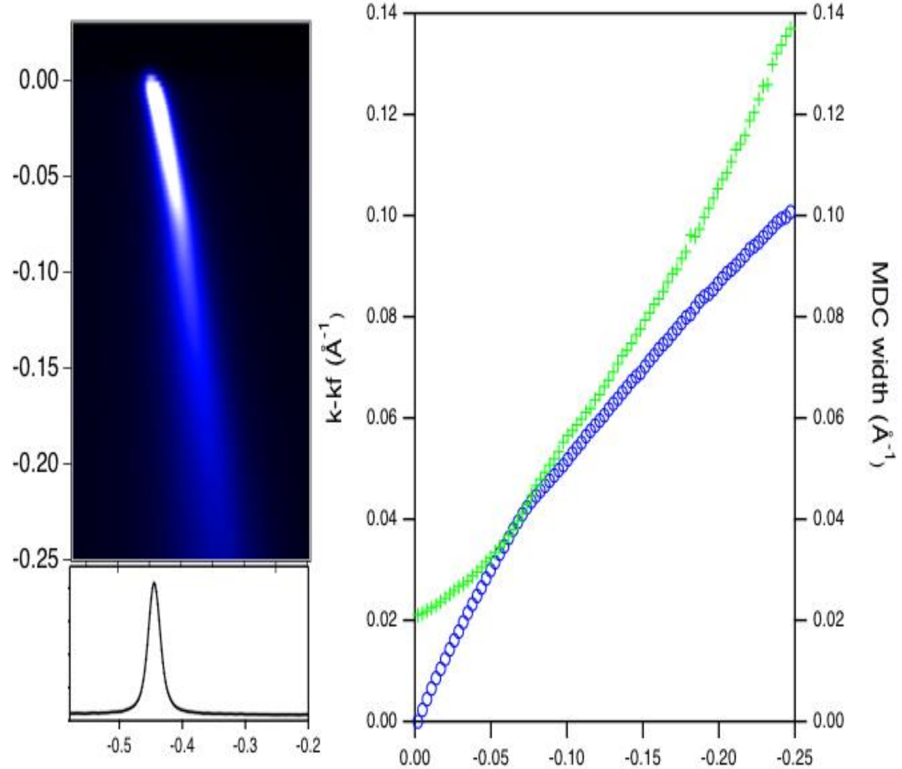


But up to optimal doping this used to be less obvious near the anti-nodes

Benchmark to the best data

- nodal direction, Pb,Bi-2201

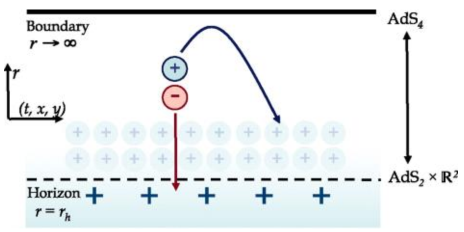


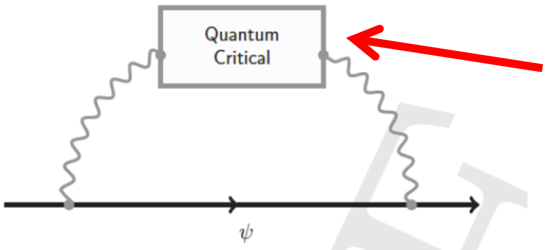


- minimal MDC width = 0.02 Å^{-1}

AdS/ARPES: the RN approaching the Fermi liquid

Liu McGreevy





This is doing the collective (transport etc) work!

