Few-body and many-body physics in a resonantly interacting two-component Fermi system

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Outline

- Introduction

- Universal 3-body physics for fermions

  SE, P. Naidon, M. Ueda, Few-body Systems **51**, 207 (2011)
  SE, P. Naidon, M. Ueda, PRA **86**, 062703 (2012)

- Novel SU(3) Trimer Phase in a 2-component mass-imbalanced Fermi system

  SE, P. Naidon, A. M. Garcia-Garcia arXiv:1507.06309 (2015)
  P. Naidon SE, A. M. Garcia-Garcia, arXiv:1507.06373 (2015)

- 3rd and 4th virial expansion of a unitary Fermi gas
  – Few-body approach to many-body physics

  *C. Gao, SE, Y. Castin, EPL **109**, 16003 (2015)*
  SE, Y. Castin, arXiv:1507.05580(2015)

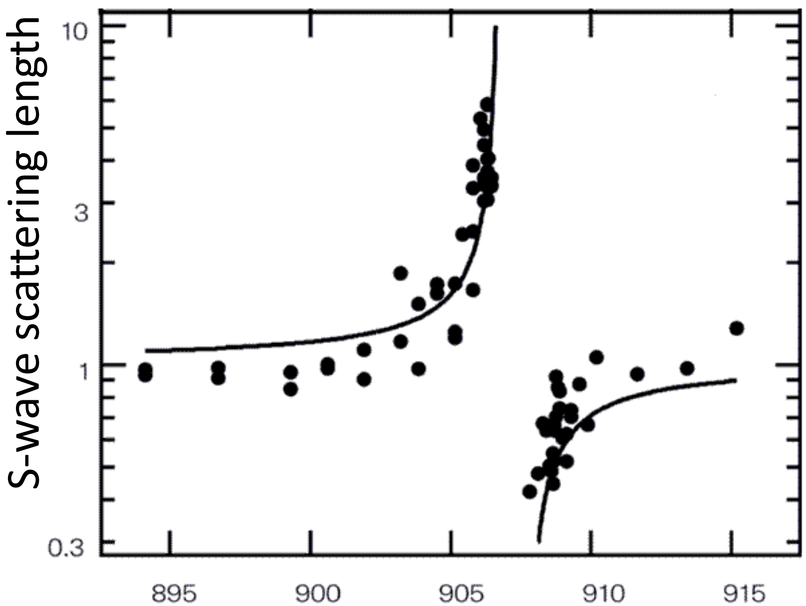
Ultracold Atoms

$$T \sim 1 - 100 \text{ nK}$$
$$k_F^{-1} \sim 100 \text{ nm}$$

- Dilute ultracold gas of neutral atoms
- Highly controllable: various systems can be explored
  - BEC, Fermi degeneracy
  - Optical Lattice ⇒ Hubbard model
  - 1D, 2D, 3D systems
  - Spin-orbit coupling
  - Non-equilibrium physics (e.g. vortices, thermalization,…)

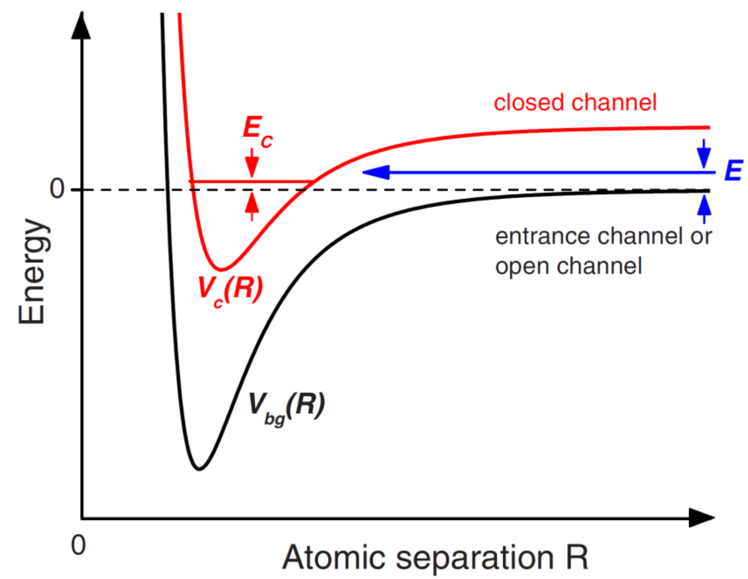
Feshbach resonance

- Control of inter-particle interaction
  - Weakly to strongly interacting systems can be explored by varying external magnetic field
  - Resonantly interacting system $a = \pm\infty$ can be prepared



External Magnetic Field [G]

S. Inouye, *et al.*, Nature **392**, 151 (1998).



C. Chin, *et al.*, Rev. Mod. Phys **82**, 1225 (2010).

BEC-BCS crossover in 2-component Fermi system

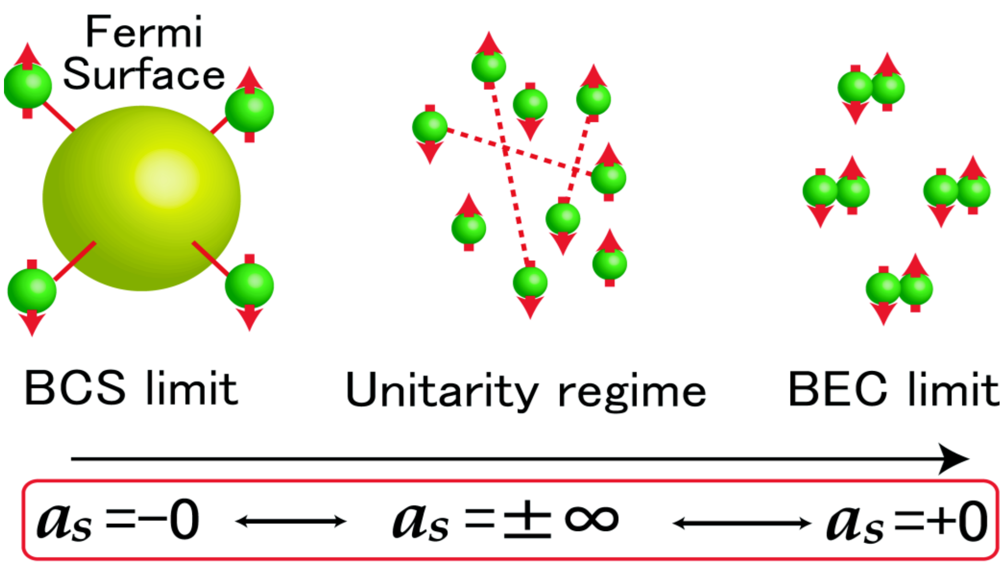
- Attractive 2-component Fermi system

  ⇒Smooth crossover from BCS to BEC superfluid

Eagles (1969), Leggett (1980), Nozieres Schmitt-Rink (1985)

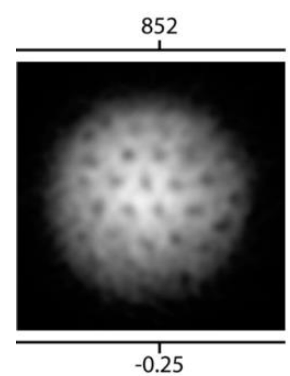
- Realized in cold atom experiments

  ⇒various universal physics explored

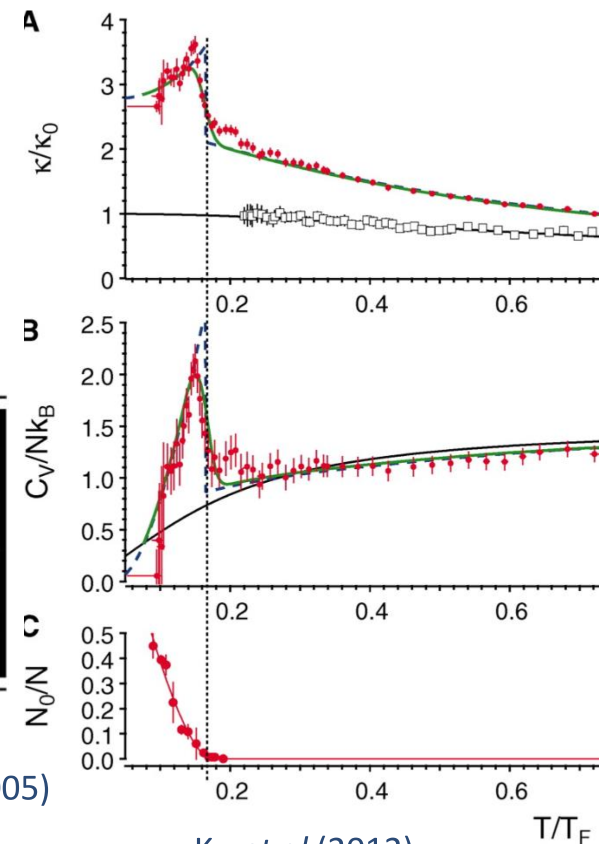


Weak attraction

Strong attraction



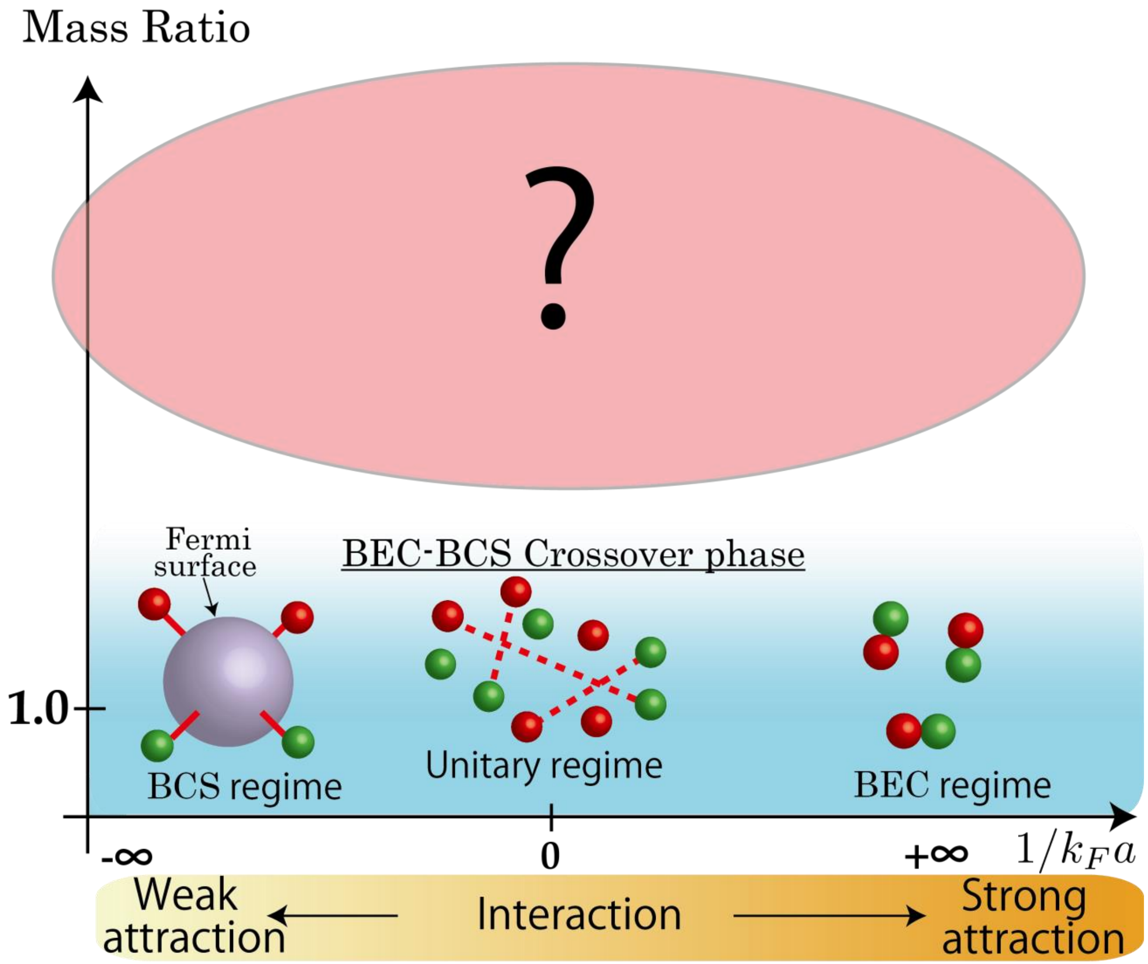
Zwierlein *et al* (2005)



Ku *et al* (2012)

Mass-imbalanced 2-component Fermi system

- Mass imbalanced fermionic mixture realized
  ^{6}Li-^{40}K, ^{6}Li-^{173}Yb
- Can we find interesting few-body and many-body physics induced by mass imbalance?



