Topological states of matter in classical and quantum magnets

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Magnetostatic spin-wave analog of integer quantum Hall states

Works done in collaboration with
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Reference

R. Shindou, et. al., Phys. Rev. B **87**, 174427 (2013)

R. Shindou, et. al., Phys. Rev. B **87**, 174402 (2013)

R. Shindou and J-i. Ohe, arXiv:1308.0199

Magnetostatic spin-wave analog of integer quantum Hall states

- **Relativistic spin-orbit interaction**

$$H_{\rm so} = \frac{\hbar}{2m^2c^2}\boldsymbol{s}\cdot\left(\nabla V(\boldsymbol{r})\times\boldsymbol{p}\right)$$

- ☐ AHE in ferromagnetic metal s
- ☐ Topological band insulators in heavy elements materials

Locking the relative rotational angle
b.t.w. the spin space and orbital space

➔ wave-functions acquire complex-valued character . .

➔ Quantum anomalous Hall effect in ferromagnetic metals, or topological surface state in topological band insulator

- **magnetic dipole-dipole interaction**

$$H_{\rm dipole} = \frac{\mu_0}{4\pi|\boldsymbol{r}-\boldsymbol{r}'|^3}\left\{3\frac{\boldsymbol{S}_{\boldsymbol{r}}\cdot(\boldsymbol{r}-\boldsymbol{r}')\ \boldsymbol{S}_{\boldsymbol{r}'}\cdot(\boldsymbol{r}-\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^2} - \boldsymbol{S}_{\boldsymbol{r}}\cdot\boldsymbol{S}_{\boldsymbol{r}'}\right\}.$$

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Content of the 1st part of my talk

- ☐ Introduction on `magnetostatic spin wave' research
- ☐ Magnetostatic spin-wave analog of integer quantum Hall state
- ☐ Chern integer and chiral edge modes for spin-wave physics
- ☐ chiral spin-wave band in ferromagnetic thin film models
- ☐ Justification via micromagnetic simulations
- ☐ Summary

☐ Magnetostatic spin wave

Spin wave : collective propagation of magnetic moments in magnets

Magnetostatic spin wave : driven by **magnetic dipole-dipole interaction**

$$\partial_t \boldsymbol{M} = \gamma \boldsymbol{H}_{\text{eff}} \times \boldsymbol{M}$$

Landau-Lifshitz equation

$$\gamma \boldsymbol{H}_{\text{eff}} = -J_{\text{exc}} a^2 \nabla^2 \boldsymbol{M} - \boldsymbol{H}_d$$

Exchange-interaction field dipolar field

Maxwell equation (magnetostatic approximation)

$$\nabla \times \boldsymbol{H}_d = 0 \frac{4\pi}{c} \boldsymbol{j} + \frac{1}{c} \partial_t \boldsymbol{D}$$

$$\nabla \cdot (\boldsymbol{H}_d + 4\pi \boldsymbol{M}) = 0$$

The dipolar field is given by magnetization itself ➔ a closed EOM for M.

□ Magnetostatic spin wave

Spin wave : collective propagation of magnetic moments in magnets

Magnetostatic spin wave : driven by magnetic dipole-dipole interaction

$$\partial_t \boldsymbol{M} = \gamma \boldsymbol{H}_{\rm eff} \times \boldsymbol{M}$$ Landau-Lifshitz equation

$$\gamma \boldsymbol{H}_{\rm eff} = -J_{\rm exc} a^2 \nabla^2 \boldsymbol{M} - \boldsymbol{H}_d$$

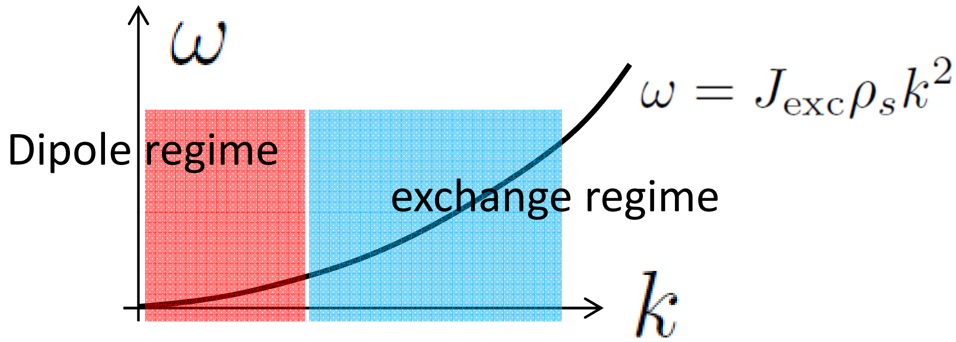
$(1/\lambda)^2$

Exchange-interaction field   dipolar field

□ Wavelength of spin waves (λ) >> exchange-interaction length

Dipolar field >> exchange-interaction field

➔ Spin wave is mainly driven by magnetic dipole-dipole intertaction.



Dipole regime

um〜sub-um

GHz〜subGHz

c.f. typical Exch.-interaction length
= several nm (iron) 〜 10nm (YIG)

☐ What is `magnetostatic (MS) spin wave' research about ?

: explore ability of spin waves to carry and/or process information

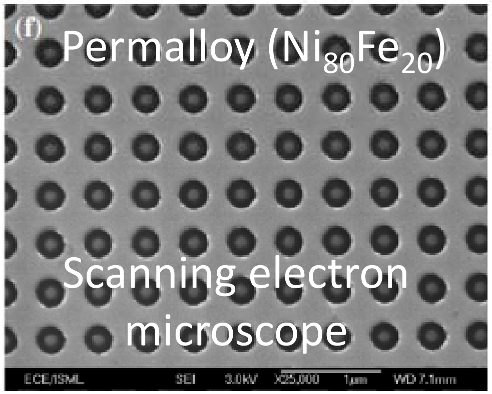
◆ An advantage over photonics, electronics, and . . .

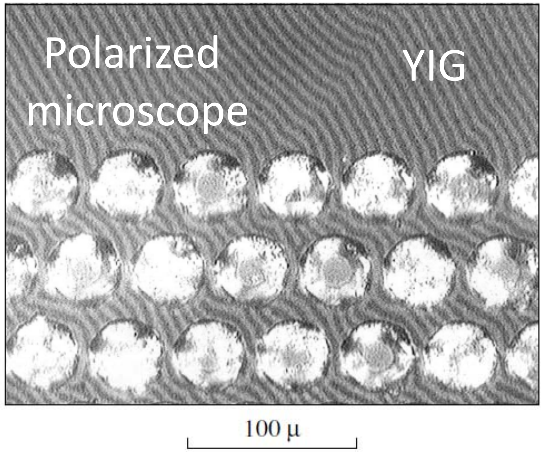
: spin-wave velocity is typically several orders slower than those of light and electron waves

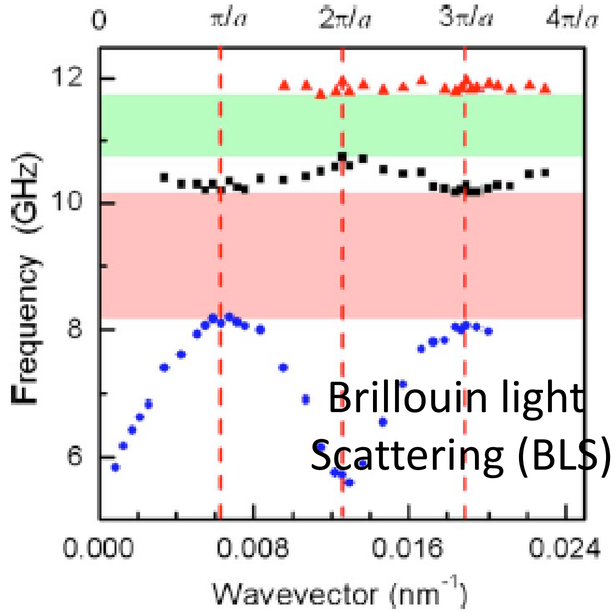
➔ Much Better prospect for ***`miniaturization' of devices***

10^{-1}ns ➔ 1cm (photonics)
10^{-1}ns ➔ 1μm～10μm (electronics)
10^{-1}ns ➔ 10^{-1}μm (magetostatic SW)

☐ periodically modulated magnetic materials







◆ Lithography technique in semiconductors engineering enables us to makes **a magnetic superlattice** in **ferromagnetic thin film.**

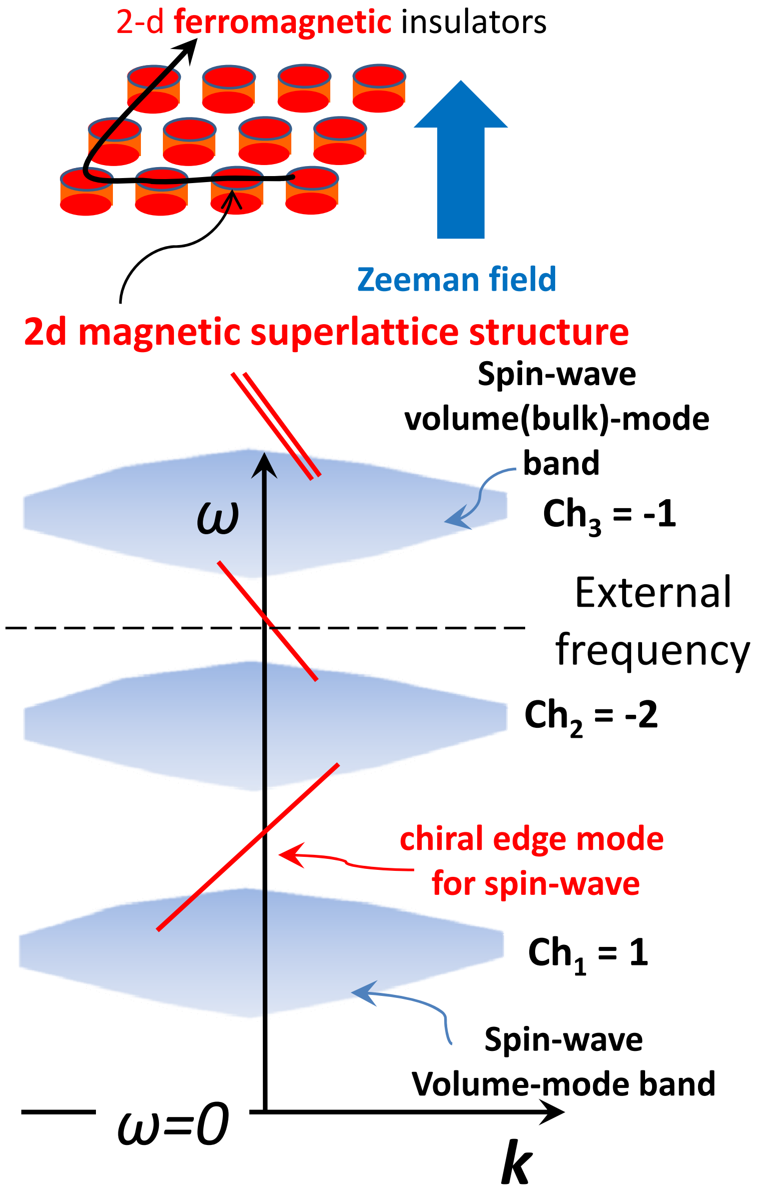
➔ **`multiple-band' character.**

Gulyaev et.al. JETP letters (2003)

Adeyeye et.al. J. Phys. D (2008)

Wang et.al. App. Phys. Letters (2009)

☐ Our Proposal = MS spin-wave analog of integer quantum Hall state



MS spin-wave analog of Integer quantum Hall state
normally magnetized `2-d' magnetic superlattice structure
magnetostatic spin-wave (boson)
multiple band character
Bloch w.f. for each band $\vert\Psi_{j,\boldsymbol{k}}\rangle$
1st **Chern integer** for each band $C_j \equiv \frac{i}{2\pi}\int_{\rm BZ} d\boldsymbol{k} \left\{ \langle \partial_{k_x}\Psi_{j,\boldsymbol{k}}\vert\boldsymbol{\sigma}_3\vert\partial_{k_y}\Psi_{j,\boldsymbol{k}}\rangle - {\rm c.c.} \right\}$
Number of chiral edge modes within a gap := sum of the Chern integers over the bands below the gap $\#_{(m,m+1)} \equiv \sum_{j=1}^{m} C_j$
chiral edge modes for spin-wave free from **static backward scatterings**

□ magnetic superlattice structure

◆ Landau-Lifshitz equation

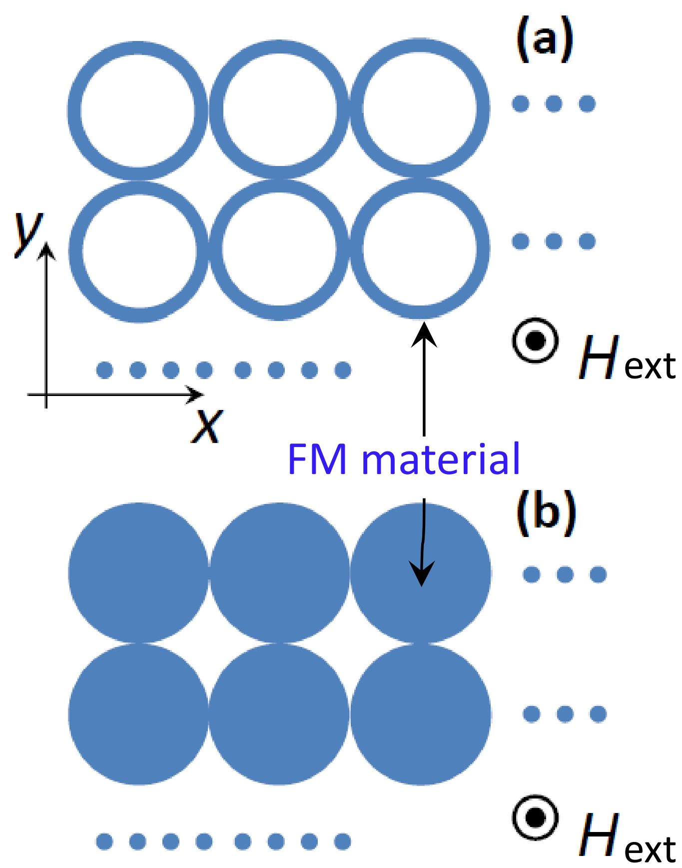
$$|\boldsymbol{M}_{\boldsymbol{r}}| = M_s$$

$$\partial_t \boldsymbol{M} = \gamma \boldsymbol{H}_{\text{eff}} \times \boldsymbol{M}$$

$$\gamma \boldsymbol{H}_{\text{eff}} = -\boldsymbol{H}_{\text{ext}} - \boldsymbol{H}_d$$

◆ Maxwell equation (magnetostatic approx.)

$$\boldsymbol{H}_d(\boldsymbol{r}) = -\frac{1}{4\pi}\sum_{\boldsymbol{r}'}\left(\frac{\boldsymbol{M}_{\boldsymbol{r}'}}{|\boldsymbol{r}-\boldsymbol{r}'|^3} - 3\frac{(\boldsymbol{r}-\boldsymbol{r}')\,(\boldsymbol{r}-\boldsymbol{r}')\cdot\boldsymbol{M}_{\boldsymbol{r}'}}{|\boldsymbol{r}-\boldsymbol{r}'|^5}\right)$$



Minimize the magnetostatic energy E_{MS}

➔ classical spin configuration $\boldsymbol{M}_0$

$$E_{\text{MS}} = -\frac{1}{2}\int d\boldsymbol{r}\,\boldsymbol{H}_d(\boldsymbol{r})\cdot\boldsymbol{M}(\boldsymbol{r}) - \boldsymbol{H}_{\text{ext}}\cdot\boldsymbol{M}$$

Transverse fluctuation around the classical spin configuration: $\boldsymbol{m}_\perp$

$$\boldsymbol{M}(\boldsymbol{r}) = \boldsymbol{M}_0(\boldsymbol{r}) + \boldsymbol{m}_\perp(\boldsymbol{r})$$

2 real-valued fields

$$m_\pm(\boldsymbol{r}) \equiv \frac{1}{\sqrt{2M_s}}\left(m_{\perp,x}(\boldsymbol{r}) \mp i m_{\perp,y}(\boldsymbol{r})\right)$$

: Holstein-Primakoff (HP) boson field

$$i\partial_t \begin{pmatrix} m_-(\boldsymbol{r}) \\ m_+(\boldsymbol{r}) \end{pmatrix} = \sum_{\boldsymbol{r}'\neq\boldsymbol{r}} (\boldsymbol{\sigma}_3)\,(\boldsymbol{H}_{2\times 2})_{\boldsymbol{r},\boldsymbol{r}'} \begin{pmatrix} m_-(\boldsymbol{r}') \\ m_+(\boldsymbol{r}') \end{pmatrix}$$

Hermite matrix

☐ magnetic superlattice structure

HP boson field

◆ Spin-wave Hamiltonian (quadratic boson Hamiltonian)

$$\mathcal{H}_{\rm sw} = \frac{1}{2}\sum_{\boldsymbol{r}\neq\boldsymbol{r}'} \left(a^{\dagger}(\boldsymbol{r}) \;\; a(\boldsymbol{r}) \right) (\boldsymbol{H}_{2\times 2})_{\boldsymbol{r},\boldsymbol{r}'} \begin{pmatrix} a(\boldsymbol{r}') \\ a^{\dagger}(\boldsymbol{r}') \end{pmatrix}$$

Because . . . .