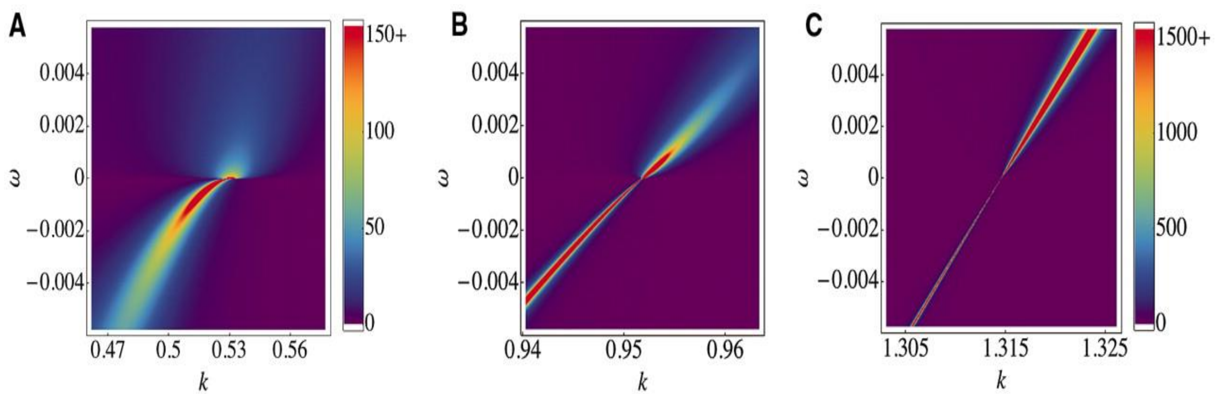
Bulk: DW fermion gas and the horizon

Boundary: would be fermions decaying in QC infrared

$$G_f(\omega, k) = \frac{1}{\omega - v_F(k - k_F) - \Sigma(k, \omega)}$$

$$\Sigma(k, \omega) \sim e^{i\phi_{k_F}} \omega^{2\nu_{k_F}}$$

$$2\nu_k \sim \sqrt{\frac{1}{\xi^2} + k^2}$$



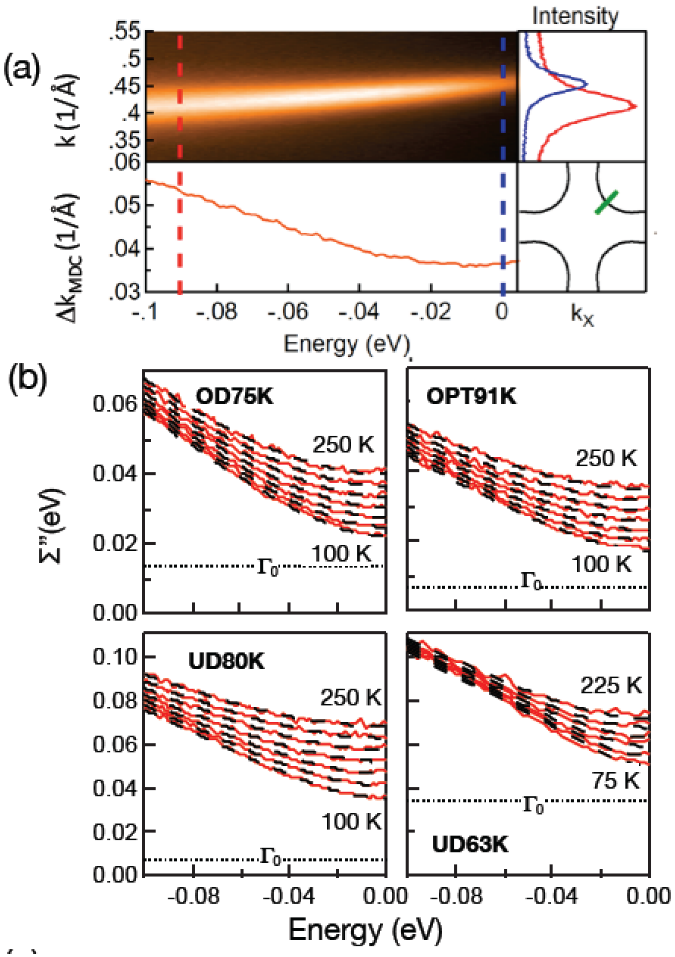
$2\nu_{k_F} < 1$ overdamped

$2\nu_{k_F} = 1$ Marginal FL

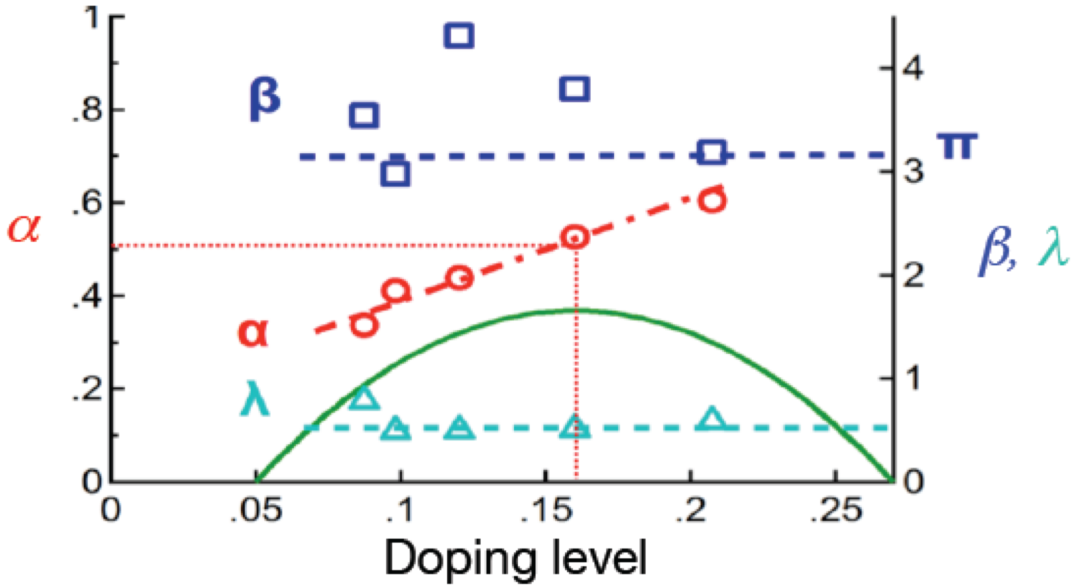
$2\nu_{k_F} > 1$ underdamped

Nodal fermion self-energies in the strange metal.

Dessau