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Hamiltonian of a mass-imbalanced Fermi gas

- Dilute & low-energy system with short-range interaction

(e.g.) cold atoms, low-filling Hubbard model, neutron stars, …

⇒Inter-particle interaction modelled well by isotropic contact interaction and become universal

$$H = \int d^3\boldsymbol{r} \left[-\psi_\uparrow^\dagger(\boldsymbol{r}) \frac{\hbar^2 \nabla^2}{2m_\uparrow} \psi_\uparrow(\boldsymbol{r}) - \psi_\downarrow^\dagger(\boldsymbol{r}) \frac{\hbar^2 \nabla^2}{2m_\downarrow} \psi_\downarrow(\boldsymbol{r}) + U \psi_\uparrow^\dagger(\boldsymbol{r}) \psi_\downarrow^\dagger(\boldsymbol{r}) \psi_\downarrow(\boldsymbol{r}) \psi_\uparrow(\boldsymbol{r}) \right]$$

$$m_\uparrow \neq m_\downarrow$$

- Attractive interaction $U < 0$ related with s-wave scattering length a

$$\frac{1}{U} = \frac{\mu_{\uparrow\downarrow}}{\hbar^2} \left[\frac{1}{2\pi a} - \sum_{\boldsymbol{k}} \frac{1}{k^2} \right]$$

$$\mu_{\uparrow\downarrow} = \frac{m_\uparrow m_\downarrow}{m_\uparrow + m_\downarrow}$$

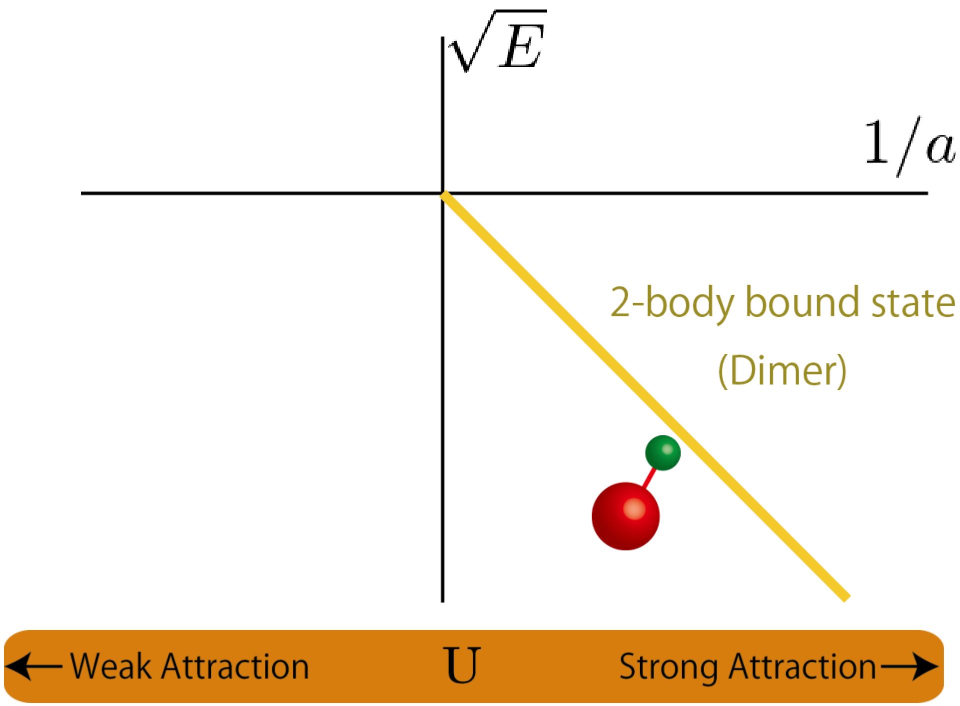
2-body problem

$$H = \int d^3\boldsymbol{r} \left[-\psi_\uparrow^\dagger(\boldsymbol{r}) \frac{\hbar^2 \nabla^2}{2m_\uparrow} \psi_\uparrow(\boldsymbol{r}) - \psi_\downarrow^\dagger(\boldsymbol{r}) \frac{\hbar^2 \nabla^2}{2m_\downarrow} \psi_\downarrow(\boldsymbol{r}) + U \psi_\uparrow^\dagger(\boldsymbol{r}) \psi_\downarrow^\dagger(\boldsymbol{r}) \psi_\downarrow(\boldsymbol{r}) \psi_\uparrow(\boldsymbol{r}) \right]$$

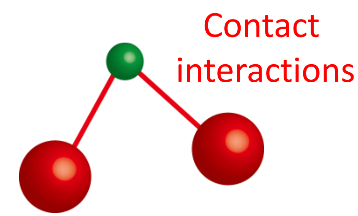
$$\frac{1}{U} = \frac{\mu_{\uparrow\downarrow}}{\hbar^2} \left[\frac{1}{2\pi a} - \sum_{\boldsymbol{k}} \frac{1}{k^2} \right]$$

- $a < 0$: no 2-body bound state
- $a > 0$: 2-body bound state (universal dimer) exists

Size $\sim a$ Energy $= -\dfrac{\hbar^2}{2\mu_{\uparrow\downarrow} a^2}$



3-body problem at unitarity 1/a=0

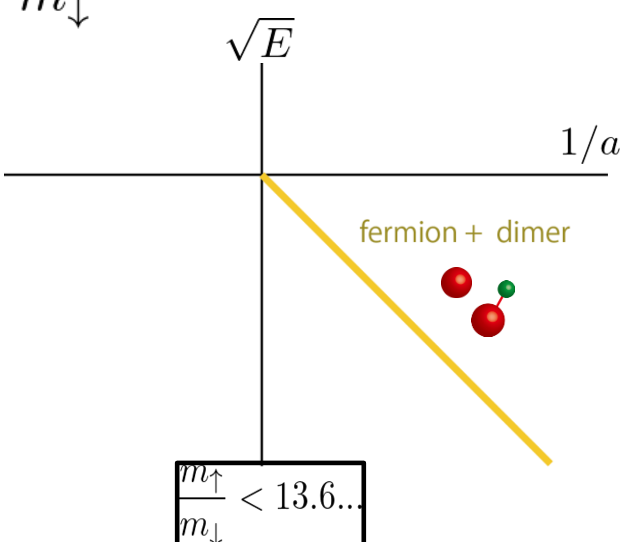


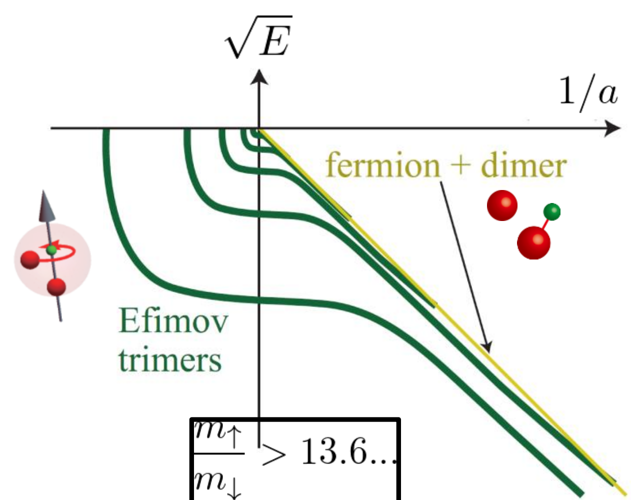
- Mediated attraction
- Antisymmetrization $\Rightarrow$ repulsion

$$V(R) = -\frac{\hbar^2\kappa^2}{2m_\downarrow R^2} + \frac{\hbar^2 L(L+1)}{2m_\uparrow R^2} \qquad L = 1$$

Born-Oppenheimer potential

- Mass ratio: knob to control 3-body physics
  - $\frac{m_\uparrow}{m_\downarrow} < 13.6...$: no 3-body bound state
  - $\frac{m_\uparrow}{m_\downarrow} > 13.6...$: infinite 3-body bound states (Efimov states)

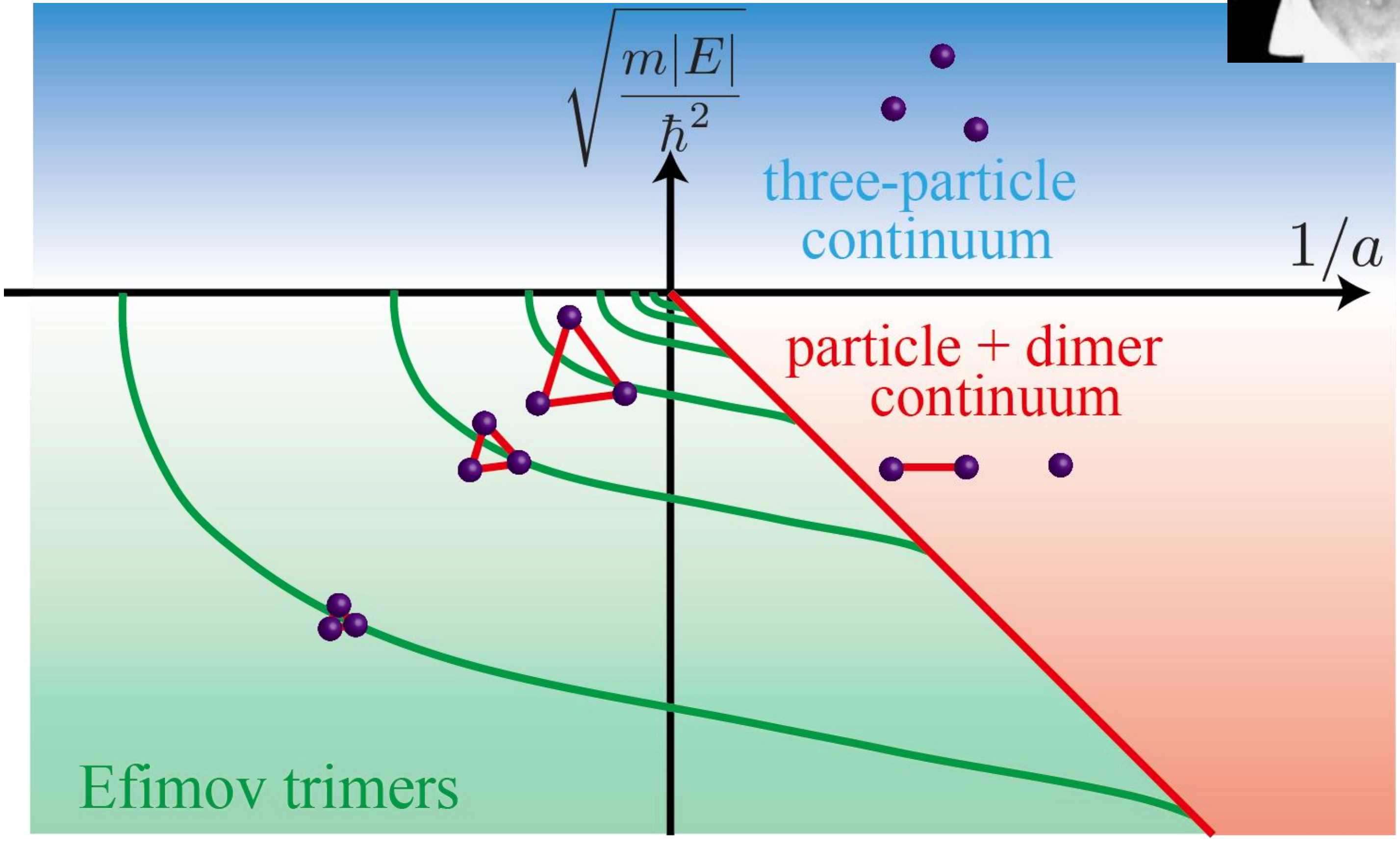




Efimov states

- Infinite 3-body bound states appear close to resonance 1/a=0
- Discrete scale invariance (RG limit cycle)
  - Binding energy $E_{n+1} = e^{-2\pi/s_0} E_n.$
  - Wave function $\Psi_{n+1}(r_i) = \Psi_n(r_i e^{-\pi/s_0}).$

V. Efimov, Phys. Lett. B **33**, 563 ((1970)



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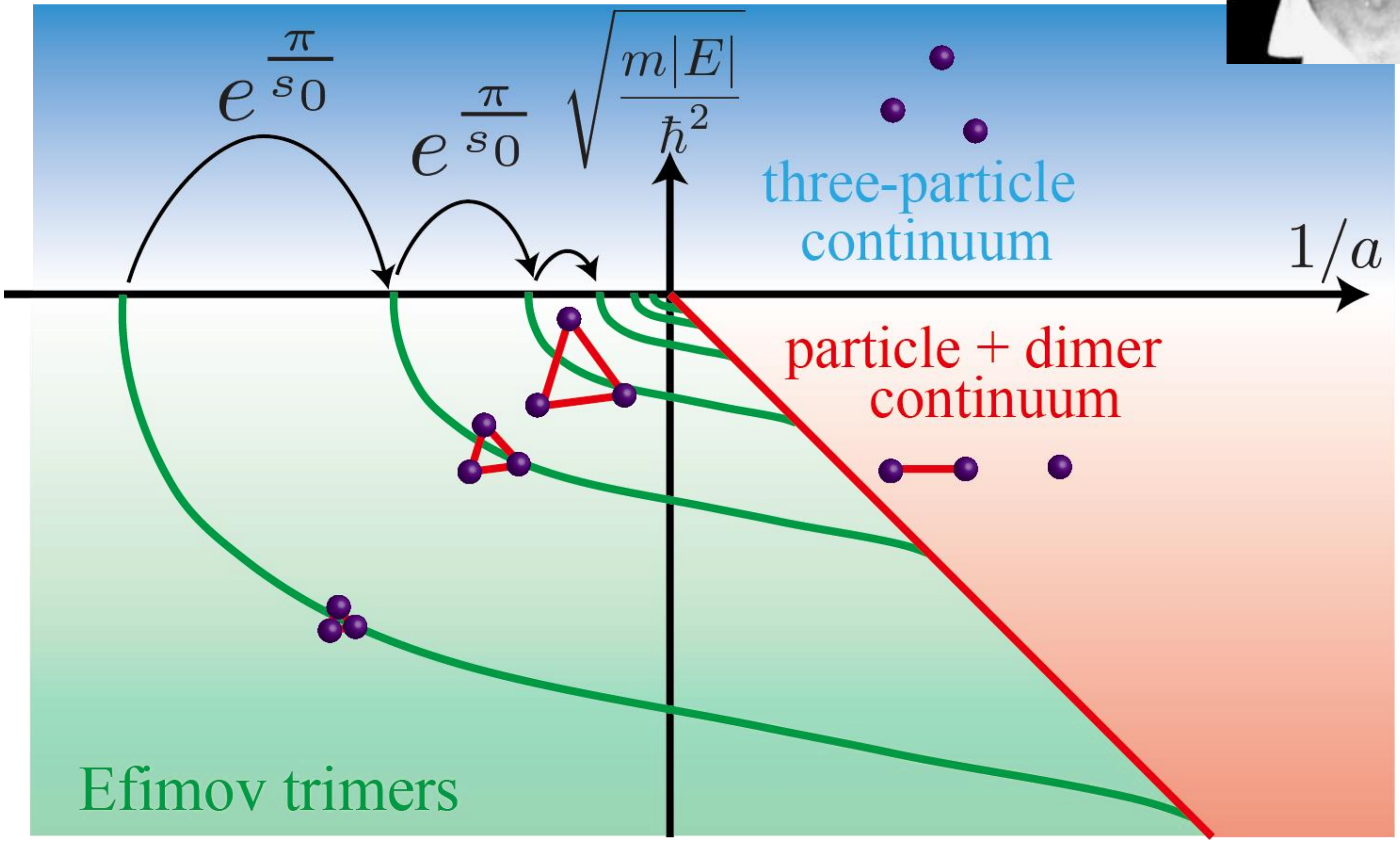