$$i\partial_t \begin{pmatrix} a(\boldsymbol{r}) \\ a^{\dagger}(\boldsymbol{r}) \end{pmatrix} = \sum_{\boldsymbol{r}'\neq\boldsymbol{r}} (\boldsymbol{\sigma}_3)\,(\boldsymbol{H}_{2\times 2})_{\boldsymbol{r},\boldsymbol{r}'} \begin{pmatrix} a(\boldsymbol{r}') \\ a^{\dagger}(\boldsymbol{r}') \end{pmatrix}^{\boldsymbol{r},\boldsymbol{r}'} \begin{pmatrix} a(\boldsymbol{r}') \\ a^{\dagger}(\boldsymbol{r}') \end{pmatrix} \Bigg]$$

$$= \sum_{\boldsymbol{r}'\neq\boldsymbol{r}} \begin{pmatrix} [a(\boldsymbol{r}), a^{\dagger}(\boldsymbol{r})] & 0 \\ 0 & [a^{\dagger}(\boldsymbol{r}), a(\boldsymbol{r})] \end{pmatrix} (\boldsymbol{H}_{2\times 2})_{\boldsymbol{r},\boldsymbol{r}'} \begin{pmatrix} a(\boldsymbol{r}') \\ a^{\dagger}(\boldsymbol{r}') \end{pmatrix}$$

-1

◆ $\boldsymbol{H}_{2\times 2}$ has a particle-particle pairing term (# of the particle is non-conserved)

🡐 Due to the spin-orbit locking nature of magnetic dipole-dipole interaction, there is no U(1) rotation symmetry in the spin-space

□ Topological Chern number from quadratic boson Hamiltonian

- BdG (Bogoliubov-de-Gennes)-type Hamiltonian

$$\mathcal{H}_{\rm sw} = \frac{1}{2}\sum_{\boldsymbol{r}\neq\boldsymbol{r}'} \left(\boldsymbol{a}^\dagger(\boldsymbol{k}) \;\; \boldsymbol{a}(\boldsymbol{k}) \right) \boldsymbol{H}_{\boldsymbol{k}} \begin{pmatrix} \boldsymbol{a}(\boldsymbol{k}) \\ \boldsymbol{a}^\dagger(\boldsymbol{k}) \end{pmatrix}$$

$2N \times 2N$ Hermite matrix

where

$$\boldsymbol{a}^\dagger(\boldsymbol{k}) \equiv \left(a_1^\dagger(\boldsymbol{k}) \;\; \cdots \;\; a_N^\dagger(\boldsymbol{k}) \right)$$

k : crystal momentum

N: # (degree of freedom within a unit cell of the magnetic superlattice)

- A bosonic BdG Hamiltonian is diagonalized in terms of para-unitary transformation $\boldsymbol{T}_k$

$$\boldsymbol{T}_{\boldsymbol{k}}^\dagger \boldsymbol{H}_{\boldsymbol{k}} \boldsymbol{T}_{\boldsymbol{k}} = \begin{bmatrix} \boldsymbol{E}_{\boldsymbol{k}} & \\ & \boldsymbol{E}_{-\boldsymbol{k}} \end{bmatrix}$$

$$a(\boldsymbol{r})a^\dagger(\boldsymbol{r}') - a^\dagger(\boldsymbol{r}')a(\boldsymbol{r}) = \delta_{\boldsymbol{r},\boldsymbol{r}'}$$

Commutation relation of boson field

$$\boldsymbol{T}_{\boldsymbol{k}}^\dagger \boldsymbol{\sigma}_3 \boldsymbol{T}_{\boldsymbol{k}} = \boldsymbol{\sigma}_3, \;\; \boldsymbol{T}_{\boldsymbol{k}} \boldsymbol{\sigma}_3 \boldsymbol{T}_{\boldsymbol{k}}^\dagger = \boldsymbol{\sigma}_3$$

$$\left[a_i(\boldsymbol{k}), \;\; a_j^\dagger(\boldsymbol{k}) \right] = \delta_{ij},$$

$$\left[a_i^\dagger(\boldsymbol{k}), \;\; a_j(\boldsymbol{k}) \right] = -\delta_{ij}$$

Orthogonality and Completeness of (new) bosonic fields

➔ $$\boldsymbol{P}_j \equiv \boldsymbol{T}_{\boldsymbol{k}} \boldsymbol{\Gamma}_j \boldsymbol{\sigma}_3 \boldsymbol{T}_{\boldsymbol{k}}^\dagger \boldsymbol{\sigma}_3$$

Projection operator filtering out the *j*-th bosonic band @k

Because this satisfies $\sum_j \boldsymbol{P}_j = \boldsymbol{1}$ and $\boldsymbol{P}_j \boldsymbol{P}_m = \delta_{jm} \boldsymbol{P}_j$

□ Topological Chern number from quadratic boson Hamiltonian

Projection operator filtering out the *j*-th bosonic band @k

$$\Rightarrow \boldsymbol{P}_j \equiv \boldsymbol{T}_{\boldsymbol{k}} \boldsymbol{\Gamma}_j \boldsymbol{\sigma}_3 \boldsymbol{T}^{\dagger}_{\boldsymbol{k}} \boldsymbol{\sigma}_3$$

◆ (First) Chern number for the *j*-th bosonic band

$$\mathrm{Ch}_1 \equiv \frac{i\epsilon_{\mu\nu}}{2\pi}\int_{\mathrm{BZ}} d\boldsymbol{k}\, \mathrm{Tr}\big[(\boldsymbol{1}-\boldsymbol{P}_j)(\partial_{k_\mu}\boldsymbol{P}_j)(\partial_{k_\nu}\boldsymbol{P}_j)\big],$$

← Avron et.al. PRL (83)

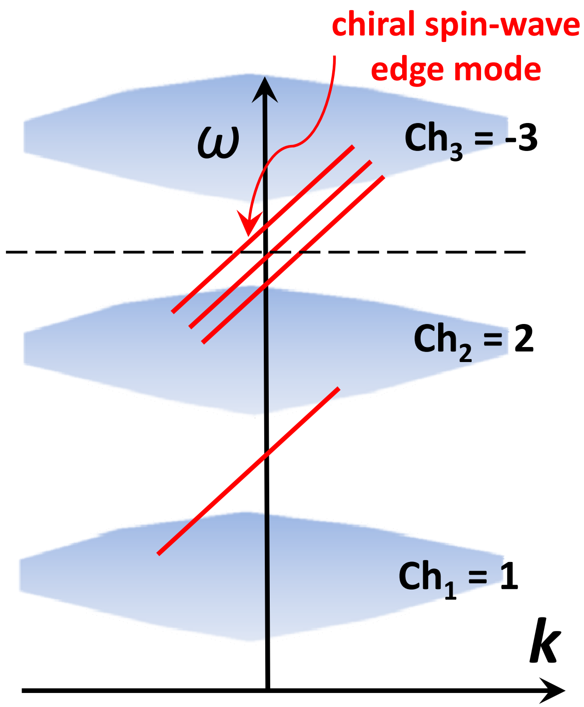
$$= \frac{i\epsilon_{\mu\nu}}{2\pi}\int_{\mathrm{BZ}} d\boldsymbol{k}\, \mathrm{Tr}\big[\boldsymbol{\Gamma}_j \boldsymbol{\sigma}_3 (\partial_\mu \boldsymbol{T}^{\dagger}_{\boldsymbol{k}}) \boldsymbol{\sigma}_3 (\partial_\nu \boldsymbol{T}_{\boldsymbol{k}})\big],$$

$$= \frac{i\epsilon_{\mu\nu}}{2\pi}\int_{\mathrm{BZ}} d\boldsymbol{k}\, \partial_\mu A_{j,\nu}.$$

→ TKNN Integer

Thouless et.al. PRL (82)

Kohmoto, Annal of Physics (85)



$$\#_{(m,m+1)} \equiv \sum_{j=1}^{m} C_j$$

Gauge field (connection)

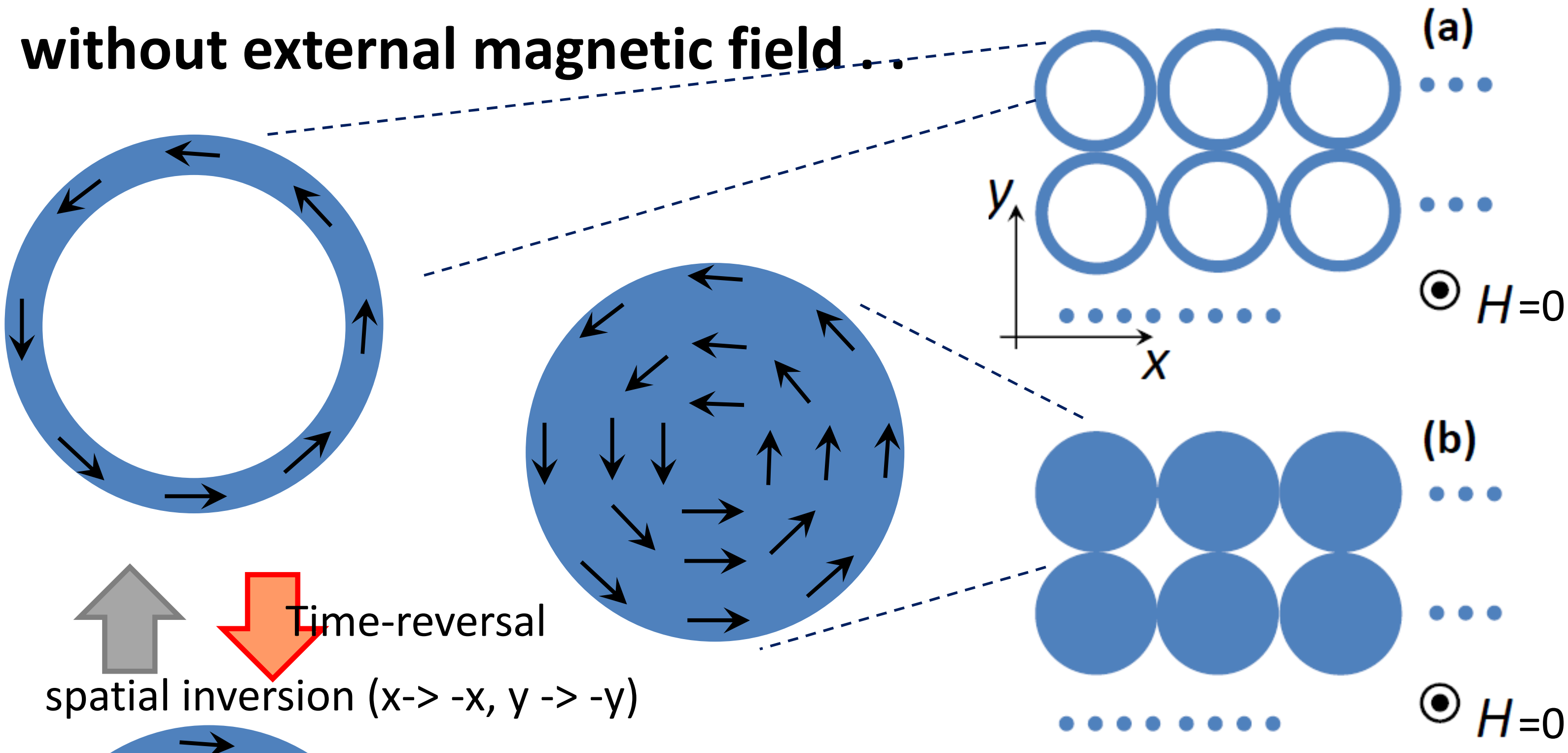
$$A_{j,\nu} \equiv i\mathrm{Tr}[\boldsymbol{\Gamma}_j \boldsymbol{\sigma}_3 \boldsymbol{T}^{\dagger}_{\boldsymbol{k}} \boldsymbol{\sigma}_3 (\partial_{k_\nu} \boldsymbol{T}_{\boldsymbol{k}})]$$

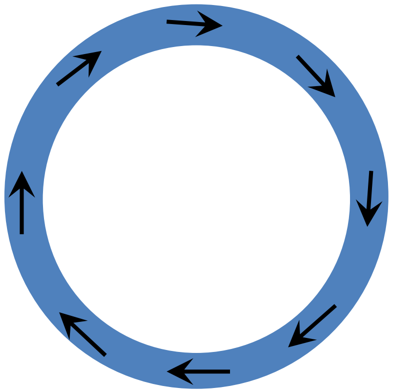
Bulk-edge correspondence

Halperin, PRB (82), . . .

Hatsugai, PRL (92), . . .

☐ without external magnetic field . .



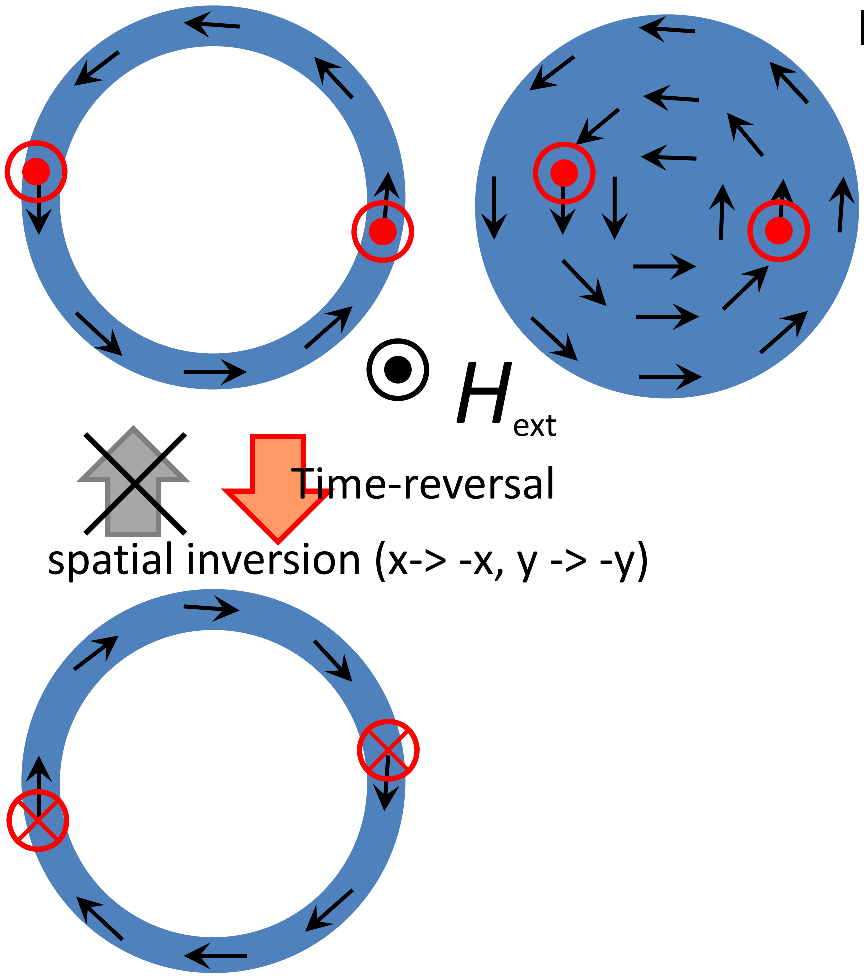


- ☐ Vortex configuration minimizes MS energy.
  - ◆ Moment lies within the x-y plane:
  - ◆ `stray-field-free' configuration:

    Moment is tangential along the boundary, while being divergence-free within the body ➔ no magnetic charge

  - ◆ Time-reversal symmetry + spatial inversion is preserved ➔ Berry curvature = 0.

☐ with external magnetic field along the out-of plane



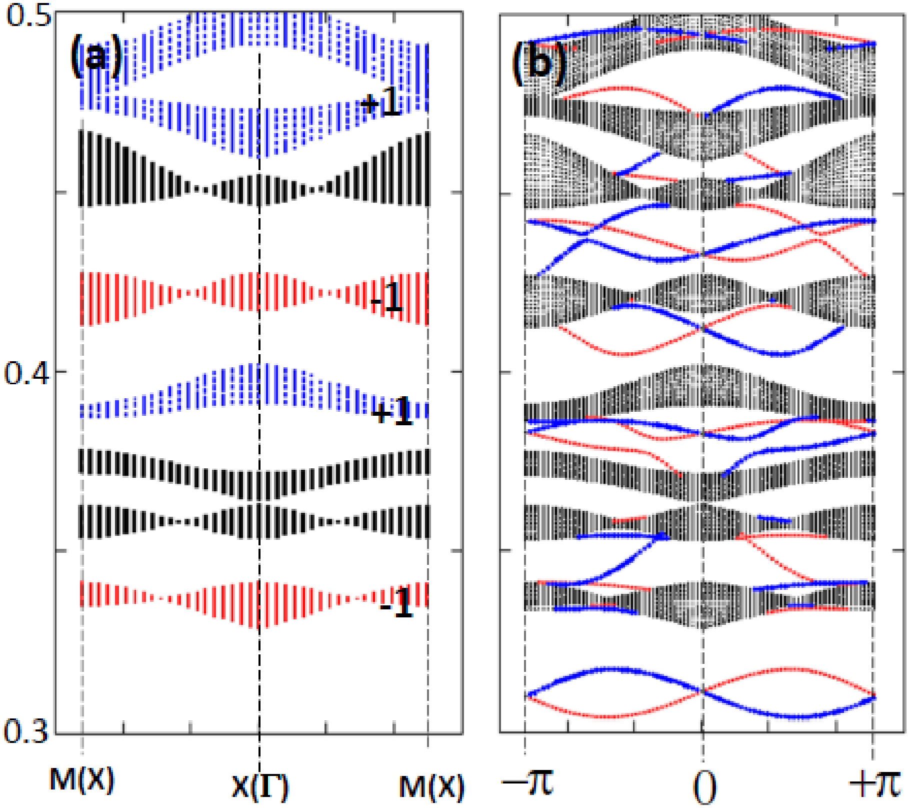
☐ Moment acquires **a finite M_z**:

- Time-reversal symmetry + spatial inversion is broken.
- mirror symmetries (e.g. (x,y) => (-x,y)) are all broken.