$$\Sigma''_{PLL}(\omega) = \Gamma_0 + \lambda \frac{\left[(\hbar\omega)^2 + (\beta k_B T)^2\right]^{\alpha}}{(\hbar\omega_N)^{2\alpha-1}}$$



arXiv:1509.01611

Ironic conclusions.

The first convincing (=dynamical) evidence for a strongly interacting quantum critical point in cuprates: RIXS and the charge order.
W.S. Lee et al., submitted to Nature.

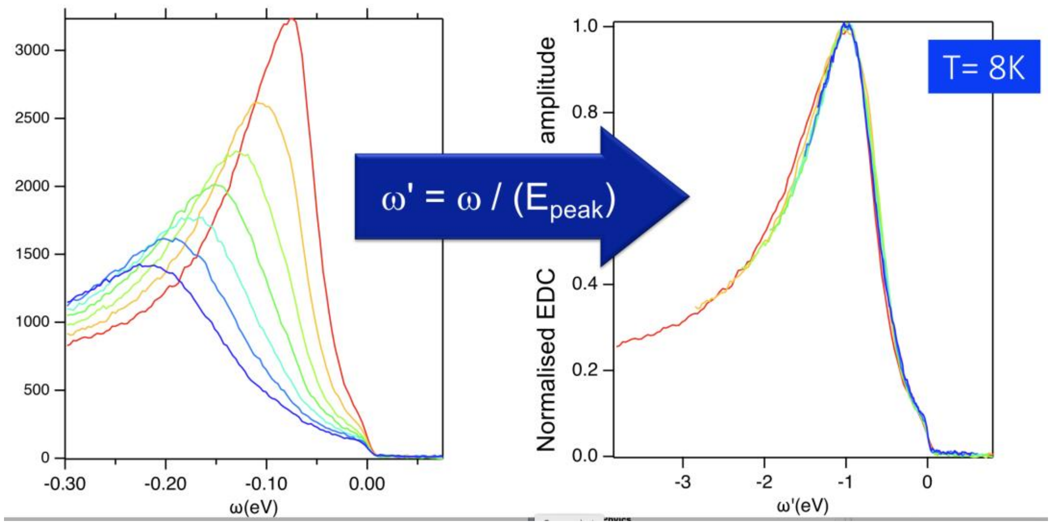
But utterly irrelevant to the strange metal: ARPES shows the ”failed first order transition” demonstrating the strange metal to be a “quantum supreme” metallic phase of matter reminiscent of a holographic strange metal.
Sudi Chen et al, accepted by Science.