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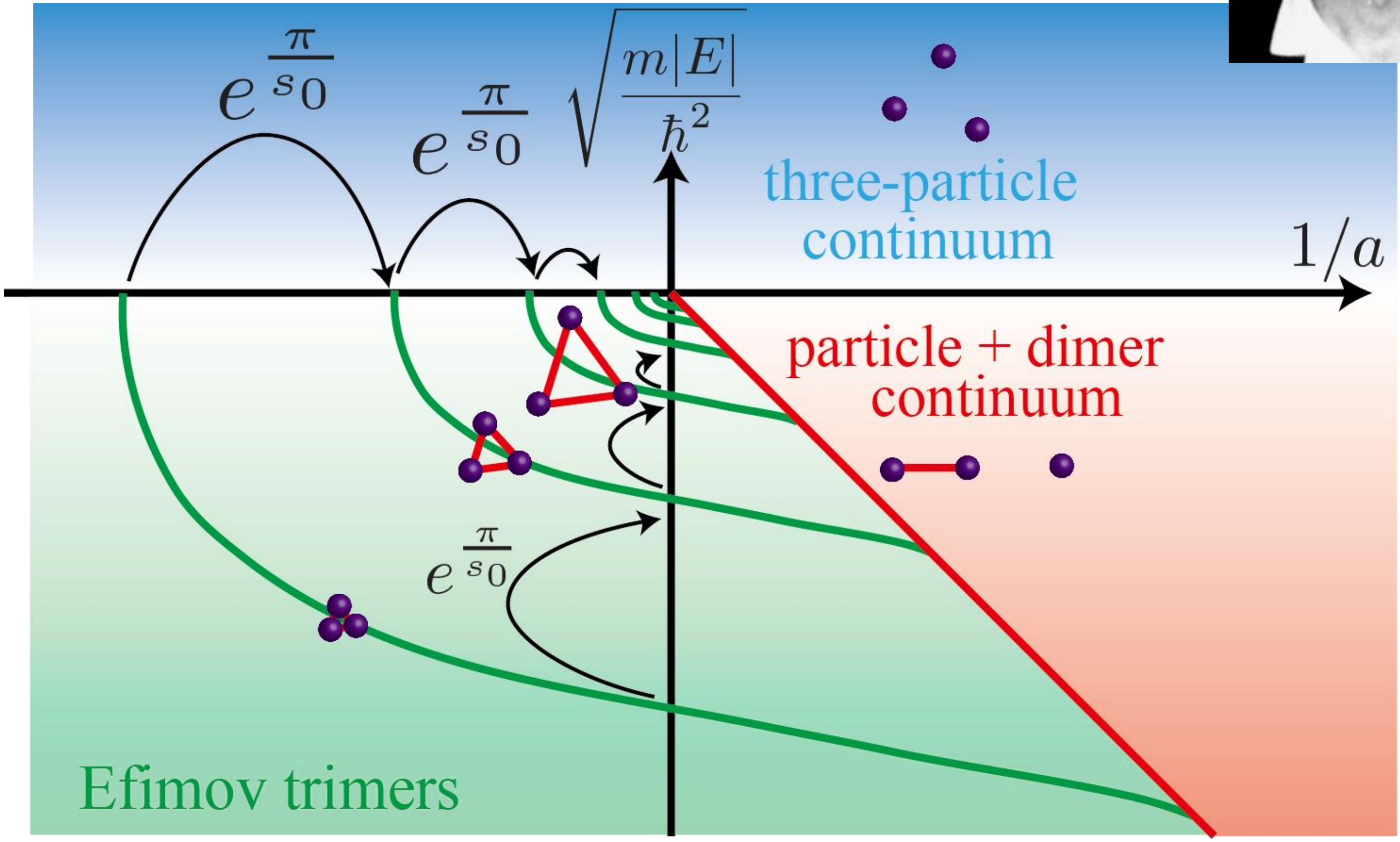

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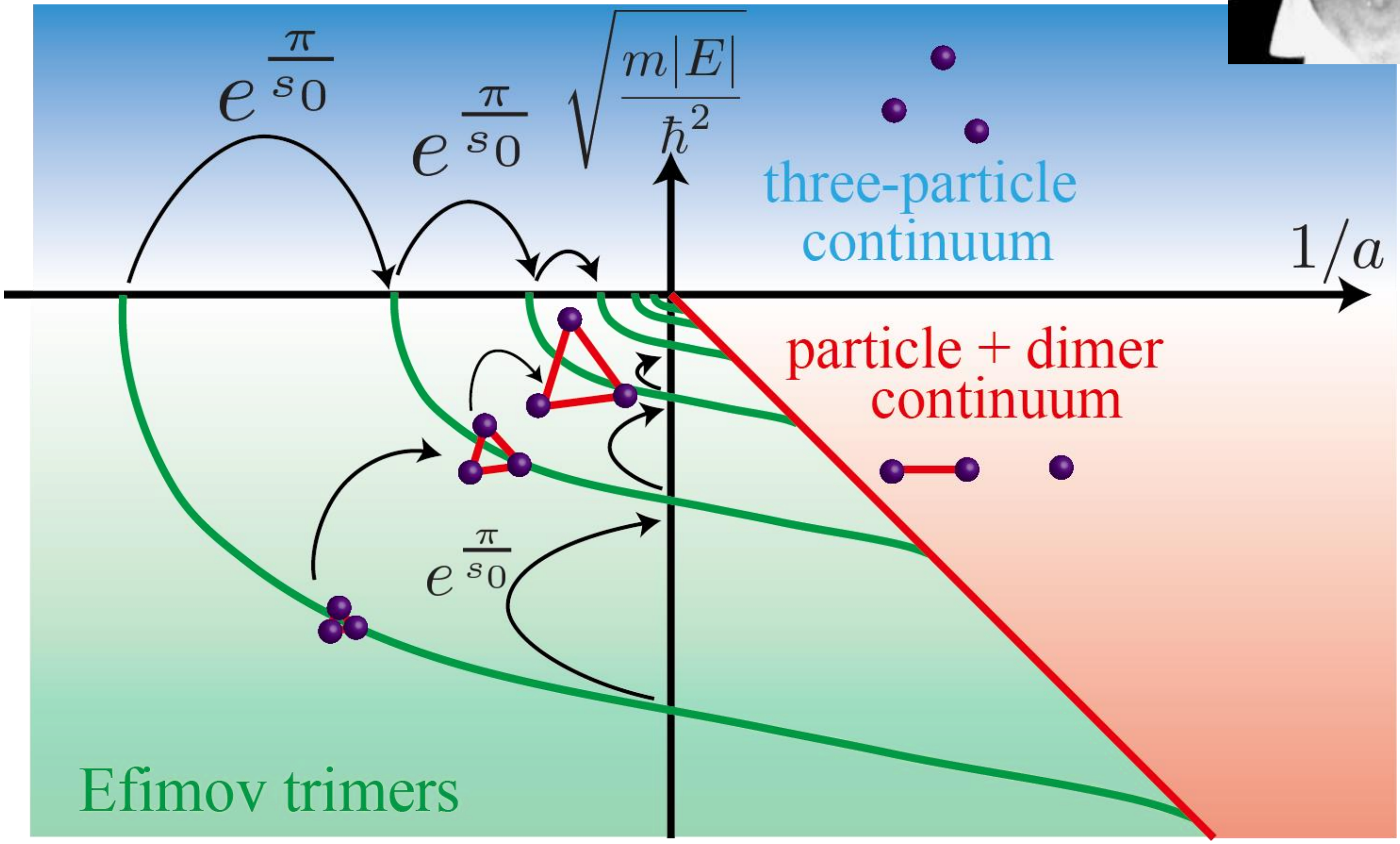
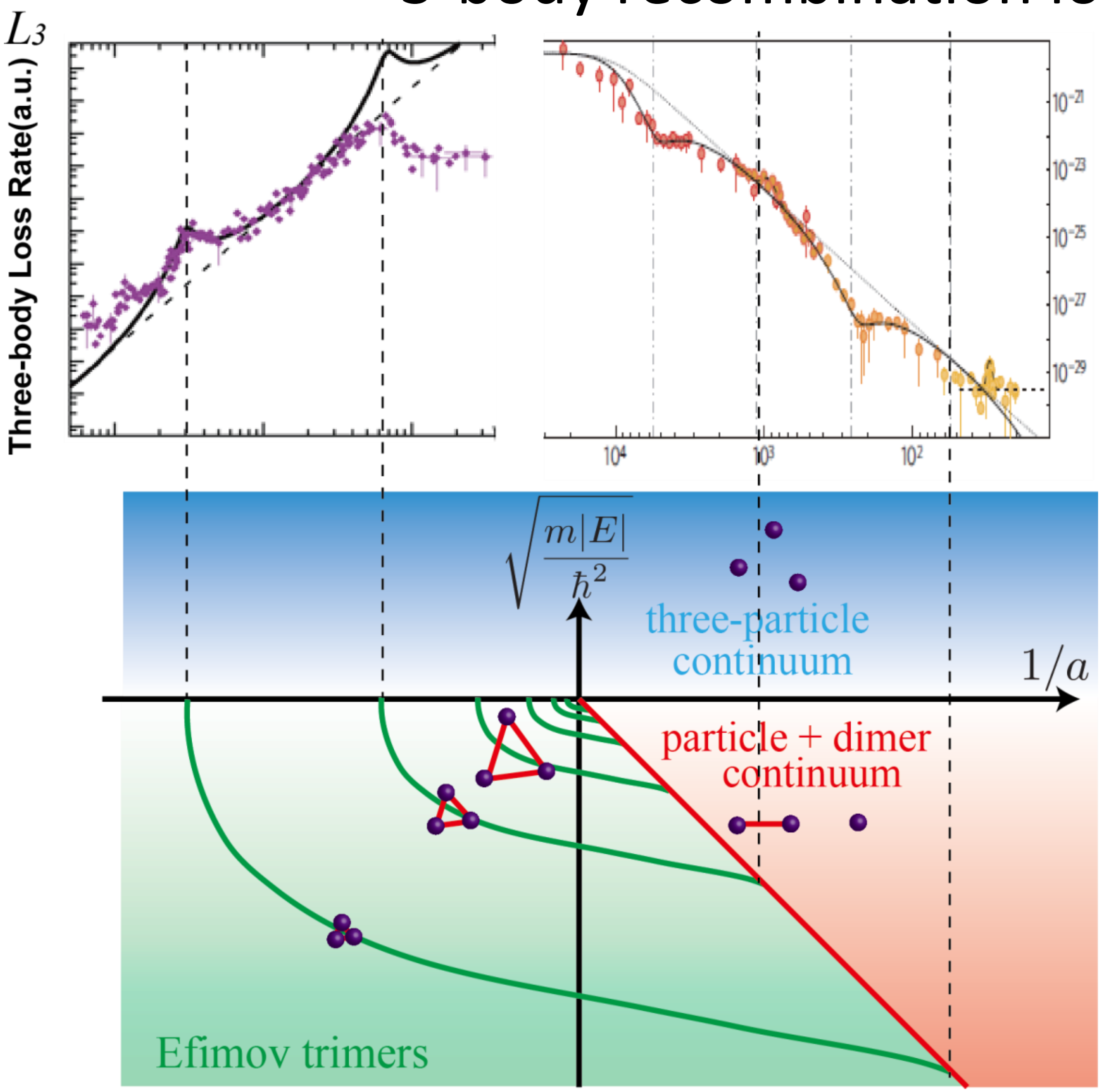


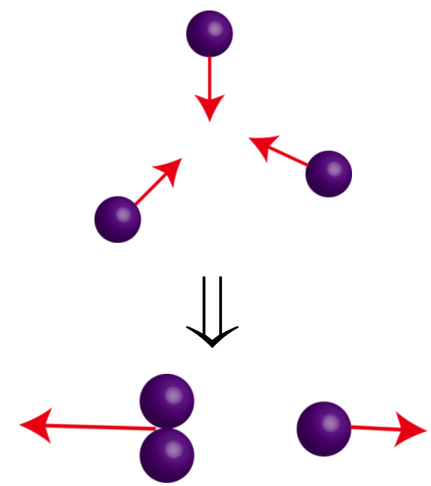
Observation of the Efimov states in cold atoms

- Efimov states: unstable in cold atom experiments due to 3-body recombination loss



Identical bosons

F. Ferlaino, R. Grimm
Physics **3**, 9 (2010)



$$\frac{dn}{dt} = -L_3 n^3 .$$

Universal trimer: stable trimers

$$8.1.. < m_\uparrow / m_\downarrow < 13.6..$$

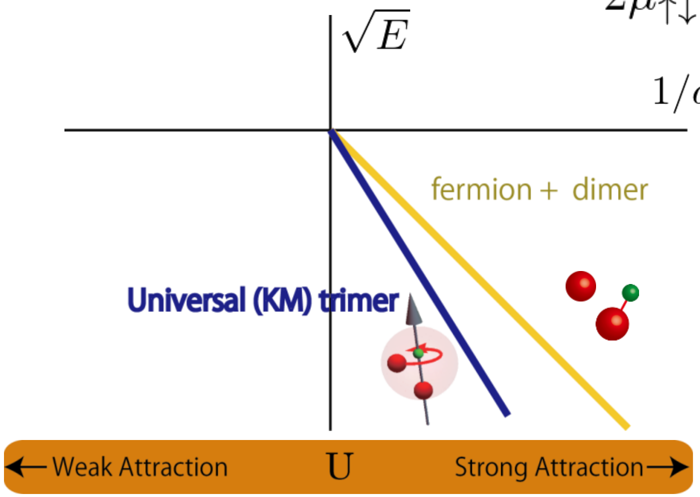
O. I. Kartavtsev, A. V. Malykh, J. Phys. B **40**, 1429 (2007)
J. Levinsen, *et al.*, Phys. Rev. Lett. 103, 153202 (2009)