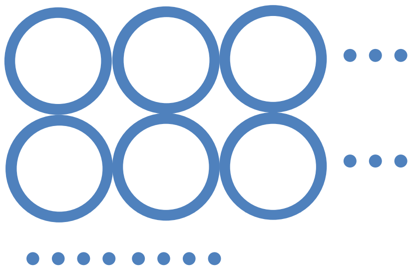
➔ Chern integer can be non-zero.

☐ Spin wave bands in the lowest frequency regime

Lowest 8 bands

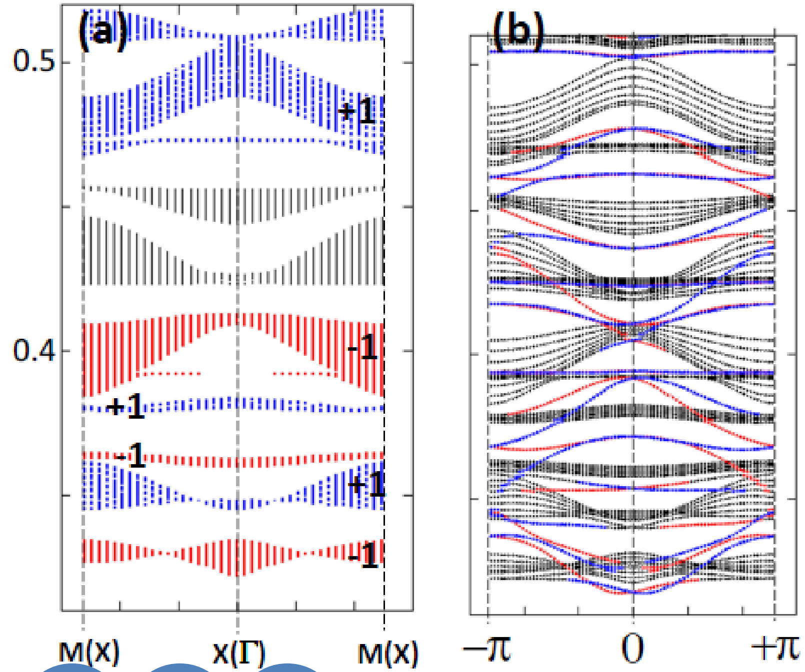


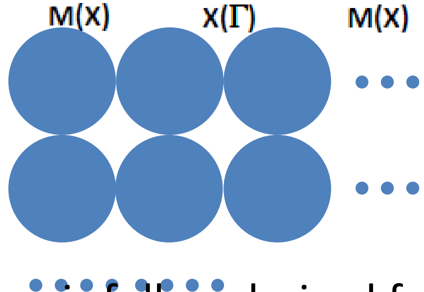
$H_{ext} = 0.94*H_s$



- The lowest bands have non-zero Chern integer **only near saturation fields.**

  → Why ?





$H_{ext} = 0.94*H_{s,1}$

H_s: Saturation field (classical spin configuration is fully polarized for $H_{ext}>H_s$)

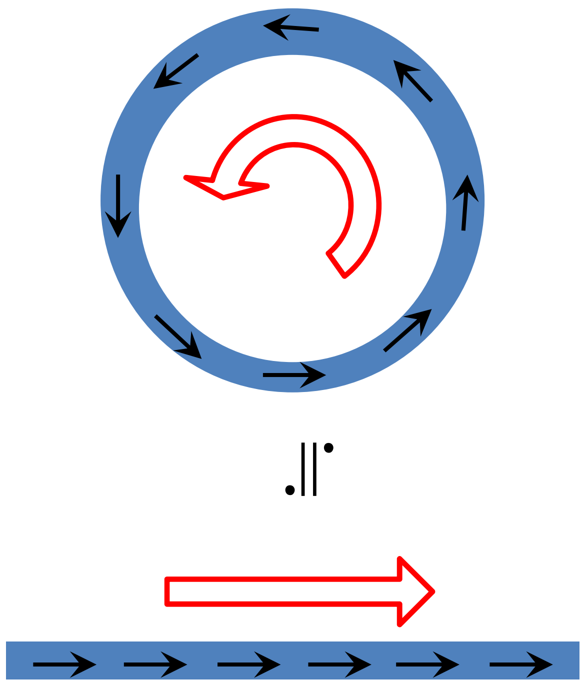
☐ spin excitations *within* a single ring . . .

➔ ``Atomic orbitals'' for ``tight-binding models''

◆ **at zero field . . .**

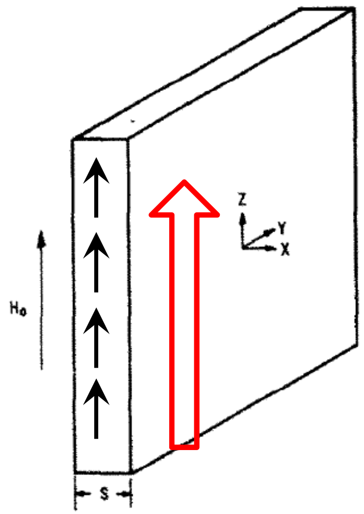
Moment is almost tangential along the ring

➔ Spin excitations along the ring becomes like the so-called **backward** volume mode in ferromagnetic thin film or thin wire.



ferromagnetic thin film or wire

Damon-Eshbach (1961), ...
Arias-Mills (2001), . . .



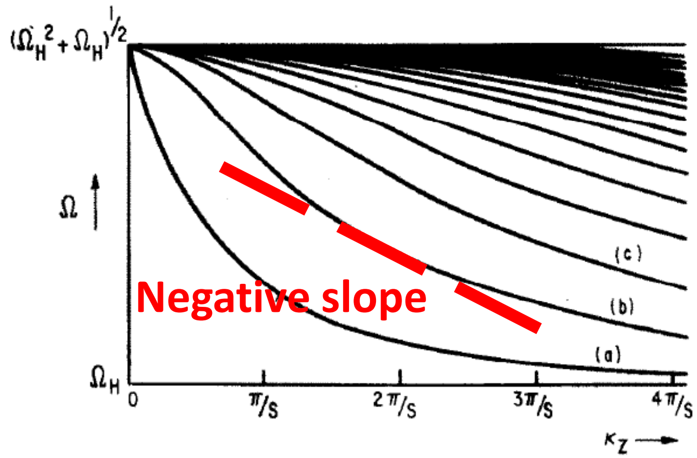
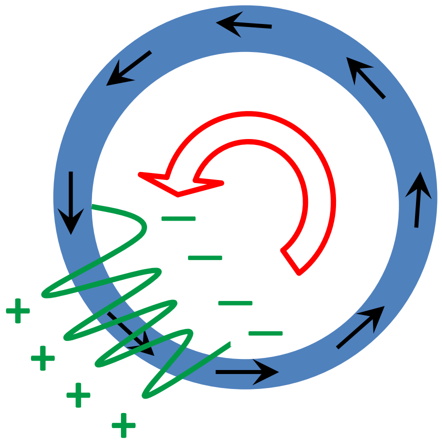


FIG. 4. Magnetostatic mode spectrum of a ferromagnetic slab magnetized in its plane for $k_y = 0$.

From Damon-Eshbach JPCS **19**, 308 (1961)

◆ Group velocity ∂ω/∂k is **antiparallel** to the vector k ➔ ``**backward**'' volume mode

☐ spin excitations within a single ring



➔ ``Atomic orbitals'' for ``tight-binding models''

◆ **near zero field . . .**

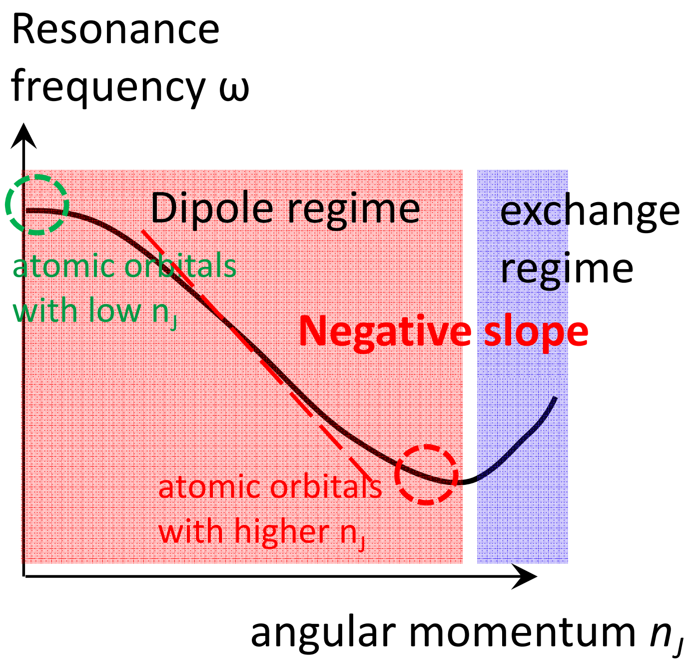
◆ Atomic orbitals with **higher** angular momenta (n_J) come in the low-frequency side of those with lower n_J (as far as the dipole regime is concerned).

◆ Atomic orbitals with higher n_J have **many nodes** along the rings. . . .

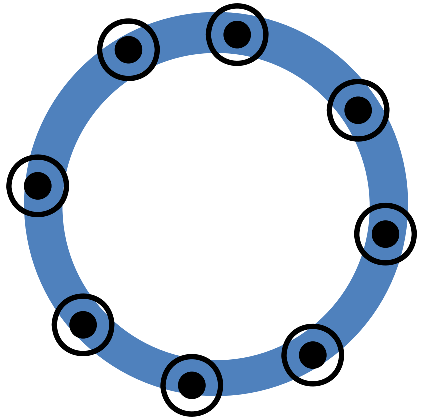
➔ The inter-ring transfer integrals between orbitals with higher n_J become ***very small***, due to the cancellation b.t.w. the opposite phases.

◆ bulk-type SW bands in the low frequency regime becomes **less dispersive and featureless .**

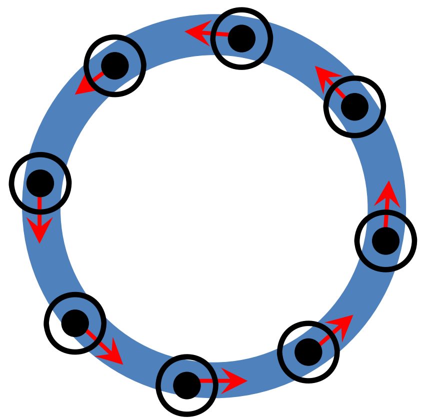
➔ Chern integers for them = 0



☐ spin excitations within a single ring



$H_{ext} > H_s$



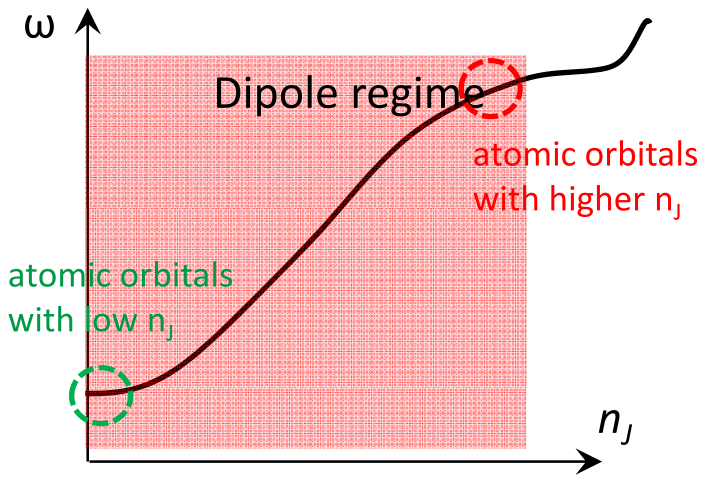
$H_{ext} < H_s$

➔ ``Atomic orbitals'' for ``tight-binding models''

- **Near the saturation field (H_s) . . .**

Moments are fully polarized above H_s, while start to acquire a finite in-plane component below H_s

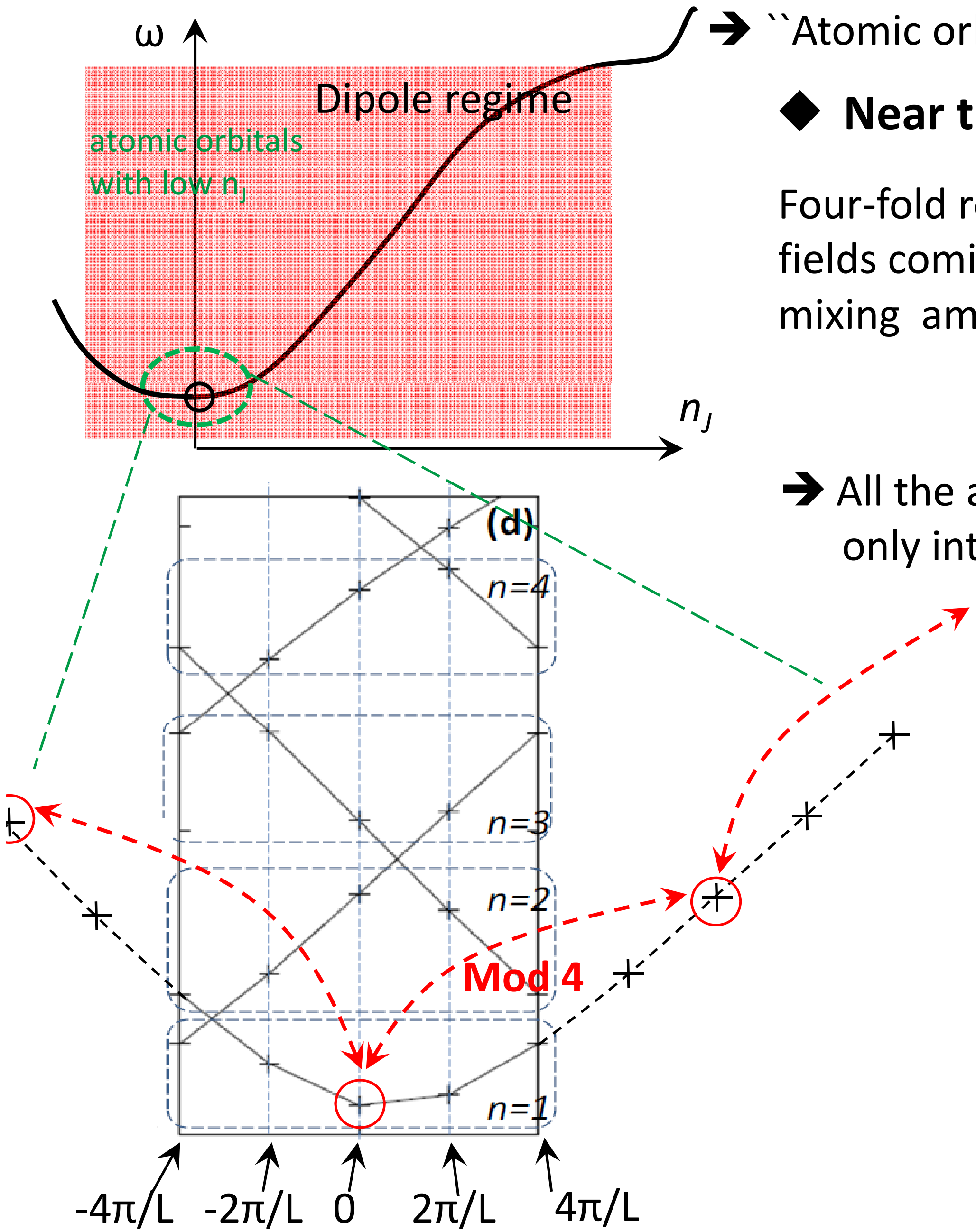
➔ The atomic orbital with **zero** angular momentum (n_J =0) becomes gapless at H_{ext} =H_s



- Bulk-type SW bands in the low frequency regime becomes **more dispersive .**

➔ chance to have non-zero Chern integers.

☐ spin excitations within a single ring



➔ ``Atomic orbitals'' for ``tight-binding models''

◆ Near the saturation field (H_s) . . .

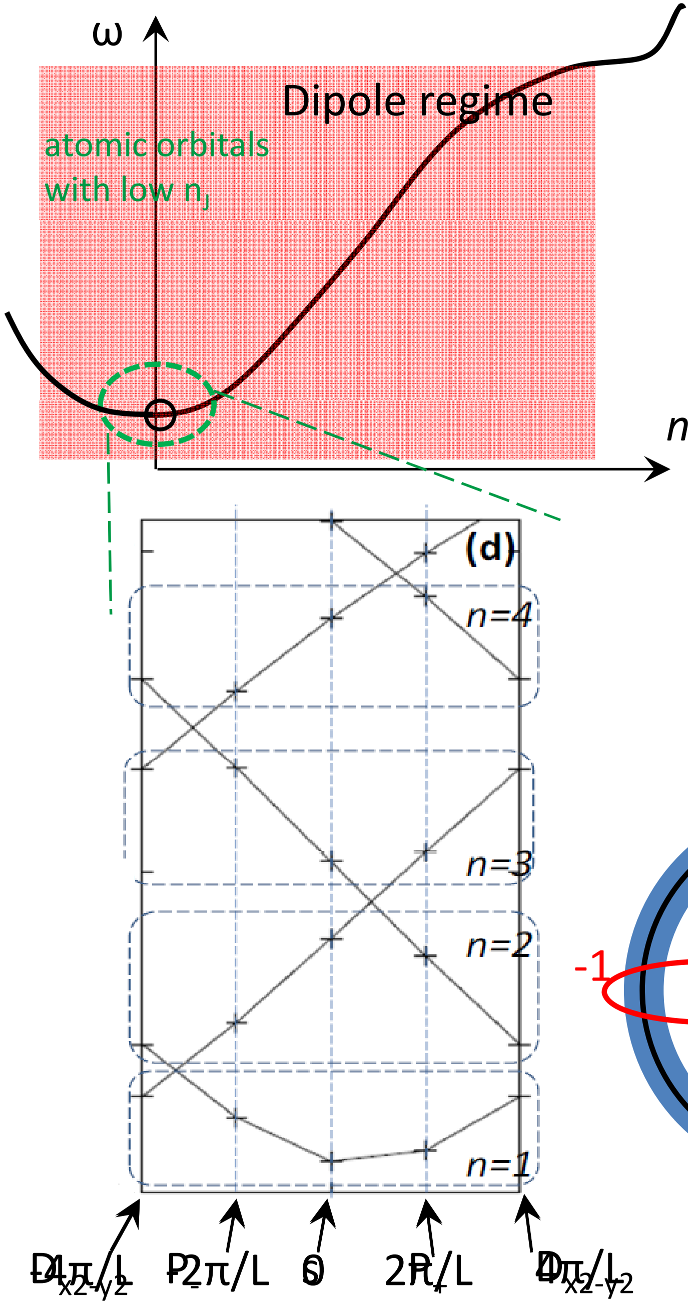
Four-fold rotational anisotropy (e.g. depolarization fields coming from neighboring rings) leads to the mixing among n_J, $n_J \pm 2\pi/L*$**4**, $n_J \pm 2\pi/L*$**8**, ...