Much more “quantum supreme matter” developments:

Intertwined order, holographic Mott insulators and the zero Hall effect phase (IOP/AC colloquium Monday).

Planckian dissipation in transport, ultrafast thermalization, see arXiv:1807.10951.

The EDC scaling collapse: the fat energy tails.



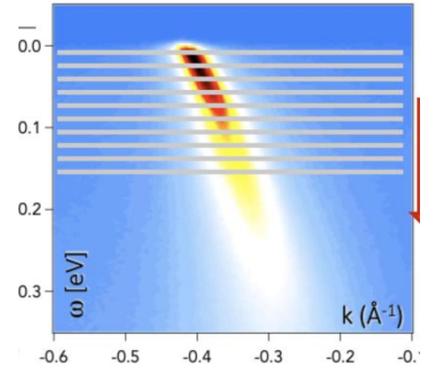
CFT propagators scale this way:

$$G = \frac{1}{\sqrt{c^2 q^2 - \omega^2}^{2\Delta}} = \frac{1}{|cq|^{2\Delta}} \frac{1}{\sqrt{1 - (\frac{\omega}{cq})^2}^{2\Delta}}$$

Holo-fermion propagators do not scale at all when $2\nu = 1.6$

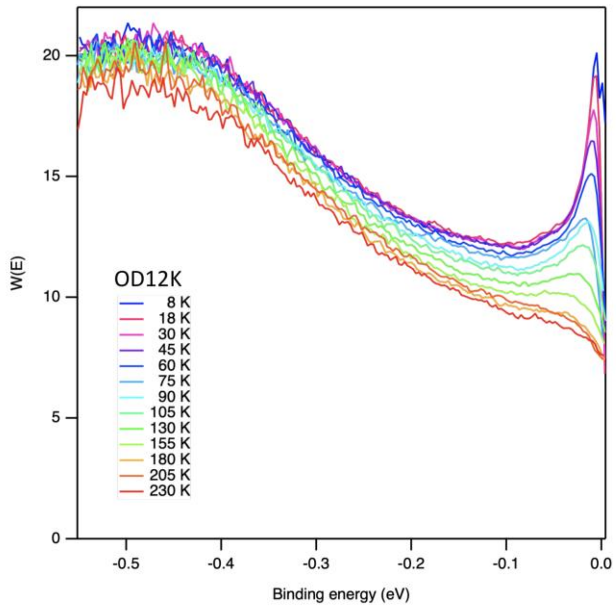
$$G = \frac{1}{E_k - \omega + i\omega^{2\nu}} = \frac{1}{E_k} \frac{1}{1 - \frac{\omega}{E_k} + i\frac{\omega^{2\nu}}{E_k}}$$

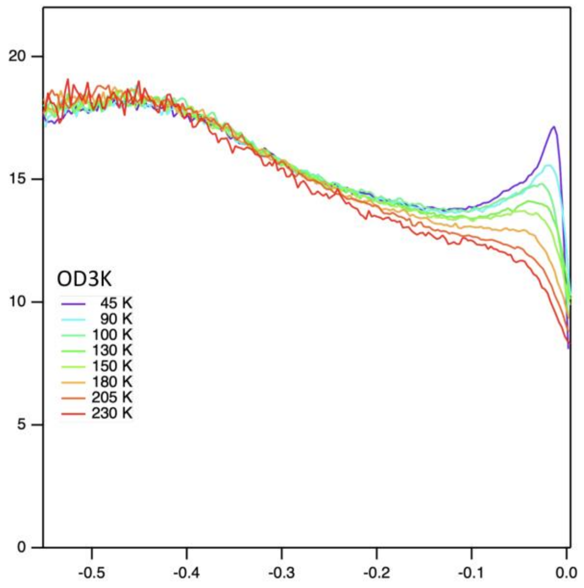
The fat energy-tail W function.



Given that the MDC's are perfect Lorentzians the only way to fit the data is:

$$A(k,\omega) = W(\omega)\,\frac{\Gamma(\omega)}{(E_k-\omega)^2+\Gamma^2(\omega)}$$





W is non-perturbative : the normal state nodal quasiparticles are a delusion!!!

Empty.