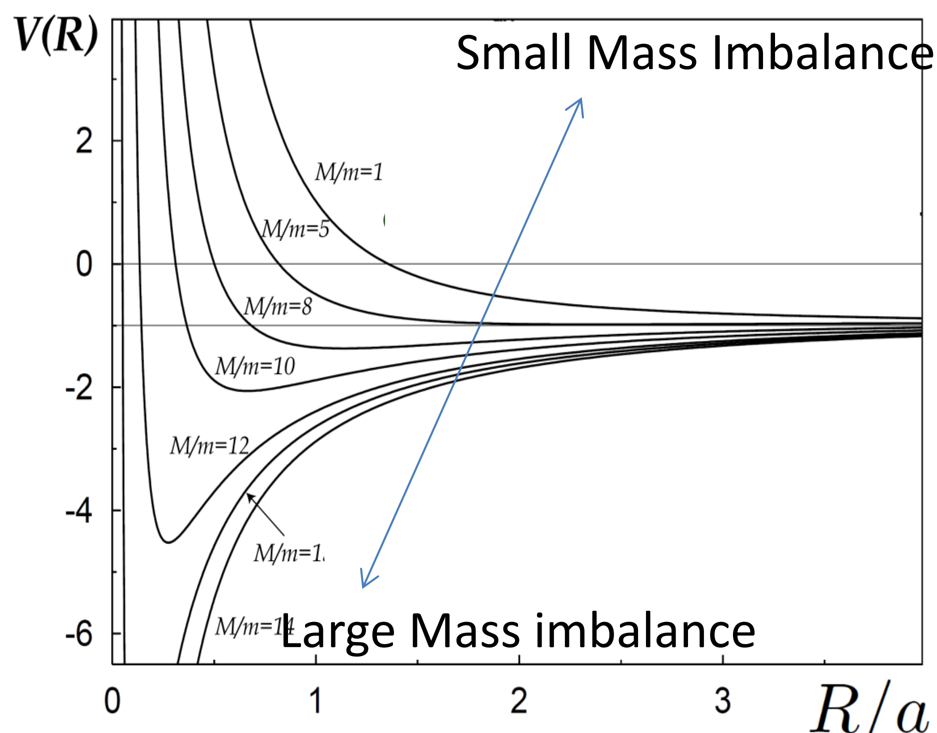
- No Efimov trimers. But another class of 3-body bound states exists for a>0

Universal trimer (Kartavtsev-Malykh trimer)

- Repulsion at short distance due to Pauli principle suppress 3-body recombination ⇒ Stable trimers
- Universally characterized by a as similar to the dimer

Size $\sim a$ Energy $\sim -\frac{\hbar^2}{2\mu_{\uparrow\downarrow}a^2}$

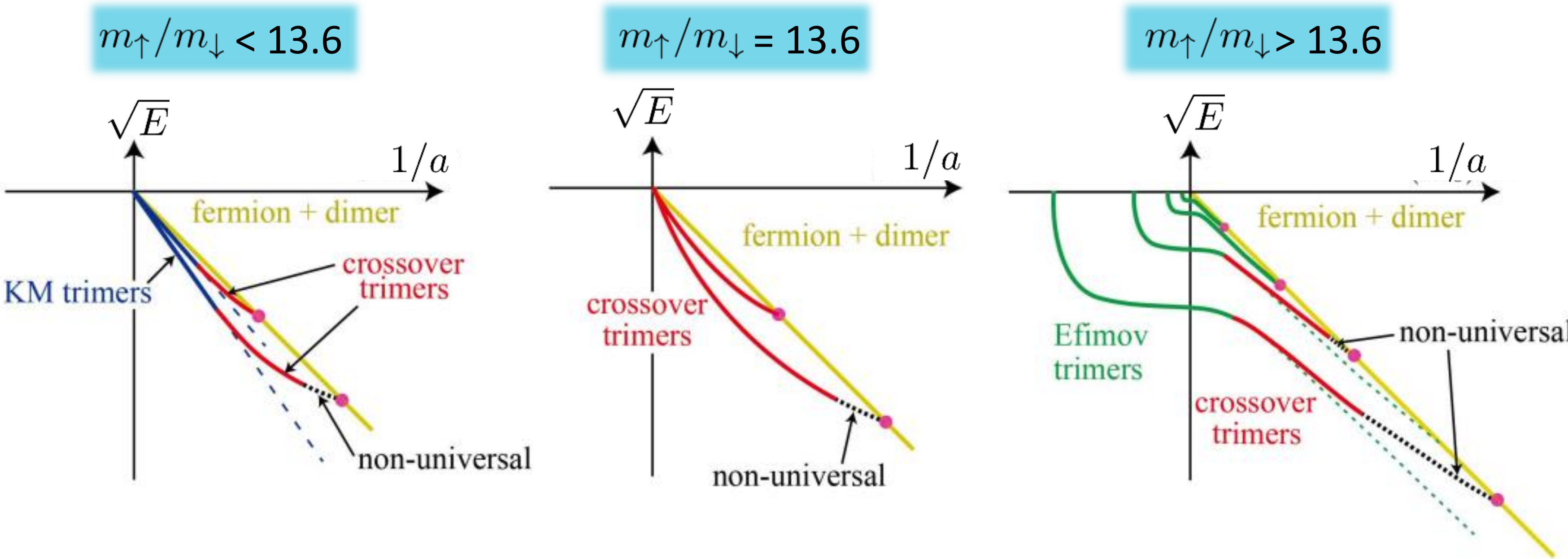




Crossover Trimer

SE, P. Naidon, M. Ueda, PRA **86**, 062703 (2012)

There exists a third trimer — crossover trimer — which smoothly connects the Efimov and KM trimers and appears away from the unitarity.



Kartavtsev-Malykh universal trimers (depend on a, **continuous scaling invariance**)

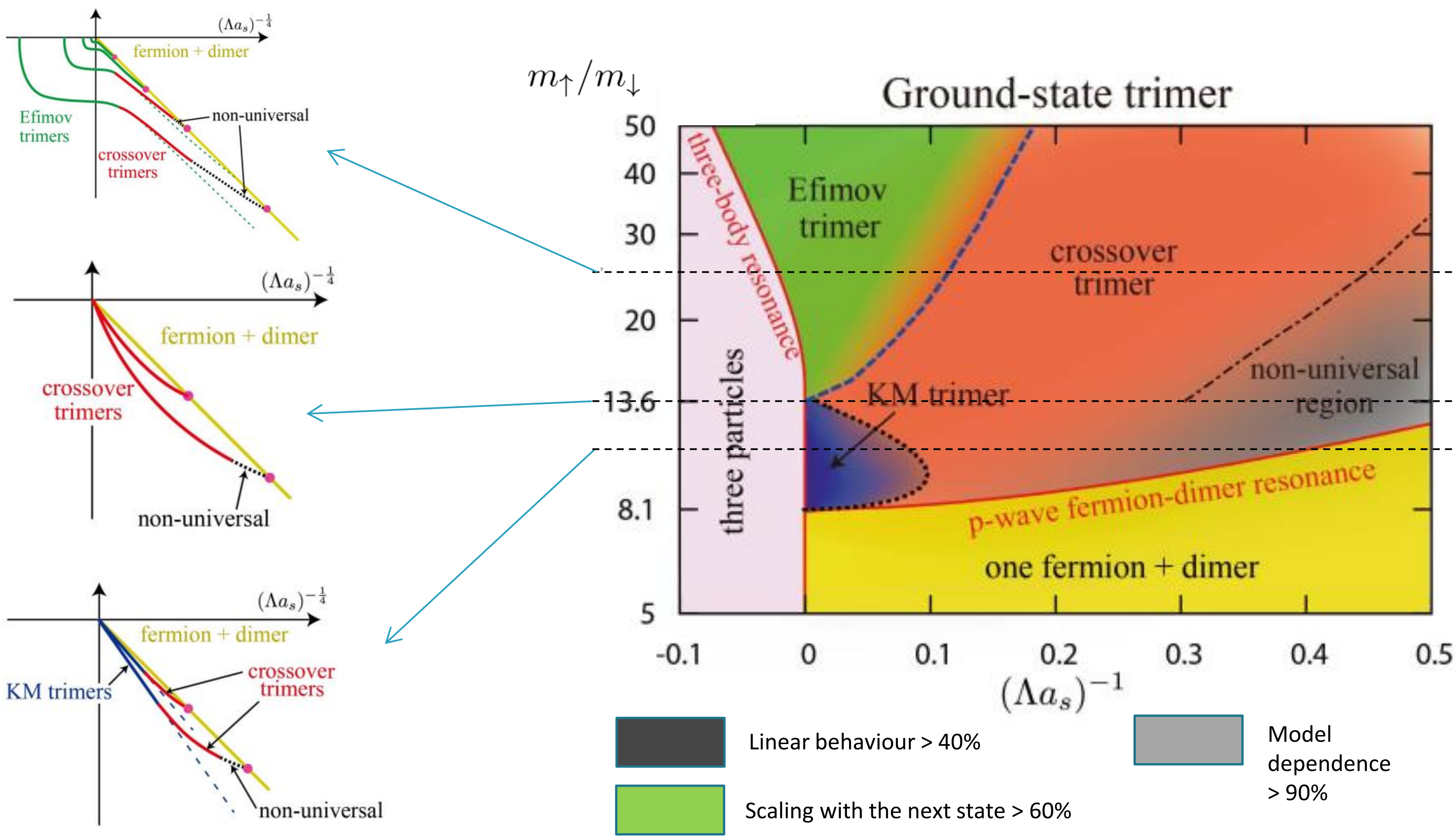
Efimov trimers (depend on a and Λ, **discrete scaling** invariance)

Crossover trimers (depend on a and Λ , **no scaling invariance**)

``Phase Diagram'' — Mass Ratio vs. *a*

SE, P. Naidon, M. Ueda, PRA **86**, 062703 (2012)

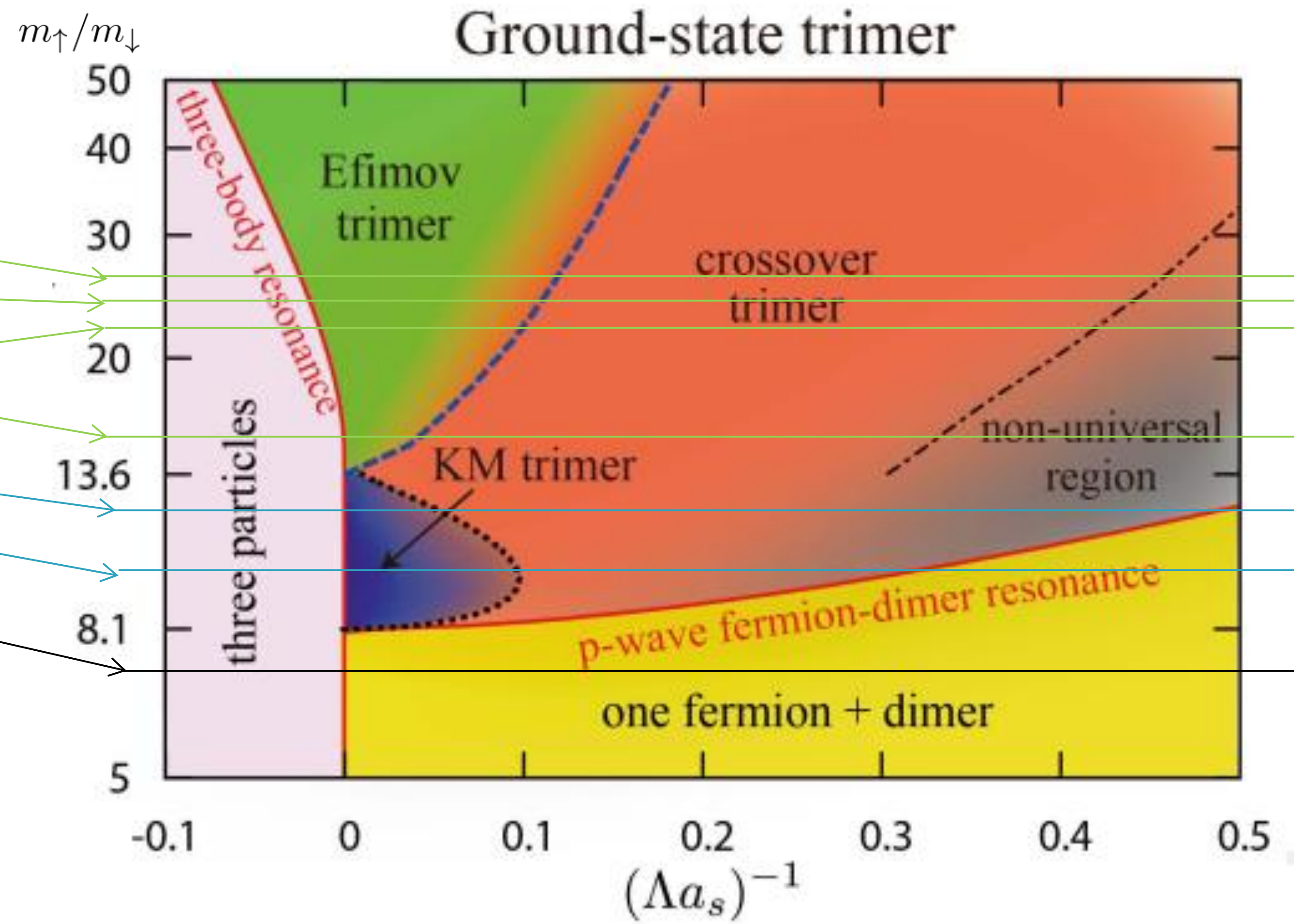
Ground-state timer



Experimental Candidates

SE, P. Naidon, M. Ueda, PRA **86**, 062703 (2012)