(L: length of the ring)

➔ All the atomic orbitals within a ring are classified only into four angular momenta;

$n_J = 0$, $\pm 2\pi/L$, $4\pi/L$.

☐ spin excitations within a single ring



➔ ``Atomic orbitals'' for ``tight-binding models''

◆ Near the saturation field (H_s) . . .

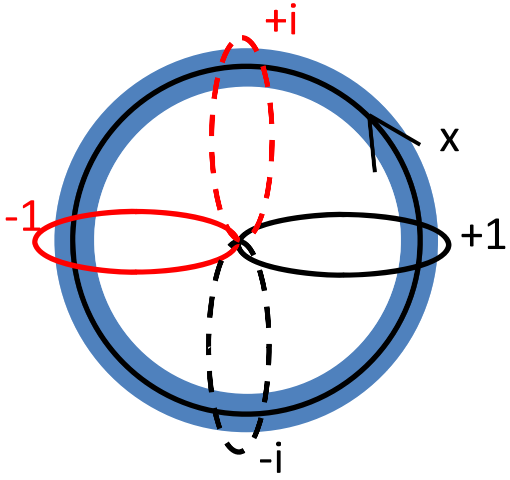
Four-fold rotational anisotropy (e.g. depolarization fields coming from neighboring rings) leads to the mixing among n_J, $n_J \pm 2\pi/L*4$, $n_J \pm 2\pi/L*8$, ...

(L: length of the ring)

➔ All the atomic orbitals within a ring are classified into four angular momenta; $n_J = 0$, $\pm 2\pi/L$, $4\pi/L$.

- Symmetry of `atomic orbitals'

$$\psi_{n_J}(x + x') = e^{i n_J x'} \psi_{n_J}(x)$$



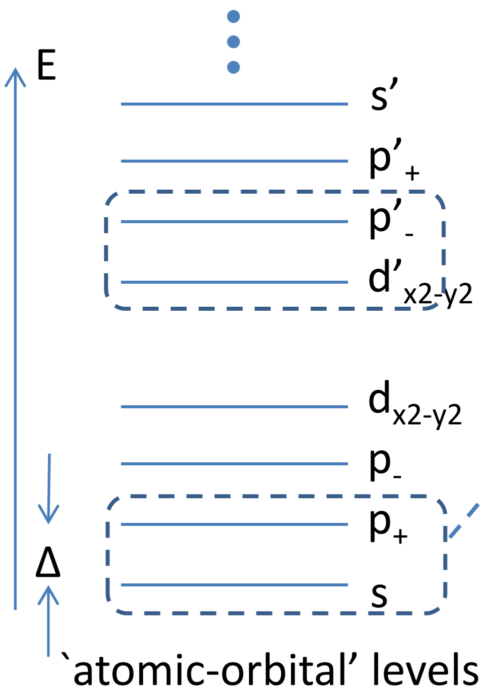
$n_J = 0$ ➔ s-wave like orbital

$n_J = 2\pi/L$ ➔ p_+-wave (p_x+ip_y) orbital

$n_J = -2\pi/L$ ➔ p_--wave (p_x-ip_y) orbital

$n_J = 4\pi/L$ ➔ d_{x2-y2}-wave orbital

☐ spin excitations within a single ring

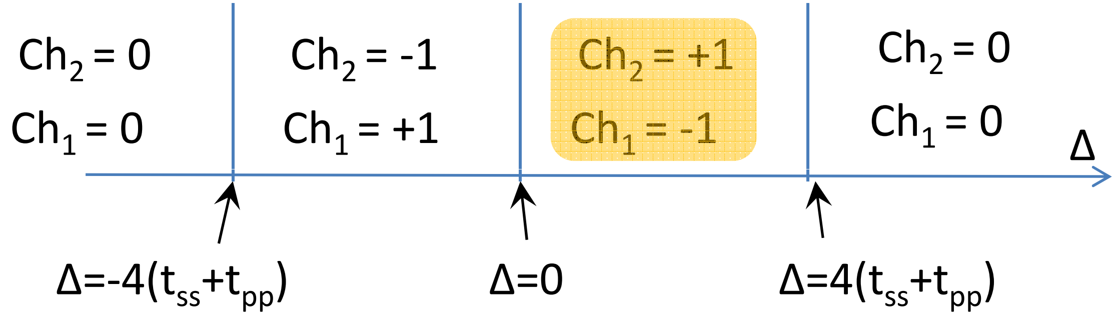


➔ ``Atomic orbitals’’ for ``tight-binding models’’

◆ **Near the saturation field (H_s) . . .**

2-bands (S-P_+) NN tight-binding model on ☐-lattice

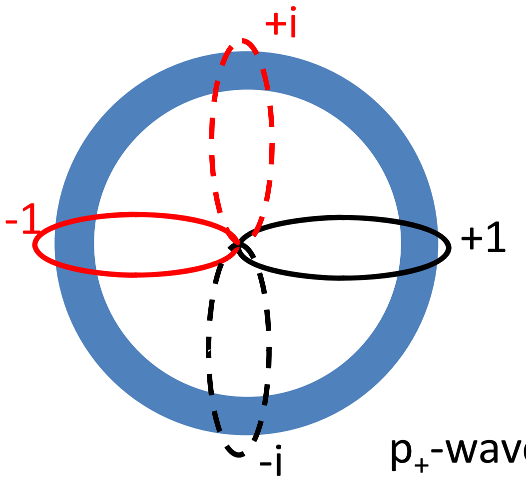
Bernevig-Hughes-Zhang, Science (2006),
Fu-Kane PRB (2007), ...



t_{ss} : NN transfer between s-orbitals

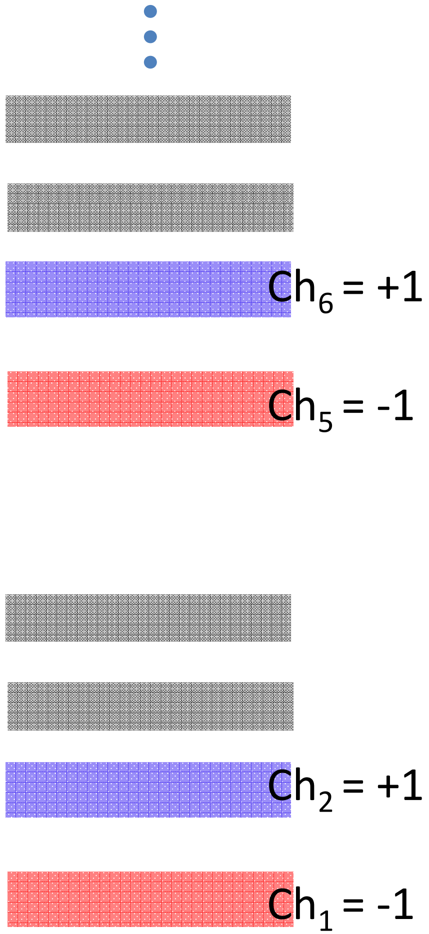
t_{pp} : NN transfer between p-orbitals

$\Delta = \varepsilon_{P+} - \varepsilon_s$



p_+-wave (p_x+ip_y) orbital

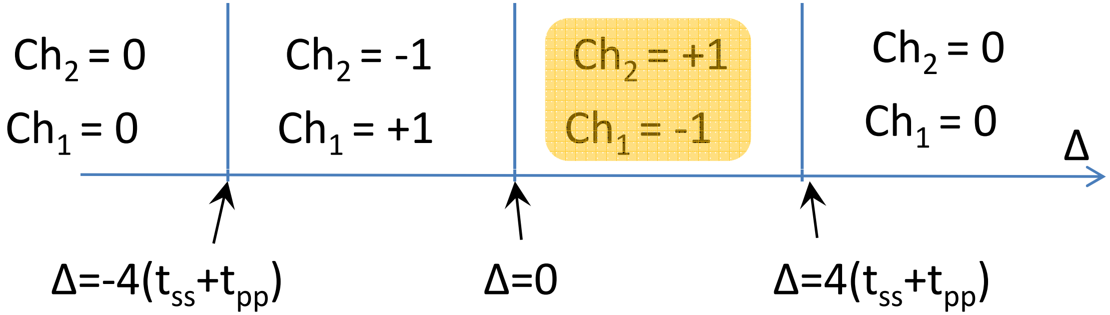
□ 2-bands NN tight-binding model on □-lattice



◆ **Near the saturation field (H_s) . . .**

2-bands (S-P_+) NN tight-binding model on □-lattice

Bernevig-Hughes-Zhang, Science (2006),
Fu-Kane PRB (2007), ...

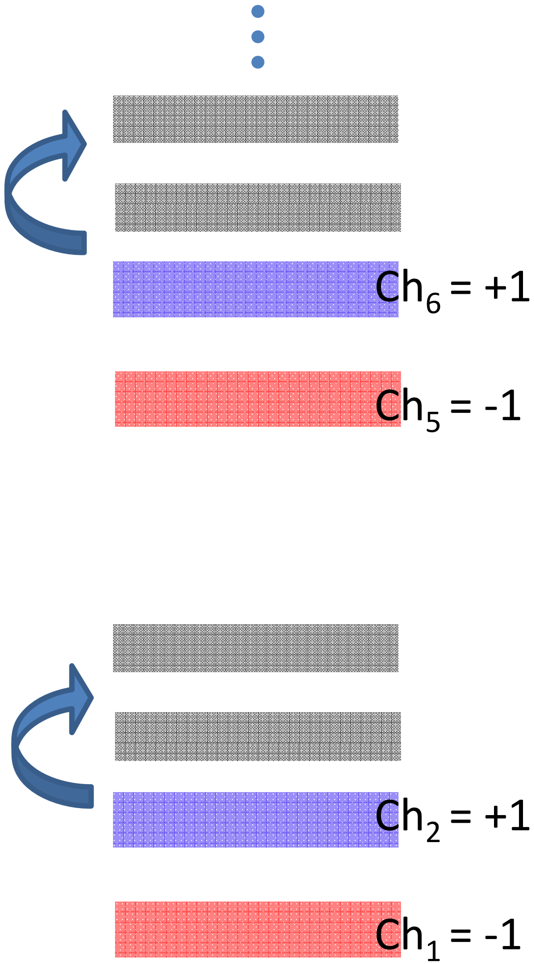


t_{ss} : NN transfer between s-orbitals

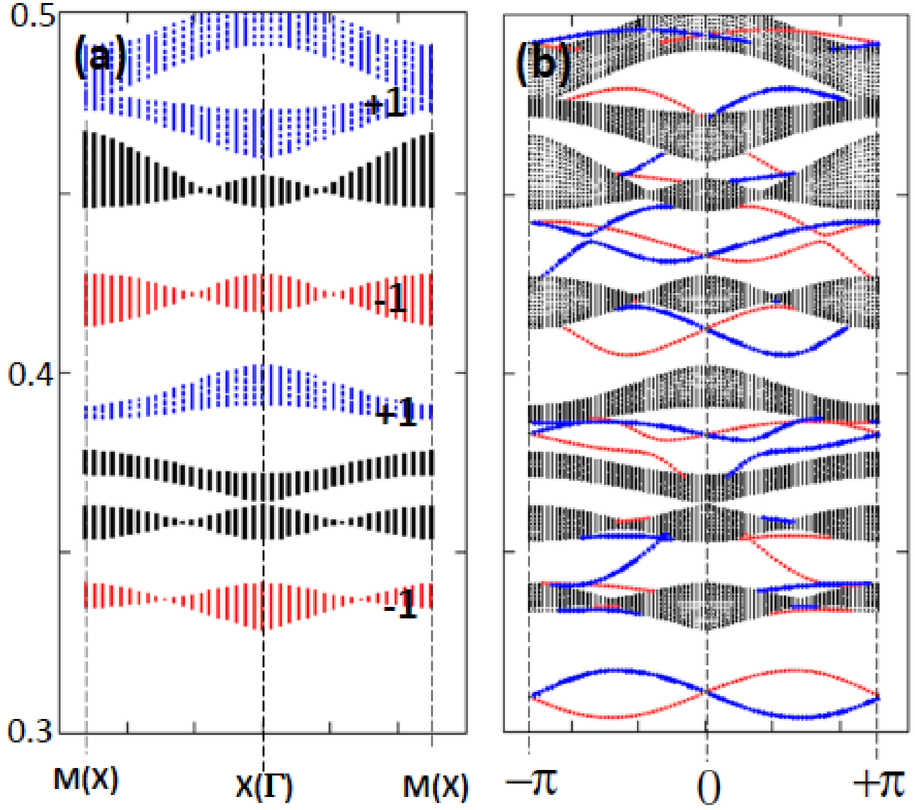
t_{pp} : NN transfer between p-orbitals

$\Delta = \varepsilon_{P+} - \varepsilon_s$

□ 2-bands NN tight-binding model on □-lattice



◆ **Near the saturation field (H_s) . . .**



$H_{ext} = 0.94*H_s$

◆ A similar interpretation is valid for the other model.

Minor details

Sometimes, coupling between 2nd lowest band and 3rd or 4th bands further transfers Ch_2=+1 into Ch_3=+1 or Ch_4=+1

Take-Out Messages of the 1st part of my talk

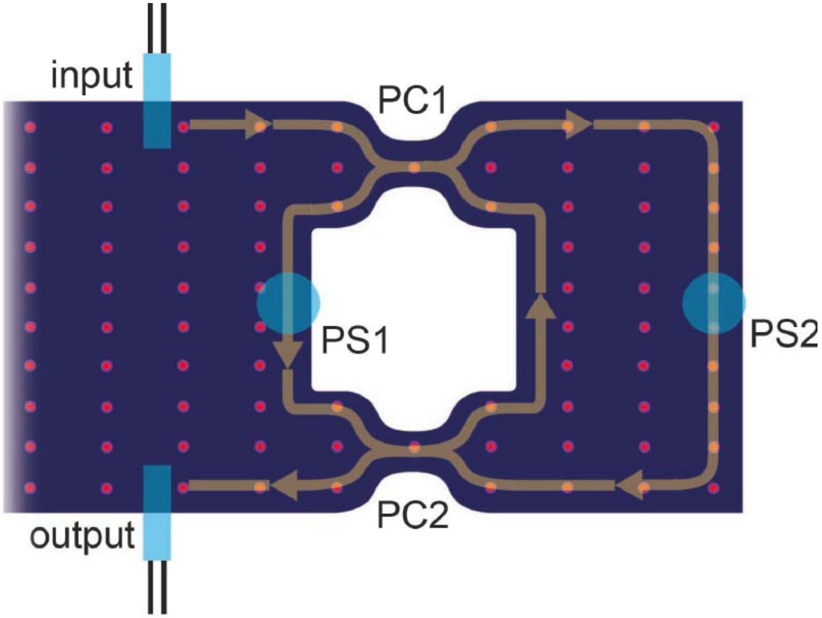
- ☐ **Magnetostatic spin-wave analog of integer quantum Hall states**
- ☐ **chiral spin-wave edge modes in dipolar regime**

➢ **Chiral edge mode is robust against elastic scatterings**

Halperin, PRB (`82)

- ☐ ***Fault-Tolerant* spin-wave devices**

◆ Spin-wave `Fabry-Perot interferometer'



PS1, PS2

Kruglyak et.al. (`10)

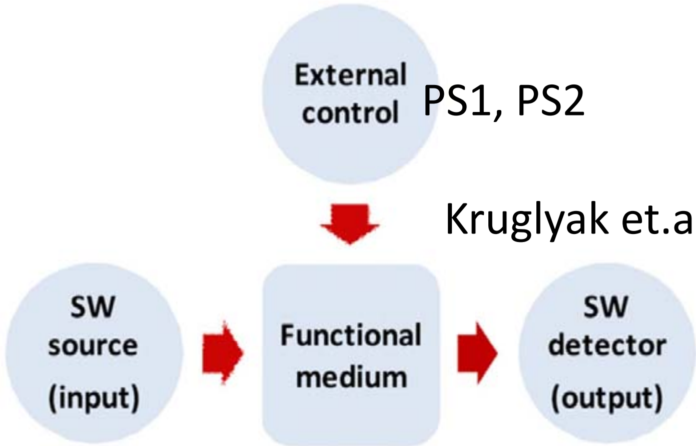


Figure 1. A block diagram of a generic magnonic device is shown.

Quantum Spin Nematic state In a quantum maget

Works done in collaboration with
Tsutomu Momoi (RIKEN) and Seiji Yunoki (RIKEN)

Reference

R. Shindou & T. Momoi, Phys. Rev. B **80**, 064410 (2009)

R. Shindou, S. Yunoki & T. Momoi, Phys. Rev. B **84**, 134414 (2011)

R. Shindou, S. Yunoki & T. Momoi, Phys. Rev. B **87,** 054429 (2013)

Content of the 2nd part of my talk

- ☐ brief introduction on quantum spin liquid (QSL)

  ---Fractionalization of magnetic excitations

  （spinon: spin ½ , charge-neutral , . . ）---

- ☐ Spin-triplet variant of QSL := quantum spin nematics (QSN)

  --- `mixed’ Resonating Valence Bond (RVB) state ---

- ☐ mixed RVB state in a quantum frustrated ferromagnet

- ☐ Mean-field theory and gauge theory of QSN

- ☐ Variational Monte Carlo studies

  --- compare them with exact diagonalization studies ---

- ☐ physical characterizations of QSN

  --- dynamical spin structure factor, NMR relaxation rate ---

- ☐ QSN can be another `route’ to a physical realization of ***fractionalizations of magnetic excitations in d>1*** .

Chanllenge in Condensed Matter Physics

A new quantum state of matter (i.e. a new form of quantum zero-point motion)

e.g. Fractional quantum (charge/spin) Hall states

Topological insulator (quantum spin Hall insulator)

Quantum spin liquid ; a quantum spin state which ***can not be characterized by any kind of spontaneous symmetry breaking*** down to T=0.

➔ **Emergent low-energy excitations: fractionalized magnetic excitations (spinons)** and **`gauge-field-like' collective excitations**

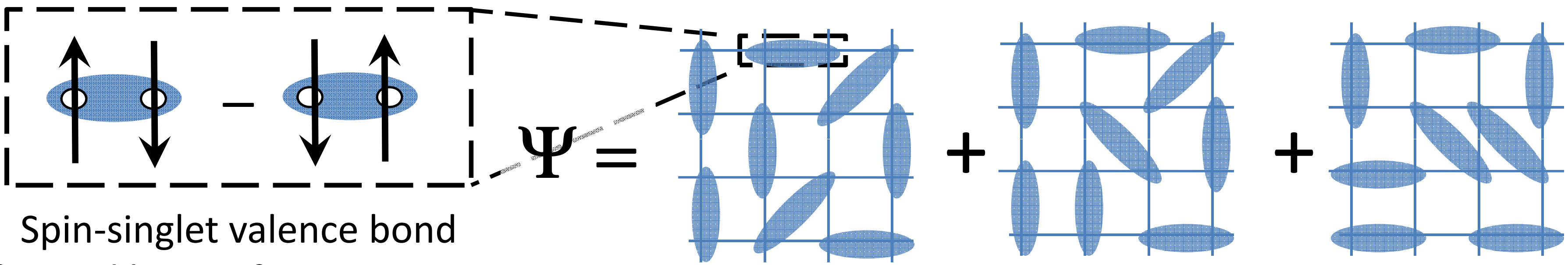
What is Quantum Spin Liquids ?

:= resonating valence bond state ; RVB state

Fazekas and Anderson (1973)

A possible ground state of S=1/2 quantum Heisenberg model on △-lattice ??

`Basic building block'



Spin-singlet valence bond (favored by Antiferromagnetic Exchange interaction)

Non-magnetic state (spin-0)

➔ quantum spin analogue of fluid-like state

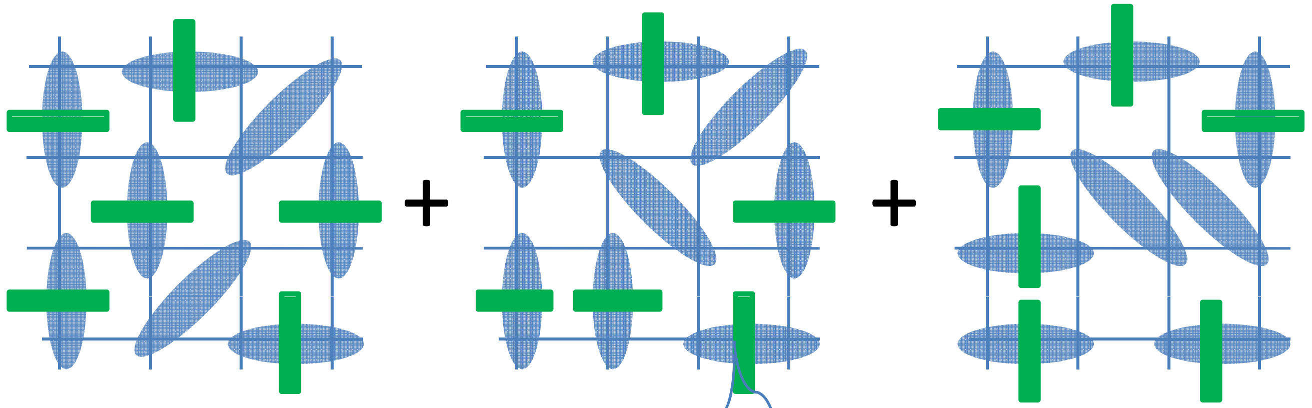
+ . . .

Possible variant of singlet RVB states ?

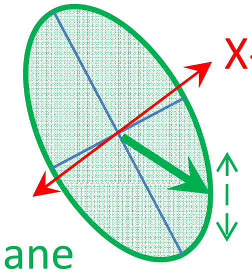
Andreev-Grishchuk (84), Chubukov (91), Chandra-Coleman-Larkin (91,92), Shannon-Momoi-Sindzingre (06), Shindou-Momoi (09)

Anderson, Baskaran, Affleck, Marston, Wen, Lee, Kotlier, Dagotto, Fradkin

- Mixed RVB states consist of singlet (and triplet valence bonds
- singlet RVB states := quantum spin liquid (emergent gauge bosons, fractionalization, topological degeneracy, . . .)

$\Psi =$  $+ \dots$

spin-singlet valence bond / spin-triplet valence bond



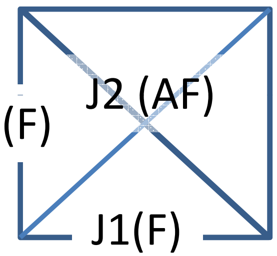
$$|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle = i[\sigma_y\sigma_0]_{\alpha\beta}|\alpha\beta\rangle \quad = 1, S_x = 0\rangle$$

$$i[\sigma_y\sigma_y]_{\alpha\beta}|\alpha\beta\rangle \propto |\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle = |S=1, S_y=0\rangle$$

$$i[\sigma_y\sigma_z]_{\alpha\beta}|\alpha\beta\rangle \propto |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle = |S=1, S_z=0\rangle$$

another maximally entangled state of two spins

An S=1 Ferro-moment is rotating within a plane.



- Ferromagnetic exchange interaction likes it.
- We also needs spin-frustrations.

  ➔ `Quantum frustrated ferromagnet' !

- Spin-triplet valence bond on NN ferro bond and Spin-singlet valence bond on NNN antiferro bond

□ billiear exchange interaction ➔ quartic term in the spinon field

$$S_{j,\mu} \equiv \frac{1}{2} f^{\dagger}_{j,\alpha}[\sigma_\mu]_{\alpha\beta} f_{j,\beta} \qquad f^{\dagger}_{j,\alpha} f_{j,\alpha} \equiv 1\,, \quad f^{\dagger}_{j,\uparrow} f^{\dagger}_{j,\downarrow} \equiv 0\,, \quad f_{j,\uparrow} f_{j,\downarrow} \equiv 0. \quad \text{for } \forall j$$

fermionic field

$$\Psi_j \equiv \begin{bmatrix} f_{j,\uparrow} & f_{j,\downarrow} \\ f^{\dagger}_{j,\downarrow} & -f^{\dagger}_{j,\uparrow} \end{bmatrix}, \qquad \Psi^{\dagger}_j \equiv \begin{bmatrix} f^{\dagger}_{j,\uparrow} & f_{j,\downarrow} \\ f^{\dagger}_{j,\downarrow} & -f_{j,\uparrow} \end{bmatrix}$$

\`Martix' analogue of Nambu-vector (Affleck et.al. (88))

Time-reversal pair

Ferro-bond → decouple in the spin-triplet space Shindou-Momoi, PRB (2009)

$$-\hat{S}_i \hat{S}_j \rightarrow \frac{1}{4} \sum_{\mu=1}^{3} \Big\{ \big(-|E_{ij,\mu}|^2 - |D_{ij,\mu}|^2 \big) + \mathrm{Tr}\big[\hat{\Psi}^{\dagger}_i \hat{U}^{\mathrm{tri}}_{ij,\mu} \hat{\Psi}_j \hat{\tau}^{t}_{\mu} \big] \Big\}$$

\`Spin-orbit hopping'

spin-triplet SU(2) link variable

$$\hat{U}^{\mathrm{tri}}_{ij,\mu} \equiv \begin{bmatrix} E^{*}_{ij,\mu} & D_{ij,\mu} \\ -D^{*}_{ij,\mu} & E_{ij,\mu} \end{bmatrix}$$

spin-triplet pairing of spinons

\`d-vector'

$$E_{ij,\mu} \equiv \langle f^{\dagger}_{i\alpha} [\sigma_\mu]_{\alpha\beta} f_{j\beta} \rangle, \quad \text{: p-h pairing}$$

$$D_{ij,\mu} \equiv \langle f_{i\alpha} [i\sigma_2 \sigma_\mu]_{\alpha\beta} f_{j\beta} \rangle \quad \text{: p-p pairing}$$

AF-bond → decouple in the spin-**singlet** space (see a Textbook by Xiao-Gang Wen)

$$\hat{S}_i \hat{S}_j \rightarrow \frac{1}{4} \Big\{ \big(-|\chi_{ij}|^2 - |\eta_{ij}|^2 \big) + \mathrm{Tr}\big[\hat{\Psi}^{\dagger}_i \hat{U}^{\mathrm{sin}}_{ij} \hat{\Psi}_j \big] \Big\}$$

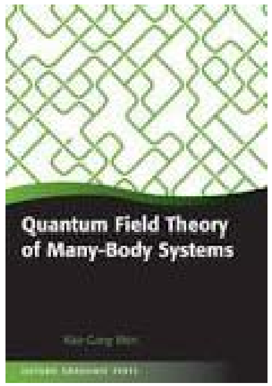
spin-singlet SU(2) link variable

$$\hat{U}^{\mathrm{sin}}_{ij} \equiv \begin{bmatrix} \chi^{*}_{ij} & \eta_{ij} \\ \eta^{*}_{ij} & -\chi_{ij} \end{bmatrix}$$

spin-singlet pairing of spinons

$$\chi_{ij} \equiv \langle f^{\dagger}_{i\alpha} f_{j\alpha} \rangle \quad \text{: p-h pairing}$$

$$\eta_{ij} \equiv \langle f_{i\alpha} [i\sigma_2]_{\alpha\beta} f_{j\beta} \rangle \quad \text{: p-p pairing}$$

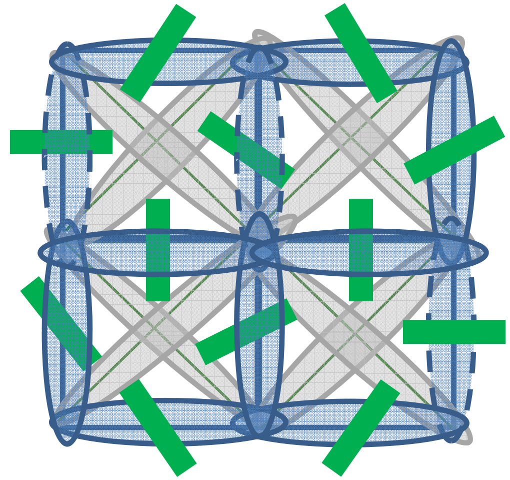


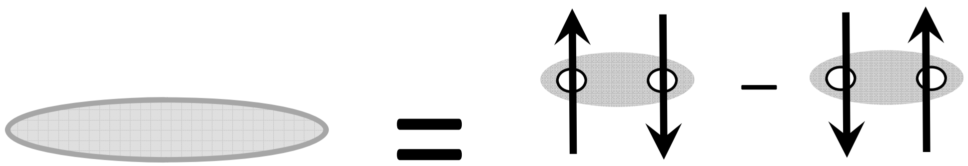
What ``pairings of spinons'' physically mean . . .

spin-singlet pairing of spinons for *antiferromagnetic* bonds

$\chi_{ij} \equiv \langle f^{\dagger}_{i\alpha} f_{j\alpha} \rangle$: p-h channel

$\eta_{ij} \equiv \langle f_{i\alpha} \left[i\sigma_2 \right]_{\alpha\beta} f_{j\beta} \rangle$: p-p channel





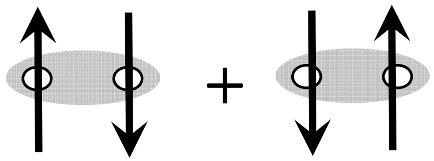
Singlet valence bond

spin-triplet pairing of spinons for ***ferromagnetic*** bonds

Shindou-Momoi PRB ('09)

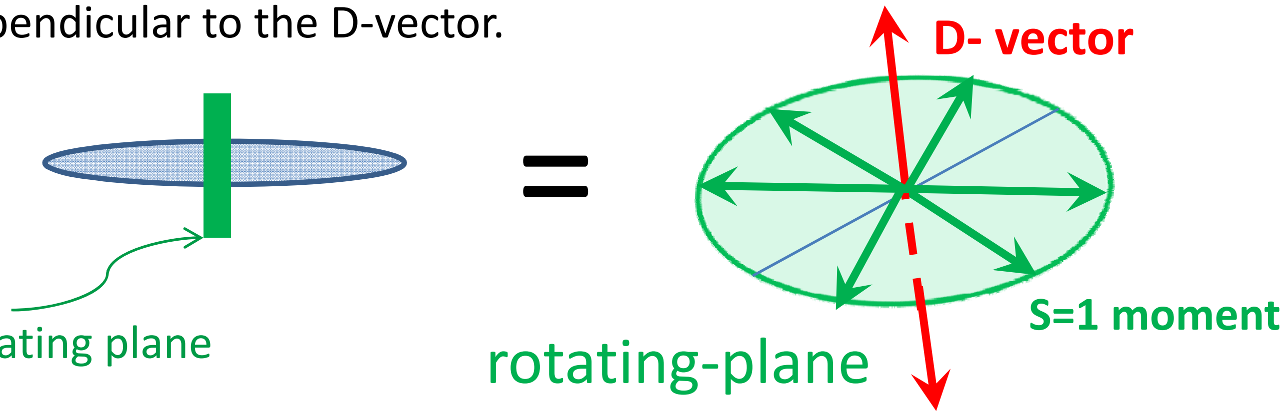
$E_{ij,\mu} \equiv \langle f^{\dagger}_{i\alpha} [\sigma_\mu]_{\alpha\beta} f_{j\beta} \rangle,$: p-h channel

$D_{ij,\mu} \equiv \langle f_{i\alpha} [i\sigma_2 \sigma_\mu]_{\alpha\beta} f_{j\beta} \rangle$: p-p channel

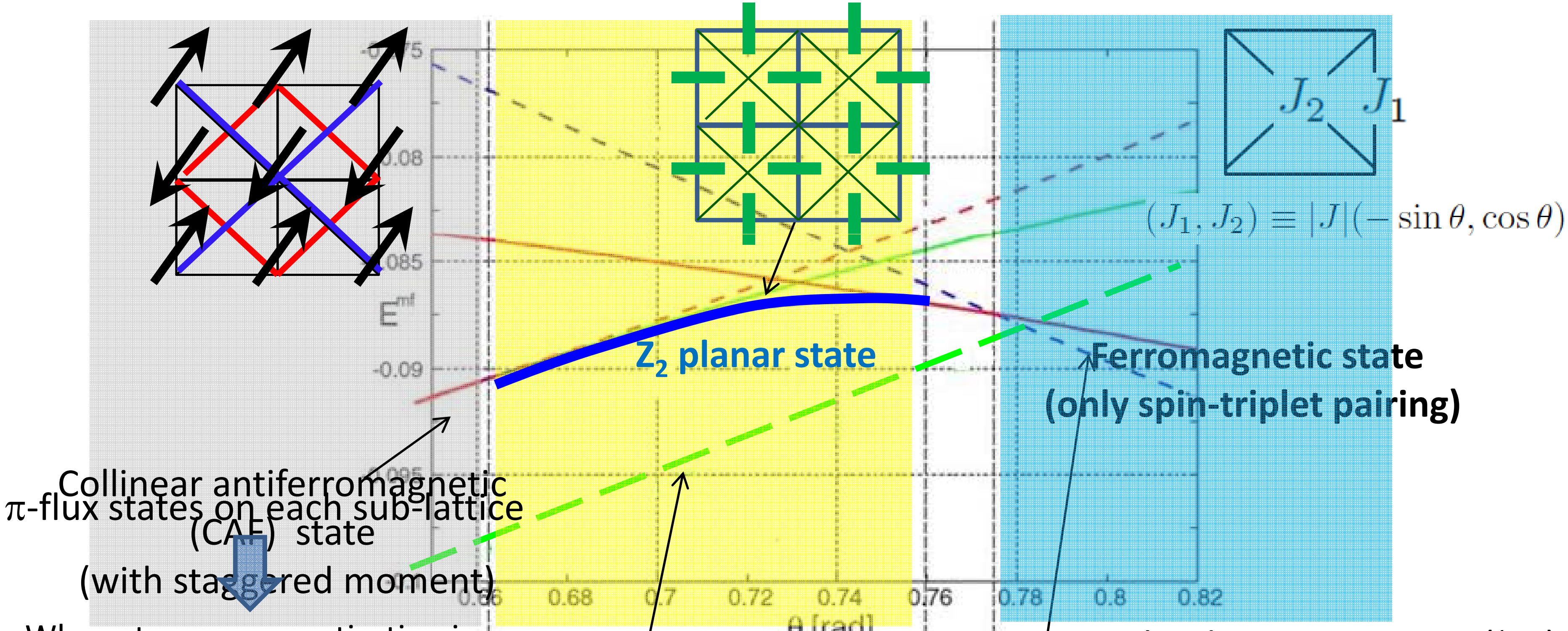


triplet valence bond

- ☐ S=1 moment is rotating within a plane perpendicular to the D-vector.