Mixture	Mass ratio
^{7}Li ^{173}Yb	24.65
^{7}Li ^{167}Er	23.79
^{7}Li ^{161}Dy	22.94
^{6}Li ^{87}Sr	14.45
^{7}Li ^{87}Sr	12.39
^{6}Li ^{53}Cr	8.80
^{7}Li ^{53}Cr	7.55
^{23}Na ^{173}Yb	7.52
^{23}Na ^{167}Er	7.26
^{23}Na ^{161}Dy	7.00
^{6}Li ^{40}K	6.64
^{7}Li ^{43}Ca	6.12
^{7}Li ^{40}K	5.70

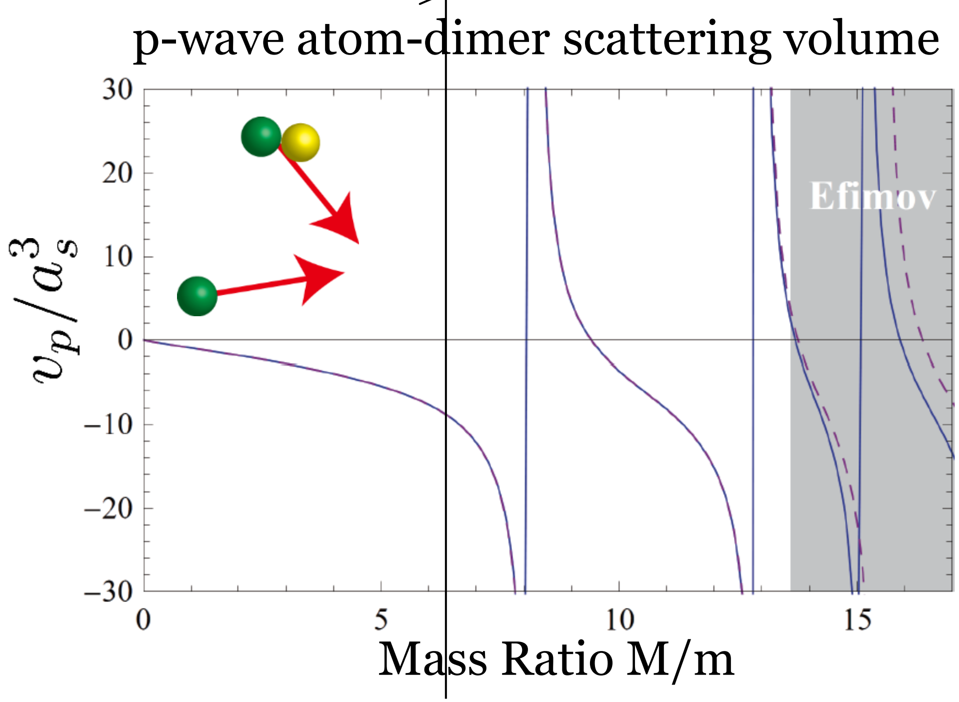


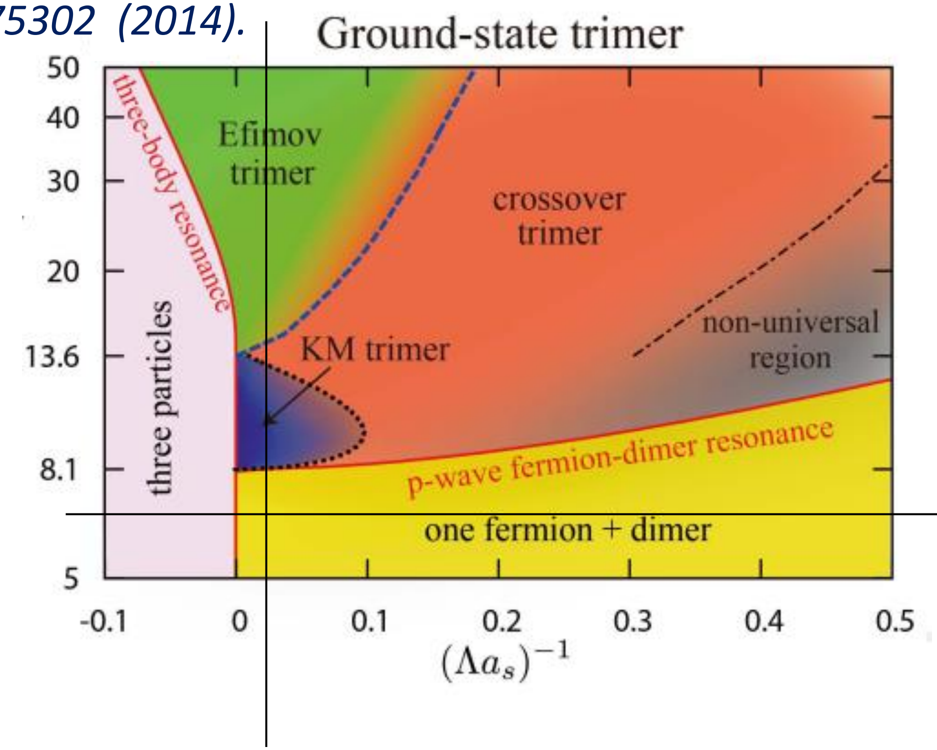
Enhanced p-wave atom-dimer scattering

- As the trimer dissociates into atom + dimer, p-wave atom dimer resonance occurs.
- The width of this resonance broad in terms of mass.
- Even for M/m < 8.1, signatures of the KM-trimer and crossover trimer can be observed from the enhanced p-wave atom-dimer scattering volume.

SE, P. Naidon, M. Ueda, Few-body Systems **51**, 207 (2011)
J. Levinsen et al., PRL. 103, 153202 (2009).
J. Michael et al. PRL. 112, 075302 (2014).

^{6}Li ^{40}K 6.64





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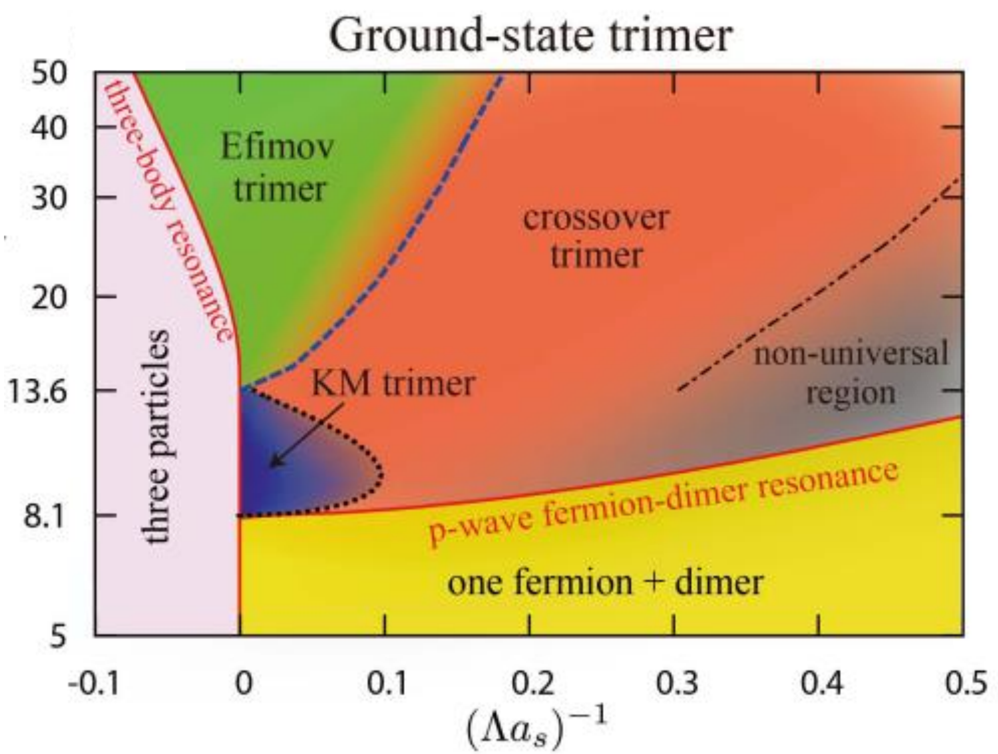
Universal trimers: stable

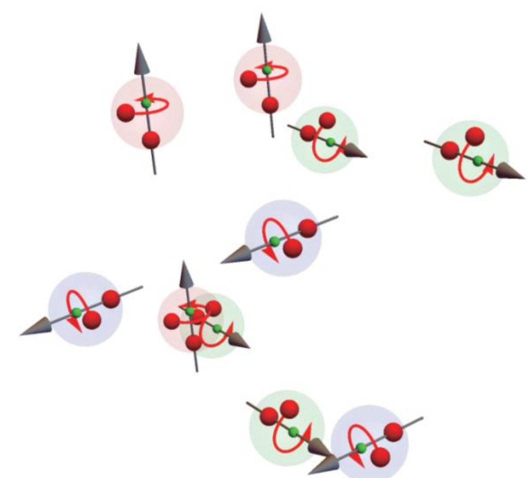
Efimov trimers ($m_\uparrow/m_\downarrow > 13.6..$)

- Unstable in cold atoms via 3-body recombination losses

Universal (Kartavtsev-Malykh) trimers ($8.1.. < m_\uparrow/m_\downarrow < 13.6..$)

- Stable $\Rightarrow$ Many-body phase composed of trimers??

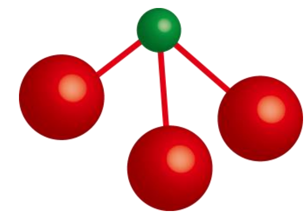




Many-body phase of universal trimers

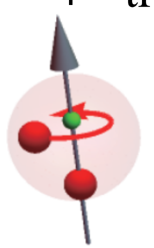
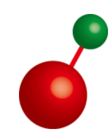
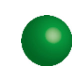
- We study many-body phase of this stable universal trimers

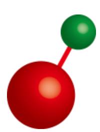
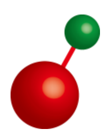
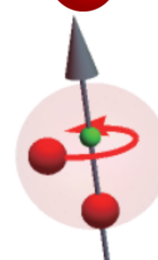
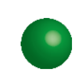
<u>Setup</u>



- $k_F\, a \rightarrow +0$: Strong 2-body attraction limit
- $8.1 < m_\uparrow / m_\downarrow < 9.5$: Only universal trimer exists
  - – Universal tetramer appears for $m_\uparrow / m_\downarrow > 9.5$ Blume (2009)
- We first investigate phase diagram in this limit by varying population imbalance.

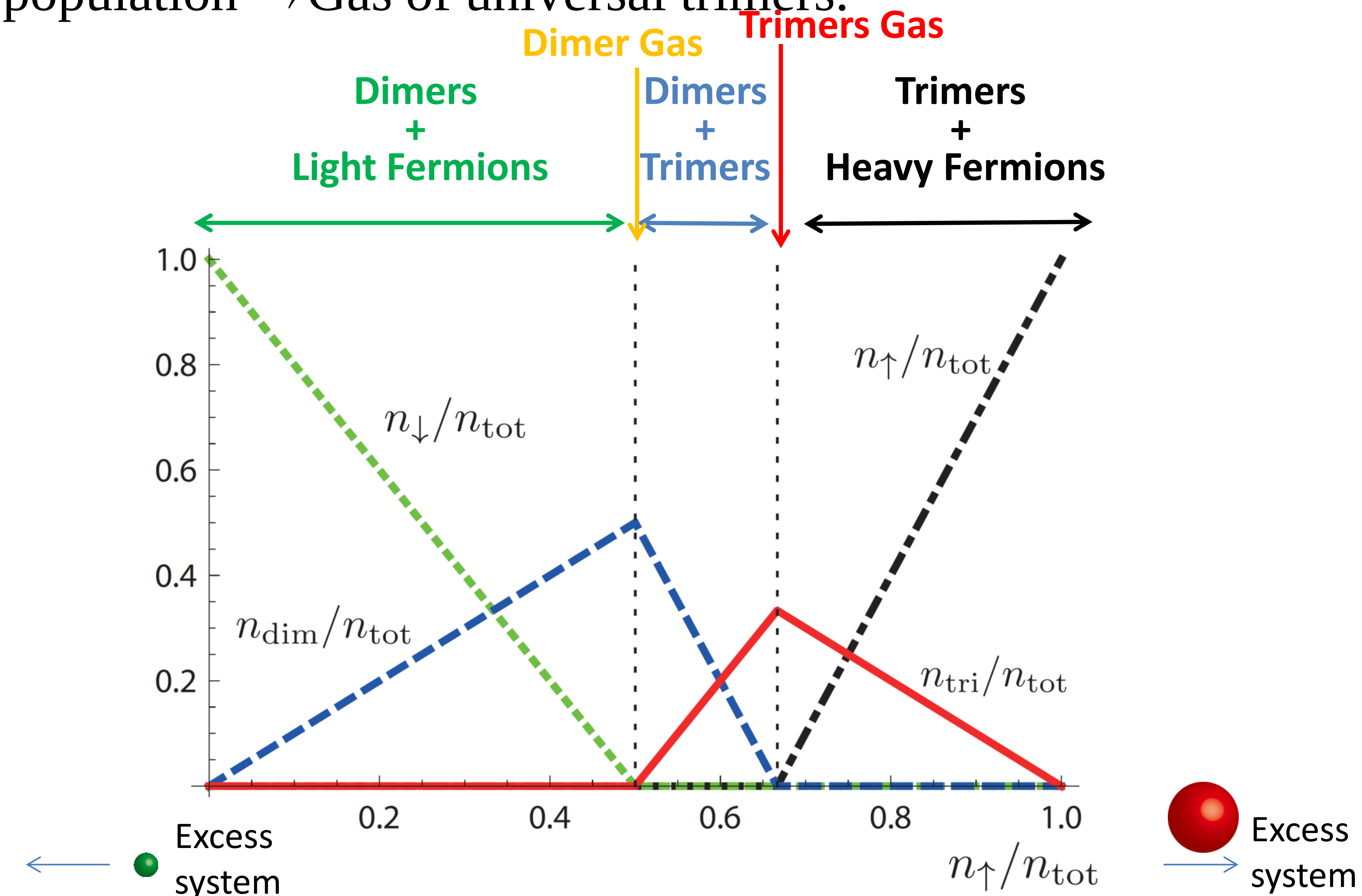
$$2|E_{dim}| > |E_{trim}| > |E_{dim}| \quad \text{when} \quad 8.1 < m_\uparrow / m_\downarrow < 9.5$$

$\Rightarrow$  more stable than  + 

$\Rightarrow$  +  more stable than  + 

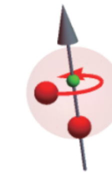
Many-body phase with universal trimers

- Population balanced⇒Gas of dimers
- 2:1 population ⇒Gas of universal trimers.