Mean-field energetics sorts out candidate pairing states at some level . . . .



Shindou-Momoi PRB ('09)

When stagger magnetization is introduced, the energy is further optimized (AF ordering)

Liang-Doucot-Anderson, PRL (88)

Only spin-triplet pairings

on NN ferro-bonds (`flat-band' state)

➔ (When projected to the spin-Hilbert space)

reduces to a fully polarized ferromagnetic state.

Isolated dimer state

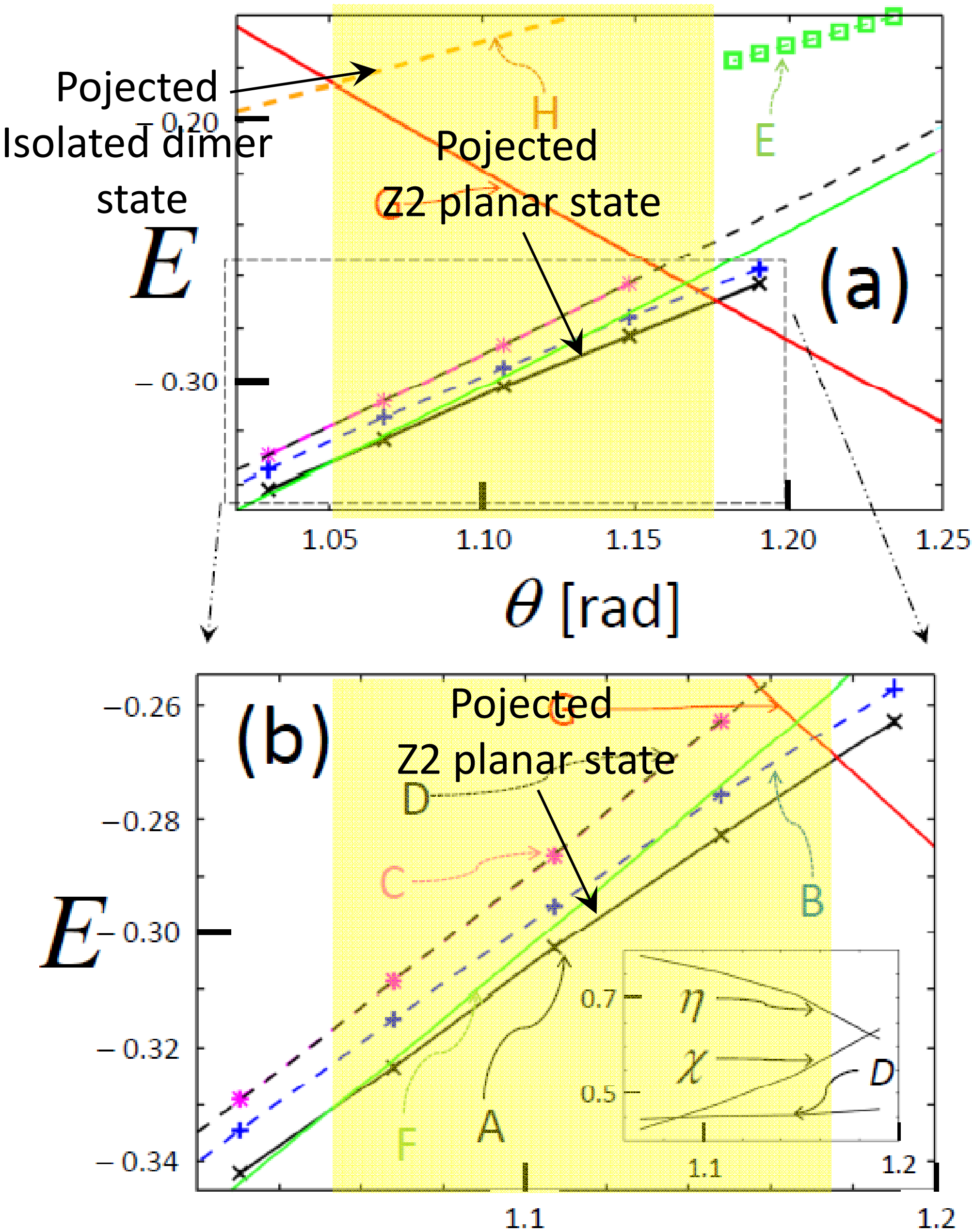
Shindou-Yunoki-Momoi PRB (2011)

- Fermionic Mean-field Theory replace the local constraint by the global one, so that pairing states do not strictly observe the local constraint generally.

$$S_{j,\mu} \equiv \frac{1}{2} f^{\dagger}_{j,\alpha} [\sigma_\mu]_{\alpha\beta} f_{j,\beta} \qquad f^{\dagger}_{j,\alpha} f_{j,\alpha} \equiv 1 \ , \quad f^{\dagger}_{j,\uparrow} f^{\dagger}_{j,\downarrow} \equiv 0 \ , \quad f_{j,\uparrow} f_{j,\downarrow} \equiv 0. \quad \text{for } \forall j$$

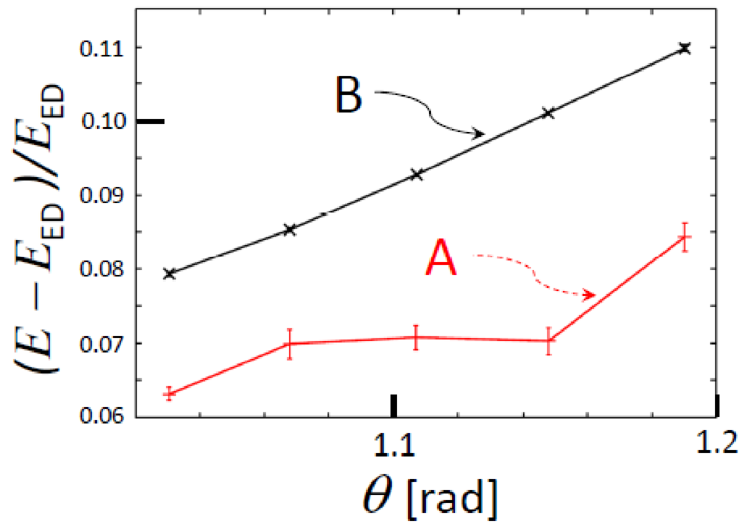
Energetics of *projected* BCS wavefunctions (V*MC* analysis)

Shindou, Yunoki, Momoi, PRB (2011)



A: $|\overline{\Psi}\rangle = \mathcal{P}_{S=0}\mathcal{P}|\Psi_{\mathrm{BCS}}\rangle$

B: $|\overline{\Psi}\rangle = \mathcal{P}_{S_z=0}\mathcal{P}|\Psi_{\mathrm{BCS}}\rangle$

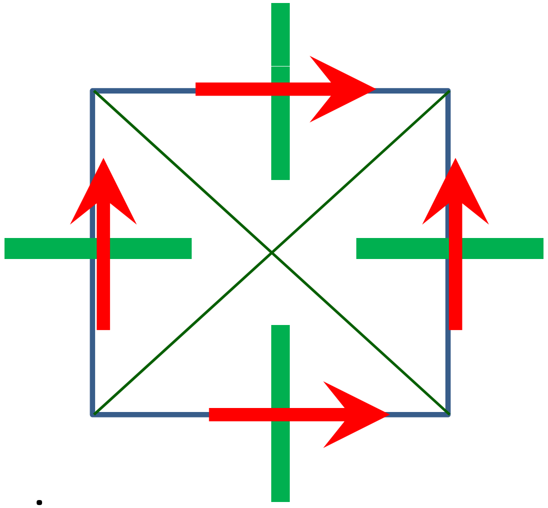


- For J1:J2=1:0.42 ∼ J1:J2=1:0.57, the projected planar state (singlet) wins over **ferro**-state and **collinear antiferromagnetic** state.
- 92%∼94% of the exact ground state with N = 36 sites.

Spin nematics character in projected planar state

$$|\overline{\Psi}\rangle = \mathcal{P}|\Psi_{\rm BCS}\rangle$$

Shindou, Yunoki, Momoi, PRB (2011)



(symmetric part of) rank-2 tensors on ferromagnetic bonds

π-rotation in the spin

$$K^{\mu\nu}_{j,m} \equiv \frac{1}{2}(S_{j,\mu}S_{m,\nu} + S_{j,\nu}S_{m,\mu}) - \frac{\delta_{\mu\nu}}{3}\langle \boldsymbol{S}_j \cdot \boldsymbol{S}_m\rangle,$$

$$\langle K^{\mu\nu}_{i,j}\rangle = -\tfrac{1}{2}\big(E_{ij,\mu}E^*_{ij,\nu} - \tfrac{1}{3}\delta_{\mu\nu}|\boldsymbol{E}_{ij}|^2\big) - \tfrac{1}{2}\big(D_{ij,\mu}D^*_{ij,\nu} - \tfrac{1}{3}\delta_{\mu\nu}|\boldsymbol{D}_{ij}|^2\big) + \text{H.c.}$$

a gauge trans.

+1

-1

Ordering of d-vectors ➔ Ordering of the quadrupole moment

$$\langle\overline{\Psi}|\boldsymbol{S}_{j,+}|\overline{\Psi}\rangle = 0.$$

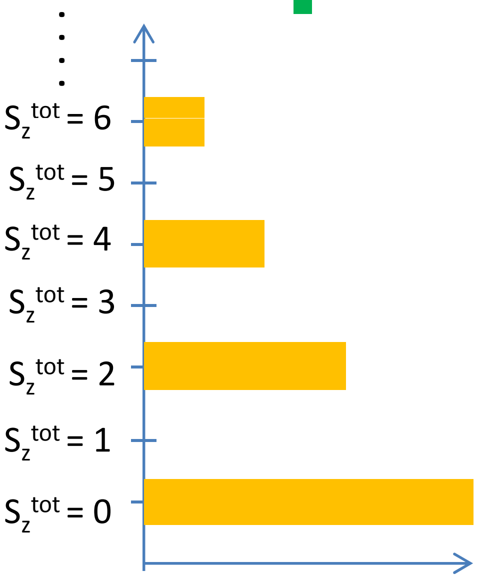
$$\langle\{\sigma_j\}|\overline{\Psi}\rangle = (-1)^{\frac{N}{2}} e^{i\pi S_z}\langle\{\sigma_j\}|\overline{\Psi}\rangle$$

$$\langle\overline{\Psi}|S_{j,+}S_{m,+}|\overline{\Psi}\rangle = J(J \cdots \neq 0$$

gauge-part

$$S_z \equiv \frac{1}{2}\sum_j \sigma_j$$

$$|\overline{\Psi}\rangle \equiv \cdots + \mathcal{P}_{S_z=-2}|\overline{\Psi}\rangle + \mathcal{P}_{S_z=0}|\overline{\Psi}\rangle + \mathcal{P}_{S_z=2}|\overline{\Psi}\rangle + \cdots$$



Weight of the projected Z2 planar state.

D-wave spin-nematics of projected Planar state

$$f\left(R_{\frac{\pi}{2}}(j-m)\right) = -f(j-m)$$

π/2-spatial rotation

D-wave ordering

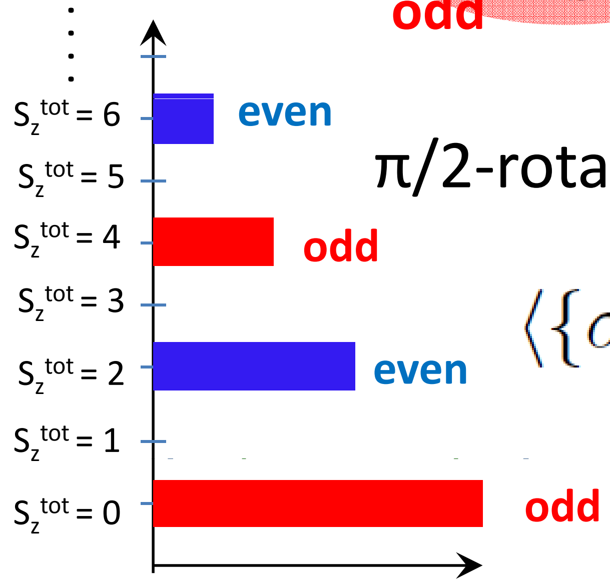
Shindou, Yunoki, Momoi, PRB (2011)

$$f(j-m) = \sum_n \langle \overline{\Psi} | \mathcal{P}_{S_z=2n+2} \boldsymbol{S}_{j,+} \boldsymbol{S}_{m,+} \mathcal{P}_{S_z=2n} | \overline{\Psi} \rangle$$

$$|\overline{\Psi}\rangle \equiv \cdots + \mathcal{P}_{S_z=-2}|\overline{\Psi}\rangle + \mathcal{P}_{S_z=0}|\overline{\Psi}\rangle + \mathcal{P}_{S_z=2}|\overline{\Psi}\rangle + \cdots$$

even, odd, even, odd

π/2-spatial rotation

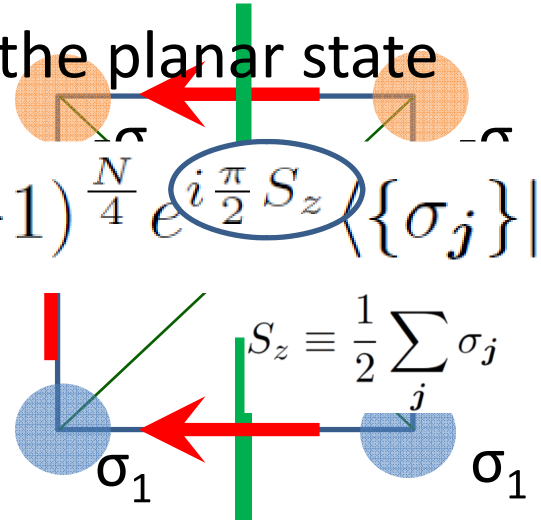


π/2-rotational symmetry in the planar state

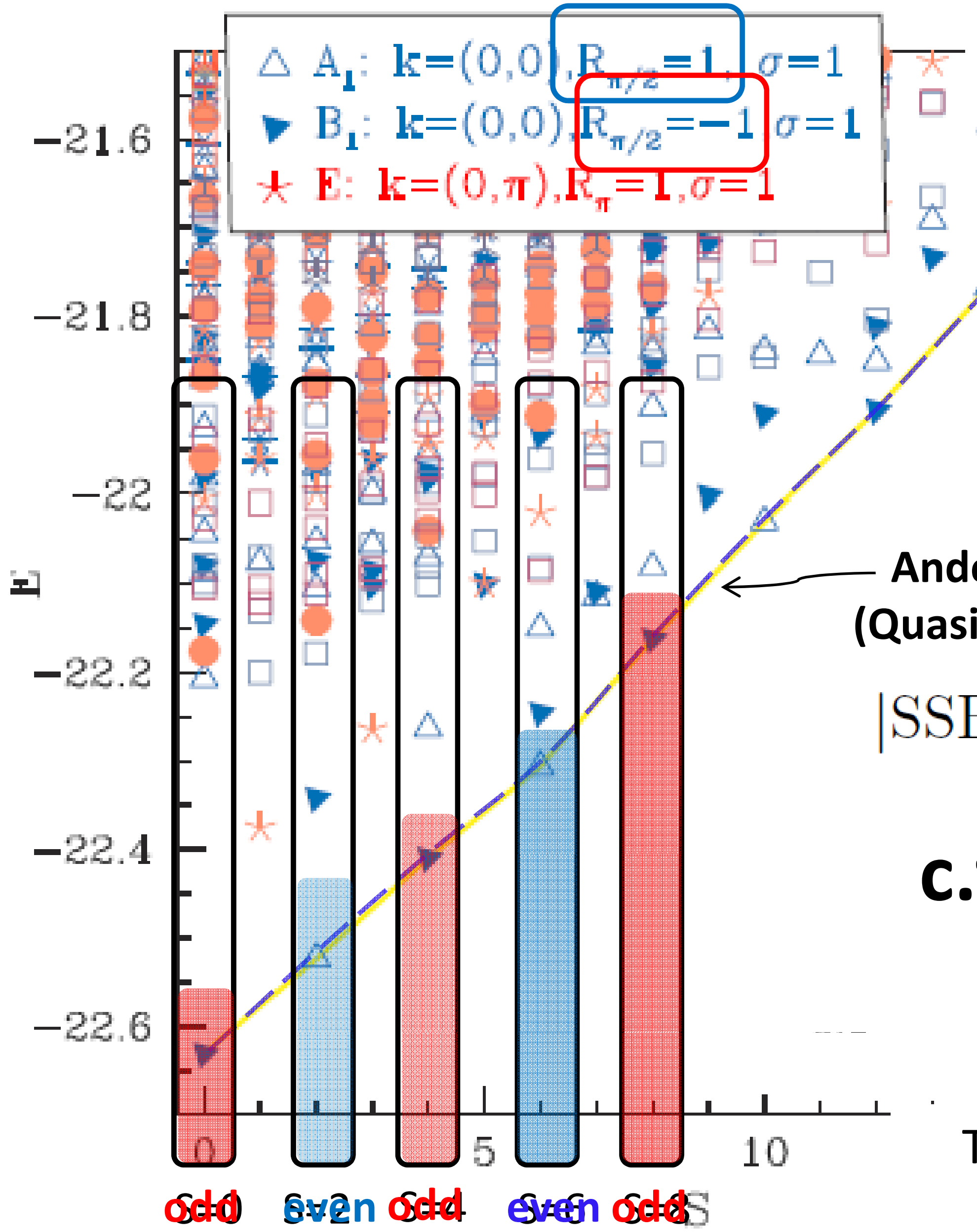
π/2-rotation in the spin space and lattice space

$$\langle \{\sigma_{R_{\frac{\pi}{2}}(j)}\} | \overline{\Psi} \rangle = (-1)^{\frac{N}{4}} e^{i\frac{\pi}{2}S_z} \langle \{\sigma_j\} | \overline{\Psi} \rangle$$

a gauge trans.



More accurately , Sz=N/2-4n➔ even, Sz=N/2-(4n+2)➔ odd



Under π/2-spatial rotation

J2=0.4*J1

Shannon, Momoi, Sindzingre PRL (2006)

ED (N = 36)

D-wave spin nematic character

Anderson's tower of state (Quasi-degenerate Joint state)

$$|\mathrm{SSB}\rangle \equiv |S=0\rangle + |S=2\rangle + |S=4\rangle + |S=6\rangle + \cdots$$

c.f.

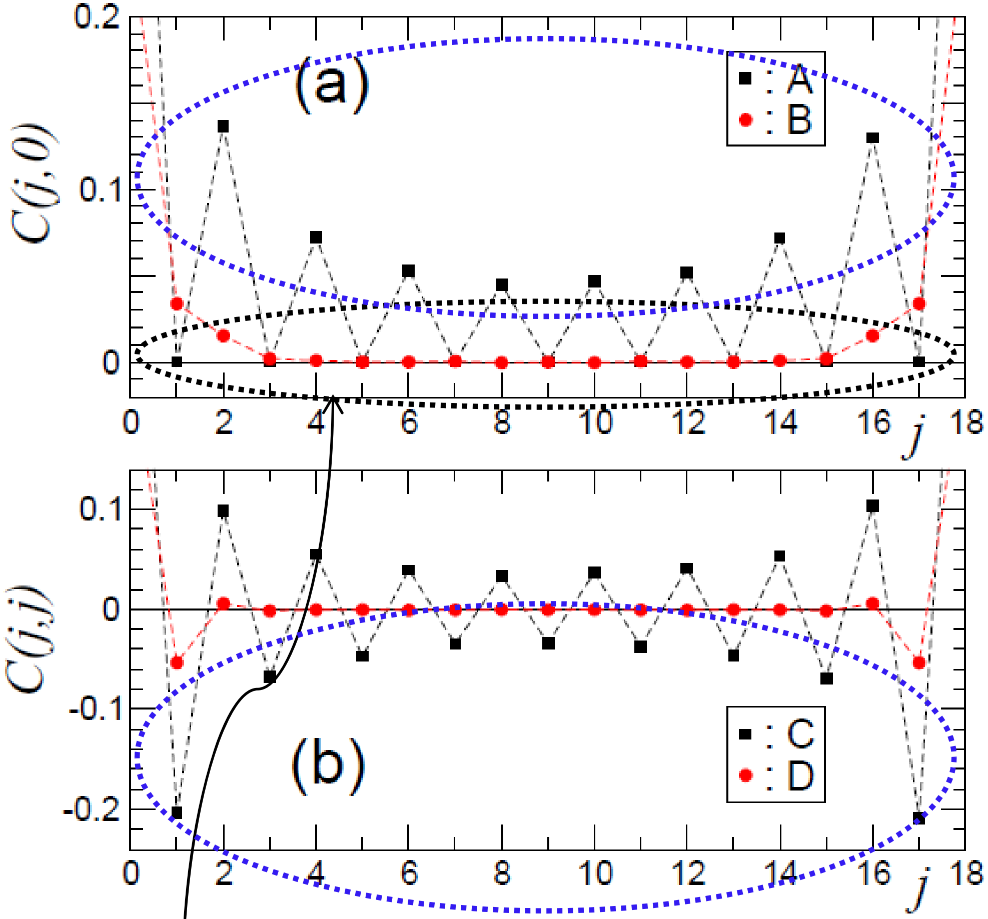
$$|\overline{\Psi}\rangle \equiv \cdots + \mathcal{P}_{S_z=-2}|\overline{\Psi}\rangle + \mathcal{P}_{S_z=0}|\overline{\Psi}\rangle + \mathcal{P}_{S_z=2}|\overline{\Psi}\rangle + \cdots$$

The projected planar state mimic the quasi-degenerate joint state with the same spatial symmetry as that of the ED study (especially under the field)

Spin correlation functions   J2=0.45*J1

Shindou, Yunoki, Momoi, PRB (2011)

$$|\overline{\Psi}\rangle = \mathcal{P}_{S_z=0}\mathcal{P}|\Psi_{\rm BCS}\rangle$$

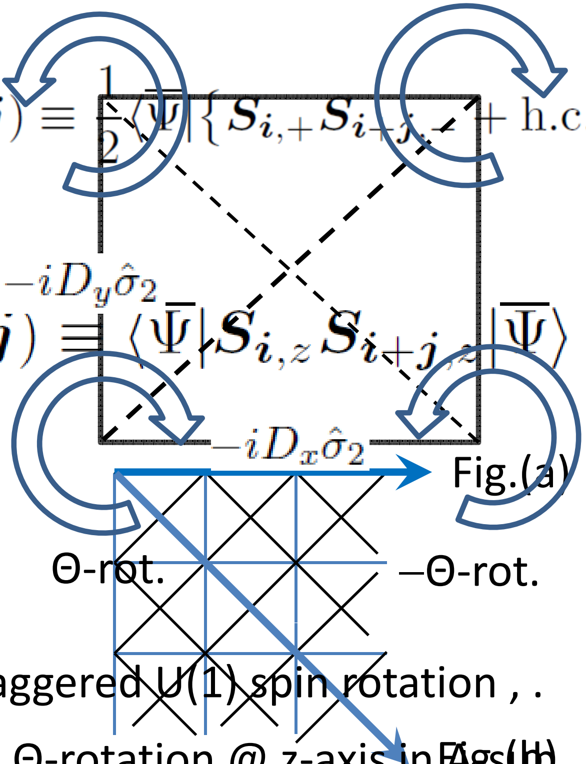


A,C

$$C_{+-}(j) \equiv \frac{1}{2}\langle\overline{\Psi}|\{S_{i,+}S_{i+j,-} + \text{h.c.}\}|\overline{\Psi}\rangle$$

B,D

$$C_{zz}(j) \equiv \langle\overline{\Psi}|S_{i,z}S_{i+j,z}|\overline{\Psi}\rangle$$



staggered U(1) spin rotation , .

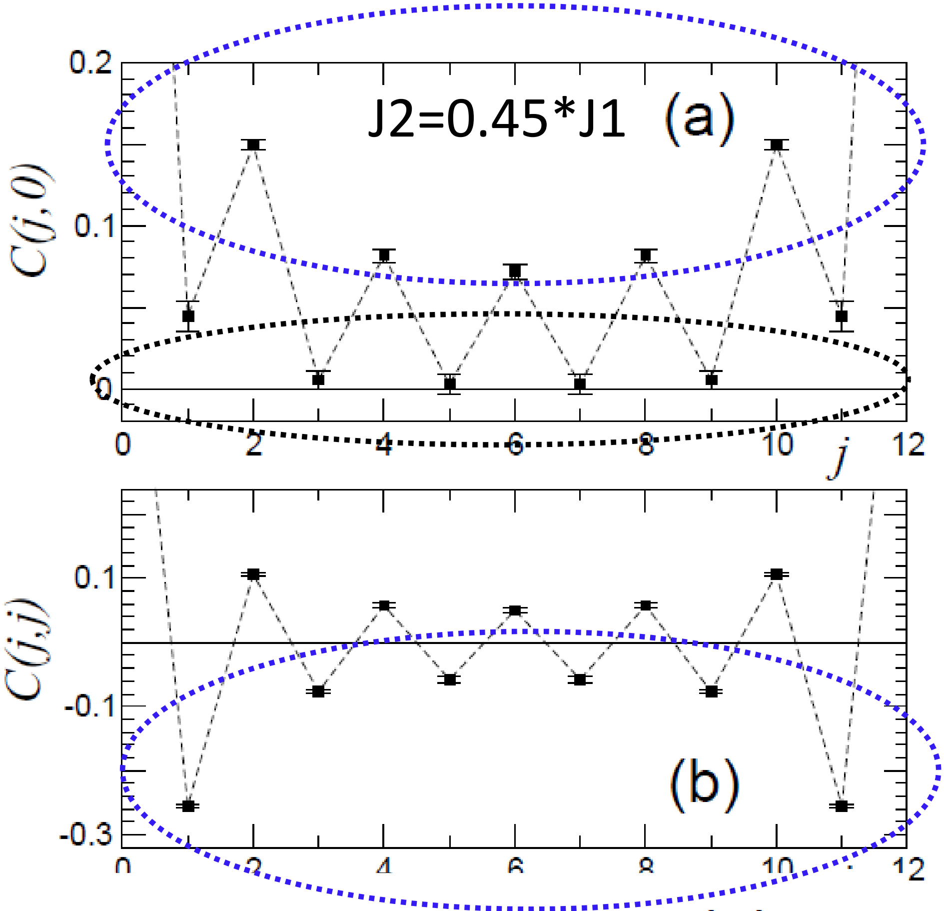
- Θ-rotation @ z-axis in A-sub.
- –Θ-rotation @ z-axis in B-sub.

- No correlation at all between the transverse spins in A-sublattice (*j*) and those in B-sublattice (*m*).

$$\langle\overline{\Psi}|\{S_{j,+}S_{m,-} + \text{h.c.}\}|\overline{\Psi}\rangle = 0$$

⇐ Staggered magnetization is conserved !

$$\langle\{\sigma_j\}|\overline{\Psi}\rangle = e^{i\theta(S_{A,z}-S_{B,z})}\langle\{\sigma_j\}|\overline{\Psi}\rangle$$

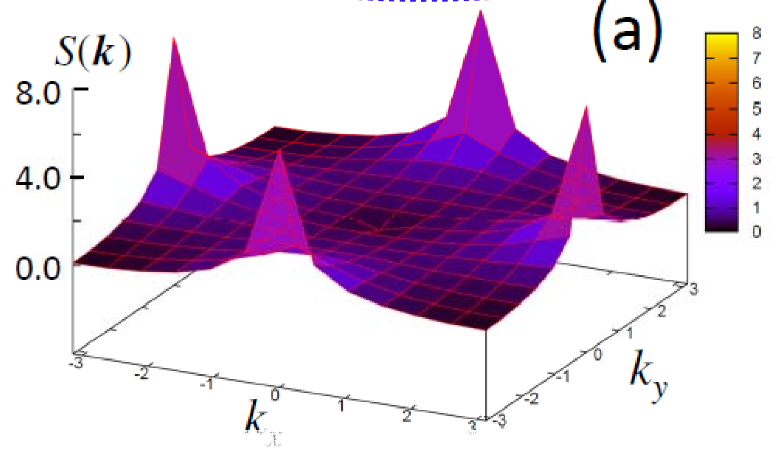


$$|\overline{\Psi}\rangle = \mathcal{P}_{S=0}\mathcal{P}|\Psi_{\rm BCS}\rangle.$$

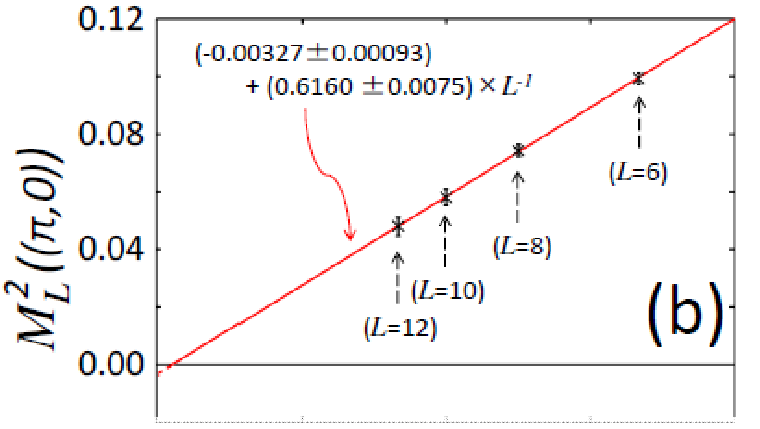
$$C(\boldsymbol{j}) \equiv \langle\overline{\Psi}|\boldsymbol{S_i}\cdot\boldsymbol{S_{i+j}}|\overline{\Psi}\rangle$$

`Interpolate' between $C_{zz}(j)$ and $C_{+-}(j)$ described so far

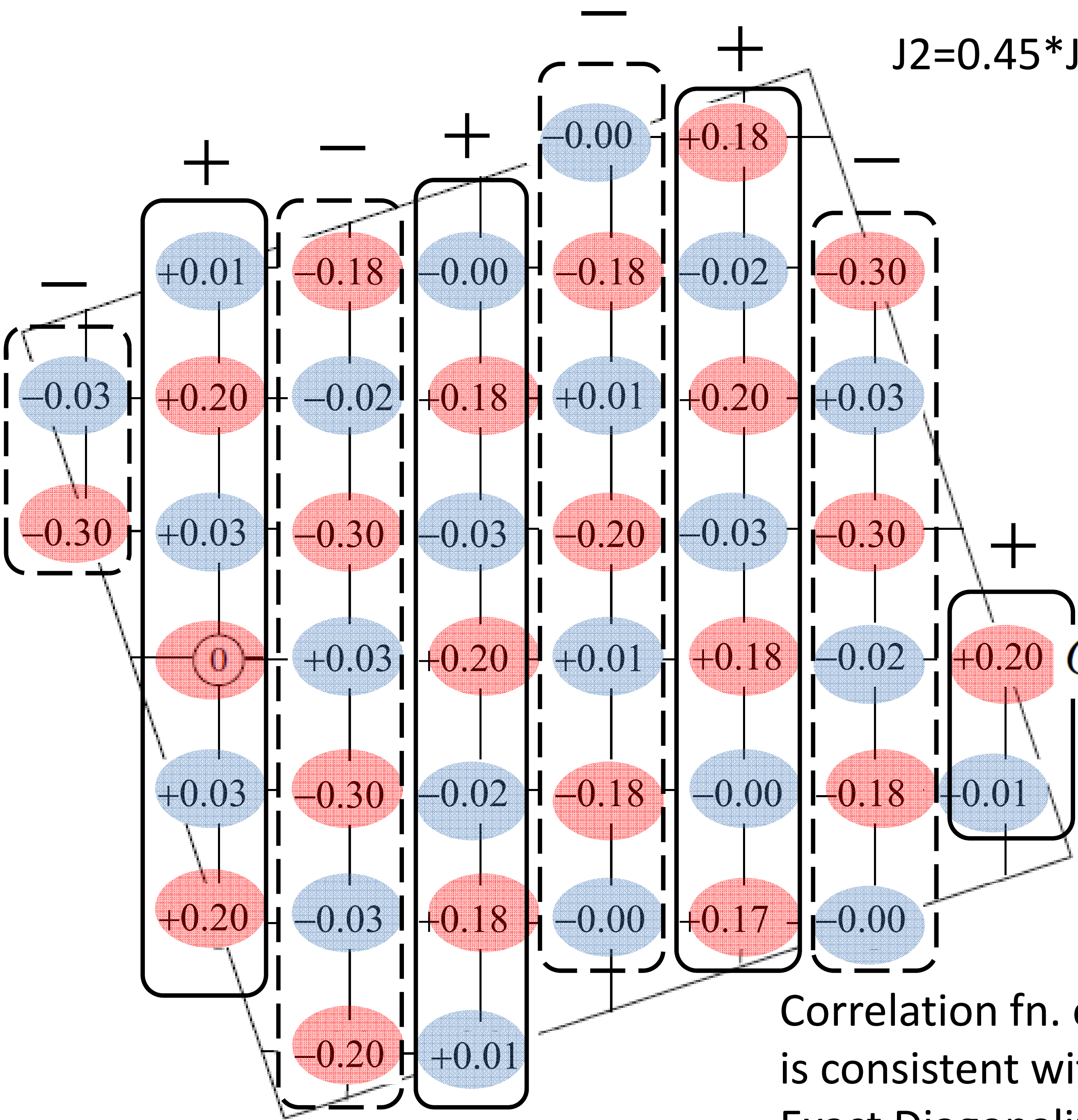
- less correlations between spins in A-sub. and those in B-sub..
- Within the same sublattice, spin is correlated antiferro.



Strong spin fluctuation at (π,0) and (0,π)



Finite size scaling suggests no ordering of Neel moment



From
Richter et.al.
PRB (2010)

ED (N = 40)

$$\langle \boldsymbol{S}_0 \cdot \boldsymbol{S}_j \rangle$$

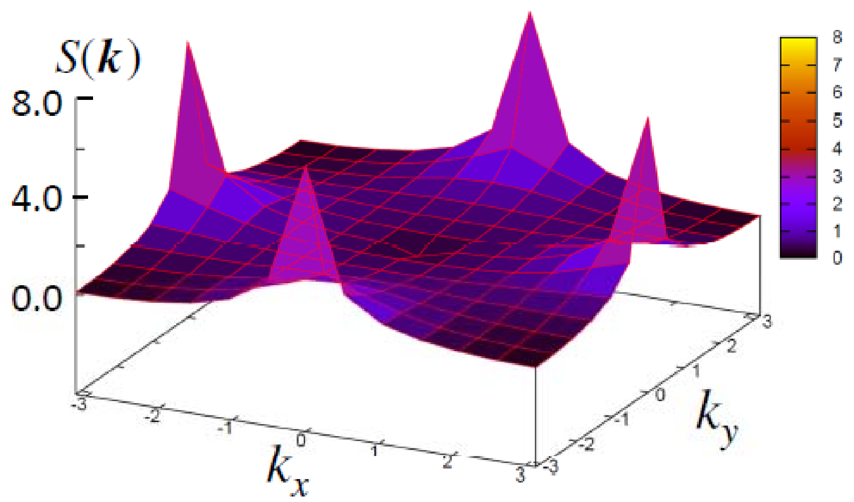
$$C(\boldsymbol{j}) \sim (-1)^{j_x} |\boldsymbol{j}|^{-\eta}$$

Strong collinear antiferromagnetic Correlations.