Trimer Phase: SU(3) Fermi system

- Universal trimers: L=1 ⇒ 3-fold degenerate



- Low-energy Hamiltonian: 3-component Fermi system

$$
\begin{aligned}
H_{\mathrm{int}}^{(\mathrm{eff})} &= g_{F=1}\int d^3\boldsymbol{r}\Big[\psi_1^\dagger(\boldsymbol{r})\psi_0^\dagger(\boldsymbol{r})\psi_0(\boldsymbol{r})\psi_1(\boldsymbol{r}) + \psi_1^\dagger(\boldsymbol{r})\psi_{-1}^\dagger(\boldsymbol{r})\psi_{-1}(\boldsymbol{r})\psi_1(\boldsymbol{r}) \\
&\qquad + \psi_{-1}^\dagger(\boldsymbol{r})\psi_0^\dagger(\boldsymbol{r})\psi_0(\boldsymbol{r})\psi_{-1}(\boldsymbol{r})\Big] \\
&= \frac{g_{F=1}}{2}\sum_{m_1 m_2}\int d^3\boldsymbol{r}\,\psi_{m_1}^\dagger(\boldsymbol{r})\psi_{m_2}^\dagger(\boldsymbol{r})\psi_{m_2}(\boldsymbol{r})\psi_{m_1}(\boldsymbol{r}),
\end{aligned}
$$

SU(3) symmetry

- Coupling constant $g_{F=1}$ is related with trimer-trimer s-wave scattering length a^{tt}

$$
\frac{1}{g_{F=1}} = \frac{\mu_{\uparrow\downarrow}}{\hbar^2}\left[\frac{1}{2\pi a^{tt}} - \sum_{\boldsymbol{k}}\frac{1}{k^2}\right]
\qquad
\mu_{\uparrow\downarrow} = \frac{m_\uparrow m_\downarrow}{m_\uparrow + m_\downarrow}
$$

- 3-component SU(3) Fermi system emerges from 2-component Fermi system.

Trimer-trimer scattering length

- $g_{F=1} \propto a^{tt} > 0$: trimer Fermi liquid
- $g_{F=1} \propto a^{tt} < 0$: superfluid pairing of trimers

$\Rightarrow$Evaluate trimer-trimer scattering length (6-body problem!!)

Trimer-trimer scattering length

- $g_{F=1} \propto a^{tt} > 0$: trimer Fermi liquid
- $g_{F=1} \propto a^{tt} < 0$: superfluid pairing of trimers

⇒Evaluate trimer-trimer scattering length (6-body problem!!)

<u>Resonating Group Method</u>

- Approximate method to solve cluster-cluster scattering
- Applied for nuclear systems.

Wheeler, Phys. Rev. (1937).
Shimizu, Rep. Prog. Phys. (1989).
Tang, LeMere, Thompsom, Phys. Rep. (1978)

Antisymmetrization

$$\Psi^{(\mathrm{RGM})} = \mathcal{A}[\phi_1(1,2,3)\phi_2(4,5,6)\psi(\boldsymbol{R})]$$

Universal trimer's wave function

Relative wave function between the trimers

$\boldsymbol{R}$:COM distance between the trimers

- Reconfiguration of the clusters by exchanging particles.
- Virtual excitations to dimers, or continuum during the collision are neglected.

Resonating Group Method for T-T scattering

- Relative Schrodinger equation for the trimers

$$\left(T_R^\ell - E\right)\psi_{\ell m}(R) + \sum_{\ell' m'} V_1^{\ell m,\ell' m'}(R)\psi_{\ell' m'}(R) = 0$$

- Direct and exchange potentials between the trimers

$$V_1^{\ell m,\ell' m'} = V_{\rm D}^{\ell m,\ell' m'} + V_{\rm EX1}^{\ell m,\ell' m'}$$

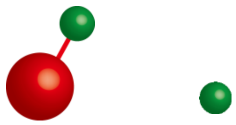
  – 9-dimensional integral with the trimer wave function

$$\begin{aligned} V_{\rm EX1}(\boldsymbol{s},\boldsymbol{s}') = \ & g\lambda \int d^3\boldsymbol{r} d^3\boldsymbol{R} \Big(\left(\bar{\phi}_x(\boldsymbol{R}_1)\phi_y(\vec{\mathcal{R}}) \mp \bar{\phi}_y(\boldsymbol{R}_1)\phi_x(\boldsymbol{\mathcal{R}})\right)^* \left(\phi_x(\boldsymbol{\mathcal{R}}_2)\phi_y(\boldsymbol{\mathcal{R}}_3) \mp \phi_y(\boldsymbol{\mathcal{R}}_2)\phi_x(\boldsymbol{\mathcal{R}}_3)\right) \\ & + \frac{2}{\kappa^3}\left(\bar{\phi}_x(\boldsymbol{R}_1')\phi_y(\boldsymbol{\mathcal{R}}) \mp \bar{\phi}_y(\boldsymbol{R}_1')\phi_x(\boldsymbol{\mathcal{R}})\right)^* \left(\phi_x(\boldsymbol{\mathcal{R}}_2')\phi_y(\boldsymbol{\mathcal{R}}_3') \mp \phi_y(\boldsymbol{\mathcal{R}}_2')\phi_x(\boldsymbol{\mathcal{R}}_3')\right) \\ & - \frac{2}{\kappa^3}\left(\phi_x(\boldsymbol{\mathcal{R}}_1'')\phi_y(\boldsymbol{\mathcal{R}}) \mp \phi_y(\boldsymbol{\mathcal{R}}_1'')\phi_x(\boldsymbol{\mathcal{R}})\right)^* \left(\phi_x(\boldsymbol{\mathcal{R}}_2'')\phi_y(\boldsymbol{\mathcal{R}}_3'') \mp \phi_y(\boldsymbol{\mathcal{R}}_2'')\phi_x(\boldsymbol{\mathcal{R}}_3'')\right) \ \Big) \end{aligned}$$

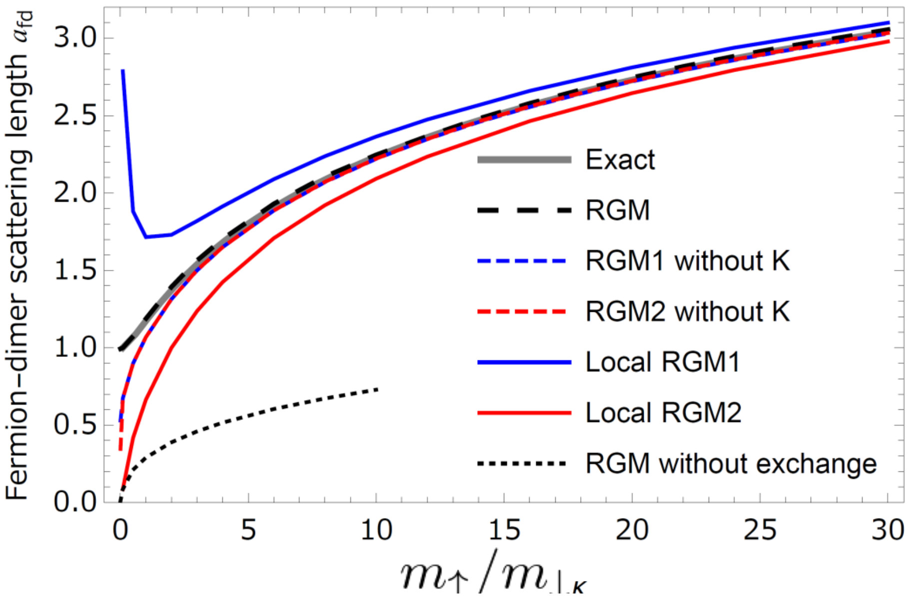
$$V_D(\boldsymbol{s}) = 2g\left(\frac{\kappa+1}{\kappa}\right)^3 \int d^3\boldsymbol{R} d^3\boldsymbol{r}' d^3\boldsymbol{R}' \left(\left|\phi_x(\boldsymbol{r}_-,\boldsymbol{R})\phi_y(\boldsymbol{r}',\boldsymbol{R}')\right|^2 + \left|\phi_x(\boldsymbol{r}',\boldsymbol{R}')\phi_y(\boldsymbol{r}_+,\boldsymbol{R})\right|^2\right)$$

Test with 3-body problem: fermion-dimer scattering

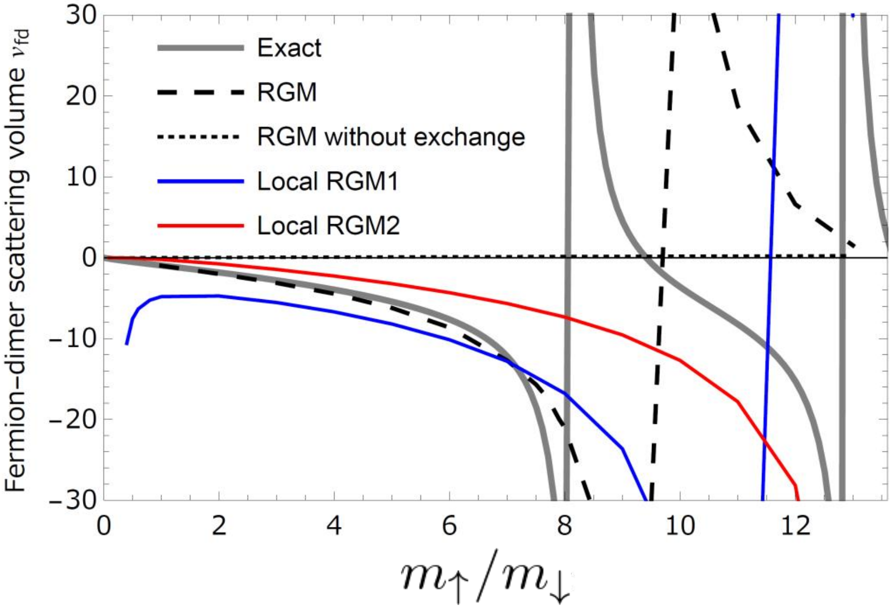
- S-wave and p-wave scatterings accurately describes by the Resonating group method
- Far more accurate than the Born approximation



S-wave scattering

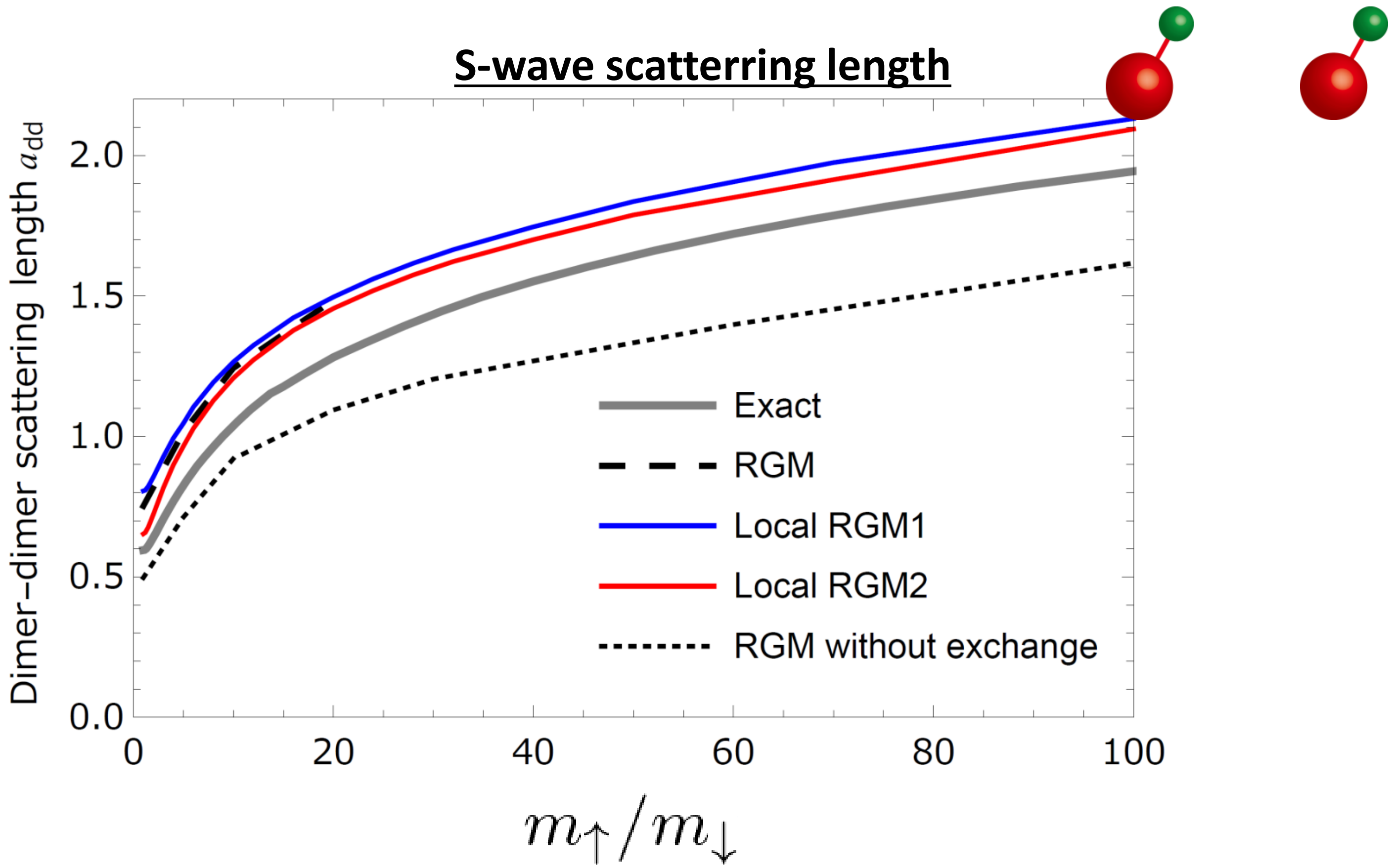


P-wave scattering

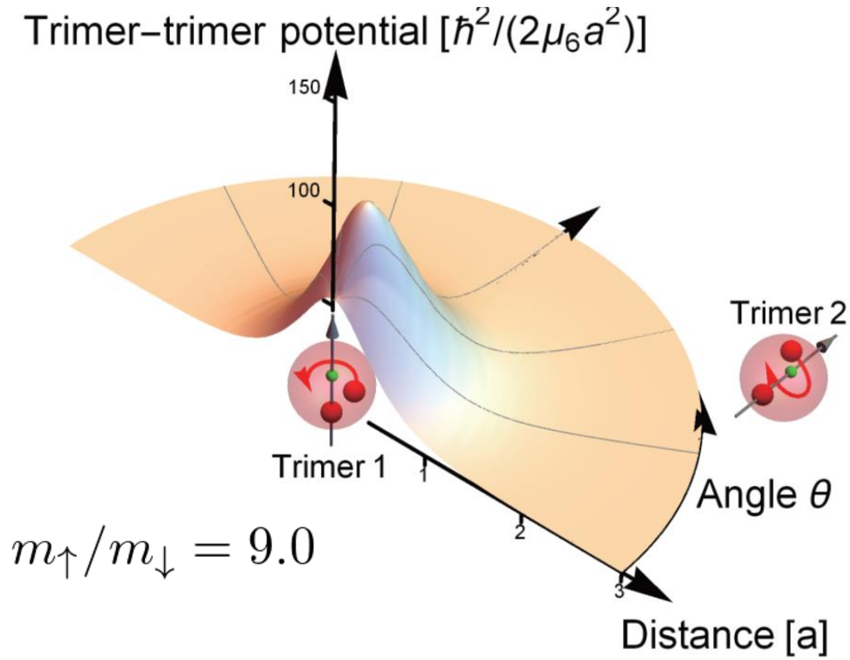


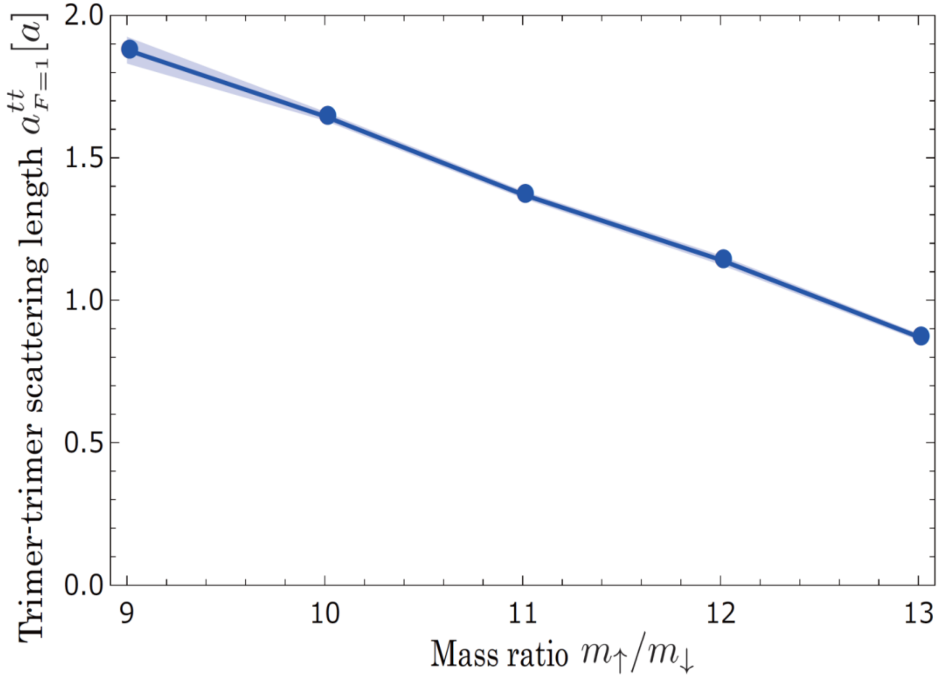
Test with 4-body problem: dimer-dimer scattering

- S-wave scattering length accurately describes by the Resonating group method
- Far more accurate than the Born approximation



Trimer-trimer potential and scattering length



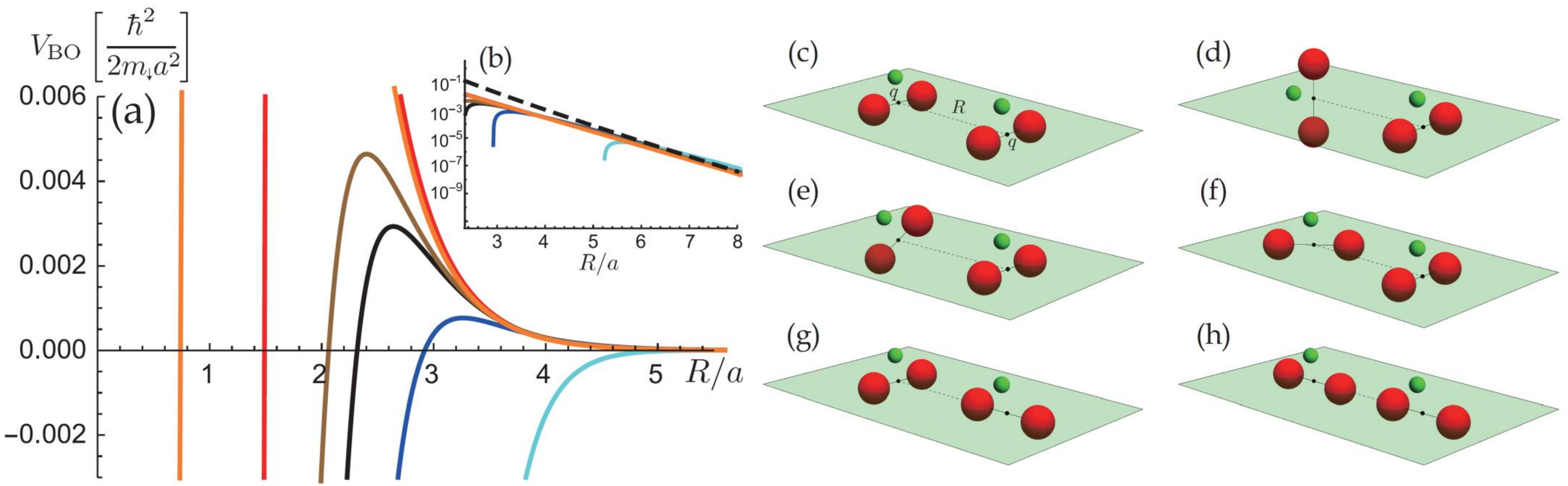


- Trimer-trimer potential: repulsive $\Rightarrow$ $g_{F=1} \propto a^{tt} > 0$
- SU(3) trimer phase is Fermi liquid, stable against recombination induced by trimer-trimer scattering.
- Trimer more tightly bound and its size gets smaller
  $\Rightarrow a^{tt}$ decreases as mass ratio gets larger

Born-Oppenheimer method

- Large distance: repulsive for all spatial configurations.

  $\Rightarrow$ Qualitatively agree with the RGM result.
- Short distance: Born-Oppenheimer method breaks down
  - Level crossings of the light fermions' solutions
  - Internal energy of the trimers become ill-defined.



Outline

- Introduction
- Universal 3-body physics for fermions

SE, P. Naidon, M. Ueda, Few-body Systems **51**, 207 (2011)
SE, P. Naidon, M. Ueda, PRA **86**, 062703 (2012)

- Novel SU(3) Trimer Phase in a 2-component mass-imbalanced Fermi system

SE, P. Naidon, A. M. Garcia-Garcia arXiv:1507.06309 (2015)
P. Naidon SE, A. M. Garcia-Garcia, arXiv:1507.06373 (2015)

- 3rd and 4th virial expansion of a unitary Fermi gas
  - Few-body approach to many-body physics

*C. Gao, SE, Y. Castin, EPL **109**, 16003 (2015)*
SE, Y. Castin, arXiv:1507.05580(2015)

Quantum Cluster (Virial) Expansion

E. Beth, G.E. Uhlenbeck (1936).

- Expand Ω via fugacity

$$\Omega = -\frac{k_B T}{\lambda_{dB}^3} \sum_{n,m} b_{n,m} e^{n\beta\mu_1} e^{m\beta\mu_2}$$

λ_{dB} : thermal de Brogie length

$b_{n,m}$: cluster (virial) coefficients

μ_i : chemical potential for i-th component

  – Good expansion at low density or high temperature

- $b_{n,m}$ corresponds to (n+m)-body physics.

  → Few-body approach to quantum many-body physics

- Revival of interests in recent cold atom experiments

  – Equation of state of the unitary Fermi gas

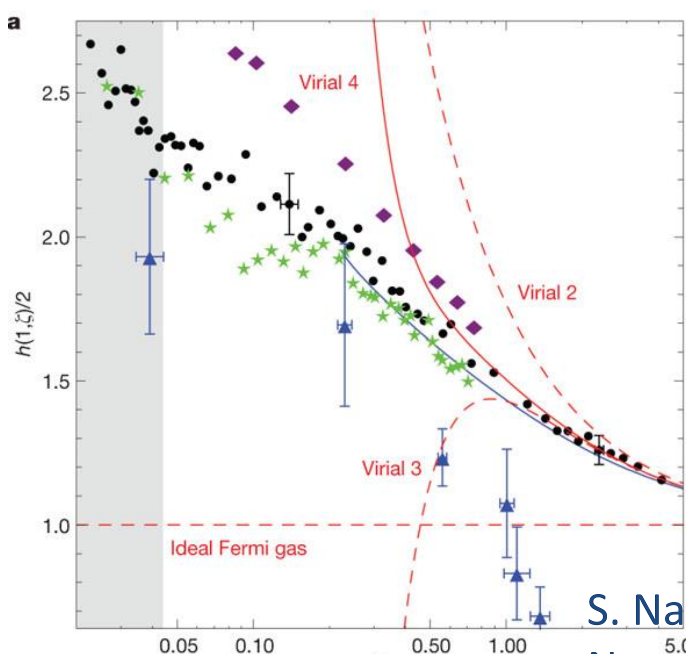
Equation of state of unitary Fermi gas

- Precise measurement of EOS $m_\uparrow = m_\downarrow$
  - Virial coefficients obtained experimentally

$\Delta b_3^{\mathrm{ENS}} = -0.35(2)$ ←Good agreement with MIT and Univ. Tokyo $\Delta b_3^{\mathrm{theory}} = -0.3551..$

$$\Delta b_4^{\mathrm{ENS}} = 0.096(15) \quad \Delta b_4^{\mathrm{MIT}} = 0.096(10)$$

$$\Delta b = b - b^{\mathrm{non-int}}$$

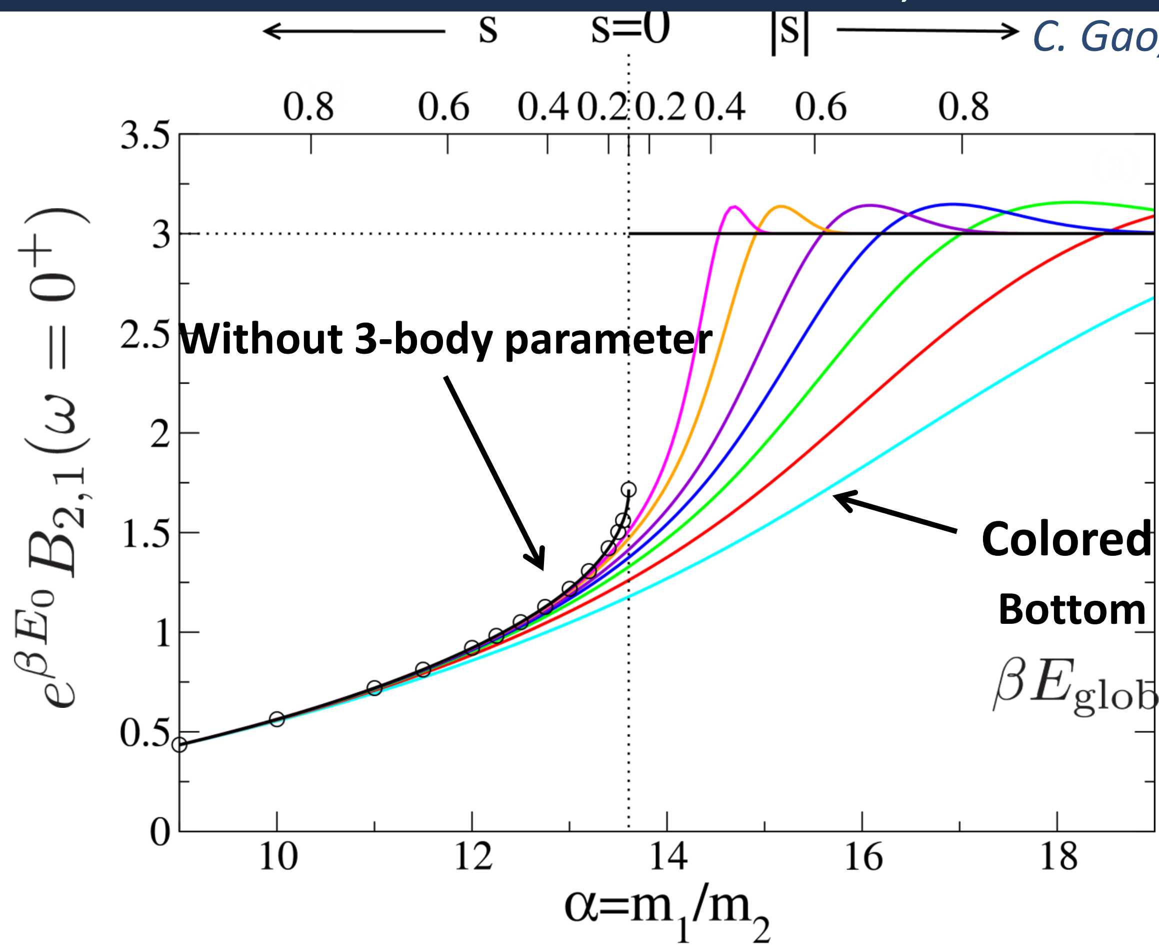


S. Nascimbène, et al. Nature (2010)

$z^{-1} = e^{-\beta\mu}$

3rd virial coefficient $B_{2,1}$ calculated analytically

*C. Gao, SE, Y. Castin, EPL **109**, 16003 (2015)*



E_0 **: Ground-state Efimov trimer energy**

$$E_0 = 0 \quad (\alpha < \alpha_c)$$

$$E_0 = E_{\mathrm{glob}} e^{-2\pi/|s|} \quad (\alpha > \alpha_c)$$

Colored curves : With 3-body parameter

Bottom (cyan) to top (purple):

$$\beta E_{\mathrm{glob}} = 10^2, 3\times 10^2, 10^3, 3\times 10^3, 10^4, 10^5, 10^6$$