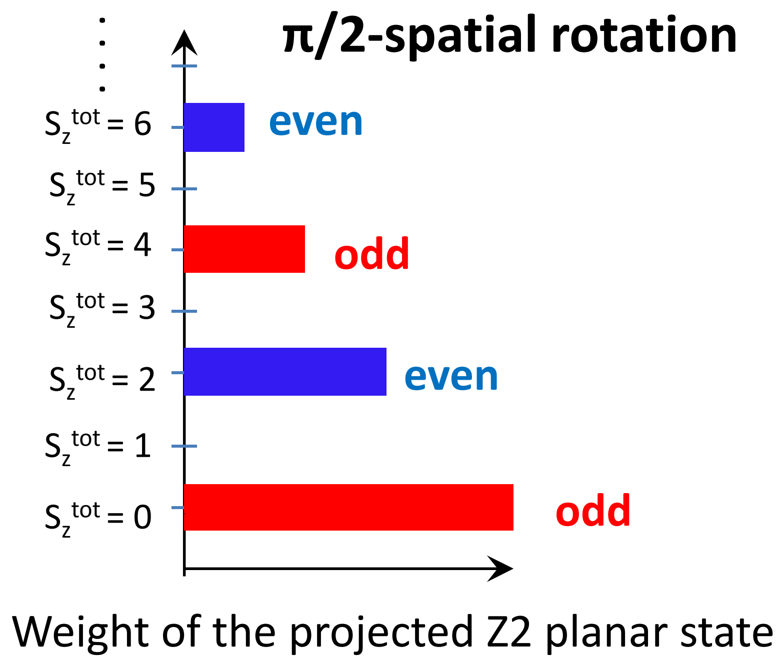
Correlation fn. of the planar state is consistent with that of the Exact Diagonalization results.

Summary of variational Monte-Carlo studies

- ☐ Energetics; Projected Z2 planar state ; $J_2 = 0.417\ J_1 \sim 0.57\ J_1$
- ☐ Spin correlation function; collinear anitferromagnetic fluctuation (But no long-ranged ordering)



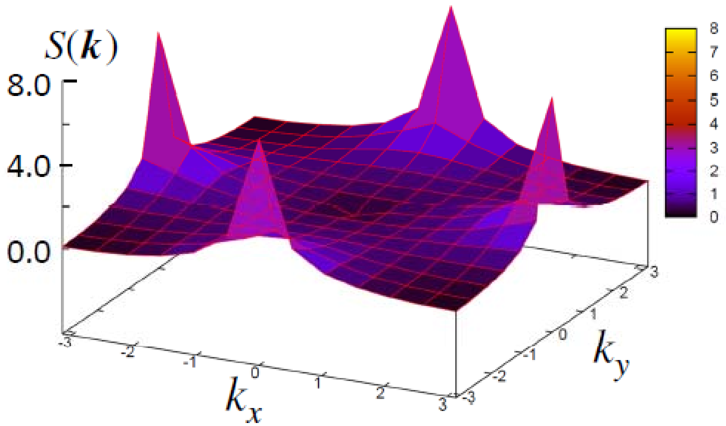
- ☐ quadruple spin moment; d-wave spatial configuration

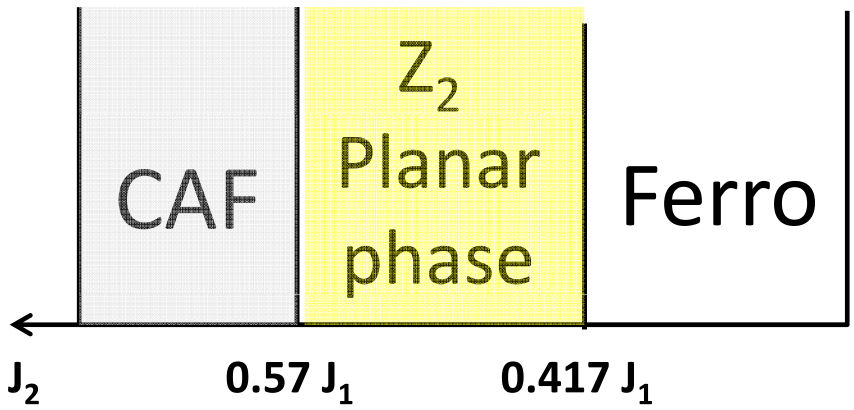


Consistent with previous exact diagonalization studies

Physical/Experimental characterization of Z_2 planar phase

- ☐ **Static spin structure takes after that of the neighboring collinear antiferromagnetic (CAF) phase**



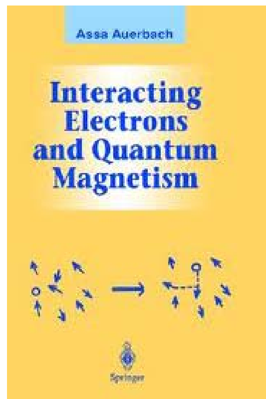


➔ How to distinguish the Z2 planar phase from the CAF phase ?

- ☐ **Dynamical spin structure factor**
- ☐ **(low) Temperature dependence of NMR $1/T_1$**

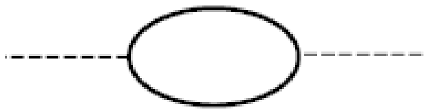
➔ Use Large-N loop expansion usually employed in QSL

consult e.g. textbook by Assa Auerbach

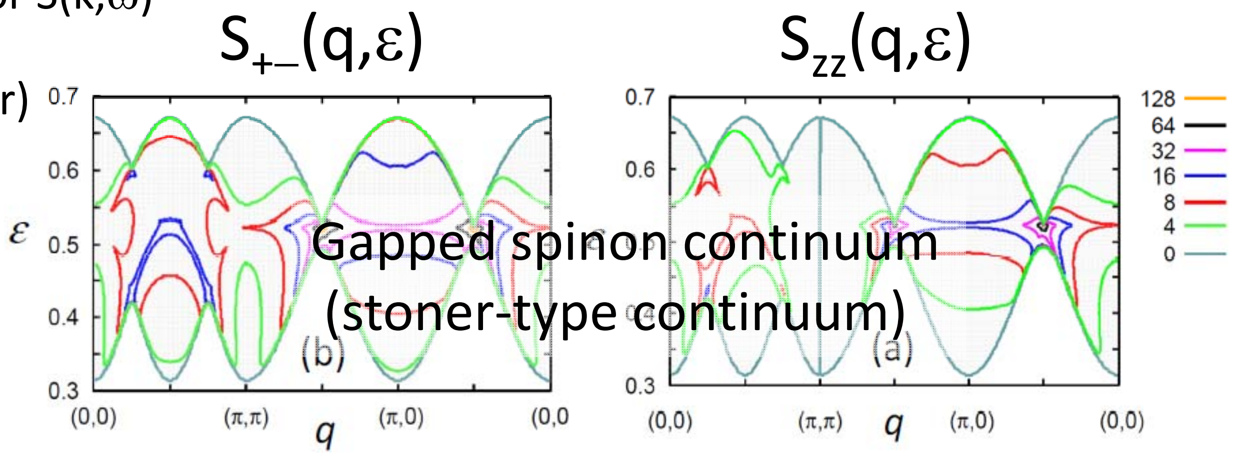


◆ Dyanmical structure factor S(k,ω)

□ Large N limit (0th order)
(individual excitation)

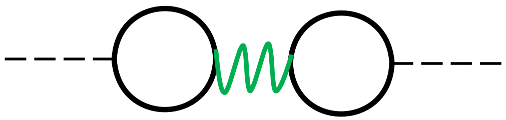


Spinon's
propagator



Gapped spinon continuum
(stoner-type continuum)

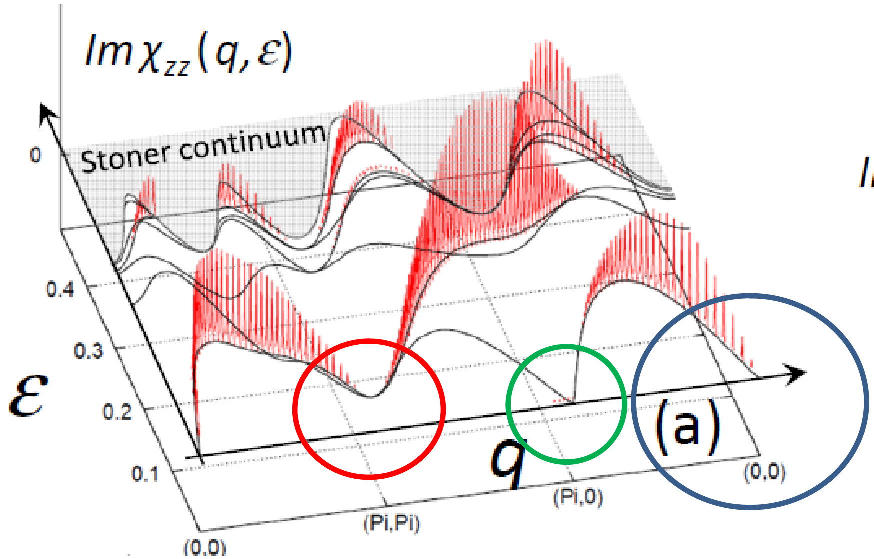
□ 1-loop correction
(collective modes: RPA-type)

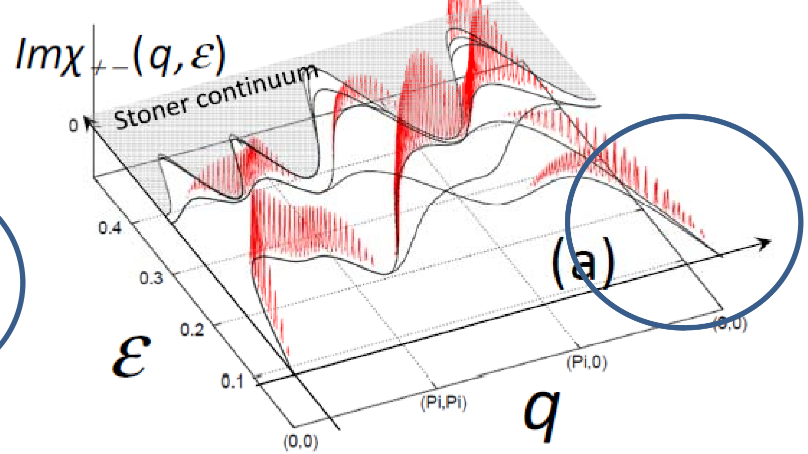


■ Spectral weight at (0,0) vanishes
as **a linear function of the momentum**.

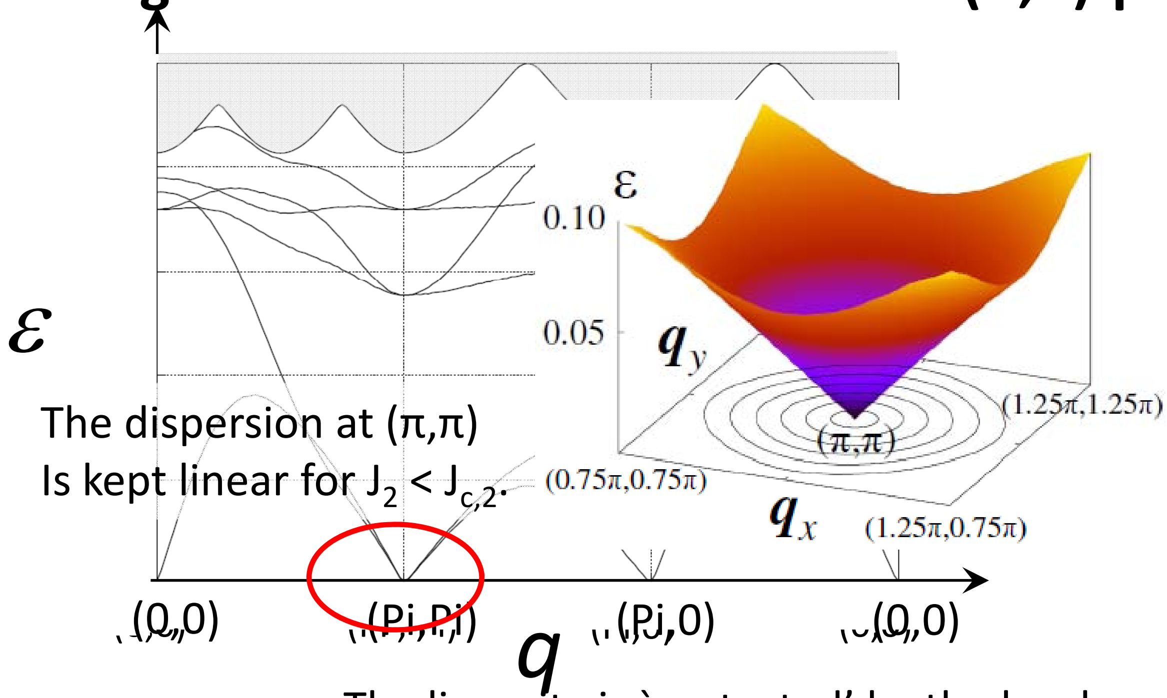
■ No weight at (π,0) and (0,π);
distinct from that of S(q,ε) in CAF phase

■ A gapped longitudinal mode at (π,π)
correpsonds to the `gapped gauge boson'
associated with the Z_2 state.

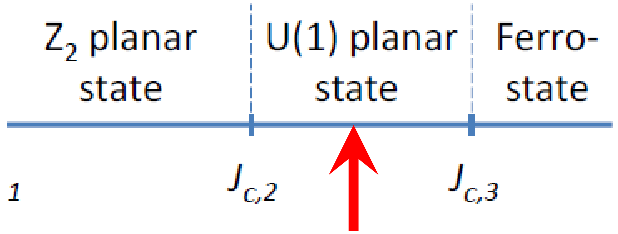




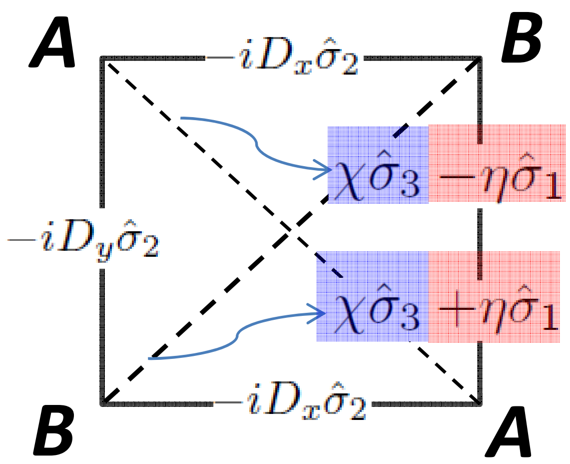
Gauge-field like collective mode at (π,π)-point



Mean-field Phase diagram



The linearity is `protected' by the local gauge symmetry.



- ☐ **singlet pairings on a NNN AF-bond**
  - ✓ p-h channel = ***s-wave***
  - ✓ p-p channel = ***d-wave***
- ☐ Global U(1) gauge symmetry

$$e^{i(-1)^{j_x+j_y}\theta\hat{\sigma}_3}\hat{U}_{jm} = \hat{U}_{jm}e^{i(-1)^{m_x+m_y}\theta\hat{\sigma}_3}$$

➔ A certain gauge boson at (π,π) should become gapless (`photon'-like)

Summary of dynamical spin structure factor

Shindou, Yunoki and Momoi, Phys. Rev. B **87**, 054429 (2013)

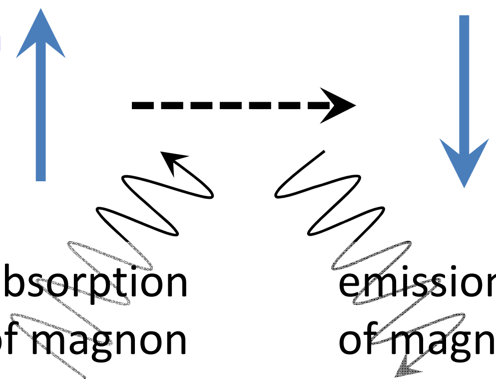
- □ No weight at (π,0) and (0,π); distinct from that of S(q,ε) in CAF phase
- □ Vanishing weight at (0,0); linear function in q

$$\mathrm{Im}\chi^{(1)}_{\mu\mu}(\boldsymbol{q},\epsilon) = a|\boldsymbol{q}|\delta(\epsilon - v|\boldsymbol{q}|) + \cdots .$$

- □ A finite mass of the (first) gapped L-mode at (π,π) describes the stability of Z_2 planar state against the confinement effect.
- □ Gapped stoner continuum at the high energy region.

Temperature dependence of NMR $1/T_1$

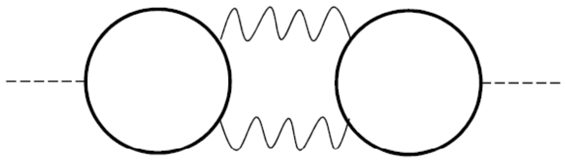
Relevant process to $1/T_1$:= Raman process



Moriya, PTP (1956)

$T_1^{-1} \propto T^{2d-3}$ (CAF phase)

Wavy lines: Gapless director-waves



$$T_1^{-1} \propto T^{2d-1}$$

d: effective spatial dimension

PRB **87**, 054429 (2013)

Take-Out Messages of the 2nd part of my talk

- Spin-triplet variant of QSL := QSN
  --- `mixed' Resonating Valence Bond (RVB) state ---
- Mean-field and gauge theory
  of QSN in a frustrated ferromagnet
- Variational Monte Carlo analysis
  --- comparison with exact diagonalization studies ---
- Physical/Experimental Characterizations of QSN
  --- dynamical spin structure factor, NMR relaxation rate ---
- QSN is a new `route' to realization of
  fractionalization of magnetic excitations in d>1

Thank you for your attention !

backup