- $b_{2,1}$ is smooth across critical mass ratio.
- Log correction by 3-body parameter even for $\alpha < \alpha_c$

⇒3-body parameter relevant even in the absence of the Efimov trimers

4th virial coefficient of unitary Fermi gas

- No agreement so far with theory and experiment $m_\uparrow = m_\downarrow$

Experiments: $\Delta b_4^{\mathrm{ENS}} = 0.096(15)$ $\Delta b_4^{\mathrm{MIT}} = 0.096(10)$

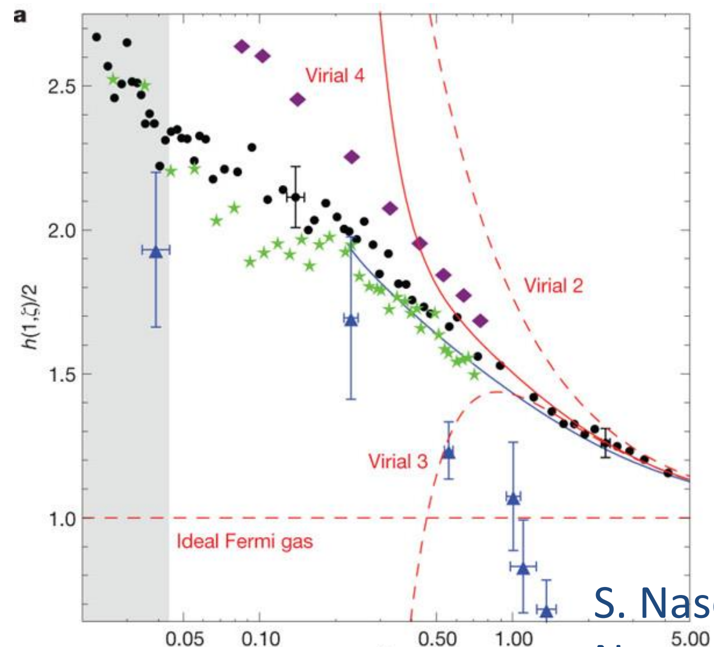
$\Delta b = b - b^{\mathrm{non-int}}$

Theories: $\Delta b_4^{\mathrm{Blume}} = -0.016(4)$ $\Delta b_4^{\mathrm{Levinsen}} \approx 0.06$

Rakshit, Daily Blume, PRA (2012) *Ngampruetikorn, Parish,Levinsen, PRA (2015)*

- We estimate Δb_4 from analytical solution of 4-body Schrodinge equation analytically at unitarity. *SE, Y. Castin, arXiv:1507.05580(2015)*

Our value: $\Delta b_4 = -0.063(1)$



$z^{-1} = e^{-\beta\mu}$

S. Nascimbène, e
Nature (2010)

Analytical solution of 2+2 fermions at 1/a=0

- Ansatz for 4-body wave function: 4 terms

$$\tilde{\psi}_{\uparrow\uparrow\downarrow\downarrow}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4)=\frac{\delta\left(\sum_{n=1}^{4}\boldsymbol{k}_n\right)}{\sum_{n=1}^{4}\frac{\hbar^2 k_n^2}{2m_n}}\left[D(\boldsymbol{k}_2,\boldsymbol{k}_4)-D(\boldsymbol{k}_2,\boldsymbol{k}_3)-D(\boldsymbol{k}_1,\boldsymbol{k}_4)+D(\boldsymbol{k}_1,\boldsymbol{k}_3)\right]$$

Zero total momentum

Kinetic Part of Hamiltonian

Antisymmetrization

- $D(\boldsymbol{k}_i,\boldsymbol{k}_j)$: wave function when ↑↓ particles get close

$$\psi_{\uparrow\uparrow\downarrow\downarrow}(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3,\boldsymbol{r}_4)\underset{r_{13}\to 0}{=}\left(\frac{1}{r_{13}}-\frac{1}{a}\right)\frac{\mu_{\uparrow\downarrow}}{2\pi\hbar^2}\mathcal{A}(\boldsymbol{r}_2-\boldsymbol{R}_{13},\boldsymbol{r}_4-\boldsymbol{R}_{13})+O(r_{13})$$

- c.f. 3+1 problem: 3 terms Castin, *et al*, PRL (2012)

$$\tilde{\psi}_{\uparrow\uparrow\uparrow\downarrow}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4)=\frac{\delta\left(\sum_{n=1}^{4}\boldsymbol{k}_n\right)}{\sum_{n=1}^{4}\frac{\hbar^2 k_n^2}{2m_n}}\left[D(\boldsymbol{k}_2,\boldsymbol{k}_3)-D(\boldsymbol{k}_1,\boldsymbol{k}_3)+D(\boldsymbol{k}_1,\boldsymbol{k}_2)\right]$$

SE, Y. Castin, arXiv:1507.05580(2015)

Analytical solution of 2+2 fermions at 1/a=0

- Obtain linear 6-dimensional integral equation after applying Bethe-Peierls boundary condition with 1/a=0

$$0 = \frac{\mu_{\uparrow\downarrow}^{3/2}}{2\pi\hbar^2}\left[\frac{(\boldsymbol{k}_2+\boldsymbol{k}_4)^2}{m_\uparrow+m_\downarrow}+\frac{k_2^2}{m_\uparrow}+\frac{k_4^2}{m_\downarrow}\right]^{1/2} D(\boldsymbol{k}_2,\boldsymbol{k}_4) + \int \frac{d^3k_1 d^3k_3}{(2\pi)^3} \frac{\delta\left(\sum_{n=1}^{4}\boldsymbol{k}_n\right)}{\sum_{n=1}^{4}\frac{\hbar^2 k_n^2}{2m_n}} [D(\boldsymbol{k}_2,\boldsymbol{k}_3) + D(\boldsymbol{k}_1,\boldsymbol{k}_4) - D(\boldsymbol{k}_1,\boldsymbol{k}_3)]$$

- Rotational and Scaling symmetries

$$D(\boldsymbol{k}_2,\boldsymbol{k}_4) = \sum_{m_z=-\ell}^{\ell} [Y_\ell^{m_z}(\boldsymbol{e}_2\cdot\boldsymbol{e}_z, \boldsymbol{e}_{4\perp 2}\cdot\boldsymbol{e}_z, \boldsymbol{e}_{24}\cdot\boldsymbol{e}_z)]^* (k_2^2+k_4^2)^{-(s+7/2)/2}(x)^{s+3/2} e^{im_z\theta_{24}/2}\Phi_{m_z}^{(\ell)}(x,u_{24})$$

$$x \equiv \ln\frac{k_4}{k_2} \qquad u_{24} \equiv \cos\theta_{24}$$

⇒ Reduce to 2-dimensional (Next slide)

- Look for s which solves the integral equation

$s \in \mathbb{R}$:No Efimov effect.

$s \in i\mathbb{R}$:Efimov effect with scale scale factor $e^{\pi/|s|}$

SE, Y. Castin, arXiv:1507.05580(2015)

Integral equation

$$0 = \left[\frac{\alpha}{(1+\alpha)^2}\left(1+\frac{u}{\operatorname{ch} x}\right) + \frac{e^{-x}+\alpha e^{x}}{2(\alpha+1)\operatorname{ch} x}\right]^{1/2} \Phi^{(\ell)}_{m_z}(x,u) + \int_{\mathbb{R}} dx' \int_{-1}^{1} du' \sum_{m_z'=-\ell}^{\ell} K^{(\ell)}_{m_z,m_z'}(x,u;x',u';\alpha)\Phi^{(\ell)}_{m_z'}(x',u')$$

$$K^{(\ell)}_{m_z,m_z'}(x,u;x',u';\alpha) =$$

$$\left(\frac{e^{x}\operatorname{ch} x'}{e^{x'}\operatorname{ch} x}\right)^{s/2}\left(\frac{e^{x+x'}}{4\operatorname{ch} x\operatorname{ch} x'}\right)^{1/4}\int_0^{2\pi}\frac{d\phi}{(2\pi)^2}\frac{e^{-im_z\theta/2}\langle \ell, m_z|e^{i\phi L_x/\hbar}|\ell, m_z'\rangle e^{im_z'\theta'/2}}{\operatorname{ch}(x-x') + \frac{1}{1+\alpha}[(u+e^{-x})(u'+e^{-x'}) + vv'\cos\phi]}$$

$$+\left(\frac{e^{-x}\operatorname{ch} x'}{e^{-x'}\operatorname{ch} x}\right)^{s/2}\left(\frac{e^{-x-x'}}{4\operatorname{ch} x\operatorname{ch} x'}\right)^{1/4}\int_0^{2\pi}\frac{d\phi}{(2\pi)^2}\frac{e^{im_z\theta/2}\langle \ell, m_z|e^{i\phi L_x/\hbar}|\ell, m_z'\rangle e^{-im_z'\theta'/2}}{\operatorname{ch}(x-x') + \frac{\alpha}{1+\alpha}[(u+e^{x})(u'+e^{x'}) + vv'\cos\phi]}$$

$$-\frac{(-1)^{\ell}}{4\pi[(u+\operatorname{ch} x)(u'+\operatorname{ch} x')\operatorname{ch} x\operatorname{ch} x']^{1/4}}\left(\frac{(u'+\operatorname{ch} x')\operatorname{ch} x'}{(u+\operatorname{ch} x)\operatorname{ch} x}\right)^{s/2}\frac{e^{im_z\gamma}\langle \ell, m_z|\ell, m_x=0\rangle\langle \ell, m_x=0|\ell, m_z'\rangle e^{-im_z'\gamma'}}{\left(\frac{e^{-x'}+\alpha e^{x'}}{1+\alpha}\right)(u+\operatorname{ch} x) + \left(\frac{e^{-x}+\alpha e^{x}}{1+\alpha}\right)(u'+\operatorname{ch} x')}$$

Integral equation

$$0=\left[\frac{\alpha}{(1+\alpha)^2}\left(1+\frac{u}{\operatorname{ch}x}\right)+\frac{e^{-x}+\alpha e^{x}}{2(\alpha+1)\operatorname{ch}x}\right]^{1/2}\Phi^{(\ell)}_{m_z}(x,u)+\int_{\mathbb{R}}dx'\int_{-1}^{1}du'\sum_{m_z'=-\ell}^{\ell}K^{(\ell)}_{m_z,m_z'}(x,u;x',u';\alpha)\Phi^{(\ell)}_{m_z'}(x',u')$$

$$\hat{M}^{(\ell)}(s)\vec{\Phi}=0$$

- Matrix $\hat{M}^{(\ell)}(s)$ has zero eigenvalue $\Leftrightarrow$ Solution s

$$\det M^{(\ell)}(s)=0$$

- 3-body: $\Lambda_\ell(s)=0$ determines the value of s

  $\Rightarrow \det \hat{M}^{(\ell)}(s)$ and $\Lambda_\ell(s)$ has similar roles

- $\Delta B_{2,1}$ can be obtained as *C. Gao, SE, Y. Castin, EPL **109**, 16003 (2015)*

$$\Delta B_{2,1}=\sum_{\ell\in\mathbb{N}}\left(\ell+\frac{1}{2}\right)\int_0^{+\infty}\frac{dS}{\pi}S\frac{d}{dS}[\ln\Lambda_l(iS)]$$

- **Assumpsion (not proved):** same formula for 4-body

$$\Delta B^{\mathrm{conj}}_{n,m}=\sum_{\ell\in\mathbb{N}}\left(\ell+\frac{1}{2}\right)\int_0^{+\infty}\frac{dS}{\pi}S\frac{d}{dS}[\ln\det M^{(\ell)}(iS)]$$

Conclusion

- Rich 3-body physics in 2-component Fermi system with mass imbalance
  - Efimov trimers, universal (Kartavtsev-Malykh) trimers, crossover trimers

SE, P. Naidon, M. Ueda, Few-body Systems **51**, 207 (2011)
SE, P. Naidon, M. Ueda, PRA **86**, 062703 (2012)

- 3-body and 4-body physics can help understanding many-body physics
  - Stable many-body quantum phase with trimers

SE, P. Naidon, A. M. Garcia-Garcia arXiv:1507.06309 (2015)
P. Naidon SE, A. M. Garcia-Garcia, arXiv:1507.06373 (2015)

  - 3rd (and possibly 4th) virial coefficient of a unitary Fermi gas

*C. Gao, SE, Y. Castin, EPL **109**, 16003 (2015)*
SE, Y. Castin, arXiv:1507.05580(2015)